Tutorials – Modern Physics & Quantum Mechanics
(2019)

1. Incident freq.
$\nu = 2.9\times10^{19}$ Hz

Scattering angle $\theta = 84^\circ$

New freq $\nu' = ?$

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\nu = \frac{c}{\lambda}$$

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0 c^2}(1-\cos\theta)$$

$$\frac{1}{\nu'} = \frac{1}{2.9\times10^{19}} + \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times(3\times10^{8})^2}(1-\cos 84^\circ)$$

$$\nu' = 2.39\times10^{19}$$

K.E of e^- = Incident photon Energy – Scattered photon energy

$$= h\nu - h\nu'$$

$$= h(\nu - \nu')$$

$$= 6.63\times10^{-34}(2.89\times10^{19} - 2.39\times10^{19})$$

$$= 3.37\times10^{-15}\ \text{J}$$

2.) Initial freq. $= 2.6\times10^{19}$ Hz

Scattering angle $\theta = 70^\circ$

New freq. $\nu' = ?$ $\quad \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$

$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0 c^2}(1-\cos\theta)$$

$$\frac{1}{\nu'} = \frac{1}{2.6\times10^{19}} + \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times(3\times10^{8})^2}(1-\cos 70^\circ)$$

$$\nu' = 2.28\times10^{19}\ \text{Hz}$$

3.

$E = 28\ keV$

$= 28 \times 10^3 \times 1.6 \times 10^{-19}\ J$

Scattering angle $\theta = 55^\circ$

$E = \frac{hc}{\lambda}$

Incident wavelength $\lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{28 \times 10^3 \times 1.6 \times 10^{-19}}$

$\lambda = 4.439 \times 10^{-11}\ m$

$\lambda' = \lambda + \frac{h}{m_0 c}(1 - \cos\theta)$

$= 4.439 \times 10^{-11} + \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8} (1 - \cos 55^\circ)$

$= 4.5434 \times 10^{-11} m$

K.E of e^- $= \frac{hc}{\lambda} - \frac{hc}{\lambda'}$

$= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$

$= 6.63 \times 10^{-34} \times 3 \times 10^8 \left[\frac{1}{4.5434 \times 10^{-11}} - \frac{1}{4.439 \times 10^{-11}}\right]$

$= 9.86 \times 10^{-17}\ J$

4.

$\lambda = 13\ pm = 13 \times 10^{-12}\ m$

K.E is max when λ' is max, $\Rightarrow$ when $\theta = 180^\circ$

$\lambda' = \lambda + \frac{h}{m_0 c}(1 - \cos\theta)$

$\lambda' = 13 \times 10^{-12} + \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8} [1 - \cos 180^\circ]$

$= 17.86 \times 10^{-12} m$

Max. K.E of e^- (at 180°) $= \frac{hc}{\lambda} - \frac{hc}{\lambda'} = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda'}\right]$

$= 4.16 \times 10^{-15}\ J$

λ' (at $\theta = 0^\circ$) $= 0$

λ' (at $\theta = 90^\circ$) $= 15.42 \times 10^{-12} m$

λ' (at $\theta = 45^\circ$) $= 14.717 \times 10^{-12} m$

5.

Compton wavelength $= \dfrac{h}{m_0 c}$

$$= \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}$$

$$= 0.0242\ \text{Å}$$

6. For e^- $\lambda = 1.2\ \text{Å}$

Group vel. v_g = particle velocity

$$v_g = \frac{p}{m} = \frac{h}{\lambda m}$$

$$= \frac{6.63\times10^{-34}}{1.2\times10^{-10}\times9.1\times10^{-31}} = 6.07\times10^{6}\ m/s$$

Phase vel. $v_p = \dfrac{v_g}{c^2} = \dfrac{6.07\times10^{6}}{(3\times10^{8})^2} = 1.48\times10^{10}\ m/s$

7. Energy of e^- = photon energy

$\lambda_{photon} = 10\,\lambda_{e^-}$

for photon $E = \dfrac{hc}{\lambda_{photon}}$

for e^- $\lambda_e = \dfrac{h}{\sqrt{2mE}}$

$$E = \frac{h^2}{2m\lambda_e^2}$$

so $\dfrac{h^2}{2m\lambda_e^2} = \dfrac{hc}{\lambda_{photon}}$

$\lambda_{ph} = 10\lambda_e$

$$\frac{h^{\cancel{2}}}{2m\lambda_e^{\cancel{2}}} = \frac{\cancel{h}c}{10\cancel{\lambda_e}}$$

$$\lambda_e = \frac{h\times10}{2mc} = 2.428\times10^{-11}\ m$$

$$E = \frac{h^2}{2m\lambda_e^2} = \frac{(6.63\times10^{-34})^2}{2\times9.1\times10^{-31}\times(2.428\times10^{-11})^2} = 4.09\times10^{-16}\ J$$

