Question bank (Electromagnetic theory)

1. Explain divergence of a vector field with a suitable example. 4/5 marks
2. Explain curl of a vector field with a suitable example. 4/5 marks
3. Explain electric flux and electric flux density (D). Mention the relation between electric flux density (D) and electric field (E). 4/5 marks
4. State and explain Gauss law in electrostatics. Express the Gauss law of electrostatics in differential form. 7/8 marks
5. Define current density. Obtain the equation of continuity for current. 4/5 marks
6. Explain magnetic field intensity (H) and magnetic flux density (B). How these two are related? 4/5 marks
7. State and explain Bio-Savart law. Express it in vector form. 4/5 marks
8. State Ampere's circuital law. Express it in differential form. 7/8 marks
9. State and explain Gauss law in magnetism. Give its Physical significance. 4/5 marks
10. State and explain Faraday's law of electromagnetic induction. Obtain it in differential form. 6/7 marks
11. What is the inconsistency or anomaly in Ampere's law? 4/5 marks
12. Explain the concept and need of displacement current. What is Maxwell-Ampere's law? 7/8 marks
13. Write the differential form of Maxwell's equations for time varying fields. How they are modified in static fields? 4/5 marks
14. What are electromagnetic waves? Derive the electromagnetic wave equation in differential form in a region free from any charges and currents. 8/9 marks

DEPARTMENT OF PHYSICS

RAMAIAH Institute of Technology

Tutorials **Electromagnetic Theory**
(Term: Aug to Dec 2019)

1. Given $\vec{D} = (2y^2z - 8xy)\hat{\imath} + (4xyz - 4x^2)\hat{\jmath} + (2xy^2 - 4z)\hat{k}$. Determine the total charge within a volume 10^{-14} m^3 at P (1,-2,3), if the divergence of $\vec{D}$ gives the charge density ρ_v.
2. Given $\vec{D} = 9x^3\hat{\imath} + 5y^2\hat{\jmath} + 2z\hat{k}$ Cm^{-2}. If the divergence of $\vec{D}$ represents the charge density ρ_v, find ρ_v at the point (1,5,9) m.
3. Given $\vec{D} = 4x\hat{\imath} + 3y^2\hat{\jmath} + 2z^3\hat{k}$ Cm^{-3}. If the divergence of $\vec{D}$ represents the charge density ρ_v, then find the total charge in a volume defined by six planes for which $1 \le x \le 2$, $2 \le y \le 3$, $3 \le z \le 4$.
4. The magnetic field intensity is given in a certain region of space as
$$\vec{H} = \frac{x+2y}{z^2}\hat{\jmath} + \frac{2}{z}\hat{k}\ \mathrm{Am^{-1}}.$$
Find the curl of the magnetic field.
5. A point charge, Q = 30nC is located at the origin in Cartesian system. Find the electric flux density and the electric field intensity at (1, 3,-4).
6. Suppose $A = x^2z^2\hat{\imath} - 2y^2z^2\hat{\jmath} + xy^2z\hat{k}$. Find $\nabla \cdot A$ at the point P (1,-1, 1).
7. A parallel plate capacitor consists of plates of area 10 cm^{-2} with separation 10mm and dielectric medium of permittivity $\epsilon = 4\,\epsilon_0$. Calculate the displacement current, if the voltage applied is 15 sin(1000t) and $\epsilon = 4\,\epsilon_0$.
8. $\vec{A} = x^2yz\hat{\imath} + xy^2z\hat{\jmath} + xyz^2\hat{k}$. Determine curl $\vec{A}$.
9. $\vec{A} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. Find div A and curl $\vec{A}$.

