Scheduling Algorithm with Transmission Power Control for Random Underwater Acoustic Networks

Prasad Anjangi and Mandar Chitre
Acoustic Research Laboratory, Tropical Marine Science Institute and
Department of Electrical & Computer Engineering
National University of Singapore, 12A Kent Ridge Road, Singapore 119223
Email: {prasad, mandar}@arl.nus.edu.sg

***Abstract*—Underwater acoustic communication networks face several challenges: large propagation delays, low bandwidth and high transmission power. Recent studies exploit the large propagation delay to allow concurrent transmissions in the network. Given a suitable network geometry, it has been shown that the network throughput increases linearly with the number of nodes, provided optimal transmission schedules are adopted. Although the throughput in an arbitrary network geometry is often more constrained, it can be increased by controlling transmission power. Power control helps reduce energy consumption and limit interference. We investigate achievable throughput in randomly deployed underwater acoustic networks by controlling transmission power. We develop joint scheduling and power control algorithm for arbitrary networks, and demonstrate performance improvement in a large number of random network geometries. We also present some network geometries for which optimal throughput is achieved by using transmission schedules with power control.**

I. Introduction

Medium access control (MAC) is a challenging problem in underwater acoustic (UWA) networks due to the large propagation delay of the acoustic channel and the half-duplex nature of the communication links [13]. The propagation speed of sound underwater is roughly five orders of magnitude lower than that of radio waves in air [14]. While propagation delay is negligible for short range radio frequency (RF) communications, it is an important parameter to be considered in the design of MAC protocols for UWA communications. The existence of spatio-temporal uncertainty in UWA networks as described in [9] due to large propagation delays, causes uncertainty in the reception time at the receiver. This problem is investigated in [6], [7], [10], [11], where the authors mitigate the effect of propagation delay. The best performance achieved using these techniques is comparable with wireless networks with negligible propagation delays. A fundamental understanding of the potential of large propagation delays in UWA networks to allow network throughput beyond that of networks with negligible propagation delays is provided in [3]. The authors present a technique to design transmission schedules that ensure that most of the interfering messages overlap in time at the unintended receivers, and the desired messages are interference-free at the intended receivers. A closely related work is presented in [8], where the authors use similar techniques as [3], and establish an upper bound on throughput for multihop grid UWA networks. More recently, a heuristic propagation delay based interference alignment algorithm for multihop UWA networks was presented in [15]. A similar idea to find schedules exploiting large propagation delays for dynamic network topologies was presented in [5].

We take advantage of the results from [3] and investigate the effect on throughput of UWA network when we limit the interference range by controlling the transmission power. In [3], the authors prove that for a fully-connected network, $\frac{N}{2}$ is the upper bound on throughput, where N is the number of nodes in the network. The authors also present some geometries and transmission schedules which can achieve this upper bound. The study considers a network where all the nodes lie in the interference range of any transmission. In fully-connected networks where the upper bound could be achieved, the transmission power control would only result in reducing the energy consumption. However, for many realistic networks with arbitrary geometries, the upper bound may not be achieved. In such cases, limiting the interference range by controlling the transmission power will cause the network throughput to increase significantly.

We consider large number of random network geometries and show through simulations that, by limiting the interference range, we can design transmission schedules with significant improvement in throughput. We also present some examples of network geometries where the upper bound $\frac{N}{2}$ is achieved due to power control, but not otherwise.

The rest of the paper is organized as follows. The system model is introduced in Section II. In Section III, we formulate the problem to choose good delay matrices for an arbitrary network geometry, and present scheduling feasibility constraints and value function for the scheduling problem, with and without power control. Simulation results are presented and discussed in Section IV and conclusions are drawn in Section V.

II. System Model

We consider an N-node network deployed randomly in a 2D space of $H \times H$ meters. Let $\mathbf{x_j}$ be the position vector of node j in the wireless network deployed such that,

$$\mathbf{x_j} = \begin{bmatrix} p \\ q \end{bmatrix}, \ \ p, q \sim U(0, H)$$

where p, q are uniformly distributed between 0 and H.

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A. Integer & Non-Integer Delay Matrices

The network geometry formed can be represented in the form of a delay matrix as also shown in [3], where each element of the delay matrix contains the propagation delay between the corresponding pair. We denote the delay matrix by $\mathbf{D}$ and the elements of $\mathbf{D}$ are written as:

$$D_{ij} = \frac{|\mathbf{x_i} - \mathbf{x_j}|}{c\tau}, \;\; i,j \in \{1,2,\ldots,N\} \tag{1}$$

where c is the speed of sound underwater and τ is the time slot length. It is important to note that the elements of the delay matrix are propagation delays between links in units of time slot length τ and can be rational numbers, i.e., $\mathbf{D}$ can be a non-integer delay matrix. But with appropriate choice of time slot length τ, the given non-integer delay matrix can be approximated by an integer delay matrix $\mathbf{D'}$ [3]:

$$D_{ij}^{'} = \left\lceil \frac{|\mathbf{x_i} - \mathbf{x_j}|}{c\tau} \right\rfloor, \;\; i,j \in \{1,2,\ldots,N\} \tag{2}$$

