ASSIGNMENT-3

Q.1 Suppose we have binary classification problem where we want to predict whether a student will pass (1) or fail (1) based on the number of hours he/she studied. We have the following training dataset:

Hours Studied	Result
2	0
3	0
4	0
6	1
8	1

Use logistic regression for 2 iterations to find the best fit line that separates the two classes. Also calculate the cost for each iteration.

Suppose the initial values of $W_0=0, W_1=0$ and learning rate $=0.01$

Ans① The logistic regression model is given by:

$$h(x) = \frac{1}{1+e^{-(W_0+W_1x)}}$$

where:

- W_0 is the bias (intercept),
- W_1 is the weight (coefficient for "Hours Studied"),
- x is the input (Hours Studied),
- $h(x)$ is the predicted probability.

→ Iteration-1

1. Compute Initial Predictions

Given:

- $W_0=0, W_1=0$
- Learning rate $\alpha = 0.01$

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For each x in the dataset, we calculate:

$$h(x) = \frac{1}{1 + e^{-(W_0 + W_1 x)}}$$

Since $W_0 = 0$ and $W_1 = 0$, the exponent becomes 0, so:

$$h(x) = \frac{1}{1 + e^0} = \frac{1}{2} = 0.5$$

for all values of x.

2. Compute Cost Function

The cost function for logistic regression is:

$$\boxed{J(W_0, W_1) = -\frac{1}{m} \sum_{i=1}^{m} [y_i \log(h(x_i)) + (1 - y_i) \log(1 - h(x_i))]}$$

Substituting $h(x_i) = 0.5$:

$$J = -\frac{1}{5} \sum_{i=1}^{5} [y_i \log(0.5) + (1 - y_i) \log(0.5)]$$

$$\Rightarrow J = -\frac{1}{5} \sum_{i=1}^{5} [\log(0.5)]$$

$$\Rightarrow J = -\frac{1}{5} \times \{5 \times (-0.693)\} =$$

$$\Rightarrow \boxed{J = 0.693}$$

Cost at iteration 0: $J = 0.693$

3. Compute Gradients:

The gradient updates are:

$$\boxed{\frac{\partial J}{\partial W_0} = \frac{1}{m} \sum (h(x_i) - y_i)} \qquad \boxed{\frac{\partial J}{\partial W_1} = \frac{1}{m} \sum (h(x_i) - y_i) x_i}$$

Substituting values:

$$\frac{\partial J}{\partial W_0} = \frac{1}{5} \Sigma (0.5 - y_i)$$

$$\Rightarrow \frac{1}{5} \left[(0.5-0) + (0.5-0) + (0.5-0) + (0.5-1) + (0.5-1)\right]$$

$$\Rightarrow \frac{1}{5} \left[0.5 + 0.5 + 0.5 - 0.5 - 0.5\right] = \frac{1}{5}(1.5-1) = \frac{0.5}{5} = 0.1$$

$$\Rightarrow \boxed{\frac{\partial J}{\partial W_0} = 0.1}$$

$$\frac{\partial J}{\partial W_1} = \frac{1}{5} \Sigma \left[h(x_i) - y_i\right] x_i$$

$$\Rightarrow \frac{1}{5} \left[0.5 \times 2 + 0.5 \times 3 + 0.5 \times 4 + (-0.5) \times 6 + (-0.5) \times 8\right]$$

$$\Rightarrow \frac{1}{5} \left[1 + 1.5 + 2 - 3 - 4\right] = \frac{1}{5}(-2.5)$$

$$\Rightarrow \boxed{\frac{\partial J}{\partial W_1} = -0.5}$$

4. Update Parameters

Using gradient descent

$$\boxed{W_0^{new} = W_0 - \alpha \frac{\partial J}{\partial W_0}} \qquad \boxed{W_1^{new} = W_1 - \alpha \frac{\partial J}{\partial W_1}}$$

$W_0 = 0 - (0.01 \times 0.1) \Rightarrow W_0 = -0.001$

$W_1 = 0 - (0.01 \times -0.5) \Rightarrow W_1 = 0.005$

5. Compute Cost for Iteration 1

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Using updated $W_0 = -0.001, W_1 = 0.005$, we compute $h(x)$ again:

$$h(x) = \frac{1}{1 + e^{-(0.001 + 0.005x)}}$$

Repeating the cost function calculation, we get:

$$J = 0.692$$

⇒ Thus after 1st iteration, the parameters are:

$$\boxed{W_0 = -0.001,\ W_1 = 0.005,\ J = 0.692}$$

→ Iteration-2

1. Compute Predictions with Updated Parameters

After the first iteration, we have:

- $W_0 = -0.001$
- $W_1 = 0.005$

Using the logistic regression hypothesis:

$$h(x) = \frac{1}{1 + e^{-(W_0 + W_1 x)}}$$

For each value of x:

1. $h(2) = \dfrac{1}{1 + e^{-(0.001 + 0.005 \times 2)}} = \dfrac{1}{1 + e^{-0.009}} \approx 0.50225$
2. $h(3) = \dfrac{1}{1 + e^{(-0.001 + 0.005 \times 3)}} = \dfrac{1}{1 + e^{-0.014}} \approx 0.50350$
3. $h(4) = \dfrac{1}{1 + e^{(-0.001 + 0.05 \times 4)}} = \dfrac{1}{1 + e^{-0.019}} \approx 0.50475$
4. $h(6) = \dfrac{1}{1 + e^{(-0.01 + 0.05 \times 6)}} = \dfrac{1}{1 + e^{-0.029}} \approx 0.50725$
5. $h(8) = \dfrac{1}{1 + e^{(-0.01 + 0.005 \times 8)}} = \dfrac{1}{1 + e^{-0.039}} \approx 0.50975$

2. Compute Cost Function

The cost function:

$$J = \frac{-1}{m} \sum_{i=1}^{m} [y_i \log(h(x_i)) + (1-y_i)\log(1-h(x_i))]$$

Substituting values:

$$J = \frac{-1}{5} [(0.\log(0.50225) + (1-0)\log(1-0.50225)) + (0.\log(0.50350) + (1-0)\log(1-0.50350)) + (1-0)\log(1-0.50475)) + (1.\log(0.50725), (1-1)\log(1-0.50725)) + (1.\log(0.50975) + (1-1)\log(1-0.50975))]$$

Computing logarithms:

$$J = \frac{-1}{5} [\log(0.49775) + \log(0.49650) + \log(0.49525) + \log(0.50725) + \log(0.50975)]$$

Approximating values:

$$J = \frac{-1}{5} [-0.698 + (-0.701) + (-0.704) + (-0.679) + (-0.674)]$$

$$\Rightarrow J = \frac{-1}{5} \times (-3.456) -$$

$$\Rightarrow \boxed{J = 0.691}$$

$\Rightarrow$ Cost at iteration 2: $J = 0.891$

B. Compute Gradients

Using the gradient formulas:

$$\frac{\partial J}{\partial w_0} = \frac{1}{m} \sum (h(x_i) - y_i) \qquad \frac{\partial J}{\partial w_1} = \frac{1}{m} \sum (h(x_i) - y_i) x_i$$

Substituting values:

$$\frac{\partial J}{\partial w_0} = \frac{1}{5} [(0.50225-0) + (0.50350-0) + (0.50425-0) + (0.50725-1) + (0.50975-1)]$$

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$$\Rightarrow \frac{\partial J}{\partial w_0} = \frac{1}{5}(0.50225 + 0.50350 + 0.50475 - 0.49275 - 0.49025)$$

$$\Rightarrow \frac{\partial J}{\partial w_0} = \frac{1}{5}(0.5275) = 0.1055$$

$$\frac{\partial J}{\partial w_1} = \frac{1}{5}[0.50225\times 2 + 0.50350\times 3 + 0.50475\times 4 + (-0.49275)\times 6 + (-0.49025)\times 8]$$

$$\Rightarrow \frac{\partial J}{\partial w_1} = \frac{1}{5}[1.0045 + 1.5105 + 2.019 - 2.9565 - 3.922]$$

$$\Rightarrow \frac{\partial J}{\partial w_1} = \frac{1}{5}[-2.3445]$$

$$\Rightarrow \boxed{\frac{\partial J}{\partial w_1} = -0.4689}$$

4. Update Parameters

Using gradient descent:

$$W_0^{new} = W_0 - \alpha \frac{\partial J}{\partial W_0} \qquad W_1^{new} = W_1 - \alpha \frac{\partial J}{\partial W_1}$$

$$W_0 = -0.001 - (0.01 \times 0.1055) \qquad W_1 = 0.005 - [0.01 \times (-0.4689)]$$

$$\Rightarrow \boxed{W_0 = -0.002055} \qquad \boxed{W_1 = 0.009689}$$

5. After Iteration-2:

$$\boxed{W_0 = -0.00205,\ W_1 = 0.009689,\ Cost = 0.691}$$

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