$(\epsilon_0 = 8.854 \times 10^{-12} Fm^{-1})$

10 Determine the constant C such that, the vector $\vec{A} = (x+ay)\hat{i} + (y+bz)\hat{j} + (x+cz)\hat{k}$ is solenoi
Ans $C = -2$

11. Given $\vec{A} = (3x^2+y+az)\hat{i} + (bx-5y^3-2z)\hat{j} + (2x+cy+3z^2)\hat{k}$
For what values of a, b & c, the $\vec{A}$ is irrotational
Ans $a = 2$ $c = -2$
$b = 1$

Unit – 5

If. $\vec{D} = (2y^2z - 8xy)\hat{i} + (4xyz - 4x^2)\hat{j} + (2xy^2 - 4z)\hat{k}$. determine the total charge with in volume $10^{-14}\ m^3$ at $P(1, -2, 3)$, if the divergence of $\vec{D}$ gives the charge density ρ_v.

$$\nabla \cdot \vec{B} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left[(2y^2z - 8xy)\hat{i} + (4xyz - 4x^2)\hat{j} + (2xy^2 - 4z)\hat{k}\right]$$

$$= \frac{\partial}{\partial x}(2y^2z - 8xy) + \frac{\partial}{\partial y}(4xyz - 4x^2) + \frac{\partial}{\partial z}(2xy^2 - 4z)$$

$$= -8y + 4xz - 4$$

$$\nabla \cdot D \text{ at } (1, -2, 3) = 16 + 12 - 4$$
$$= 24\ C$$

Total charge in vol. $10^{-14}\ m^3 = 24 \times 10^{-14}\ C$

2. $\vec{D} = 9x^3\hat{i} + 5y^2\hat{j} + 2z\hat{k}$

$$\nabla \cdot \vec{D} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot (9x^3\hat{i} + 5y^2\hat{j} + 2z\hat{k})$$

$$= \frac{\partial}{\partial x} 9x^3 + \frac{\partial}{\partial y} 5y^2 + \frac{\partial}{\partial z} 2z$$

$$= 27x^2 + 10y + 2$$

At $(1, 5, 9) = 27 + 50 + 2$
$= \cancel{86}\ 79\ C/m^3$

3. $\vec{D} = 4x\hat{i} + 3y^2\hat{j} + 2z^3\hat{k}$

$$\nabla \cdot \vec{D} = \rho .$$

$$\int_V \rho \, dV = Q = \text{Total charge}$$

$$\int_V \nabla \cdot \vec{D} \, dV = \int_V \rho \, dV = Q$$

$$\int_1^2 \int_2^3 \int_3^4 \nabla \cdot \vec{D} \, dV = \oint_1^2$$

$$\nabla \cdot \vec{B} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left(4x\hat{i} + 3y^2\hat{j} + 2z^3\hat{k}\right)$$

$$= 4 + 6y + 6z^2$$

$$\int_V \nabla \cdot \vec{B} \, dV = \int_1^2 \int_2^3 \int_3^4 (4 + 6y + 6z^2) \, dx \, dy \, dz$$

$$= \int_1^2 \int_2^3 \left(4z + 6yz + \frac{6z^3}{3}\right)\Big|_3^4 dx \, dy$$

$$= \int_1^2 \int_2^3 \left[(16 + 24y + 128) - 12 - 18y - 54\right] dy \, dx$$

$$= \int_1^2 \int_2^3 (6y + 78) \, dy \, dx$$

$$= \int_1^2 \left[\frac{6y^2}{2} + 78y\right]_2^3 dx$$

$$= \int_1^2 \left(27 + 234 - 12 - 156\right) dx$$

$$= 93 \text{ C}$$

156
15
171
234
171

POCO
SHOT ON POCO M2 PRO

$$\vec{H} = \left(\frac{x+2y}{z^2}\right)\hat{j} + \frac{2}{z}\hat{k}$$

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \frac{x+2y}{z^2} & \frac{2}{z} \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}\left(\frac{2}{z}\right) - \frac{\partial}{\partial z}\left(\frac{x+2y}{z^2}\right)\right] - \hat{j}\left[\frac{\partial}{\partial x}\left(\frac{2}{z}\right) - \frac{\partial}{\partial z}(0)\right]$$
$$+ \hat{k}\left[\frac{\partial}{\partial x}\left(\frac{x+2y}{z^2}\right) - \frac{\partial}{\partial y}(0)\right]$$

$$= \hat{i}\left[0 + 2z^{-3}(x+2y)\right] - \hat{j}\left[0\right] + \hat{k}\left[\frac{1}{z^2} - 0\right]$$

$$= \frac{2}{z^3}(x+2y)\hat{i} + \frac{1}{z^2}\hat{k}$$

5. Electric field intensity $\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$

~~$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$~~

(0,0,0) (1,3,-4)