where by $\lceil a \rfloor$ we denote the closest integer to the real value a. In reality, the values of time slot length τ are constrained to those allowed by the underwater acoustic modems. To be more precise, the packet lengths are constrained by the modem configuration and capability. These constraints translate to restrictions on time slot lengths in order to efficiently utilize the slots. We denote the minimum and maximum possible time slot lengths that can be set by $\tau_{\min}$ and $\tau_{\max}$ respectively. The difference between the corresponding elements of the integer and non-integer delay matrix provides us information that can be used to select the time slot length that enables efficient utilization of slots. The maximum packet length that can be transmitted in a time slot with length τ, given ρ^+ and ρ^- is $\tau(1-\rho^- - \rho^+)$ [3], where ρ^+ and ρ^- are given by:

$$\rho^+ = \max_{ij}(D_{ij} - D'_{ij}) \tag{3}$$

$$\rho^- = -\min_{ij}(D_{ij} - D'_{ij}) \tag{4}$$

where $i,j \in \{1,2,\cdots,N\}$ for a fully-connected network.

B. Communication and Interference Range

We adopt a protocol channel model and denote by α, the ratio of interference range to the communication range. These ranges are function of the transmission power P. Let d_I denote the interference range and d_C, the communication range in the acoustic channel for a given transmission power P. We can express α as:

$$\alpha = \frac{d_I}{d_C} = \frac{(\frac{d_I}{c\tau})}{(\frac{d_C}{c\tau})} = \frac{D_I}{D_C} \tag{5}$$

where D_I and D_C are the propagation delays corresponding to the interference and communication range respectively in the acoustic medium, in units of time slot length τ. In wireless radio networks, the interference range is often considered to be approximately twice the communication range [1], [4]. We assume the same factor in UWA network and use $\alpha = 2$ in our simulations.

C. Schedules

A schedule is denoted by matrix $\mathbf{W}$ which determines the time slots in which each node in the network transmits and receives messages. It can be elucidated as follows:

1) If $W_{j,t} = i > 0$, then node j transmits a message to node i in time slot t.
2) If $W_{j,t} = -i < 0$, then node j receives a message from node i in time slot t.
3) If $W_{j,t} = 0$, then node j is idle during time slot t.

If $W_{j,t+T} = W_{j,t} \;\forall\; j,t$, then the schedule is periodic with period T. It can be written as a matrix of order $N \times T$ denoted by $\mathbf{W}^{(T)}$.

$$W_{j,t} = W_{j,(t \mod T)}^{(T)}$$

1) Necessary Condition for Transmission: Node j transmits a message to node i during time slot t only if node i is able to successfully receive the message during time slot $t + D_{ij}$, i.e.,

$$W_{j,t} = i \Leftrightarrow W_{i,t+D_{ij}} = -j \;\; \forall \;\; i \neq j \tag{6}$$

2) Necessary Condition for Successful Reception: To ensure successful reception at time slot t of a transmitted message from node j, it is required that no other nodes transmit messages that arrive at node i during time slot t. Therefore,

$$W_{i,t} = -j \Rightarrow W_{k,t-D_{ik}} \leq 0 \;\; k \neq i \tag{7}$$

D. Throughput

The average throughput S of a schedule with period T can be computed by counting the number of receptions in the schedule $\mathbf{W}^{(T)}$.

$$S = \frac{1}{T}\sum_t \sum_j \mathbb{1}(W_{j,t}^{(T)} < 0) \tag{8}$$

where $\mathbb{1}(E)$ is the indicator function of an event E, with value of 1 if E is true and 0 otherwise. In the case where the period of the schedule computed is not known, the approximate throughput is computed by counting the number of receptions over a large number of time slots $T^{'}$. In that case, the approximate throughput $S^{'}$, computed over $T^{'}$ slots is given by:

$$S^{'} = \frac{1}{T'}\sum_{t=1}^{T^{'}} \sum_{j=1}^{N} \mathbb{1}(W_{j,t} < 0) \tag{9}$$

The throughput defined in the above (8) & (9) only count the number of receptions but do not take into account the utilization of the time slots. ρ-throughput denoted by S_ρ, and defined as:

$$S_\rho = S(1 - \rho^+ - \rho^-) \tag{10}$$

takes into account the time slot utilization [3].

E. Example Delay Matrix & Schedule

The delay matrix, schedule and throughput for a three node equilateral triangle are given below:

$$\mathbf{D} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \mathbf{W}^{(4)} = \begin{bmatrix} 2 & 3 & -3 & -2 \\ -3 & -1 & 1 & 3 \\ -2 & 1 & -1 & 2 \end{bmatrix}$$

The above-mentioned delay matrix represents a network geometry where the nodes are placed such that they make an equilateral triangle with the link propagation delays as one unit of time slot length. The schedule can be interpreted as follows: In first time slot, node 1 transmits a message to node 2, and in the second time slot, node 2 receives a message from node 1 and so on. Also, note that the period of the schedule in this example is $T = 4$ and the schedule repeats itself for every 4 time slots. The above schedule example is taken from [3] for illustration.

III. Scheduling Problem

Before formulating the scheduling problem, we present a method for selecting the time slot length to compute the closest integer delay matrix representing the random network geometry generated. This delay matrix is used to compute the schedules, with and without power control.

