# PRINCIPLES AND TECHNIQUES OF SCHLIEREN IMAGING SYSTEMS

Amrita Mazumdar
Columbia University
New York, NY
am3210@columbia.edu
July 2011

July 12, 2011

## ABSTRACT

This paper presents a review of modern-day schlieren optics system and its application. Schlieren imaging systems provide a powerful technique to visualize changes or nonuniformities in refractive index of air or other transparent media. For over two hundred years, schlieren systems have been implemented for a wide variety of non-intrusive studies of flow. With the popularization of computational imaging techniques and widespread availability of digital imaging systems, Schlieren systems provide a novel method of viewing transparent flow. Although schlieren systems were popularized in the early 19th century, the recent improvements in optics production and computing power have given rise to a wider range of modifications and applications for the technique. Innovations such as background-oriented schlieren, rainbow schlieren interferometry, and synthetic schlieren imaging have built upon the fundamentals of schlieren imaging to provide new, less ambiguous, quantifiable studies of Schlieren systems. This paper presents a historical background of the technique, describes the methodology behind the system, presents a mathematical proof of schlieren fundamentals, and lists various recent applications and advancements in schlieren studies.

## 1 INTRODUCTION

Schlieren imaging systems have been used since the early 1800's to visualize fluctuations in optical density [1]. A dynamic and straightforward utility, these systems are primarily applied to conduct qualitative visualizations. Schlieren optics provide an informative, non-intrusive method for studying transparent and optical media [2]. It is advantageous to use them in fluid dynamics studies because they are sensitive to changes and do not interfere with flow [1, 3]. They can also be used to study optical materials and changes in refractive index within the material [4, 2]. Most commonly, schlieren systems have been applied to visualize diverse subjects such as striations in blown glass, inhalation in humans and animals, shock waves from a plane in flight, and heat emanating from a system [4].

Exploration within the field of schlieren imaging became stagnant in the 1900s, largely a cause of the delicate nature of aligning a Schlieren system, the exorbitant cost of creating a system large enough to study everyday objects, and the stationary nature of the system [5, 6, 7]. With the development of powerful digital cameras and computer systems, however, new methodologies for studying refractive media based on Schlieren optics have emerged, increasing the amount of optical and quantitative information captured by modern cameras. These new applications have value in the fields of computer vision, biology, physics, and environmental engineering [8, 9, 10, 11]. In this paper, we explore the history and fundamentals of Schlieren imaging, the design of a basic Schlieren system, advancements in computational Schlieren systems, and future applications and analysis techniques implementing Schlieren imaging concepts.

# 2 HISTORY

Schlieren optics techniques have been applied for hundreds of years, prior to their use as a fluid dynamics tool, to study optical components in telescopes and microscopes. The word schlieren itself derives from the German word schliere, or "striae," and refers to the streak-like appearance of fluid flow visualized through the system [4, 1]. Rudimentary schlieren techniques have been implemented for far longer than scientists have been given credit for. Although most sources cite schlieren optics as originating in the 19th century, notable observations of schlieren techniques were discussed as early as the 17th century by Robert Hooke, who presented his findings before the Royal Society and discussed his work in the book Micrographia, and Christiaan Huygens, who concurrently developed a schlieren technique to visualize veins, or striae, in optical components [1]. Leon Foucault, in the mid-19th century, developed a knife-edge test for telescope mirrors which visualized air-flow, although he did not recognize it himself [12, 1]. The first to officially recognize and present the schlieren imaging technique was August Toepler, who devised a functional apparatus for schlieren imaging as well as an in-depth procedure on how to design a simple schlieren system [1, 3]. During the early 20th century, Hubert Schardin conducted extensive research on the applications of schlieren imaging systems, developing numerous new techniques, some of which are only being explored today [4, 1]. Many physicists and fluid dynamics engineers applied the techniques proposed by Toepler and Schardin to study shock waves from aircraft, the mixing of liquids and gases, and applied color filters to their systems to obtain quantifiable data [5]. These methods were largely advanced by the advent of modern computer systems, which provided many different ways of capturing and quantifying schlieren images, and will be discussed in more detail in later sections.

# 3 OPTICAL THEORY

The basis for schlieren imaging emerges from Snell's Law, which states that light slows upon interaction with matter. If media is homogeneous, such as in a vacuum, or space, light travels uniformly, at a constant velocity. When encountering inhomogenous media, such as fluids in motion, light rays refract and deflect from their continuous path, resulting in schliere. To apply appropriate quantitative analysis to schlieren optics systems, it is necessary to outline the rules governing schlieren images. This section was compiled with significant guidance from the work of Schardin, Settles, Sutherland et al., Mitra et al., Raskar et al., Agarwal et al., and Hosch et al [4, 1, 13, 12, 6, 14, 15].

