KS

RAMAIAH
Institute of Technology

DEPARTMENT OF PHYSICS

Tutorials Vibrations

(Term: Aug to Dec 2019)

1. The amplitude of a particle executing damped oscillations falls from 56cm to 3cm after 80 oscillations. If the time period of oscillation is 1.8s, determine its (i) relaxation time, (ii) damping constant and (iii) time in which energy falls to $1/10^{th}$ of initial value.
2. The amplitude of an oscillator of frequency 250Hz falls to 1/10th of its initial value after 800 cycles. Calculate its (i) relaxation time (ii) quality factor (iii) time in which its energy falls to 1/15th of its initial value (iv) damping constant (v) logarithmic decrement.
3. If the quality factor of an under damped tuning fork of frequency 256Hz is 1000, calculate the time in which its energy is reduced to 1/e times its original energy. How many oscillations will the tuning fork make in this time? Also calculate the damping constant and relaxation time.
4. A spring suspended from a rigid support carries a mass of 100g at its lower end and oscillates with a frequency of 10Hz. The amplitude reduces to half its initial value in 1minute. Calculate the damping constant and relaxation time.
5. A damped harmonic oscillator has its first amplitude 16cm and it reduces to 1cm after 20 oscillations. If the period of oscillation is 6.9s, calculate the relaxation time.
6. The energy of a piano string of frequency 256 Hz reduces to half of its initial value in 2 s. What is the quality factor of the string?
7. If the quality factor of an undamped tuning fork of frequency 256 is 103, calculate the time in which its energy is reduced to (1/5) of its original energy. Calculate time in which the tuning fork will make 200 oscillations.
8. The Q value of a spring loaded with 0.5 kg is 120. It vibrates with a frequency of 10 Hz. Calculate the force constant and mechanical resistance.
9. The Q value of a spring loaded with 0.3 kg is 60. It vibrates with a frequency of 2 Hz. Calculate the force constant and mechanical resistance.
10. The quality factor of a spring loaded with 0.8 kg vibrating with frequency of vibration 10 Hz is 80. Calculate mechanical resistance.

11. An under-damped oscillator has a time period of 2s and the amplitude of oscillation goes down by 10% in 1 oscillation. How much is the logarithmic decrement λ of the oscillator? Determine the damping coefficient b. What would be the time period of this oscillator if there was no damping? (iv) What should be damping coefficient b such that time period is increased to 4s?

12. A massless spring of constant 12 N/m is suspended from a rigid support and carries a load of 0.15 kg at its lower end. If the energy of system decays to 1/e of its initial value in 35s, calculate the resistive force constant 'r', angular frequency of the oscillator and quality factor.

13. A massless spring of spring constant 10 N/m is suspended from a rigid support and carries a mass of 0.1 kg at its lower end. The system is subjected to a resistive force r(dy/dt), where r is the resistive force constant and dx/dt is the velocity. It is observed that the system performs damped oscillatory motion and its energy decays to 1/e of its initial value in 50 s.

a) What is the value of resistive force constant r ?

b) What is natural angular frequency of the oscillator?

c) What is quality factor?

d) What is the percentage change in frequency due to damping?

14. A pendulum oscillates 100 times in a second. The quality factor of the pendulum is 1000. Calculate the time in which the amplitude of the pendulum decays to $1/e^4$ of its initial value.

15- A Critically damped oscillator with $b = 2/s$ is initially at $x = 0$ with velocity 6 m/s. What is the farthest distance the oscillator moves from the origin?