8)

For (He^4) He nucleus $m_{He} = 4 m_p$

$$m_{He} = 4 \times 1.6 \times 10^{-27}\ kg$$

$$q_{He} = 2e$$
$$= 2 \times 1.6 \times 10^{-19}\ C$$

$$\lambda = 0.75 \times 10^{-12}\ m$$

Group vel.

$$v_g = \frac{h}{m_{He}\lambda} = \frac{6.63 \times 10^{-34}}{4 \times 1.6 \times 10^{-27} \times 0.75 \times 10^{-12}}$$
$$= 1.32 \times 10^{5}\ m/s$$

phase vel.

$$v_p = \frac{c^2}{v_g} = 6.806 \times 10^{11}\ m/s$$

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

P. difference

$$V = \frac{h^2}{2mq\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 4 \times 1.6 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times (0.75 \times 10^{-12})^2}$$
$$= 182.06\ V$$

9. de-Broglie wavelength of e^- $\lambda_e = \frac{h}{m_e v_e} = \frac{h}{\sqrt{2m_e E}}$

for proton $\lambda_p = \frac{h}{m_p v_p} = \frac{h}{\sqrt{2m_p E}}$

(i) $v_e = v_p = v$

$$\frac{\lambda_e}{\lambda_p} = \frac{\frac{\cancel{h}}{m_e \cancel{v}}}{\frac{\cancel{h}}{m_p \cancel{v}}} = \frac{m_p}{m_e} = \cancel{42.8}\ 1836.5$$

(ii) K.E of e^- = K.E of proton = E

$$\frac{\lambda_e}{\lambda_p} = \frac{\frac{\cancel{h}}{\sqrt{2m_e \cancel{E}}}}{\frac{\cancel{h}}{\sqrt{2m_p \cancel{E}}}} = \sqrt{\frac{m_p}{m_e}} = 42.87$$

8) For (He^4) α He nucleus $m_{He} = 4\,m_p$

$$m_{He} = 4 \times 1.6 \times 10^{-27}\ kg$$

$$q_{He} = 2e$$
$$= 2 \times 1.6 \times 10^{-19}\ C$$

$$\lambda = 0.75 \times 10^{-12}\ m$$

Group vel.
$$v_g = \frac{h}{m_{He}\lambda} = \frac{6.63 \times 10^{-34}}{4 \times 1.6 \times 10^{-27} \times 0.75 \times 10^{-12}}$$
$$= 1.32 \times 10^5\ m/s$$

phase vel.
$$v_p = \frac{c^2}{v_g} = 6.806 \times 10^{11}\ m/s$$

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

P. difference
$$V = \frac{h^2}{2mq\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 4 \times 1.6 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times (0.75 \times 10^{-12})^2}$$
$$= 182.06\ V$$

9. de-Broglie wavelength of e^- $\lambda_e = \frac{h}{m_e v_e} = \frac{h}{\sqrt{2m_e E}}$

for proton $\lambda_p = \frac{h}{m_p v_p} = \frac{h}{\sqrt{2m_p E}}$

(i) $v_e = v_p = v$

$$\frac{\lambda_e}{\lambda_p} = \frac{\frac{\cancel{h}}{m_e \cancel{v}}}{\frac{\cancel{h}}{m_p \cancel{v}}} = \frac{m_p}{m_e} = \cancel{42.8}\ 1836.5$$

(ii) K.E of e^- = K.E of proton = E

$$\frac{\lambda_e}{\lambda_p} = \frac{\frac{\cancel{h}}{\sqrt{2m_e \cancel{E}}}}{\frac{\cancel{h}}{\sqrt{2m_p \cancel{E}}}} = \sqrt{\frac{m_p}{m_e}} = 42.87$$