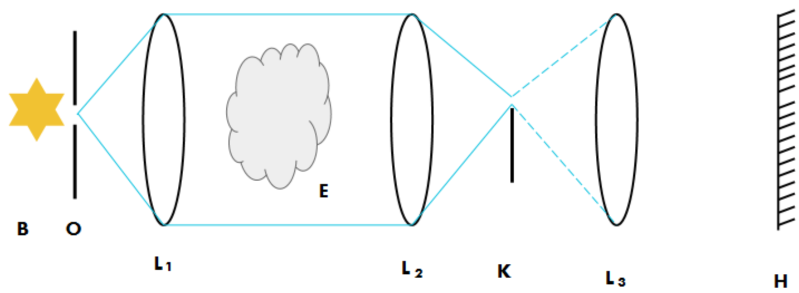



**Figure 1:** A typical lens-based schlieren optics system

## 3.1 BASICS OF LIGHT PROPAGATION IN SCHLIEREN SYSTEMS

The most rudimentary schlieren system implements a straight-line arrangement of lenses to visualize schliere, as shown in Figure 1. A parallel beam of light, emanating from a light source $B$ and passing through slit $O$ is focused by lens $L_1$.This parallel beam is then deflected by the schlieren test subject $E$ and passed through a second lens $L_2$, where it is focused on the knife-edge cut-off $K$. Lenses $L_2$ and $L_3$ (a projection lens) focus the schlieren image onto a screen or photographic sensor, depicted as $H$.

## 3.2 REFRACTING LIGHT THROUGH AN OBJECT

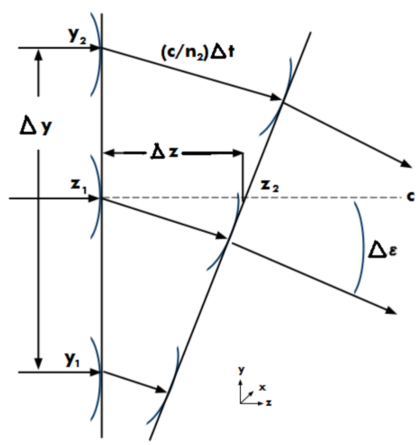



**Figure 2:** Diagram of light-ray deflection by a refractive-index gradient $dn/dy$

To present a more simple deconstruction of schlieren light refraction, we assume a negative vertical refractive-index gradient $\partial n/\partial y < 0$, and no gradient in the x- or z-directions. A planar light wave, initially vertical, becomes displaced after propagating through a schlieren object. If it covers a differential distance in differential time, $\Delta z/\Delta t$, it is refracted through the differential angle $\Delta\varepsilon$. The refractive index is defined as $n = c/v$, where $n$ is the refractive index, $c$ is the speed of light in a vacuum, and $v$ is the local light speed. Examining Figure 2, we find:

$$\Delta\varepsilon = \frac{c/n_2 - c/n_1}{\Delta y}\Delta t$$

Combining this expression with an expression for differential time, $\Delta t = \Delta z \frac{n}{c}$, and simplifying terms, we find:

$$\Delta\varepsilon = \frac{n}{n_1 n_2}\frac{(n_1 - n_2)}{\Delta y}\Delta z$$

We can simplify this by letting all finite differences approach zero and simplifying $\frac{n}{n_1 n_2}$ to $1/n$, and obtain

$$\frac{d\varepsilon}{dz} = \frac{1}{n}\frac{dn}{dy}$$

If we use small angle approximation to postulate $d\varepsilon$ is equal to $dy/dz$, we obtain

$$\frac{\partial^2 y}{\partial z^2} = \frac{1}{n}\frac{\partial n}{\partial y} \tag{1}$$

relating the curvature of a refracted ray to the magnitude of the refractive-index gradient. Schlieren images can be found by integrating the ray curvature of light rays in optical inhomogeneities in the appropriate directions:

$$\varepsilon_y = \frac{1}{n}\int \frac{\partial n}{\partial y}\partial z \tag{2}$$

These equations can also be used to find the component of the 2-D gradient in the x-direction by replacing the $y$'s with $x$'s.

Equation 1 indicates it is not the refractive index $n$ causing ray deflection, but the gradient $\partial n/\partial y$ of this refractive index. Additionally, Eqns. 1 and 2 show light ray deflections bend towards regions of higher refractive index.

# 4 RELATED SYSTEMS

Slight deviations in a conventional schlieren set-up can result in systems with distinct but informative optical representations of flow. Two notable methods, shadowgraphy and interferometry, have developed as parallel but unique paths of observation in flow visualization. Both systems are similar to schlieren imaging in that they all visualize changes in refractive index of transparent media and visualize flow density in a manner which qualitatively resembles an embossed image, but each set-up produces uniquely remarkable images worth mentioning [3].

## 4.1 SHADOWGRAPHY

Shadowgraphy is an optical technique which projects the shadow of an optical image onto a viewing plane to be recorded [1]. The method of collecting shadowgrams is similar to how humans visualize fluctuations in air density with their eyes, e.g. from the hood of an overheated car or gas rising from a barbecue grill [6]. A basic shadowgraphy system consists of a light source and a flat screen to project shadowgrams onto for observation, as shown in Figure 3. Because shadowgrams are simply shadows of a schliere object, they can be magnified in accordance to their distance from the screen [1]. Additionally, it is very simple to modify a schlieren system into a shadowgraph collection system, demonstrated in Figure 4, and the exercise can prove useful during alignment and testing in a laboratory setting.