1. Tutorials – Vibrations

$T = 1.8\ s$

$t = 80 \times 1.8$
$= 144\ s$

$A = 3\ cm$
$A_0 = 56\ cm$

$A = A_0 e^{-bt}$

$3 = 56\ e^{-144b}$

$\ln\left(\frac{3}{56}\right) = -144b$

$b = 2.0324 \times 10^{-2}\ /s$

Relaxation time $\tau = \frac{1}{2b} = 24.6\ s$

Damping const $= 2.0324 \times 10^{-2}\ /s$

$E = E_0 e^{-2bt}$

$\frac{E_0}{10} = E_0 e^{-2bt}$

$t = 54.64\ s$

2. Q

$f = 250\ Hz$

$T = \frac{1}{250}\ s$

$t = \frac{1}{250} \times 800 = 3.2\ s$

$E = E_0\ e^{-2bt}$

$\frac{E_0}{15} = E_0\ e^{-2bt}$

$t = 1.88\ s$

$A = A_0\ e^{-bt}$

$\frac{A_0}{10} = e^{-bt}$

$b = 0.7195\ /s$

$\tau = \frac{1}{2b} = 0.694\ s$

$Q = \omega\tau = 1091.7$

logarithmic decrement $\lambda = bT$
$= 2.878 \times 10^{-3}$

3. $Q = 1000$

$f = 256$ Hz

$E = E_0 e^{-2bt}$

$\frac{E_0}{e} = E_0 e^{-2bt}$

$t = \frac{1}{2b} = \tau$

$Q = \omega \tau$

$\frac{1000}{2 \times 3.14 \times 256} = \frac{1}{2b}$

$\tau = \frac{1}{2b} = 0.62$

$t = 0.62$ s

$b = \frac{1}{2 \times 0.62} = 0.8064/s$

No. of oscillation in τ s $= \tau \times f$

$= 158.72$

4. $m = 100$ g

$f = 10$ Hz

$A = A_0 e^{-bt}$

$\frac{A_0}{2} = A_0 e^{-b \times 60}$

$-0.693 = -60b$

$b = 0.01155/s$

$\tau = 43.28$ s

5. $A = A_0 e^{-bt}$

$1 = 16 e^{-b138}$

$\frac{1}{16} = e^{-138b}$

$b = 0.02009/s$

$T = 6.9$ s

$t = 20 \times 6.9$

$= 138$ s

③

$\tau = \frac{1}{2b} = 24.88\ s$

6.

$f = 256\ Hz$

$t' = 2\ s$

$E = E_0 e^{-2bt}$

$\frac{E_0}{2} = E_0 e^{-2b \times 2}$

$0.6931 = 4b$

$b = 0.1732/s$

$Q = \omega \tau$

$= 2\pi f \frac{1}{2b} = 4641.108$

7.

$f = 256\ Hz$

$Q = 103$

$E = E_0 e^{-2bt}$

$Q = \omega \tau$

$Q = 2\pi f \frac{1}{2b}$

$b = \frac{2\pi f}{2Q} = 7.8043/s$

$\frac{E_0}{5} = E_0 e^{-2bt}$

$\frac{1}{5} = e^{-2 \times 7.8043 t}$

$t = 0.103\ s$

$f = 256\ Hz$

$T = \frac{1}{256}$

Time for 200 oscillation $= 200 \times \frac{1}{256} = 0.781\ s$

8.

$Q = 120$

$m = 0.5\ kg$

$f = \ Hz$

$f = \frac{1}{2\pi} \sqrt{\frac{\mu}{m}}$

$\mu = 4\pi^2 f^2 m$

$= 1973.89$ N/m

$2b = \frac{r}{m}$

Mechanical resistance

$r = m\,2b$

$= \frac{m}{\tau} = \frac{m\omega}{Q}$ as $Q = \omega\tau$

$= \frac{0.5 \times 2\pi f}{120}$

$= 0.2616$ Kg/s

9. $Q = 60$

$m = 0.3$ kg

$f = 2$ Hz

$f = \frac{1}{2\pi}\sqrt{\frac{\mu}{m}}$

$\mu = 47.37$ N/m

$r = 2b \times m$

$Q = \omega\tau$

$60 = 2\pi \times 2 \times \frac{1}{2b}$

$2b = \frac{4\pi}{60}$

$r = \frac{4\pi}{60} \times 0.3$

$= 0.06282$ kg/s

10. $Q = 80$

$m = 0.8$ kg

$f = 10$ Hz

$\mu = 4\pi^2 f^2 m$

$= 3155.07$ N/m

$r = 2b\ m$

$r = \left(\frac{\omega}{Q}\right) m$

$= \frac{2\pi \times 1\cancel{0}}{8\cancel{0}} \times 0.8$

$= 0.628$ kg/s

(11)

$T = 2\,s$

$A = \frac{90}{100} A_0$ Amplitude decreases by 10%.

