PEU 356: Mathematical Physics II

Numerical Analysis Assignment 1: **(100 points)**
Due: Thursday, March 31, 2022

Reading Assignment

Chapters 1 and 2, An Introduction to Numerical Computation, Wen Shen.

Problem 1 (Number Representation in Different Bases) (25 points)

a) Convert the following binary numbers to decimal numbers
   i) $(110111001.101011101)_2$ ii) $(1001100101.01101)_2$

b) Convert the following decimal number to binary. Keep 10 fractional points
   i) $(100.01)_{10}$ ii) $(64.625)_{10}$

c) For the second number in (b) part (ii) above, determine how it will look like in 32-bit single-precision floating point representation.

Problem 2 (Floating Point Representation and Error Propagation) (20 points)

Perform a detailed study for the error propagation in the computation $z = xy$. Let $fl(x) = x(1 + \delta x)$ and $fl(y) = y(1 + \delta y)$, where $fl(x)$ and $fl(y)$ are the floating point representations of x and y, respectively. Find an expression for the absolute error and the relative error in the answer $fl(z)$.

Problem 3 (Taylor Series with Remainder) (30 points)

Using Maclaurin series, and up to 6 decimal digits of accuracy, write a code that computes an approximate value for the integral

$$\int_0^{0.5} \cos(x^2)\, dx$$

Notes:
- You are allowed to use any programming language of your preference.
- Don't use ready-made packages, write your own detailed algorithm.
- Submit both the source code file and a screenshot of this code's output.
- The code should output both the final result and the value of the expansion at each iteration.

Hint:
Use the condition $|T(n+1) - T(n)| < \varepsilon$, where ε is the desired precision, in order to check that you have added enough terms.

Problem 4 (Polynomial Interpolation) **(25 points)**

Write a code that find the polynomial interpolating function forthe following data points:

x_i	0	5	10	15
y_i	3	8	-2	9

using two different methods: a) By solving a linear system of equations, and b) Using Lagrange interpolation formula. Plot in both cases the points and the interpolating function.

Notes:

- You are allowed to use any programming language of your preference.
- Don't use ready-made packages, write your own detailed algorithm.
- You are allowed to use a ready-made function in whatever framework for solving a system of linear equations, no need to write a code for it.
- You are also allowed to use plotting modules of any framework you use.
- Submit both the source code file and a screenshot of the code's output.

Bonus Problem (Splines) **(15 points)**

Let $L(x)$ be the linear splines that interpolates the data in problem 4 above. Describe what $L(x)$ consists of, and what conditions it has to satisfy. Find $L(x)$ and compute the value for $L(12)$.

Hint:
You may need to read the part on splines in Chapter 3 of Wen Shen's book.