HQ18

a) // Given an undirected graph $G(V,E)$ in which each vertex has a label $L(v)$ where the label is some integer from $\{0,1,2,\ldots,9\}$.

We will construct another graph $H(V',E')$ using graph reduction based approach as follows:

Defining vertices V'

Each vertex in V' will represent a node in H which corresponds to vertices in G, ~~but~~ ~~without labels~~

Number of vertices

Number of vertices in worst case can be $O(V)$

This is so because while traversing from s to t all the vertices in graph G is considered for the given pattern.

Defining Edges E'

From the edge set E of graph G, will take all edges (u,v) in set E' such that u has a label of $(i \% 3)$ and v has a label of $((i+1) \% 3)$ where $i \in [1,2,3]$, and v is not already present in V'.

Number of Edges

We have defined $E' \subseteq E$, so we are not considering any extra edges which are not present in E.

Therefore in worst case the number of edges in H will be $O(E)$.

Graph problem to solve on $H(V',E')$

We need to solve a reachability problem from the given source vertex s to desired target vertex t on the reduced graph H.

Algorithm

RLV(G, s, t) problem can be solved by running BFS(H, s).

We are constructing the graph H at runtime only, because each new neighbour vertices in H can be directly determined from edgeset of G only.

Time Complexity

BFS() for an undirected graph from source (s) to target (t) takes $O(V+E)$. Mapping this complexity with our solution $|V'| = O(V)$ and $|E'| = O(E)$

So time complexity of this problem is $O(V+E)$.

b) // Given an undirected graph $G(V,E)$ in which every edge is labeled where $L(e) \in [0,1,2,\dots,9]$.

We will construct another ~~vertex~~ graph $S(V_0, E_0)$ using graph reduction based approach as follows:

Defining vertices V_0

Each vertex V_0 in S corresponds to an edge of graph and the label of edges of E will be added to its corresponding vertex in vertex set V_0.

Number of vertices

Number of vertices in $S(V_0, E_0)$ will be $O(E)$ as there exists an vertex in S for each edge of G.

Defining Edges E_0 :

For every pair of edges (u,v) and (v,w) in G, such that (u,v) edge has label $(i \% 3)$ and (v,w) edge has label $((i+1) \% 3)$ where $i \in [1,2,3]$, we have already created two vertices in $S(V_0, E_0)$ denoting (u,v) as first and (v,w) as second vertex, and here we will simply add an edge between them.

Number of Edges

Number of edges in worst-case will be $O(V)$.

ALGORITHM

RLE(G, s, t) :

  for every vertex $u \in V_0$ where $u.label = 1$:

    ~~RLV(G, s, t)~~

    RLV(S, u, t)

Time Complexity

RLV( ) takes $O(V+E)$ as explained in part-(a).

here we are calling RLV( ) $O(V_0)$ time and $|V_0| = O(E)$

So time complexity of this problem $= O(E * (V+E))$