$T = 2\,s$

Here $t = T = 2\,s$

$A = A_0 e^{-bt}$

$\frac{9}{10} \cancel{A_0} = \cancel{A_0} e^{-2b}$

$b = 0.0526/s$

$\therefore \lambda = bT$

$= 0.105$

Here damping

for damping $T = \frac{2\pi}{\sqrt{\omega^2 - b^2}} = 2$

$\pi = \sqrt{\omega^2 - b^2}$

$\omega = \cancel{3.3444}\ 3.14$

Time period without damping $T = \frac{2\pi}{\omega}$

$= 2\,s$

It means damping is less here

$T_{\text{without damping}} = T_{\text{with damping}}$

If $T = 4\,s$

$T = \frac{2\pi}{\sqrt{\omega^2 - b^2}} = 4$

$b = 2.738/s$

$m = 0.15$ kg

$t = 35$ s

$E = E_0 e^{-2bt}$

$b = \frac{1}{70} = 0.01428/s$

$\omega = \sqrt{\frac{\mu}{m}} = 8.9442$ rad/s

$r = 2bm$

$= 4.285 \times 10^{-3}$ kg/s

$\omega =$ natural angular freq.

freq. of damped oscillation $\omega' = \sqrt{\omega^2 - b^2}$

$= 8.9439$ rad/s

$Q = \omega \tau$

$= 313.04$

13.

$\mu = 10$ N/m

$m = 0.1$ kg

$t = 50$ s

$E = E_0 e^{-2bt}$

$b = 0.01/s$

$\omega = \sqrt{\frac{\mu}{m}} = 10$ rad/s

$r = 2bm$

$= 2 \times 10^{-3}$ kg/s

$Q = \omega \tau$

$= 10 \times 50 = 500$

freq. of damped oscillator $\omega' = \sqrt{\omega^2 - b^2}$

$$= \sqrt{10^2 - (0.01)^2}$$

$$= 9.999 \text{ rad/s}$$

% change in freq $= \dfrac{\omega - \omega'}{\omega}$

$$= \left[1 - \frac{\omega'}{\omega}\right]$$

$$= \left(1 - \frac{\sqrt{\omega^2 - b^2}}{\omega}\right)$$

$$= \left[1 - \left(1 - \frac{b^2}{\omega^2}\right)^{1/2}\right]$$

as $b \ll \omega$

$$= \cancel{1} - \cancel{1} + \frac{1}{2}\frac{b^2}{\omega^2}$$

$$= \frac{1}{2}\frac{b^2}{\omega^2} = 5 \times 10^{-7}$$

% change $= 5 \times 10^{-5}$ %.

14.

$f = 100$

$Q = 1000$

$A = A_0 e^{-bt}$

$\dfrac{A}{e^4} \dfrac{1}{} \dfrac{A_0}{\cancel{A_0}} = e^{-bt}$

$e^{-4} = e^{-bt}$

$4 = bt$

$t = \dfrac{4}{b} = 12.738$ s

$Q = \omega \tau$

$1000 = \cancel{2}\pi 100 \times \dfrac{1}{\cancel{2}b}$

$b = 0.314$ /s

15) $b = 2\,s^{-1}$

$x(0) = 0$

$v = \left.\frac{dx}{dt}\right|_{\text{at } t=0} = 6\,m/s$

For critically damped oscillator $y = e^{-bt}(p+qt)$

Here $x(t) = e^{-bt}(p+qt)$

at $t=0$, $x=0$

so $0 = (p+q(0))\, e^{-b\times 0}$

$$\boxed{0 = p}$$

At $t=0$, $v = \frac{dx}{dt} = 6\,m/s$

$x(t) = e^{-bt}(p+qt)$

$\frac{dx}{dt} = -b\, e^{-bt}(p+qt) + e^{-bt}(q)$

At $t=0$

$6 = -b\, e^{0}(0+q\times 0) + e^{0} q$

$6 = 0+q$

$$\boxed{q=6}$$

as $b=2$, $p=0$, $q=6$

so $x = e^{-2t}\, 6t$

for farthest distance $x = x_{max}$

$\frac{dx}{dt} = 0$

$\frac{dx}{dt} = e^{-2t}\, 6 + 6t\,(-2)\, e^{-2t}$

$0 = e^{-2t}\, 6 - 12t\, e^{-2t}$

$0 = 6e^{-2t}(1-2t)$

$1-2t = 0$

$t = \frac{1}{2}\,s$

So $x_{max} = e^{-2\times\frac{1}{2}}\left(6\times\frac{1}{2}\right)$

$= e^{-1}\, 3$

$= 1.1\, m$