HQ 19

Reduction algorithm

// S[ ] array of elements in the SUBSET-SUM problem
// T target sum in the SUBSET-SUM problem
// M[ ] array of marks in the WILLPASS problem
// D[ ] array of ~~marks in~~ deadlines in the WILLPASS problem
// t[ ] array of time required for each homework in the WILLPASS problem.
// P the minimum passing mark in WILLPASS problem.
// WILLPASS problem's parameters ie M[ ], D[ ], t[ ] contain the respective values corresponding to each homework in order.
// assume there are n elements in array S[ ], starting with 1 index.

```
def Reduce (S, T) :
    for( i = 1 to n)
        M[i] = S[i]
        t[i] = S[i]
        D[i] = T
    P = T
    return (M, D, t, P) =
        parameters ~~in~~ of Will Pass problem.
```

Explanation

The Reduce ( ) reduces an array S of n elements with target sum T to the parameters of WillPass problem i.e an array of n integers with marks M[ ] of each homework, t[ ] time required to complete such n homeworks, D[ ] deadline for each n homeworks and P, the minimum passing marks. We map the array elements directly to marks and time array. Next we set the deadline of all n homeworks to the target value T. Also the minimum passing mark is set to T. We then return all the required parameters of WillPass problem.

Example

An example of a YES instance of Subset Sum and the output of the reduction on this instance.

Let $S = \{3, 2, 1, 6\}$ be an array of elements where we want a target sum of $T = 8$.

Clearly Subset Sum problem will return a YES instance as a subset exist with sum exactly equal to 8, $\{2, 6\}$

now, reducing this problem to Will Pass problem we will have the following from the Reduce( ) defined before —

$m = \{ 3, 2, 1, 6 \}$

$t = \{ 3, 2, 1, 6 \}$

$D = \{ 8, 8, 8, 8 \}$

$P = 8$.

~~now Will Pass will first~~ find out set of all ~~possible marks of home work~~ which are possible ~~to be done within given time and deadline~~ i.e ~~of homeworks~~ possible to do within ~~the 8 deadline~~.

now in Will Pass we get a sequence of home work where time = 2 and 6 thus it is within its respective deadline 8. ($2 \leq 8$, $2+6 \leq 8$) ~~and~~ Adding up the corresponding marks of the two home works we get $6+2 = 8$. now marks $8 \geq 8 = P$, thus Will Pass will also return a YES instance as a home work sequence was possible whose markses were greater than or equal to passing marks.

An example of a NO instance of Subset Sum and the output of the reduction on this instance.

Let $S = \{5, 4, 9, 2\}$ be an array of elements where we want a target sum of $T = 3$.

Clearly subset sum problem will return a NO instance as there exist no subset whose sum will be 3.

reducing the problem to willpass problem we will have the following from the Reduce() defined above —

$m = \{5, 4, 9, 2\}$

$D = \{3, 3, 3, 3\}$

$t = \{5, 4, 9, 2\}$

$p = 3$

now here in Willpass as all the deadlines are less than the time required to complete any of the homework, so no homework schedule is possible. Thus marks is equal to 0 : Willpass will also return a NO instance as 0 is not ≥ 3, the passing mark p.

Complexity analysis

Time complexity required $O(n)$ due to the ~~loop~~ for loop in Reduce( ) defined above. Rest all are constant time operations. Reduction algorithm has a polynomial time complexity.

Lemma

We will get a YES instance on subset-sum problem iff we get a YES instance on WillPass problem.

Proof of Lemma

Let there exist a possible subset of elements from set S with target sum exactly equal to T, thus subset sum will have a Yes instance. Now we are marks and time to array elements and setting all deadlines and ~~time to~~ passing mark to T.

Whenever a homework schedule is produced the time of all such homeworks should be within ($\leq$) the deadline, as deadline for all homeworks is T so time of all homeworks should be $\leq T$. Now time and marks are the same here so marks produced will also be $\leq T$. Now while checking if marks produced is $\geq P$ i.e $\geq T$ ~~will~~ will ensure always that marks so obtained from a possible homework schedule is equal to T (exactly) due to the two

constraint of time ~~(and marks)~~ being $\leq T$
and marks (which is same as time) being $\geq T$.
Thus will pass will also return a YES instance.

~~Let WillPass returns a YES instance ie a possible homework schedule exist s.t the marks so obtained are $\geq P$, the minimum passing mark. Now we know that in subsetsum the array elements~~

Suppose Will Pass returns a YES instance ie a possible schedule exist s.t the marks so obtained are $\geq P$, the minimum passing mark. Taking the corresponding array elements in subset sum which are the marks and time array elements and T as the deadline ~~or~~ and P, we will also a get YES instance on Subset Sum because the total marks obtained from the homeworks are always $\leq$ deadline ie $\leq T$ and the total marks also have to be $\geq P$ ie again $\geq T$. The ~~two~~ 2 constraints makes total marks always $= T$. Thus a possible selection of elements ie subset is guaranteed ~~and~~ (since from the same set of elements, time was picked up) with target sum exactly equal to T