10. Pot. diff

$V = 300\ V$

$$\lambda = \frac{h}{\sqrt{2mqV}} = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times300}}$$

$$= 7.09\times10^{-11}\, m$$

$$v_g = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times7.09\times10^{-11}}$$

$$= 1.027\times10^{7}\ m/s$$

$$v_p = \frac{c^2}{v_g} = 8.74\times10^{9}\ m/s$$

11 * Phase vel. $v_p = \sqrt{\frac{2\pi S}{\lambda\rho}}$

$$v_g = v_p - \lambda\frac{dv_p}{d\lambda}$$

$$= \sqrt{\frac{2\pi S}{\lambda\rho}} - \lambda\frac{d}{d\lambda}\left(\sqrt{\frac{2\pi S}{\lambda\rho}}\right)$$

$$\frac{d}{d\lambda}\sqrt{\frac{2\pi S}{\lambda\rho}} = \sqrt{\frac{2\pi S}{\rho}}\frac{d}{\lambda d}\lambda^{-1/2}$$

$$= -\frac{1}{2}\sqrt{\frac{2\pi S}{\rho}}\,\lambda^{-3/2} = -\frac{1}{2}\sqrt{\frac{2\pi S}{\lambda\rho}}\cdot\frac{1}{\lambda}$$

$$v_g = \sqrt{\frac{2\pi S}{\lambda\rho}} - \cancel{\lambda}\left[-\frac{1}{2\cancel{\lambda}}\sqrt{\frac{2\pi S}{\lambda\rho}}\right]$$

$$= \sqrt{\frac{2\pi S}{\lambda\rho}} + \frac{1}{2}\sqrt{\frac{2\pi S}{\lambda\rho}} = v_p + \frac{1}{2}v_p$$

$$\boxed{v_g = \frac{3}{2}v_p}$$

12. Phase vel.

$$v_p = \sqrt{\frac{g\lambda}{2\pi}}$$

$$v_g = v_p - \lambda \frac{d}{d\lambda} v_p$$

$$\frac{d}{d\lambda} v_p = \frac{d}{d\lambda} \sqrt{\frac{g\lambda}{2\pi}}$$

$$= \sqrt{\frac{g}{2\pi}} \frac{d}{d\lambda} \lambda^{1/2}$$

$$= \frac{1}{2} \sqrt{\frac{g}{2\pi}} \lambda^{-1/2}$$

$$v_g = v_p - \lambda \left[\frac{1}{2\lambda^{1/2}} \sqrt{\frac{g}{2\pi}} \right]$$

$$= v_p - \frac{1}{2} \sqrt{\lambda} \sqrt{\frac{g}{2\pi}}$$

$$= v_p - \frac{1}{2} \sqrt{\frac{g\lambda}{2\pi}} = v_p - \frac{1}{2} v_p$$

$$\boxed{v_g = \frac{1}{2} v_p}$$

13.

$\Delta x = 0.1\ nm$ $\Delta\lambda = 4.5 \times 10^{-4} Å$

$E = ?$ $= 4.5 \times 10^{-14} m$

$\lambda = ?$

$$p = \frac{h}{\lambda}$$

$$|\Delta p| = \frac{1}{\lambda^2} h \Delta\lambda$$

$$\lambda^2 = \frac{h \Delta\lambda}{|\Delta p|} = \frac{h \Delta\lambda}{\frac{h}{4\pi\Delta x}}$$

[As $\Delta p \Delta x \geq \frac{h}{4\pi}$, $\Delta p \geq \frac{h}{4\pi\Delta x}$]

$$\lambda^2 = 4\pi \Delta x \Delta\lambda$$

$$\lambda = 7.08 \times 10^{-12} m$$

$$E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (7.08 \times 10^{-12})^2} = 4.807 \times 10^{-15} J$$

14. $E = 160\, eV$

$$m = \frac{0.6\, MeV}{c^2} = \frac{0.6\times10^{6}\times1.6\times10^{-19}}{(3\times10^{8})^2}$$

$$= 1.066\times10^{-30}\ kg$$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.663\times10^{-34}}{\sqrt{2\times1.066\times10^{-30}\times160\times1.6\times10^{-19}}}$$

$$= 9.265\times10^{-11}\ m$$

$$v_g = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}}{1.066\times10^{-30}\times9.265\times10^{-11}}$$

$$= 6.709\times10^{6}\ m/s$$

$\cancel{v_p = \frac{c^2}{v_g}}$ $v_g = v_{particle}$

$$\Delta v = 3\%\ of\ v$$

$$= \frac{3}{100}\times6.709\times10^{6}$$

$$= 1.9127\times10^{5}\ m/s$$

$$\Delta x \geq \frac{h}{4\pi m\Delta v}$$

$$\Delta x \geq \frac{6.63\times10^{-34}}{4\times3.14\times1.066\times10^{-30}\times1.912\times10^{5}}$$

$$\Delta x \geq 2.38\times10^{-10}\ m$$

17. 15.

