User Guide

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File main.m finds the inverse solution to "the gravitational inverse problem of obtaining the shape of a frontier $z(w)$ between two media of different densities"[1]. It is the example of an ill-posed non-linear problem from [2]. Considering discrete data with continuous parameter case, authors of [1] use the following back propagation where they have used $z(w)$ with two iterations k-th and $(k+1)$-th but without discretizing the w.

$$\hat{z}_{k+1}(w) = z_0 + \int dw' \sum_i \sum_j C_{p_0}(w, w') G_k^i(w')(S^{-1})^{ij}[d_0^j - g^j(\hat{\mathbf{z}}_k) + \int dw'' G_k^j(w'').[\hat{z}_k(w'') - z_0(w'')] \tag{1}$$

where they introduce the concept of using integration operator due to continuous parameter. We are coding the above equation(1). The covariance matrix of the above equation is defined by

$$C_p(w, w') = \sigma^2 exp[-\frac{1}{2}\frac{(w-w')^2}{\Delta^2}] \tag{2}$$

and

$$G_k^i(w) = \frac{2(H - z_k(w))}{(x^i - w)^2 + [H - z_k(w)]^2} \tag{3}$$

This is the derivative of $g(x^i, w, z_k(w))$ at the point z_k. Note that here $\mathbf{G}^T$ in the continuous setting is the more general adjoint G^*. That adjoint G^* is defined by $\langle Gr, t\rangle = \langle t, G^* g\rangle$ for all r, t in a Hilbert space[1]. In our code we have coded the above equation(1) and plot it.

In my code, I have considered the following inputs and output.

Inputs:

n : Total number of data

d : Data file

σ : Prior uncertainty which is changeable and is the reason of having different priori covariance. K : Number of iteration that we want. This is also changeable.

Output:

$z_h at$: Frontier

References

[1] A. Tarantola, B. Valette. *Generalized Nonlinear Inverse Problems Solved Using the Least Squares Criterion.* Institut de Physique du Globe de Paris, 75005 Paris, France. Reviews of Geophysics and Space Physics, Vol. 20, No. 2, pages 219 – 232; May 1982.

[2] A. Tikhonov, V. Arsenine. *Methodes de resolution de problemes mal poses.* Editions MIR, moscow, 1976.