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RAMAIAH
Institute of Technology
(Autonomous Institute, Affiliated to VTU)
(Approved by AICTE, New Delhi & Govt. of Karnataka) Accredited by NBA & NAAC with 'A' Grade

SEMESTER END EXAMINATIONS – JANUARY 2019

Course & Branch	: **B.E. : Common to all branches**	**Semester**	: **I**
Subject	: **Engineering Mathematics - I**	**Max. Marks**	: **100**
Subject Code	: **MA11/MAT101**	**Duration**	: **3 Hrs**

Instructions to the Candidates:

- Answer one full question from each unit.

UNIT- I

1. a) Define Jacobian of $u, v, w,$ with respect to x,y,z. CO1 (02)

b) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $xu_x + yu_y + zu_z = 0$. CO1 (04)

c) If $u = e^{\left(\frac{x^3 y^3}{x^2+y^2}\right)}$ show that $xu_x + yu_y = 4u \log u$. CO1 (07)

d) Find the angle of intersection between the following pair of curves: $r = a(1+\sin\theta)$ and $r = a(1-\sin\theta)$. CO1 (07)

2. a) State Euler's theorem on homogeneous function of two independent variables. CO1 (02)

b) Find the Pedal equation of $r^n = a^n \cos n\theta$. CO1 (04)

c) If $u = e^{xy} \sin(yz)$, where $x = t^2$, $y = t-1$, $z = \frac{1}{t}$ then du/dt at t=1 by partial differentiation. CO1 (07)

d) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$ show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$. CO1 (07)

UNIT- II

3. a) Write the expression to find the area bounded by the cartesian curve. CO2 (02)

b) Evaluate $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right) dx$. CO2 (04)

c) Find the volume of revolution of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$, when rotated with respect to x- axis. CO2 (07)

d) Trace the curve $x^3 + y^3 = 3axy,\ a > 0$. CO2 (07)

4. a) Write the expression to find the surface area of the solid for a polar curve when rotated about the initial line and the line $\theta = \pi/2$. CO2 (02)

	b)	Find the area of the cardioid $r = a(1+\cos\theta),\ a>0$.	CO2	(04)
	c)	Find the length of one arc of the cycloid: $x = a(\theta - \sin\theta), y = a(1-\cos\theta)\ a>0, 0\le\theta\le 2\pi$.	CO2	(07)
	d)	Trace the curve $y^2(2a-x) = x^3, a>0$.	CO2	(07)

UNIT- III

5.	a)	Write the relation between cartesian and spherical polar coordinate systems.	CO3	(02)
	b)	Evaluate $\int_0^1\int_0^2\int_0^3 xyz\,dx\,dy\,dz$.	CO3	(04)
	c)	If R is the region bounded by $x=0, y=0$ and $x+y=1$ then show by using the transformation $x+y=u,\ x-y=v$ that $\iint_R \sin\left(\frac{x-y}{x+y}\right)dx\,dy = 0$.	CO3	(07)
	d)	Using double integration, find the area enclosed by the parabolas x^2 and $y^2 = x$.	CO3	(07)
6.	a)	Convert the integral $\int_0^a \int_0^{\sqrt{a^2-x^2}} f(x,y)\,dy\,dx$ into polar coordinates.	CO3	(02)
	b)	Find the limits by changing the order of integration $\int_1^2\int_1^{x^2}(x^2+y^2)\,dy$.	CO3	(04)
	c)	Find the volume of the ellipsoid $x^2+y^2+z^2=a^2$ using triple integration.	CO3	(07)
	d)	Evaluate $\iint_R dxdy$ where R is the region bounded by the curves $y=x^2$ and $y=x$. Interpret the answer obtained.	CO3	(07)

UNIT- IV

7.	a)	Explain the geometrical meaning of gradient of a scalar field.	CO4	(02)
	b)	If $r = \sec t\,\hat{i} + \tan t\,\hat{j}$ is the position vector of a moving particle then find the velocity and acceleration at $t=\pi/6$.	CO4	(04)
	c)	Find the values of the constants a, b, c such that $F = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$ is conservative. Also find its scalar potential.	CO4	(07)
	d)	Prove that $\nabla\times(\varphi A) = \varphi(\nabla\times A) + (\nabla\varphi\times A)$	CO4	(07)
8.	a)	Define Laplacian operator.	CO4	(02)
	b)	Show that $\vec{f} = \frac{x\hat{i}+y\hat{j}}{x^2+y^2}$ is a solenoidal vector.	CO4	(04)
	c)	Find the angle between the surfaces: $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2,-1,2)$.	CO4	(07)

	d)	Find the directional derivative of $\phi = y^2 x + yz^3$ at the point (2,-1, 1) in the direction of the normal to the surface $x \log z - y^2 = -4$ taken at the point (-1,2,1).	CO4	(07)

UNIT- V

9.	a)	Give the physical interpretation of $\int_c F \cdot dr$, if F is force on a particle moving along C.	CO5	(02)
	b)	By using Green's theorem, evaluate $\int_c (x^2 + xy)dx + (x^2 + y^2)dy$ where C is the square formed by the lines $x = \pm 1;\ y = \pm 1$.	CO5	(04)
	c)	Evaluate $\int_s F \cdot \hat{n}\, ds$ where $F = yz\,\hat{i} + 2y^2\,\hat{j} + xz^2\,\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 9$ contained in the first octant between $z = 0$ and $z = 2$ using Gauss – divergence theorem.	CO5	(07)
	d)	Evaluate $\int_c F \cdot dr$ where $F = y\,\hat{i} + z\,\hat{j} + x\,\hat{k}$ where C is the boundary of the upper half of the sphere $x^2 + y^2 + z^2 = 1$ using Stoke's theorem.	CO5	(07)
10	a)	State Gauss – divergence theorem.	CO5	(02)
	b)	Find the circulation of F round the curve C where $F = (x - y)\hat{i} + (x + y)\hat{j}$ and C is the circle $x^2 + y^2 = 4,\ z = 0.$	CO5	(04)
	c)	State and prove Green's theorem in a plane.	CO5	(07)
	d)	Evaluate Stoke's theorem for $F = -y^3\,\hat{i} + x^3\,\hat{j}$ where s is the circular disk $x^2 + y^2 \le 1,\ z = 0.$	CO5	(07)

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