A. Choosing Time Slot length

We know from [3] that the utilization efficiency of a time slot of length τ is given by:

$$\eta = \frac{\tau(1 - \rho^+ - \rho^-)}{\tau} = 1 - \rho^+ - \rho^-$$

A simple one dimensional optimization problem needs to be solved in order to select the time slot length τ which would provide the closest integer delay matrix yielding maximum utilization of time slots. We use a brute-force method to find an optimal time slot length. The problem is formally written as follows:

$$\begin{aligned} &\underset{\tau}{\text{minimize}} && \rho^+ + \rho^- \\ &\text{subject to} && \tau = \{\tau_{\text{min}} + iv \ ; \ i \in \mathrm{Z}^+ \cap \left[1, \lfloor \frac{\tau_{\text{max}} - \tau_{\text{min}}}{v} \rfloor \right]\} \end{aligned}$$

where v is the smallest incremental step size in which the time slot length can be altered in the modem. We use $v = 1$ ms in our simulations, but it may be varied to model different modems. For each value of time slot length between τ_{min} and τ_{max}, the delay matrices $\mathbf{D}$ and $\mathbf{D}^{'}$ are computed using (1) and (2) and the corresponding values of ρ^+ and ρ^- are computed using (3) and (4) further to find the value of the objective function in the above optimization problem. The optimal time slot length τ^* found is then used to compute the delay matrix $\mathbf{D}$ as given below:

$$D_{ij} = \frac{|\mathbf{x_i} - \mathbf{x_j}|}{c\tau^*}$$

The delay matrix $\mathbf{D}$ computed is then rounded off to the closest integer delay matrix $\mathbf{D}'$ as shown in (2).

B. Link Scheduling Optimization Problem

The scheduling problem is formulated as a sequential decision problem (SDP) [3], where at each time slot t, the decisions are taken on which nodes should transmit to which other node in the network and the schedule is updated accordingly. A deterministic SDP is defined by the state space, action space, the transition function f which describes how the state changes as a result of the actions, and the reward function, which evaluates the immediate performance of the action taken.

1) State Space: The state of the decision problem is denoted by $\mathbf{W}^{\{t,u\}}$, which represents the partial schedule containing all the transmissions between time slots t and $t - \alpha G$ and $u - 1$ transmission decisions already taken in time slot t. The propagation delay corresponding to the maximum transmission range in the network is denoted by G, and hence αG is the propagation delay corresponding to maximum interference range in the network. The transmission on this link would affect αG slots in future. Therefore, in order to take decision at time slot t, it is enough to consider transmissions which occurred in the past till $t - \alpha G$ slots. All possible partial matrices that $\mathbf{W}^{\{t,u\}}$ can take, form the state space of the decision problem and is denoted by $\mathcal{W}$.

2) Action Space: The u^{th} action to be taken in time slot t is denoted by $\mathbf{x}^{\{t,u\}}$. The u^{th} action $\mathbf{x}^{\{t,u\}}$ taken is a tuple (j, k), which denotes node j transmitting to node k. Let M_t be the total number of transmissions during time slot t. Since we have N nodes, $M_t \leq N$. The action space from which the u^{th} action $\mathbf{x}^{\{t,u\}}$ is chosen at time slot t is denoted by $\mathcal{X}^{\{t,u\}}$.

3) Transition Function: As a result of the u^{th} transmission decision taken in time slot t in the state $\mathbf{W}^{\{t,u\}}$, the state changes according to the transition function $f : \mathcal{W} \times \mathcal{X}^{\{t,u\}} \rightarrow \mathcal{W}$:

$$\mathbf{W}^{\{t,u+1\}} = f(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}}) \ \forall \ u < M_t \tag{11}$$

$$\mathbf{W}^{\{t+1,1\}} = f(\mathbf{W}^{\{t,M_t\}}, \mathbf{x}^{\{t,M_t\}}) \tag{12}$$

4) Reward Function: Due to the action taken when in a particular state, an immediate scalar reward denoted by $r(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}})$ is obtained according to the reward function $r : \mathcal{W} \times \mathcal{X}^{\{t,u\}} \rightarrow \mathbb{R}$. Note that the reward due to any action is the number of successful transmissions added to the schedule due to that action. Since every action (j, k) is just one transmission, the reward is 1 every time an action is taken. The reward evaluates the immediate effect of the transition from one state to another but does not say anything about its long-term effect.

5) Policy: The actions are chosen according to a policy $\pi : \mathcal{W} \rightarrow \mathcal{X}^{\{t,u\}}$, using:

$$\mathbf{x}^{\{t,u\}} = \pi(\mathbf{W}^{\{t,u\}}) \tag{13}$$

Usually, the goal is to find an optimal policy that maximizes the return, starting from any initial state. The return is the sum of rewards along a trajectory starting at some initial state. It represents the reward obtained due to the sequence of decisions taken in the long run.

6) Q-Value Function: A way to characterize policies is by using their value functions. Two types of value functions exist – state value functions (V-functions) and state-action value functions (Q-functions) [2]. The Q-function $Q^\pi : \mathcal{W} \times \mathcal{X}^{\{t,u\}} \rightarrow \mathbb{R}$ of a policy π gives the return obtained when starting from a given state, applying a given action, and following policy π thereafter. The optimal Q-function is defined as the best Q-function that can be obtained by any policy:

$$Q^*(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}}) = \max_{\pi} \; Q^\pi(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}}) \qquad (14)$$

Any policy π^* that selects at each state, an action with the largest optimal Q-value, i.e., that satisfies:

$$\pi^*(\mathbf{W}^{\{t,u\}}) \in \underset{\mathbf{x}^{\{t,u\}}}{\operatorname{argmax}} \; Q^\pi(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}}) \qquad (15)$$

is optimal. In general, for a given Q-function, a policy π that satisfies:

$$\pi(\mathbf{W}^{\{t,u\}}) \in \underset{\mathbf{x}^{\{t,u\}}}{\operatorname{argmax}} \; Q(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}}) \qquad (16)$$

is said to be greedy in Q. So finding an optimal policy can be done by first finding Q^*, and then using (15) to compute a greedy policy in Q^* [2].

