Discrete Mathematics and Algorithms (CSE 611)
Monsoon 2018
Assignment Set 3
Total Marks: 100

Deadline: September 18, 2018, 1:00 PM (Class Room)
Course Instructor: Dr. Ashok Kumar Das

Numeric Functions, Generating Functions and Recurrence Relations

1. Consider an air traffic-control system in which the desired altitude of an aircraft, a_r, is computed by a computer every second and is compared with the actual altitude of the aircraft, b_{r-1}, determined by a tracking radar 1 second earlier. Depending on whether a_r is larger or smaller than b_{r-1}, the altitude of the aircraft will be changed accordingly. Specifically, the change in altitude at the r-th second, $b_r - b_{r-1}$, is proportional to the difference $a_r - b_{r-1}$. That is,

   $$b_r - b_{r-1} = K(a_r - b_{r-1})$$

   where K is a proportional constant.
   (a) Determine b_r, given that $a_r = 1000(\frac{3}{2})^2$, $K = 3$, and $b_0 = 0$.
   (b) Determine b_r, given that

   $$a_r = \begin{cases} 1000(\frac{3}{2})^r, 0 \leq r \leq 9 \\ 1000(\frac{3}{2})^{10}, r \geq 10 \end{cases}$$

   $K = 3$, and $b_0 = 0$.

   [10 + 10 = 20]

2. Using the generating function, show that solution of the following recurrence relation

   $$a_k - 7a_{k-1} + 10a_{k-2} = 3^k$$

   with initial conditions $a_0 = 0$ and $a_1 = 1$, is

   $$a_k = \frac{8}{3}2^k - \frac{9}{2}3^k + \frac{11}{6}5^r$$

   [20]

3. Consider the multiplication of bacteria in a controlled environment. Let a_r denote the number of bacteria there are on the r-th day. We define the rate of growth on the r-th day to be $a_r - 5.a_{r-1}$. It is known that the rate of growth at r-th day is three times the growth of the $(r-1)$-th day. Determine a_r, given that $a_0 = 1$.

   [20]

4. Interest for money deposited in a savings account is paid at a rate of 0.5 percent per month, with interest compounded monthly. Suppose $\$50$ is deposited into a savings account each month for a period of five years. What is the total amount in the account four years after the first deposit? Twenty years after the first deposit?

[20]

5. Let

$$\begin{aligned} a_r &= \begin{cases} 1, r = 0 \\ 3, r = 1 \\ 2, r = 2 \\ 0, r \geq 3 \end{cases} \\ c_r &= 5^r \text{ for all } r \end{aligned}$$

Given that $c = a * b$, that is c is the convolution of numeric functions a and b. Show that $b_r = \frac{25}{42}5^r - \frac{1}{6}(-1)^r + \frac{4}{7}(-1)^r 2^r, r \geq 0$.

[20]

Submission Instructions

Copying in assignments leads to award ZERO marks in assignment marks. Also, the source from which you have copied, that source student will be treated under the same rule.

Please submit the assignment in hard copy stating the following at the top after the class on the deadline date only:

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submitted by
Name: XYZ
Roll No: abc