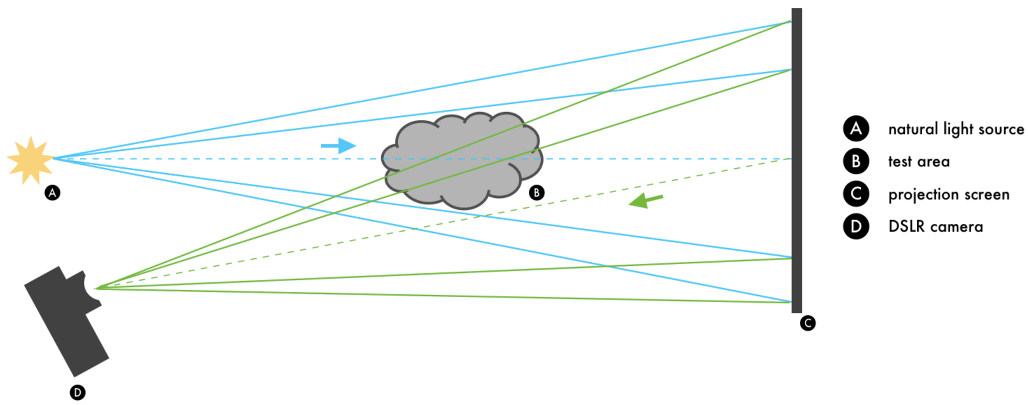



**Figure 3:** A basic shadowgraphy set-up

**Figure 4:** Rudimentary shadowgraph captured with experimental schlieren system

Because schlieren techniques produce actual images of the schlieren object rather than a shadow, they may seem to be more desirable to implement. Shadowgrams, however, have proven to be useful in that they are more easily observable and collectible [3]. In addition, the size and flexibility of a shadowgraphy system is advantageous for visualizing large-scale or extremely sensitive schlieren objects [1]. Although shadowgraphy is known to be less sensitive than schlieren methods for gradual flow, shadowgrams have been known to produce more dramatic images of shock waves and turbulent flows [6, 1]. This can be attributed to the shadowgram's correspondence to the Laplacian of the schliere's refractive index, $\partial^2 n/\partial x^2$, compared to a schlieren image corresponding with $\partial n/\partial x$ the first spatial derivative of striations [16].

## 4.2 INTERFEROMETRY

Schlieren interferometry techniques, although bearing closer ties to interferometry than schlieren imaging, does utilize some basics of the visualization technique, and should be accounted for. Schlieren interferometry was introduced and is most commonly used as a means to apply some form of quantitative analysis to what was otherwise a primarily visual observation technique, but has since given rise to more efficient and informative methods of measurement [6].

The typical schlieren interferometry system is set up like a basic two-lens schlieren system, but with the replacement of the knife-edge cut-off with some sort of prism or fine wire to induce interference or diffraction in the schlieren pattern, which can then be analyzed [1]. The Wollaston-Prism shearing interferometer replaces the knife-edge with a birefringent Wollaston polarizers sandwiched between crossed polarizers, resulting in fringe interferometry bearing close resemblance to schlieren images [17]. Color schlieren interferometry replaces the cut-off with a color filter, and then applies particle-image velocimetry diagnostic techniques to obtain quantitative data from a schlieren image [18]. This interpolation on schlieren techniques coupled with advancements in computing power have given rise to new fields of quantitative schlieren imaging, discussed in more detail in later sections [11].

# 5 EXPERIMENTAL IMPLEMENTATION

## 5.1 SET-UP

A single-mirror coincident schlieren system was constructed (see Figure 5). The system implements an LED flashlight with a structured pinhole to create a point light source, condenser lens, half mirror, 12î diameter f/8 spherical mirror, and a series of metal and wooden blocks to use as stands. Despite losing half of the light intensity at the beam-splitter, images proved to be illuminated adequately up to shutter speeds of 1/1000. Images were captured by a Canon EOS 20D DSLR camera with a 50mm f/1.8 lens.