The Q-functions Q^π and Q^* are recursively characterized by the Bellman equations: [2], [12]:

$$\begin{aligned} Q^*(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}}) &= r(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}}) \\ &+ \max_{\mathbf{x}^{\{t,u+1\}}} Q(f(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}}), \mathbf{x}^{\{t,u+1\}}) \end{aligned} \qquad (17)$$

Usually, the second term above is preceded by a discounting factor if the infinite horizon time is considered in computing Q-Value function. The discounting ensures that the return will always be bounded if the rewards are bounded. However, since we look at a finite time horizon in future as will be explained in the next section, we do not consider discounting in this formulation. The reward for a single transmission decision is always 1, and hence it does not affect the decision making, therefore,

$$Q^*(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}}) = \max_{\mathbf{x}^{\{t,u+1\}}} Q(f(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}}), \mathbf{x}^{\{t,u+1\}}) \qquad (18)$$

Note that if we have optimal Q-values for each state-action pair possible, we can take decisions optimally in every state. Hence, finding an optimal Q-value is of great interest to us. Solving the above problem shown in (17) using traditional methods like Q-value iteration is computationally intensive due to the large state space and action space [2], [12] . Instead, we follow a similar approach used in [3] where an approximate value function is defined for each state. If we denote the approximate Q-value function for each state-action pair by $Q^\dagger(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}})$, then the optimal decision is taken as follows:

$$\mathbf{x}^{*\{t,u\}} = \; \arg\max_{\mathbf{x}^{\{t,u\}}} Q^\dagger(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}}) \qquad (19)$$

7) Approximate Q-Value Function: At each state, based on the particular action (j, k) chosen, the partial schedule is updated and lands in a different state. An approximate Q-Value function thus needs to be defined for the state-action pair $Q^\dagger(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}})$. The Q-Value function should capture the ability of accommodating future potential transmissions when it is in the current state and a particular action is taken. As we want throughput to be maximized, the decision which supports maximum number of future potential transmissions is considered a good decision. The approximate Q-Value function is written as:

$$Q^\dagger(\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}}) = \sum_{j=1}^{N} \sum_{k=1}^{N} \sum_{\zeta=0}^{\alpha G} Z_{jk\zeta}(f((\mathbf{W}^{\{t,u\}}, \mathbf{x}^{\{t,u\}}))) \qquad (20)$$

where $Z_{jk\zeta}(\mathbf{W}^{\{t,u\}})$ is a *transmission indicator function* with value 1, if a transmission from node j to node k is allowed between time slots t and $t + \alpha G$, and 0 otherwise, given the partial schedule $\mathbf{W}^{\{t,u\}}$.

8) Transmission Indicator Function: We list here the feasibility constraints under which the transmission from node j to node k is allowed or disallowed at time slot $t + \zeta$, where ζ ranges from 0 to αG, to capture the potential of accommodating future transmissions till αG slots ahead from time slot t.

a) Self transmissions are not allowed, i.e., a node is not permitted to transmit a message to itself.

$$Z_{jk\zeta}(\mathbf{W}^{\{t,u\}}) = 0 \;\text{ if }\; j = k \qquad (21)$$

b) If node j in time slot $t + \zeta$ is already scheduled to transmit or receive a message from some transmission that occurred in earlier time slot then node j is not permitted to transmit a message to node k and hence

$$Z_{jk\zeta}(\mathbf{W}^{\{t,u\}}) = 0 \;\text{ if }\; W^{\{t,u\}}_{j,t+\zeta} \neq 0 \qquad (22)$$

c) If node k is already receiving a message in time slot $t + \zeta + D'_{jk}$ from some other node in the network, then the transmission from node j to node k should not be permitted in time slot $t + \zeta$, which implies

$$Z_{jk\zeta}(\mathbf{W}^{\{t,u\}}) = 0 \;\text{ if }\; W^{\{t,u\}}_{k,t+\zeta+D'_{jk}} \neq 0 \qquad (23)$$

d) If there exists a node i which has transmitted in such a slot $t + \zeta + D'_{jk} - D'_{ik}$, that its reception, even though not intended at node k, interferes with the transmission from node j to node k then node j must not be permitted to transmit to node k. This can be written as

$$Z_{jk\zeta}(\mathbf{W}^{\{t,u\}}) = 0 \;\text{ if }\; \exists \; i \;\text{ s.t. }\; W^{\{t,u\}}_{i,t+\zeta+D'_{jk}-D'_{ik}} > 0 \qquad (24)$$

e) If there exists a link from node l to node i in the network such that the transmission from node l and its reception at node i happen in such a slot that the transmission from node j to node k would be interfering at the node i, then node j must not be permitted to transmit.

