**PEU 356: Mathematical Physics II**

Numerical Analysis Assignment 2: **(100 points)**
Due: Sunday, May 8, 2022

**Reading Assignment**

Chapters 1, section 1.6, An Introduction to Numerical Computation, Wen Shen, 1st Edition.

Chapter 4, sections 4.1 and 4.2, Numerical Analysis, Richard L. Burden and J. Douglas Faires, 9th Edition.

**Problem 1 (Difference Schemes to Compute Derivatives)** **(30 points)**

Write a code to get an approximation for the derivatives of the function $f\ (x)\ =\ sin(x)$ at $x\ =\ \pi/6$. Try steps $h\ =\ 10^{-1}, 10^{-2}, \ldots, 10^{-6}$. Compute the derivative using the following methods:

a) First-order derivative using Richardson's extrapolation in $O(h^6)$,
b) Second-order derivative in order $O(h^4)$,
c) Third-order derivative using any method of your choice,
d) Fourth-order derivative using any method of your choice.

*Notes:*
- You are allowed to use any programming language of your preference.
- Don't use ready-made packages, write your own detailed algorithm.
- Submit both the source code file and a screenshot of this code's output.
- The code should output each derivative for each $h$.

**Problem 2 (5-Point Formula and its Error Bound)** **(30 points)**

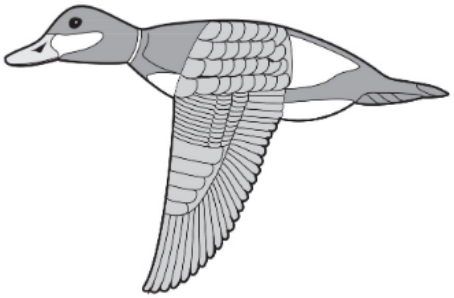
Given the following data points

| $x_i$ | 1.05 | 1.10 | 1.15 | 1.20 | 1.25 | 1.30 |
|---|---|---|---|---|---|---|
| $y_i$ | -1.709847 | -1.373823 | -1.119214 | -0.9160143 | -0.7470223 | -0.6015966 |

Compute an approximation for the first-order derivative at each point using a 5-point formula and compute the error bound for this approximation.

**Problem 3 (Lagrange Interpolation)** **(40 points)**

To approximate the top profile of the duck in the image, we have chosen points along the curve through which we want the approximating curve to pass. The points are as follows:



| $x_i$ | 0.9 | 1.3 | 1.9 | 2.1 | 2.6 | 3.0 | 3.9 |
|---|---|---|---|---|---|---|---|
| $y_i$ | 1.3 | 1.5 | 1.85 | 2.1 | 2.6 | 2.7 | 2.4 |

| $x_i$ | 4.4 | 4.7 | 5.0 | 6.0 | 7.0 | 8.0 | 9.2 |
|---|---|---|---|---|---|---|---|
| $y_i$ | 2.15 | 2.05 | 2.1 | 2.25 | 2.3 | 2.25 | 1.95 |

| $x_i$ | 10.5 | 11.3 | 11.6 | 12.0 | 12.6 | 13.0 | 13.3 |
|---|---|---|---|---|---|---|---|
| $y_i$ | 1.4 | 0.9 | 0.7 | 0.6 | 0.5 | 0.4 | 0.25 |

Using these points and Lagrange interpolation, write a code that computes an approximating polynomial then plot along the points given. What is the problem with the resulting interpolating polynomial?

*Notes:*

- You are allowed to use any programming language of your preference.
- Don't use ready-made packages, write your own detailed algorithm.
- Submit both the source code file and a screenshot of this code's output.
- The code should output a plot of the Lagrange interpolating curve for the given points.