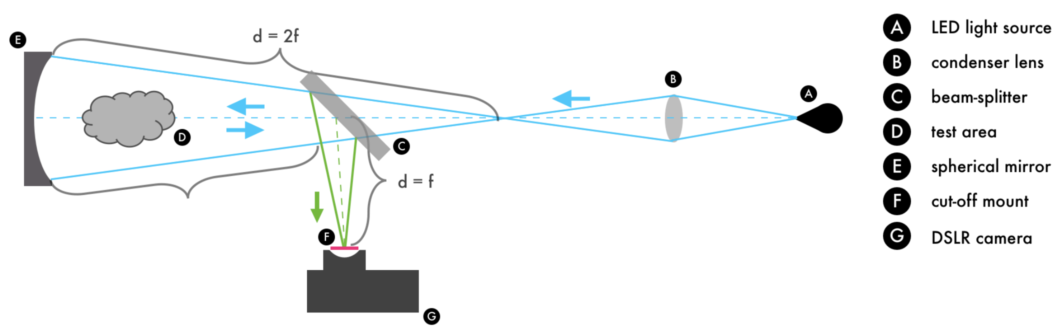



**Figure 5:** Single-mirror coincident system design

The system was assembled by first arranging components as outlined in Figure 5, and then precisely adjusting the positioning for optimal results. The light was precisely focused so as to avoid aberrations and misalignment of the optical components, and each of the components was carefully aligned to pass the light without interference or distortion. An image of the final set-up is shown in Figure 6. After positioning all of the components, the test object was then introduced to the system, and the camera focused its lens on the test subject. Test photos were then taken to ensure proper exposure of the mirror, before introducing the cut-off filter. The cut-off filters were mounted onto wooden blocks, to facilitate substitution, and the wooden blocks were placed on a precise x-y translation stand. The camera on a tripod was positioned directly behind the cut-off filter. In order to obtain a uniform background and proper distribution of

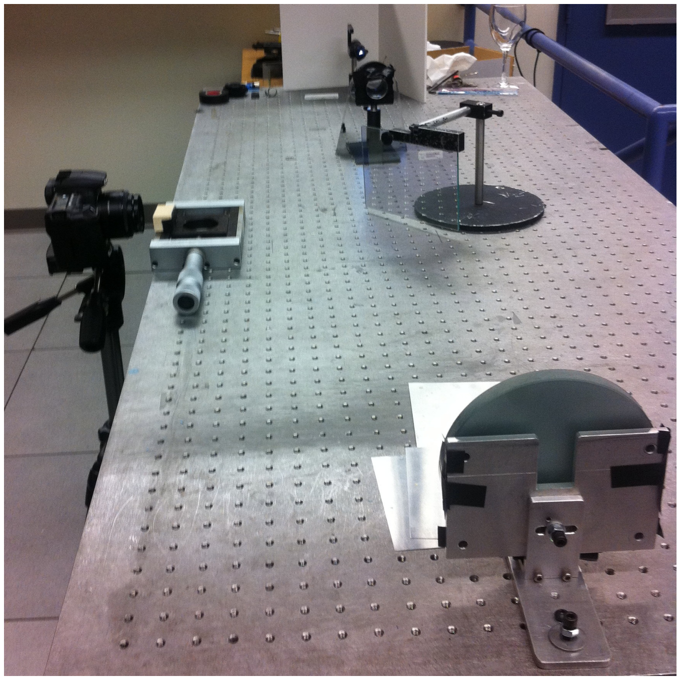


**Figure 6:** The aligned schlieren system

light, it was imperative the cut-off filter be positioned exactly at the focus of the reflected light. Once this uniform cut-off was positioned properly, Schlieren images could be visualized and recorded with the camera control software.

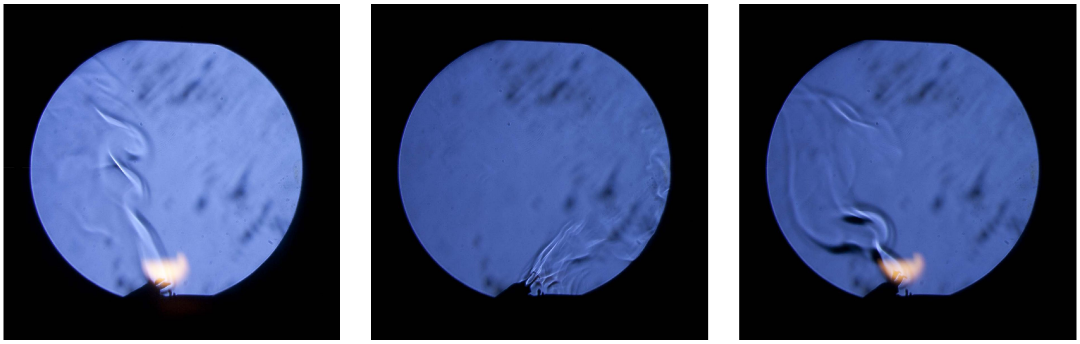
## 5.2 CAPTURED IMAGES

After constructing the system, images were captured. Test subjects included different types of common candles, optical components such as filters and glass panes, and human inhalation and exhalation airflows.

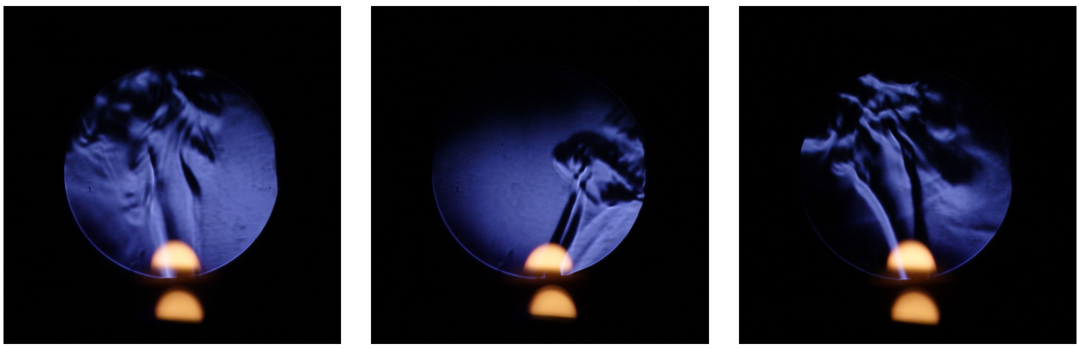
Table 1 demonstrates the variances in contrast and light intensity which can be modulated through use of a knife-edge cut-off. By adjusting the height of the knife-edge in the image plane, the number of schlieren rays deflected from the image increase, emphasizing the contrast between schliere in the image. A higher knife-edge also increases the sensitivity of the schlieren system, but at the cost of losing light in the image. These photos were taken with a Canon 50mm lens at f/1.8, with a shutter speed of 1/125 seconds and ISO 100.