30nC

$$\vec{r} = \hat{i} + 3\hat{j} - 4\hat{k}$$

$$|\vec{r}| = \sqrt{1^2+3^2+4^2} = \sqrt{26}$$

$$\hat{r} = \frac{\vec{r}}{|r|} = \frac{\hat{i}+3\hat{j}-4\hat{k}}{\sqrt{26}}$$

$$\vec{E} = \frac{30\times10^{-9}}{4\times\pi\times 8.85\times10^{-12}\times 26}\left(\frac{\hat{i}+3\hat{j}-4\hat{k}}{\sqrt{26}}\right)$$

$$= 2.034\,(\hat{i}+3\hat{j}-4\hat{k})$$

$$|\vec{E}| = 10.375 \text{ V/m}$$

$\vec{D} = \epsilon \vec{E}$

for free space ϵ_0

$\vec{D} = \epsilon_0 \vec{E} = \frac{Q}{4\pi r^2} \hat{r}$

$= 9.18 \times 10^{-11} \frac{(\hat{i} + 3\hat{j} - 4\hat{k})}{\sqrt{26}}$

$= 1.8 \times 10^{-11} (\hat{i} + 3\hat{j} - 4\hat{k})$

$|\vec{D}| = 9.18 \times 10^{-11} \ C/m^2$

6. $\vec{A} = x^2 z^2 \hat{i} - 2y^2 z^2 \hat{j} + xy^2 z \hat{k}$

$\nabla \cdot \vec{A} = \frac{\partial}{\partial x}(x^2 z^2) + \frac{\partial}{\partial y}(-2y^2 z^2) + \frac{\partial}{\partial z}(xy^2 z)$

$= 2xz^2 - 4yz^2 + xy^2 \Big|_{at\ (1,-1,1)}$

$= 2 + 4 + 1$

$= 7$

8. $\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2yz & xy^2z & xyz^2 \end{vmatrix}$

$= \hat{i}\left[\frac{\partial}{\partial y}(xyz^2) - \frac{\partial}{\partial z}(xy^2z)\right] - \hat{j}\left[\frac{\partial}{\partial x}(xyz^2) - \frac{\partial}{\partial z}(x^2yz)\right]$

$+ \hat{k}\left[\frac{\partial}{\partial x}(xy^2z) - \frac{\partial}{\partial y}(x^2yz)\right]$

$= \hat{i}(xz^2 - xy^2) - \hat{j}(yz^2 - x^2y) + \hat{k}(y^2z - x^2z)$

$$\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\nabla \cdot \vec{A} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z)$$

$$= 1 + 1 + 1$$

$$= 3$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y} z - \frac{\partial}{\partial z} x\right] - \hat{j}\left[\frac{\partial}{\partial x} z - \frac{\partial}{\partial z} x\right] + \hat{k}\left[\frac{\partial}{\partial x} y - \frac{\partial}{\partial y} x\right]$$

$$= 0$$

7. Area $= 10 \times 10^{-4}\ m^2$

distance $= 10\,mm = 10 \times 10^{-3}\ m$

Voltage $= 15 \sin(1000t)$

$\epsilon = 4\epsilon_0$ $\quad I_d = ?$

$I_d = \epsilon \frac{d\phi}{dt}$ where $\phi = EA$

$$= \epsilon A \frac{dE}{dt}$$

But $E = V/d$

$$= \frac{\epsilon A}{d} \frac{dV}{dt}$$

$$= \frac{\epsilon A}{d} \frac{d}{dt}(15 \sin 1000t)$$

$$= \frac{\epsilon A}{d}\left[1000 \times 15 \cos(1000t)\right]$$

$$= 15000 \frac{\epsilon A}{d} \cos 1000t$$

$$= \frac{15000 \times 4\epsilon_0 \times 10 \times 10^{-4}}{10 \times 10^{-3}} \cos(1000t)$$

$$I_d = 5.295 \times 10^{-8} \cos(1000t) \text{ Ampers}$$