$$Z_{jk\zeta}(\mathbf{W}^{\{t,u\}}) = 0 \text{ if } \exists\ l, i \text{ s.t. } W^{\{t,u\}}_{l,t+\zeta+D'_{ji}-D'_{li}} = i \tag{25}$$

f) Finally if none of the above constraints are satisfied, then node j should be permitted to transmit to node k in time slot $t+\zeta$ and hence

$$Z_{jk\zeta}(\mathbf{W}^{\{t,u\}}) = 1 \tag{26}$$

9) Limiting Interference Range by Power Control: Now we shall consider controlling the transmission power in order to limit the interference range, and revisit the above-listed scheduling feasibility constraints. We assume that minimum transmission power P_{jk}, is used to transmit a message from node j to node k. If α is the ratio of interference range to communication range, then by transmitting at minimum power between node j to node k, the message would be heard by all the nodes in the interference range of node j which is $\alpha(|\mathbf{x_j} - \mathbf{x_k}|)$. This implies that the message would be heard till αD_{jk} time slots in future starting from the time slot in which node j transmitted. Note that the interference constraint due to α should be applied to the non-integer version of the delay matrix $\mathbf{D}$ instead of the integer delay matrix $\mathbf{D}'$. The reason is that, even if some fraction of the time slots get affected due to the interference, they become unavailable to be used for reception.

We note that the constraints (21),(22) and (23) remain the same even if we limit the interference range. Consider the constraint (24) and assume that while searching for node i we find such a node in the network satisfying the constraint (24). This implies $W^{\{t,u\}}_{i,t+\zeta+D'_{jk}-D'_{ik}} > 0$. This also implies that node i is transmitting to node $W^{\{t,u\}}_{i,t+\zeta+D'_{jk}-D'_{ik}}$ in time slot $t+\zeta+D'_{jk}-D'_{ik}$. Since the transmission link found is between node i and node $W^{\{t,u\}}_{i,t+\zeta+D'_{jk}-D'_{ik}}$, the following must be true if node k lies in the interference range of transmission from node i:

$$t+\zeta+D'_{jk}-D'_{ik}+\lceil \alpha D_{i,W^{\{t,u\}}_{i,t+\zeta+D'_{jk}-D'_{ik}}} \rceil \geq t+\zeta+D'_{jk}$$

$$\Rightarrow D'_{ik} \leq \lceil \alpha D_{i,W^{\{t,u\}}_{i,t+\zeta+D'_{jk}-D'_{ik}}} \rceil \tag{27}$$

Therefore, the constraint (24) should be now modified as follows:

$$Z_{jk\zeta}(\mathbf{W}^{\{t,u\}}) = 0 \text{ if } \exists\ i \text{ s.t.}$$
$$W^{\{t,u\}}_{i,t+\zeta+D'_{jk}-D'_{ik}} > 0 \ \& \ D'_{ik} \leq \lceil \alpha D_{i,W^{\{t,u\}}_{i,t+\zeta+D'_{jk}-D'_{ik}}} \rceil \tag{28}$$

Now we consider constraint (25) and examine it in the case of limiting the interference range. The condition must include not only that there exists such nodes l and i, but also that node i must lie in the interference range of node j's transmission to node k , i.e.,

$$D'_{ji} \leq \lceil \alpha D_{jk} \rceil \tag{29}$$

Therefore, constraint (25) should now be modified as follows:

$$Z_{jk\zeta}(\mathbf{W}^{\{t,u\}}) = 0 \text{ if } \exists\ l, i \text{ s.t.}$$
$$W^{\{t,u\}}_{l,t+\zeta+D'_{ji}-D'_{li}} = i \ \& \ D'_{ji} \leq \lceil \alpha D_{jk} \rceil \tag{30}$$

To put it all together, the transmission indicator function including power control is summarized below:

$$Z_{jk\zeta}(\mathbf{W}^{\{t,u\}}) = \begin{cases} 0 \text{ if } j = k \\ 0 \text{ if } W^{\{t,u\}}_{j,t+\zeta} \neq 0 \\ 0 \text{ if } W^{\{t,u\}}_{k,t+\zeta+D'_{jk}} \neq 0 \\ 0 \text{ if } \exists\ i \text{ s.t. } W^{\{t,u\}}_{i,t+\zeta+D'_{jk}-D'_{ik}} > 0 \ \& \\ \quad D'_{ik} \leq \lceil \alpha D_{i,W^{\{t,u\}}_{i,t+\zeta+D'_{jk}-D'_{ik}}} \rceil \\ 0 \text{ if } \exists\ l, i \text{ s.t.} W^{\{t,u\}}_{l,t+\zeta+D'_{ji}-D'_{li}} = i \ \& \\ \quad D'_{ji} \leq \lceil \alpha D_{jk} \rceil \\ 1, \text{otherwise} \end{cases} \tag{31}$$

10) Algorithm to take Transmission decisions: The optimal time slot length τ^* to be used is computed by solving the optimization problem as presented in the section III-A. We can compute the corresponding closest integer delay matrix as follows :

$$D'_{ij} = \left\lceil \frac{|\mathbf{x_i} - \mathbf{x_j}|}{c\tau^*} \right\rceil$$

The algorithm that summarizes the procedure to take decisions at each slot is presented below:

Algorithm 1 Algorithm to take u^{th} transmission decision at time slot t with power control