Table 2 presents a number of photos taken using a rainbow color filter in place of a conventional cut-off. By adjusting the position of the filter within the plane, light intensities can be encoded with different colors depending on the position of the filter on the image plane. With this set-up, light intensity can only be modulated through the use of an aperture at the condenser lens (component B in Figure 5) or at the sensor level through the camera's settings. These photos were taken with a Canon 50mm lens at f/1.8, with a shutter speed of 1/40 seconds and ISO 100.

**Row 1:** Candle plume captured with normal knife-edge positioning



**Row 2:** Candle plume with adjusted knife-edge for increased contrast



**Row 3:** Human exhalation and candle plume with normal knife-edge positioning

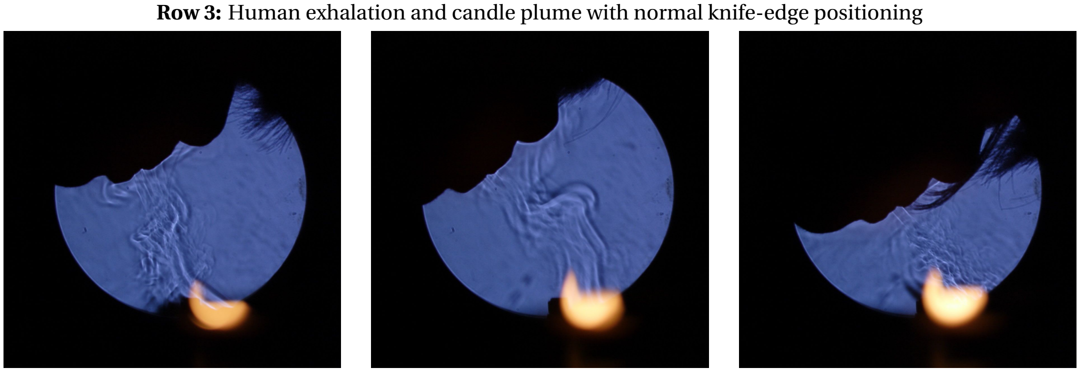


**Table 1:** Knife-edge cut-off schlieren images of a small standing candle

## 5.3 IMPLEMENTATION AND VERIFICATION

Alignment of the initial system was not difficult, and once the correct adjustments were made to accomodate the constraints of the half-mirror and condensor lens heights, the system produced quite clear schlieren images. Implementing the cut-off, however, was extremely straightforward. Determining the precise height and location of the focus of the image plane to place the cut-off was difficult, even after using measurement tools to find the right position. If the cut-off was a few millimeters closer or further from the focus, dark shadows would obscure the schlieren image, rather than