$E = 4\,keV = 4\times10^{3}\times1.6\times10^{-19}\,J$

$p = \sqrt{2mE}$

$= \sqrt{2\times 9.1\times10^{-31}\times4\times10^{3}\times1.6\times10^{-19}}$

$p = 3.41\times10^{-23}\ kg\,m/s$

$\Delta p \geq \frac{h}{4\pi\Delta x}$

$\Delta x = 0.15\times10^{-9}\,m$

$\Delta p \geq \frac{6.63\times10^{-34}}{4\times3.14\times0.15\times10^{-9}}$

$\Delta p \geq 3.517\times10^{-25}\ kg\,m/s$

% Uncertainty in momentum

$\frac{\Delta p\times100}{p} = 1.03\%$

16. For e^- $(\Delta v)_e \geq \frac{h}{4\pi m_e\Delta x}$

for proton $(\Delta v)_p \geq \frac{h}{4\pi m_p\Delta x}$

Δx same

$\frac{(\Delta v)_e}{(\Delta v)_p} = \frac{m_p}{m_e} = 1835$

17.

$$m = \frac{0.5\ MeV}{c^2}$$

$$= \frac{0.5\times10^6\times1.6\times10^{-19}}{(3\times10^8)^2} = 8.9\times10^{-31}\ kg$$

$$E = 160\ eV = 160\times1.6\times10^{-19}$$
$$= 2.56\times10^{-17}\ J$$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63\times10^{-34}}{\sqrt{2\times8.9\times10^{-31}\times2.56\times10^{-17}}}$$

$$= 9.83\times10^{-11}\ m$$

$$v_g = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}}{8.9\times10^{-31}\times9.83\times10^{-11}}$$

$$= 7.51\times10^6\ m/s$$

For e^- $\Delta v = 1.4\%$ of v

Give for e^- $v = 6\times10^5\ m/s$

$$\Delta v = \frac{1.4}{100}\times6\times10^5 = 8.4\times10^3\ m/s$$

$$\Delta x \geq \frac{h}{4\pi m\Delta v}$$

$$\Delta x \geq \frac{6.63\times10^{-34}}{4\times3.14\times9.1\times10^{-31}\times8.4\times10^3}$$

$$\Delta x \geq 6.89\times10^{-9}\ m$$

18.

$$E = 0.5\ keV$$
$$= 0.5\times10^{3}\times1.6\times10^{-19}\ J$$

for e⁻ $p = \sqrt{2mE}$

$$= \sqrt{2\times9.1\times10^{-31}\times0.5\times10^{3}\times1.6\times10^{-19}}$$
$$= 1.207\times10^{-23}\ kg\,m/s$$

$$\Delta x = 0.5\ \mathring{A}$$

$$\Delta p \geq \frac{h}{4\pi\,\Delta x}$$

$$\Delta p \geq \frac{6.63\times10^{-34}}{4\times3.14\times0.5\times10^{-10}}$$

$$\Delta p \geq 1.05\times10^{-24}\ kg\,m/s$$

% Uncertainty in momentum $= \frac{\Delta p}{p}\times100$

$$= 8.79\%$$

19. Lowest energy of particle trapped in 1D box

$$E = \frac{n^2h^2}{8ma^2} \qquad n = 1$$

$$E_1 = \frac{1^2h^2}{8ma^2} = 1.7\ eV$$

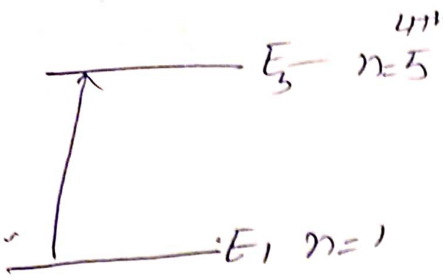
$$E_2 = \frac{2^2h^2}{8ma^2} = 4E_1 = 4\times1.7\ eV$$
$$= 6.8\ eV$$

$$E_3 = \frac{3^2h^2}{8ma^2} = 9E_1 = 15.3\ eV$$

$a = ?$ $E_1 = \frac{h^2}{8ma^2} = 1.7\times1.6\times10^{-19}$

$$a = \frac{h}{\sqrt{2mE_1}} = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.7\times1.6\times10^{-19}}}$$
$$= 4.7\times10^{-10}\ m$$

20.