Require: $\mathbf{D}'$, $\mathbf{D}$, current state of the schedule $\mathbf{W}^{\{t,u\}}$,
  $\mathbf{W} \leftarrow \mathbf{W}^{\{t,u\}}$
  while true **do**
    Compute Z from $\mathbf{W}$ using (31)
    $\mathcal{X} \leftarrow \{(j,k),\ \forall j,k\ s.t.\ Z_{jk0} = 1\}$
    if $\mathcal{X}$ is empty **then**
      return $\mathbf{W}^{\{t+1,1\}} \leftarrow \mathbf{W}, M_t \leftarrow u$
    end if
    Compute $Q^\dagger(\mathbf{W}, \mathbf{x})$ using (20)
    $\mathbf{x}^* = \text{argmax } Q^\dagger(\mathbf{W}, \mathbf{x})$
    $\mathbf{W} = f(\mathbf{W}, \mathbf{x}^*)$
  end while

IV. Performance Evaluation

We evaluate and compare the performance of the proposed idea of limiting the interference range through power control using simulations. We compare throughput, computed when the schedules are found, with and without power control. We also look at different network geometries found through simulations, for which the upper bound on throughput is achieved only due to power control.

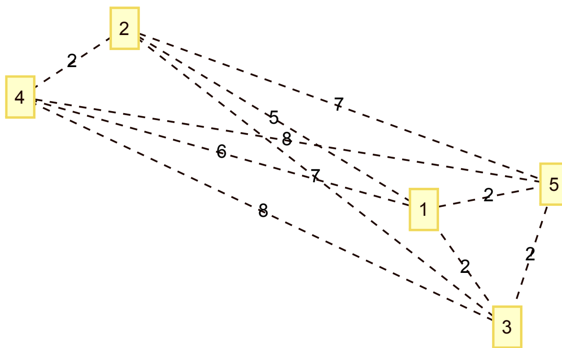


Fig. 1. 5-node trivial random network geometry with throughput $S = 2.5$ and ρ-throughput $S_\rho = 1.99$

We consider 200 randomly generated N-node networks in a 2D space of $H \times H$ meters, where the number of nodes N considered range from 2 to 8 nodes. We set H = 3000 m, $\tau_{\text{min}} = 45$ ms, $\tau_{\text{max}} = 3000$ ms and $v = 1$ ms. The closest integer delay matrix is computed for each randomly generated network geometry, after selecting an appropriate time slot length τ, by solving the optimization problem presented in section III-A. Algorithm 1 is used to compute the schedules with power control. Note that in Algorithm 1, while computing transmission indicator function $Z(\cdot)$, if we do not limit the interference range, i.e., we use the constraints (24) & (25) instead of constraints (28) & (30), then we compute schedules without applying power control.

A. Geometries Found with Throughput Gain

Some of the network geometries found are shown in Fig. 1, 2 & 3. Each network geometry plotted is on a different scale and is only representative of the shape of the network. Also note that the edges are labelled with approximated values of the propagation delays between nodes, which are taken from the corresponding integer delay matrix of the network geometry.

1) Trivial Geometries with $\frac{N}{2}$ throughput: Consider a network geometry with the following delay matrix which was found while 5-node random network geometries were generated (see Fig. 1) and an appropriate time slot length $\tau = 168$ ms was chosen to approximate it to the closest integer delay matrix. The delay matrix $\mathbf{D}$ computed is given below:

$$\mathbf{D} = \begin{bmatrix} 0 & 5.26 & 2.16 & 6.30 & 2.00 \\ 5.26 & 0 & 7.30 & 1.88 & 6.88 \\ 2.16 & 7.30 & 0 & 8.11 & 2.26 \\ 6.30 & 1.88 & 8.11 & 0 & 8.15 \\ 2.00 & 6.88 & 2.26 & 8.15 & 0 \end{bmatrix}$$

The closest integer delay matrix, when used with the scheduling algorithm with power control, generates a schedule with

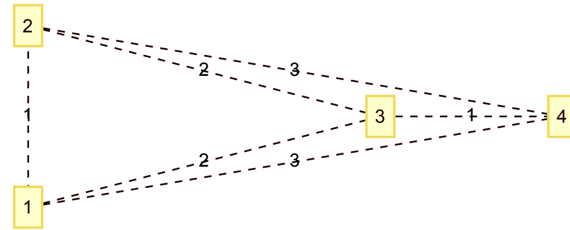


Fig. 2. 4-node non-trivial network geometry with throughput $S = 2$ and ρ-throughput $S_\rho = 2$

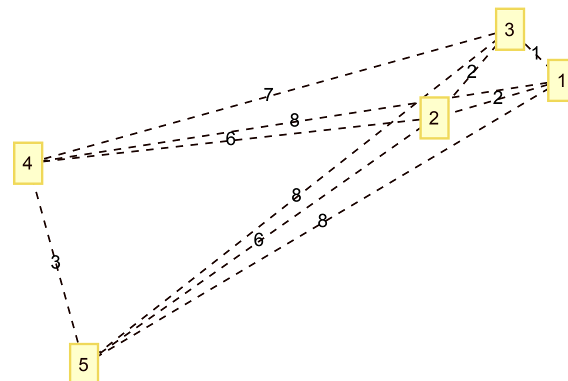


Fig. 3. 5-node non-trivial random network geometry with throughput $S = 2.5$ and ρ-throughput $S_\rho = 1.45$

period $T = 8$, which is shown below:

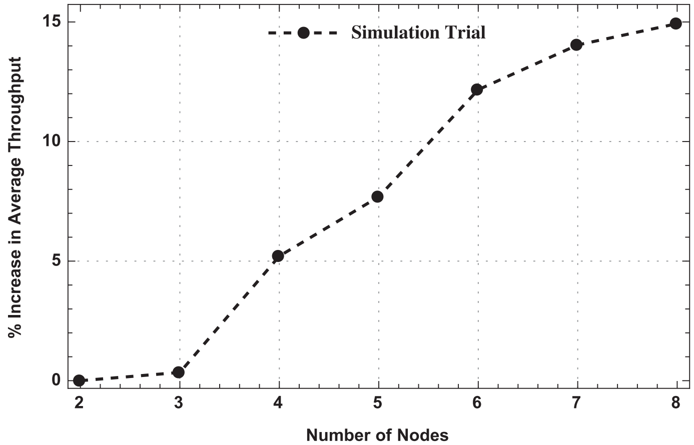
$$\mathbf{W}^{(8)} = \begin{bmatrix} 5 & 5 & -5 & -5 & -3 & -3 & 3 & 3 \\ 4 & 4 & -4 & -4 & 4 & 4 & -4 & -4 \\ -1 & -1 & 1 & 1 & 5 & 5 & -5 & -5 \\ 2 & 2 & -2 & -2 & 2 & 2 & -2 & -2 \\ 1 & 1 & -1 & -1 & 3 & 3 & -3 & -3 \end{bmatrix}$$

Although counting the number of successful receptions in one period of the above schedule would result in a throughput of 2.5, we need to be mindful of the approximations in propagation delays made while computing schedules. However, we can see that due to power control, two sub-networks are formed with links involving nodes $2, 4$ and nodes $1, 3, 5$ forming an equilateral triangle. The ρ-throughput can be computed by adding the ρ-throughput of each sub-network as given below:

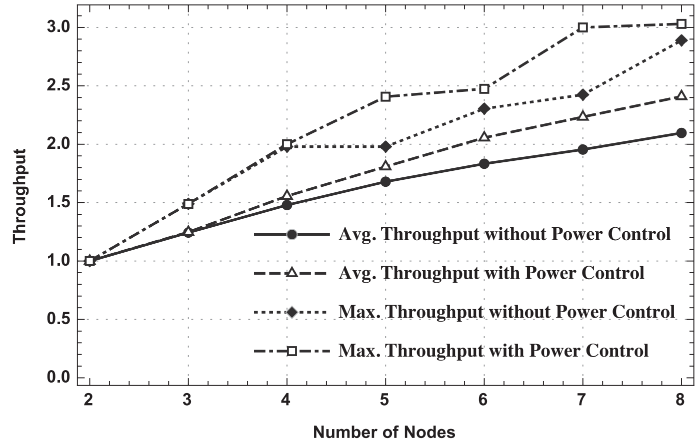
$$S_\rho = 1(1 - 0.12) + 1.5(1 - 0 - 0.26) = 1.99$$

Also note that the links which are used in the schedule are those which cause minimum interference.

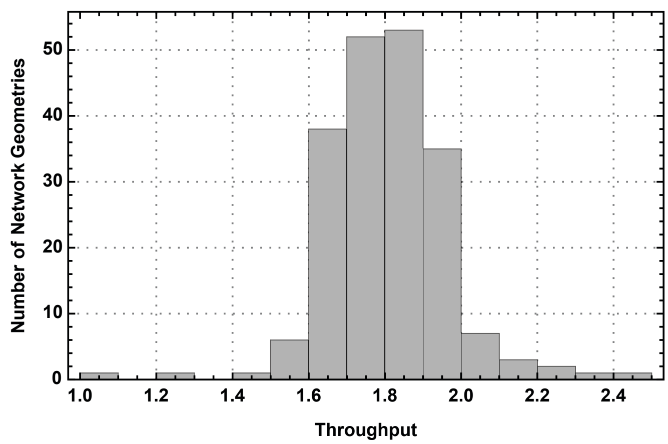
2) Non-Trivial Geometries with $\frac{N}{2}$ throughput: The above example of network geometry do not give insight into how power control results in schedules with higher throughput, when we cannot spatially separate the network into smaller sub-networks. Consider a 4-node network geometry (see Fig.



(a) Percentage increase in average throughput due to power control



(b) Average & Maximum throughput over 200 random network geometries



(c) Histogram of throughput with power control for 200 randomly generated 5-node networks

Fig. 4. Effects of limiting the interference range on throughput of randomly generated networks

2) with the following delay matrix:

$$\mathbf{D} = \begin{bmatrix} 0 & 1 & 2 & 2.9788 \\ 1 & 0 & 2 & 2.9788 \\ 2 & 2 & 0 & 1 \\ 2.9788 & 2.9788 & 1 & 0 \end{bmatrix}$$

For this network geometry, the schedule computed without power control results in a schedule with period $T = 7$ as shown below:

$$\mathbf{W}^{(7)} = \begin{bmatrix} -4 & 3 & -4 & 2 & -2 & 2 & -2 \\ 3 & 0 & 3 & 1 & -1 & 1 & -1 \\ 0 & -4 & -2 & -1 & -2 & 0 & -4 \\ 3 & 0 & 0 & 0 & 1 & 3 & 1 \end{bmatrix}$$

