

1. Recursive algorithm to merge two sorted arrays of equal size.

// A is a sorted array with n elements

// B is a sorted array with n elements

// C is an empty array with space for $2n$ elements

// given two sorted arrays A and B Merge method merges two sorted arrays into C.

// assume no duplicates anywhere.

Merge (A, B, C, n):

$i = 1$

$j = 1$

PutElement($2n$)

// assume A, B, C are global arrays and i, j variables are global

// PutElement(k) puts the correct element at $C[k]$, taking it from either A or B

PutElement(k):

if $k = 0$: return

PutElement($k-1$)

if $i > n$

$C[k] = B[j]$

$j = j + 1$

else if $j > n$

$C[k] = A[i]$

$i = i + 1$

else if $A[i] < B[j]$

$C[k] = A[i]$

$i = i + 1$

else

$C[k] = B[j]$

$j = j + 1$

2. Time complexity analysis of recursive Merge algorithm.

Let- $T(k)$ denote the time complexity of $\text{PutElement}(k)$.

$$\begin{aligned}T(k) &= T(k-1) + c \\&= T(k-2) + c + c \\&= T(k-3) + c + c + c \\&\vdots \\&= T(k-m) + mc\end{aligned}$$

for the base case, $k-m=0 \Rightarrow m=k$

$$T(k) = T(0) + kc$$

$$T(k) = 1 + kc$$

$$T(k) \cong O(k)$$

Now, from $\text{Merge}()$ method we know that- $k = 2n$

and time complexity of $\text{Merge}()$ method \cong time complexity of $\text{PutElement}()$

Therefore, the time complexity of $\text{Merge}()$ method is $O(n)$.