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RAMAIAH
Institute of Technology

(Autonomous Institute, Affiliated to VTU)
(Approved by AICTE, New Delhi & Govt. of Karnataka) Accredited by NBA & NAAC with 'A' Grade

SEMESTER END EXAMINATIONS – JANUARY 2020

Program	:	**B.E. : Common to all Programs**	**Semester**	:	**I**
Course Name	:	**Engineering Mathematics - I**	**Max. Marks**	:	**100**
Course Code	:	**MA11/MAT101**	**Duration**	:	**3 Hrs**

Instructions to the Candidates:

- Answer one full question from each unit.

UNIT- I

1. a) Define homogeneous function for two variables. CO1 (02)

   b) Find the length of perpendicular from pole to the tangent for the curve $r = a(1-\cos\theta)$ at $\left(a, \frac{\pi}{2}\right)$. CO1 (04)

   c) If $z = f(x, y)$ where $x = r\cos\theta$ and $y = r\sin\theta$, then show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$. CO1 (07)

   d) Find the Jacobian of $x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi,\ z = r\cos\theta$ w.r.t (r, θ, ϕ). CO1 (07)

2. a) State the condition for the functions u and v are in terms of x and y are functionally dependent. CO1 (02)

   b) If $u = x^4 y + y^2 z^3$, where $x = rse^{-t}$, $y = rs^2 e^{-t}$ and $z = r^2 s\sin t$, then find $\frac{\partial u}{\partial s}$ at $r = 2,\ s = 1\ \&\ t = 0$. CO1 (04)

   c) Find the angle of intersection of pairs of curves: $r = \frac{a\theta}{1+\theta}$ and $r = \frac{a}{1+\theta^2}$. CO1 (07)

   d) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then show that CO1 (07)

      (i) $xu_x + yu_y = \sin 2u$, (ii) $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = \sin 4u - \sin 2u$.

UNIT- II

3. a) Write the expression to find the volume of the solid for a polar curve, when rotated about the initial line and the line $\theta = \frac{\pi}{2}$. CO2 (02)

   b) Evaluate : CO2 (04)

      $\int_0^{\pi} x\sin^8 x\cos^6 x\, dx.$

   c) Trace the curve $y^2(2a - x) = x^3,\ a > 0$. CO2 (07)

   d) Find the surface area of the solid obtained when cycloid $x = a(\theta - \sin\theta),\ y = a(1-\cos\theta),\ a > 0,\ 0 \le \theta \le 2\pi$ is rotated about its base. CO2 (07)

4.	a)	State any two properties while tracing cartesian curves.	CO2	(02)
	b)	Find the perimeter of the asteroid $x^{2/3}+y^{2/3}=a^{2/3},\ a>0.$	CO2	(04)
	c)	Prove that: $\int_0^{2a} x^n\sqrt{2ax-x^2}\,dx=\pi a^2\left(\frac{a}{2}\right)^n\frac{(2n+1)!}{(n+2)!\,n!}.$	CO2	(07)
	d)	Find the volume of the solid generated by revolution of the cardioid $r=a(1+\cos\theta),\ a>0$ about the initial line.	CO2	(07)

UNIT- III

5.	a)	Define velocity and acceleration of a vector point function of single variable.	CO3	(02)
	b)	Find $a,\ b,\ c$ such that $F=(x+2y+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+cy+2z)\hat{k}$ is conservative .	CO3	(04)
	c)	Prove the vector identity $\nabla\times(\nabla\times A)=\nabla(\nabla.A)-\nabla^2 A$.	CO3	(07)
	d)	Find the directional derivative of the function $\phi=xyz$ along the direction of the normal to the surface $xy^2+yz^2+zx^2=3$ at the point $(1,\ 1,\ 1)$.	CO3	(07)
6.	a)	Give the physical meaning of curl of a vector field.	CO3	(02)
	b)	At any point on the curve $r=3\cos t\,\hat{i}+3\sin t\,\hat{j}+4t\,\hat{k}$, find normal vector.	CO3	(04)
	c)	Find the values of $a\ \&\ b$ so that the surfaces $ax^2-byz=(a+2)x$ and $4x^2y+z^3=4$ may intersect orthogonally at the point $(1,-1,\ 2)$.	CO3	(07)
	d)	Find the value of the constant 'a' such that $A=y(ax^2+z)\hat{i}+x(y^2-z^2)\hat{j}+2xy(z-xy)\hat{k}$ is solenoidal. For this value of 'a' show that $curl A$ is also solenoidal.	CO3	(07)

UNIT- IV

7.	a)	Write the transformation equations from cartesian to cylindrical polar coordinates.	CO4	(02)
	b)	Evaluate: $\int_0^{\pi}\int_0^{a(1+\cos\theta)} r\,dr\,d\theta.$	CO4	(04)
	c)	Evaluate: $\int_0^1\int_{\sqrt{y}}^{2-y} xy\ dx\,dy$ by changing the order of integration.	CO4	(07)
	d)	Find the volume tetrahedron bounded by the planes $x=0, y=0, z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$	CO4	(07)
8.	a)	With the help of a neat diagram, mark the region of integration of the double integral	CO4	(02)

$\int_0^3 \int_{-x}^{x} f(x,y)\,dy\,dx$.

b) Evaluate $\iint_R \frac{\sin x}{x}\,dx\,dy$, where R is the triangle in the $xy-$plane bounded by the $x-$axis, the line $y=x$ and the line $x=1$. CO4 (04)

c) Evaluate $\int_0^\infty \int_0^\infty \int_0^\infty \frac{dxdydz}{(1+x^2+y^2+z^2)^2}$ using spherical polar coordinates. CO4 (07)

d) Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$ by changing the order of integration. CO4 (07)

UNIT- V

9. a) Define work done in moving a particle along a path C. CO5 (02)

b) Using Green's theorem evaluate $\int_c (x^2-2xy)dx+(x^2y+3)dy$ around the boundary of the region defined by $y^2=8x$ and $x=2$. CO5 (04)

c) Evaluate $\int_s F.\hat{n}\,ds$, where $F=4x\hat{i}-2y^2\,\hat{j}+z^3\,\hat{k}$ and S is the surface bounded by $x^2+y^2=4$ and the planes $z=0$ and $z=1$ using Gauss divergence theorem. CO5 (07)

d) Evaluate $\int_c F.dr$, where $F=(2x-y)\hat{i}-yz^2\,\hat{j}-y^2z\,\hat{k}$ and C is the boundary of the upper half of the sphere $x^2+y^2+z^2=1$ using Stoke's theorem. CO5 (07)

10. a) State Stoke's theorem. CO5 (02)

b) Find the total work done in moving particle by a force field $F=3xy\,\hat{i}-5z\,\hat{j}+10x\,\hat{k}$ along the curve $x=t^2+1$, $y=2t^2$ and $z=t^3$ from $t=1$ to $t=2$. CO5 (04)

c) State and prove Green's theorem in a plane. CO5 (07)

d) Using Gauss divergence theorem evaluate $\int_s F.\hat{n}\,ds$, where $F=2xy\,\hat{i}+yz^2\,\hat{j}+xz\,\hat{k}$ and S is the rectangular parallelepiped bounded by $x=0, y=0, z=0, x=2, y=1, z=3.$ CO5 (07)

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