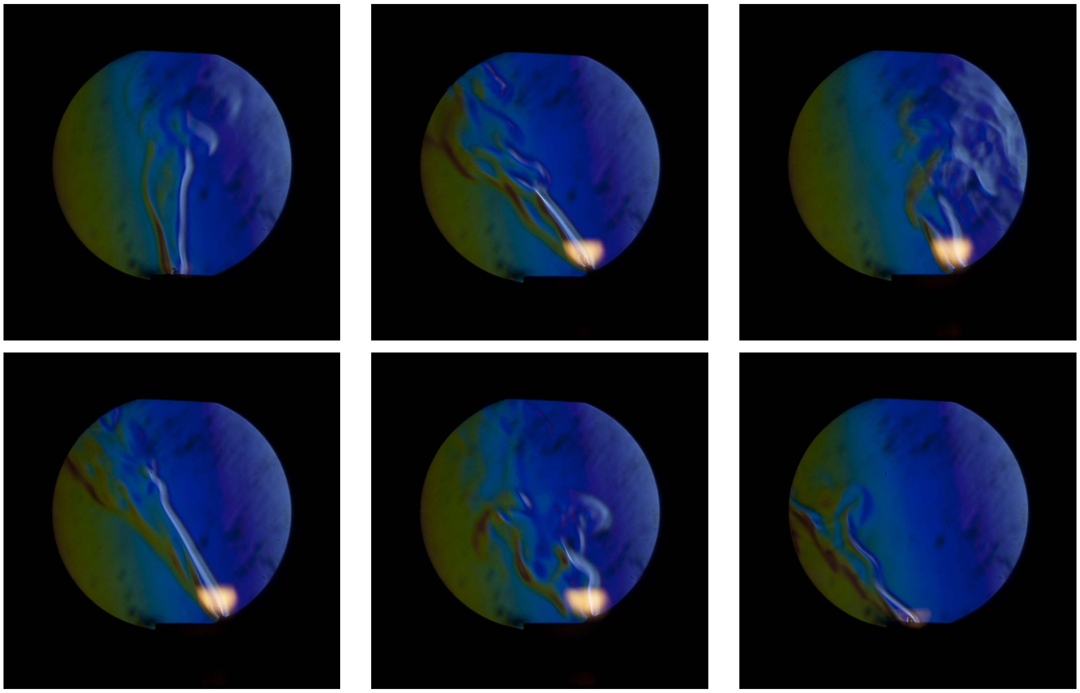


**Table 2:** Color-filter Cut-off Schlieren Images

uniformly reducing or increasing the intensity of the background [10]. This difficulty was compounded by the fact that slight misalignments in a knife-edge cut-off's placement would result in duplicate schlieren images appearing on the image plane, as demonstrated in Figure 7.

An experimental verification technique of our schlieren system's alignment was designed and implemented, albeit unsuccessfully. The plan was to reproduce and predict the duplicate-image phenomena described. Using Fourier analysis and measurements of our set-up to calculate the distance between a static object and its duplicate, we were going to slightly modify the angle of the half-mirror many times and capture images, comparing those results to our predicted distances to find the "accuracy" of the system. Because of misalignment during the implementation of this test, however, the duplication effects could not be reproduced, and the test was not conducted.

## 5.4 LIMITATIONS

The most obvious limitations of the presented system are size and portability. The size limitations are twofold: the size of the system itself is dependent on the size and focal-length of the mirror, meaning a smaller mirror would be more compact; a compact system, however, sacrifices image size, as the image captured is dependent on the size of the mirror reflecting it [2]. Additionally, with a system of this design, all subjects must be stationary, or small enough to move within the field of view of the system, as the system itself is not significantly flexible or portable.

The constraints of mirror size and system invariance limited the types of subjects we could actually test. A Melles-Griot optical bench was used to ensure static positioning of the components. Once implemented, however, we realized many of the subjects we wished to test required moving the system, either to a larger area or a different room, and either

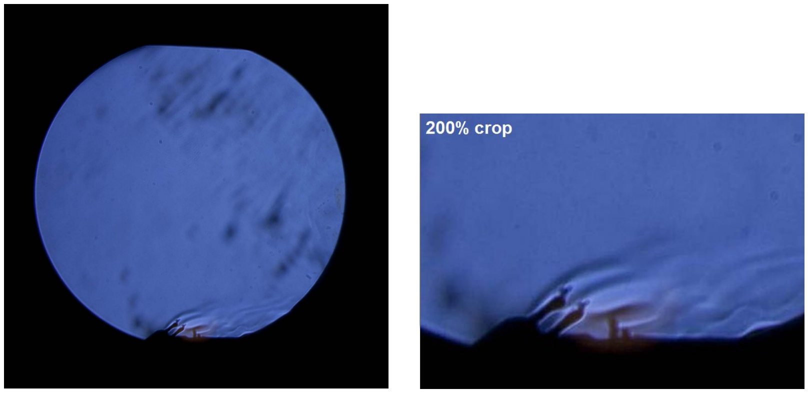



**Figure 7:** Captured image and 200% crop of duplicate candle wicks as viewed through camera lens. Although only one wick is photographed, two appear, one appearing a small distance away from the other. This could be attributed to slight misalignments with the half-mirror and knife-edge cut-off, especially because this discrepancy disappears in color-filter schlieren images.

would require recalculations and a significant time commitment for deconstructing and reconstructing a system.

Another considerable limitation is the cost of constructing a basic schlieren system. Although most of the parts can be found in any physics or optics lab, schlieren systems require high-quality, polished field mirrors and lenses, as any defects in optical components can result in aberrations and distortions in the schlieren images [1]. As outlined in the next section, schlieren techniques become more valuable and applicable once they are unconstrained by the laboratory setting or the quality of the components used.

# 6 ADVANCED SCHLIEREN TECHNIQUES

With respect to the established schlieren systems and the merits and limitations of the system outlined above, it bears mentioning that many innovations on schlieren techniques for visualizing refractive inhomogenities have been published in recent years. These systems, adopting the fundamentals of schlieren imaging while incorporating advancements in computing power and production capabilities. This section reviews a few of the more popular innovations to schlieren imaging, all of which have contributed greatly to the power and information able to be collected through schlieren techniques.

## 6.1 BACKGROUND-ORIENTED SCHLIEREN

Background-oriented schlieren (BOS)systems are an interation on basic schlieren techniques minimizing the number of precisely aligned optical components [5]. The most simple BOS systems can be implemented with just a camera and a proper background image, with a computer for post-processing [5]. BOS systems detect refractive-index deflections by comparing an image of an undistorted background with a background obstructed by refracting flow and performing a per-pixel analysis based on the density of gas given by the Gladstone-Dale equation [8, 3]. Because background-oriented schlieren is dependent only on the size and detail of the background image used, BOS has an advantage above other flow visualization tools of possessing a virtually unlimited field of view [5]. These systems are also referred to as synthetic schlieren systems [19].