$a = 15\,\mathring{A}$

Excitation energy

$E_5 - E_1$

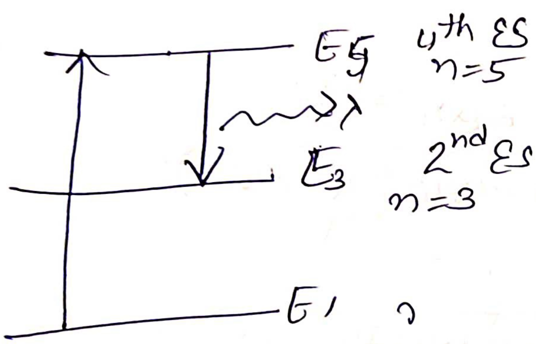
$$E_5 - E_1 = \frac{5^2 h^2}{8ma^2} - \frac{1^2 h^2}{8ma^2}$$

$$= \frac{h^2}{8ma^2}(25-1)$$

$$= \frac{24 h^2}{8ma^2}$$

$$= \frac{24 \times 6.63 \times 10^{-34}}{8 \times 9.1 \times 10^{-31} \times (15 \times 10^{-10})^2}$$

$$= 6.44 \times 10^{-19}\ J$$



Energy of photon Emitted

$$\frac{hc}{\lambda} = E_5 - E_3$$

$$\frac{hc}{\lambda} = \frac{5^2 h^2}{8ma^2} - \frac{3^2 h^2}{8ma^2}$$

$$\frac{hc}{\lambda} = \frac{h^2}{8ma^2}(25-9)$$

$$\frac{\cancel{h}c}{\lambda} = \frac{\overset{2}{\cancel{16}}\, h^{\cancel{2}}}{\cancel{8}ma^2}$$

$$\lambda = \frac{ma^2 c}{2h}$$

$$\lambda = 4.639 \times 10^{-7}\ m$$

21)

5 antinodes

$n = 5$

$E = 260\ eV$

$$E = \frac{n^2 h^2}{8 m a^2}$$

$$260 \times 1.6 \times 10^{-19} = \frac{5^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times a^2}$$

$$a^2 = \frac{25 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 260 \times 1.6 \times 10^{-19}}$$

$$a = 1.904 \times 10^{-10}\ m$$

22

$a = 5\ Å$

2nd ES $\quad n = 3$

$$E_3 = \frac{3^2 h^2}{8 m a^2} = \frac{9 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (5 \times 10^{-10})^2}$$

$$= 2.17 \times 10^{-18}\ J$$

$$p = \sqrt{2mE} = \sqrt{2 \times 9.1 \times 10^{-31} \times 2.17 \times 10^{-18}}$$

$$= 1.99 \times 10^{-24}\ kg\ m/s$$

$$\cancel{\lambda = \frac{2L}{n} = 2 \times 5 \times 10^{-10}}$$

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{1.99 \times 10^{-24}} = 3.33 \times 10^{-10}\ m$$

23) Lowest Energy $E_1 = \frac{1^2 h^2}{8ma^2}$

$a = 10^{-14} m$

. Same for both neutron & e^-

For neutron $(E_1)_n = \frac{h^2}{8m_n a^2}$

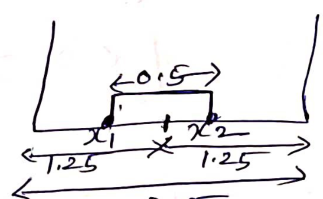
For e^- $(E_1)_e = \frac{h^2}{8m_e a^2}$

$$\frac{(E_1)_n}{(E_1)_e} = \frac{\frac{\cancel{h^2}}{8m_n \cancel{a^2}}}{\frac{\cancel{h^2}}{8m_e \cancel{a^2}}}$$

$$= \frac{m_e}{m_n} = 5.44 \times 10^{-4}$$

24. $a = 2.5 nm$

We know $\psi = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$



In Ground state $n = 1$

$\psi_1 = \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a}$

$x_1 = 1.25 - 0.25 = 1 nm$

$x_2 = 1.25 + 0.25 = 1.5 nm$

$$P = \int_{x_1}^{x_2} |\psi|^2 dx$$

$$= \frac{2}{a} \int_{x_1}^{x_2} \sin^2 \frac{n\pi x}{a} dx$$

$$= \frac{\cancel{2}}{a}\int_{x_1}^{x_2}\left(\frac{1-\cos\frac{2n\pi x}{a}}{\cancel{2}}\right)dx$$

