

Q) Divide and conquer algorithm to return the set of all corner points.

1. `cornerPoints (P, left, right) :`
2. `if $|P| \leq 1$: return P`
3. `m = medianPoint (P)`
4. `L = cornerPoints (P[left...m], left, m)`
5. `R = cornerPoints (P[m+1...right], m+1, right)`
6. `i = 1`
7. `l = L[i]`
8. `h = R[i]`
 // consider the points in L in ^{decreasing} ~~increasing~~ order of y coordinate
9. `while (i < |L|)`
10. `if (l.y > h.y)`
11. `remove l from L`
12. `i = i + 1`
13. `l = L[i]`
14. `S = all unique points in L and R`
15. `return S`

The time complexity of `cornerPoints()` is as follows :

$$T(n) = \begin{cases} 2T(n/2) + cn & \text{if } n \geq 2 \\ c & \text{if } n < 2 \end{cases}$$

using master's theorem we can easily obtain

$$T(n) = \Theta(n \log n)$$

Proof of Correctness

Base case: If size of set P is equals to 1, then only one corner point exists.

Induction Hypothesis: Assume that $\text{cornerPoints}(P[l \dots m], l, m)$ will give all the corner points in the left-partition of the 2D plane, and $\text{cornerPoints}(P[m+1 \dots h], m+1, h)$ will give all the corner points in the right-partition of the 2D plane, where the partition is imaginarily assumed with respect to a median point $p \in P$.

Induction Claim: To show that $\text{cornerPoints}(P[l \dots h], l, h)$ will give all the corner points in the 2D plane.

Proof: By IH, L will contain all the corner points in left partition w.r.t median point and R will contain all the corner points of the right-partition. Now there might be some points in L which have a point above/right of it in the right partition. Hence viewing from those points of the right-partition R , few points of L may not be considered as corner points anymore. Thus, a point p in L will not be a corner point of the whole 2D-plane, if there exists a point q in R , such that $p.y \leq q.y$ (i.e., y -component of p is less than that of q). Such points are removed from L , by comparison in the while loop in line 10. Now all of the corner points of L are being compared with the point in R which has highest y component. If any point in the left partition L lies below or left w.r.t any point in R , then obviously such points of L can be removed using the 1st point of R only, as it has the highest y -component (points in R considered in decreasing order of y -component). Thus we will get all corner points of the 2D plane from $(L \cup R)$.