Discrete Mathematics and Algorithms (CSE 611)
Monsoon 2018
Assignment Set 4

Divide and Conquer:

1. Let us assume that **n** people are voting to nominate a candidate (number of candidates might be more than 2). Also assume that the number of votes received by each candidate forms a sequence. A person wins the election if he/she receives majority of the votes.

    - Propose a Divide & Conquer algorithm to determine whether a candidate received majority of votes, if so determine the candidate.

    - Give an upper bound on the number of comparisons needed by the algorithm.

2. Let $\beta > 0$. If $f(n) \leq 2f(n/2) + c.n^{1+\beta}$ for $n > 1$ and $f(1) \leq c$ for $n \leq 1$, for some constant c, prove that:

$$f(n) \leq c.n^{1+\beta}$$

3. In Quicksort let $n/7^{th}$ smallest element is chosen using an $O(n^2)$ algorithm as the pivot. Calculate the worst case time complexity of the algorithm.

4. Let $T = \{x_1\ x_2\ ...\ x_n\} \subset Z$ where the entries might not be in sorted order. Assume $n = 2^k$ and $k \geq 1$. How many comparisons are required to find the minimum in S in worst case?

Probability:

5. Two coins are tossed. For the first coin, if the head comes, then the second head comes up with a probability of 0.6. If at first, the tail comes up, then the probability of tail coming up again is 0.6. Find the probability for
    a. The first toss is head
    b. The second toss is a head
    c. State whether events in (a) and (b) are independent or not.

6. Let X be a discrete random variable with the following Probability Mass Function:

$$P_x(X) = \begin{cases} 0.1 & x = 0.2 \\ 0.2 & x = 0.4 \\ 0.2 & x = 0.5 \\ 0.3 & x = 0.8 \\ 0.2 & x = 1 \\ 0 & \text{otherwise} \end{cases}$$

   a. Find R_x , the range of X
   b. Find P(x <= 0.5)
   c. Find P(0.25 < x < 0.75)
   d. Find P(x = 0.2 | x < 0.6)

7. The number of customers arriving at a grocery store is a Poisson Random Variable. On an average, 10 customers arrive per hour. Let x be the number of customers arriving from 10 AM to 11:30 AM. What is P(10 < x < 15).

8. Let X be the Continuous Random Variable with Probability Density Function given by:

$$fx(X) = \tfrac{1}{2}\,(e^{-|x|}),\ \forall\ x \in R$$

   If $Y = X^2$, find CDF of Y.