**Figure 8:** Laser-source schlieren image for background-oriented schlieren applications

Bearing many similarities to interferometry, the technique became usable only once computing power became strong enough to quickly process ray deflection data, and have since been applied to study turbulent flows, supersonic jets, and sound waves [3, 5]. The sensitivity of the BOS technique is directly related to the experimental setup geometry, especially the distance from the schlieren object to the background [20]. Depending on the object being observed, appropriate backgrounds for background-oriented schlieren can be either natural or synthetic. Nature-based surroundings have a wide range and availability of viable backgrounds for BOS imaging, such as forest tree-lines or sunlit cornfields, and present enough detail and randomness to visualize both high-speed and low-speed schliere [1, 9]. For more controllable studies, the use of grids or stripes, sometimes augmented by color gradients to provide more detailed quantification, can be used to provide significant contrast for simple schliere visualization, such as ballistics or optical testing [4, 1, 6]. A diffused laser light source can also produce an appropriate background for smaller flows, as shown in Figure 8, and is employed for microscope-based BOS studies [1].

A more recent and notable advancement in background-oriented schlieren techniques is the development of light-field BOS (LF-BOS. Light-field BOS uses a 4D light probe, rather than a static image, to capture schlieren images. The rays coming from the probe are optically encoded with color and intensity variations, resulting in an extremely exact way to quantify and analyze ray deflection from a schlieren object. Advantages presented by LF-BOS are the facility and functional portability of the hand-held system, as well as the low cost and minimal calibration required for observation [6].

## 6.2 RESOLUTION IMPROVEMENTS

As a a consequence of advancing computational imaging tools used in conjunction with schlieren systems, many researchers have sought out modifications and algorithms to extract quantitative information from schlieren images. These increases in processing, however, come at the cost of spatial resolution and dynamic range. Accordingly, a good deal of recent work in schlieren imaging has been devoted to improving the quality of schlieren images and recovering resolution losses after processing.

### 6.2.1 Spatial Resolution

Spatial resolution, or the clarity of lines in an image, is imperative to preserving the quality of schlieren objects in an image. Maintaining spatial resolution across high- and low-sensitivity systems is a major concern for studies involving extremely large or small schlieren objects, because the sensitivity of a system tends to fall off as spatial resolution is increased [20]. From an implementation point-of-view, spatial resolution can be moderately improved by simply choosing a more-powerful, larger-size LED as the light source, providing finer spatial coherency [20]. One technique for achieving higher spatial resolution is using a sharp-focus, or multi-source schlieren system. These systems arrange three separate point light sources to focus on the image at the same point, providing a 3-dimensional encoding of the schlieren object. The images produced are then processed with Fourier analysis to produce information about the object [21]. Light-field BOS, mentioned earlier, is a similar technique for maintaining spatial resolution by encoding the light rays to be recovered later in processing [6].

### 6.2.2 Dynamic Range

The dynamic range, or difference in luminesence values, of schlieren images is typically large, as the discrepancies between static and fluid parts of a schlieren object are what form the image. Adjustments to the dynamic range of the system are made simply by modifying the cut-off before the image is recorded [1]. The typical cut-off is a knife edge or color filter, as they are simple to implement and adjust, but these filters may prove to degrade resolution too much for use with certain applications, such as microscopic schlieren techniques [7]. Hoffman modulation contrast filters can be used to contrast this problem, as they not only provide suitable dynamic range for visualizing schliere, but also encode information about the phase object, allowing more information about the schlieren object to be recorded in one image [7]. Another method of improving dynamic range in schlieren images involves superimposing photos from a pulsed ultrasound probe and measuring the diffracted light differences between the two images [22].

### 6.2.3 Noise

Noise in schlieren imaging is unavoidable, especially from digital camera sensors operating at high ISO settings. For extremely fine schlieren objects, for which high sensitivity is needed, noise can detract from the quality of the image and subsequent visual and quantitative analysis [23]. Additionally, noise can appear in photos from extraneous density gradients obstructing the schliere to be observed [24]. Methods for improving spatial resolution are shown to improve noise quality in schlieren images, but individual steps can be taken to reduce noise levels. One technique involves using a pulsed laser to record multiple schlieren images and then superimposing them, while another involves using a dynamic synthetic background to combat noise [15, 19].

# 7 CONCLUSION

Although conceived hundreds of years ago, refractive-index visualization techniques have newfound value in the modern fields of fluid analysis and computer vision and graphics research. This paper shows the progression and evolution of the centuries-old schlieren imaging technique, and why it has become popular again. Employing and improving upon these optical systems could give rise to more informative photographs and new ways of analyzing and interpreting photos, especially for fields studying fluid transport or light ray deflection. A great deal of literature has been published on the topic of schlieren imaging techniques, especially regarding their applications and uses, but there is still a great deal of work to be done in the design and implementation of schlieren systems. As technology and computing power evolve, schlieren imaging will evolve with it, helping to visualize the invisible.