$$P = \frac{1}{a}\left[x - \frac{a}{2n\pi}\sin\frac{2n\pi x}{a}\right]_{x_1}^{x_2}$$

for ground state $n=1$, $a = 2.5\ nm$

Here $x_1 = 1\ nm$

$x_2 = 1.5\ nm$

$$= \frac{1}{2.5}\left[\left(1.5 - \frac{2.5}{2\pi}\sin\frac{2\pi\times1.5}{2.5}\right) - \left(1 - \frac{2.5}{2\pi}\sin\frac{2\pi\times1}{2.5}\right)\right]$$

$$= \frac{1}{2.5}\left[0.5 - \frac{2.5}{2\pi}\sin\frac{3\pi}{2.5} + \frac{2.5}{2\pi}\sin\frac{2\pi}{2.5}\right]$$

$$= \cancel{[0.5 - 0.796\times\sin216^\circ + 0.796\times\sin144^\circ]}$$

$$\cancel{[0.5 + 0.4678 + 0.4678]}$$

$$= 0.2 - \frac{1}{2\times3.14}\left[\sin\frac{3\pi}{2.5} - \sin\frac{2\pi}{2.5}\right]$$

$$= 0.2 - \frac{1}{6.283}\left[\sin216 - \sin144^\circ\right]$$

$$= 0.2 - \frac{1}{6.283}\left[-0.5878 - 0.5878\right]$$

$$= 0.2 + 0.1871$$

$$= 0.3871$$

25)

$$m = 1.67 \times 10^{-27} kg$$

$$a = 0.1 \times 10^{-9} m$$

2^{nd} ES $n = 3$

$$E_3 = \frac{3^2 h^2}{8ma^2} = \frac{9 \times 6.63 \times 10^{-34}}{8 \times 1.67 \times 10^{-27} \times (0.1 \times 10^{-9})}$$

$$= 2.961 \times 10^{-20} J$$

$$p = \sqrt{2mE}$$

$$= \sqrt{2 \times 1.67 \times 10^{-27} \times 2.961 \times 10^{-20}}$$

$$= 9.944 \times 10^{-24} kg\,m/s$$

$$P = \int_0^{a/3} |\psi|^2 dx$$

$$= \int_0^{a/3} \left(\sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a}\right)^2 dx$$

$$= \frac{1}{a}\left[x - \frac{a}{2n\pi} \sin\frac{2n\pi x}{a}\right]_0^{a/3}$$

$n = 3$

$$= \frac{1}{a}\left[\left(\frac{a}{3} - 0\right) - \frac{a}{2 \times 3\pi} \sin\frac{6\pi \cancel{a}}{3\cancel{a}} - 0\right]$$

$$= \frac{1}{a}\left[\frac{a}{3} - \frac{a}{6\pi} \underbrace{\sin 2\pi}_{\to 0}\right]$$

$$= \frac{1}{a} \times \frac{\cancel{a}}{3} = \frac{1}{3}$$

26 $\psi = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$

$$P = \int_{x_1}^{x_2} |\psi|^2 dx$$

$$= \frac{2}{a} \int_{x_1}^{x_2} \sin^2 \frac{n\pi x}{a} dx$$

$$= \frac{\cancel{2}}{a} \int_{x_1}^{x_2} \left(\frac{1 - \cos \frac{2n\pi x}{a}}{\cancel{2}} \right) dx$$

$$= \frac{1}{a} \left[x - \frac{a}{2n\pi} \sin \frac{2n\pi x}{a} \right]_{x_1}^{x_2}$$

(i) $x_1 = 0$ $x_2 = L/2$ Ground state $n = 1$

width of box is L

$$P = \frac{1}{L} \left[x - \frac{L}{2n\pi} \sin \frac{2\pi x}{L} \right]_0^{L/2}$$

$$= \frac{1}{L} \left[\left(\frac{L}{2} - 0 \right) - \frac{L}{2n\pi} \sin \pi - 0 \right]$$

$$= \frac{1}{2}$$

(ii) $x_1 = 0$ & $x_2 = L/4$ First ES $n = 2$

$$P = \frac{1}{L} \left[x - \frac{L}{2 \times 2\pi} \sin \frac{4\pi x}{L} \right]_0^{L/4}$$

$$= \frac{1}{L} \left[\left(\frac{L}{4} - 0 \right) - \frac{L}{4\pi} \sin \pi - 0 \right]$$

$$= \frac{1}{4}$$