The above schedule yields throughput, $S = \frac{11}{7} = 1.56$. However, if the schedule is computed after limiting the interference range, we can achieve a throughput $S = 2$ as shown below:

$$\mathbf{W}^{(2)} = \begin{bmatrix} 2 & -2 \\ 1 & -1 \\ 4 & -4 \\ 3 & -3 \end{bmatrix}$$

We note that node 3 is in the interference range of transmission on links $(1, 2)$ & $(2, 1)$ and also nodes 1 & 2 are in the interference range of the link $(3, 4)$. Controlling the transmission power at nodes 1 & 2, results in no interference at node 4 and hence we can design the transmission schedule which can achieve a throughput of 2 as shown above. The ρ-throughput of the above network is also 2. Note that the set of interfering links for this network, when used with the schedule computed with power control, is $\mathcal{I} := \{(1, 2), (2, 1), (2, 3), (3, 2), (1, 3), (3, 1), (3, 4), (4, 3)\}$. Hence while computing ρ^+ and ρ^- we can ignore the approximations in the propagation delay of the links $(2, 4)$ and $(1, 4)$. This results in the values of ρ^+ and ρ^- to be 0. Consider another example, a 5-node network geometry as shown in Fig. 3 with the following delay matrix when used with time slot length $\tau = 241$ ms:

$$\mathbf{D} = \begin{bmatrix} 0 & 1.88 & 1.03 & 7.63 & 7.85 \\ 1.88 & 0 & 1.64 & 5.77 & 6.05 \\ 1.03 & 1.64 & 0 & 7.06 & 7.65 \\ 7.63 & 5.77 & 7.06 & 0 & 2.95 \\ 7.85 & 6.05 & 7.65 & 2.95 & 0 \end{bmatrix}$$

and the schedule is computed with a period of 12 and with throughput $S = 2.5$ as shown below:

$$\mathbf{W}^{(12)} = \begin{bmatrix} 3 & 2 & -2 & 2 & -2 & -3 & -3 & -3 & 2 & -2 & 3 & 3 \\ 1 & 3 & 1 & -1 & -3 & -1 & 3 & 1 & 3 & -3 & -1 & -3 \\ -1 & -1 & 2 & -2 & 1 & 1 & 1 & 2 & -2 & 2 & -2 & -1 \\ 5 & 5 & 5 & -5 & -5 & -5 & 5 & 5 & 5 & -5 & -5 & -5 \\ 4 & 4 & 4 & -4 & -4 & -4 & 4 & 4 & 4 & -4 & -4 & -4 \end{bmatrix}$$

Similar to the example above, while computing ρ-throughput, the set of interfering links is enumerated and the values of ρ^+ and ρ^- are computed. It results in $\rho^+ = 0.0549$ and $\rho^- = 0.3627$, and hence $S_\rho = 2.5(1 - 0.0549 - 0.3627) = 1.456$.

B. Simulation Results

1) Percentage Increase in Average Throughput: As we see in Fig. 4(a), the percentage increase in the average throughput computed is plotted as a function of the number of nodes in the network. If we denote the approximate throughput $S^{'}$, computed over $T^{'}$ time slots without power control, by $S^{'}_{\text{wopc}}$, and the throughput, computed with power control, by $S^{'}_{\text{wpc}}$, then the percentage increase is computed by:

$$\beta = \frac{S^{'}_{\text{wpc}} - S^{'}_{\text{wopc}}}{S^{'}_{\text{wopc}}} \times 100$$

From the plot, it is clear that limiting the interference range results in improving the average throughput. For an 8-node network the percentage increase in average throughput is close to 15%. Fig. 4(b) shows the average throughput and maximum throughput computed as a function of number of nodes. It is clearly seen that the throughput values computed with power control, are no less than the values computed without power control, and hence it is always better to limit the interference range.

2) Histogram of Throughput of Random Geometries: A histogram of throughput values computed for 5-node random network geometries is shown in Fig. 4(c). We can see that the maximum throughput values computed lie between 1.7 and 1.9 and hence it is no surprise to see that the average throughput value from Fig. 4(b) is in the same range for $N = 5$. We can also see how often the network geometries occur for which the upper bound is achieved. It is clear from Fig. 4(c), that there is a non-negligible probability of finding networks for which optimal throughput can be achieved.

V. Conclusion

We have shown through simulation that limiting the interference range by controlling the transmission power results in significant throughput increase. Limiting the interference range allows more transmission opportunities in the network and hence results in throughput that is closer to the upper bound $\frac{N}{2}$, when compared with the throughput that for a fully-connected network. We have also shown instances of random network geometries for which the throughput upper bound is achieved due to power control, but not without. The network geometries presented demonstrate that power control along with exploitation of propagation delay can result in schedules which yield high throughput. We also showed that the computation of ρ^+ and ρ^- need not take into account the propagation delay between all links in the network. This results in better ρ-throughput than when power control is not applied.

The work presented here does not consider node or link fairness, simply scheduling transmissions on links to maximize total throughput assuming a saturated network. In a multihop network, where messages need to be delivered over a set of links from source to destination, this may be sub-optimal. In extreme cases, this may result in fragmented networks where certain sources cannot reach certain destinations. We intend to address this problem of fairness and multihop networks in the future.

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