## References


[1] G. S. Settles, *Schlieren and Shadowgraph Techniques: Visualizing Phenomena in Transparent Media*. Springer, 2 ed., 2006.

[2] H. G. Taylor and J. M. Waldram, “Improvements in the schlieren method,” *Journal of Scientific Instruments*, vol. 10, no. 12, pp. 378–389, 1933.

[3] H. Richard and M. Raffel, “Principle and applications of the background oriented schlieren (BOS) method,” *Measurement Science and Technology*, vol. 6, pp. 1576–1585, 2001.

[4] H. Schardin, “Schlieren methods and their applications,” *Ergeb. Exakten Naturewiss*, vol. 20, no. 303, 1942.

[5] G. E. A. Meier, “Computerized background-oriented schlieren,” *Experiments in Fluids*, vol. 33, no. 1, pp. 181–187, 2002.

[6] G. Wetzstein, R. Raskar, and W. Heidrich, “Hand-held schlieren photography with light field probes,” in *IEEE International Conference on Computational Photography ICCP*, 2011.

[7] A. Vogel, I. Apitz, S. Freidank, and R. Dijkink, “Sensitive high-resolution white-light schlieren technique with a large dynamic range for the investigation of ablation dynamics,” *Optics Letters*, vol. 31, no. 12, pp. 1812–1814, 2006.

[8] B. Atcheson, W. Heidrich, and I. Ihrke, “An evaluation of optical flow algorithms for background oriented schlieren imaging,” *Experiments in Fluids*, vol. 46, no. 3, pp. 467–476, 2009.

[9] M. Hargather and G. S. Settles, “Natural-background-oriented schlieren imaging,” *Experiments in Fluids*, vol. 48, pp. 59–68, 2010.

[10] V. Gopal, J. L. Klosowiak, R. Jaeger, T. Selimkhanov, and M. J. Z. Hartmann, “Visualizing the invisible: the construction of three low-cost schlieren imaging systems for the undergraduate laboratory,” *European Journal of Physics*, vol. 29, no. 3, pp. 607–617, 2008.

[11] S. Dalziel, G. Hughes, and B. Sutherland, “Whole-field density measurements by ’synthetic schlieren’,” *Experiments in Fluids*, vol. 28, no. 4, pp. 322–335, 2000.

[12] S. Mitra, M. Chaskar, and S. Phadke, “Design and fabrication of a simple schlierenscope,” *American Journal of Physics*, vol. 49, no. 2, pp. 158–161, 1981.

[13] B. R. Sutherland, S. B. Dalziel, G. O. Hughes, and P. Linden, “Visualisation and measurement of internal waves by synthetic schlieren, part 1: Vertically oscillating cylinder,” *Journal of fluid mechanics*, vol. 390, pp. 93–126, 1999.

[14] S. Agarwal, S. Mallick, D. Kriegman, and S. Belongie, “On refractive optical flow,” in *European Conference on Computer Vision*, Springer Verlag, 2004.

[15] J. W. Hosch and J. P. Walters, “High spatial resolution schlieren photography,” *Applied Optics*, vol. 16, pp. 473–482, Feb 1977.

[16] D. G. Gregory-Smith, A. R. Gilchrist, and P. Senior, “A combined system for measurements of high-speed flow by interferometry, schlieren and shadowgraph,” *Measurement Science and Technology*, vol. 1, no. 5, pp. 419–424, 1990.

[17] T. P. Davies, “Schlieren photography–short bibliography and review,” *Optics and Laser Technology*, vol. 13, no. 1, pp. 37–42, 1981.

[18] G. S. Settles, “Colour-coding schlieren techniques for the optical study of heat and fluid flow,” *International Journal of Heat and Fluid Flow*, vol. 6, no. 1, pp. 3–15, 1985.

[19] R. R. La Foy, “Adaptive synthetic schlieren imaging,” Master’s thesis, Massachusetts Institute of Technology, 2009.

[20] G. S. Settles, “Important developments in schlieren and shadowgraph visualization during the last decade,” in *Internatioanl Symposium on Flow Visualization*, Springer Verlag, 2004.

[21] A. Hanenkamp and W. Merzkirch, “Investigation of the properties of a sharp-focusing schlieren system by means of fourier analysis,” *Optics and Lasers in Engineering*, vol. 44, no. 3-4, pp. 159–169, 2006.

[22] N. Kudo, H. Ouchi, K. Yamamoto, and H. Sekimizu, “A simple schlieren system for visualizing a sound field of pulsed ultrasound,” *Journal of Physics: Conference Series*, vol. 1, no. 1, pp. 146–149, 2004.

[23] C. Huang, J. W. Gregory, and J. P. Sullivan, “A modified schlieren technique for micro flow visualization,” *Measurement Science and Technology*, vol. 18, no. 5, pp. N32–N34, 2007.

[24] S. H. Collicott and T. R. Salyer, “Noise reduction properties of a multiple-source schlieren system,” in *AIAA Journal*, pp. 1683–1688, 1994.