Amrita Shah AMS 332 Spring 2023

Unit 3: Modeling Cellular Physiological Systems with Spatial Variation

Project 3 Report

Part A: A diffusing autoregulatory gene

1. An ODE model was implemented for a simple autoregulatory gene. A reaction-diffusion model was used to account for spatial variation in the system.

   A system was simulated where the initial concentrations of X_{prot} and X_{rna} are 0.5mM everywhere in space. The total time was set to 30 seconds.

   Figure A1a: Protein and RNA concentrations in mM were plotted as a function of time at the first three positions in space.

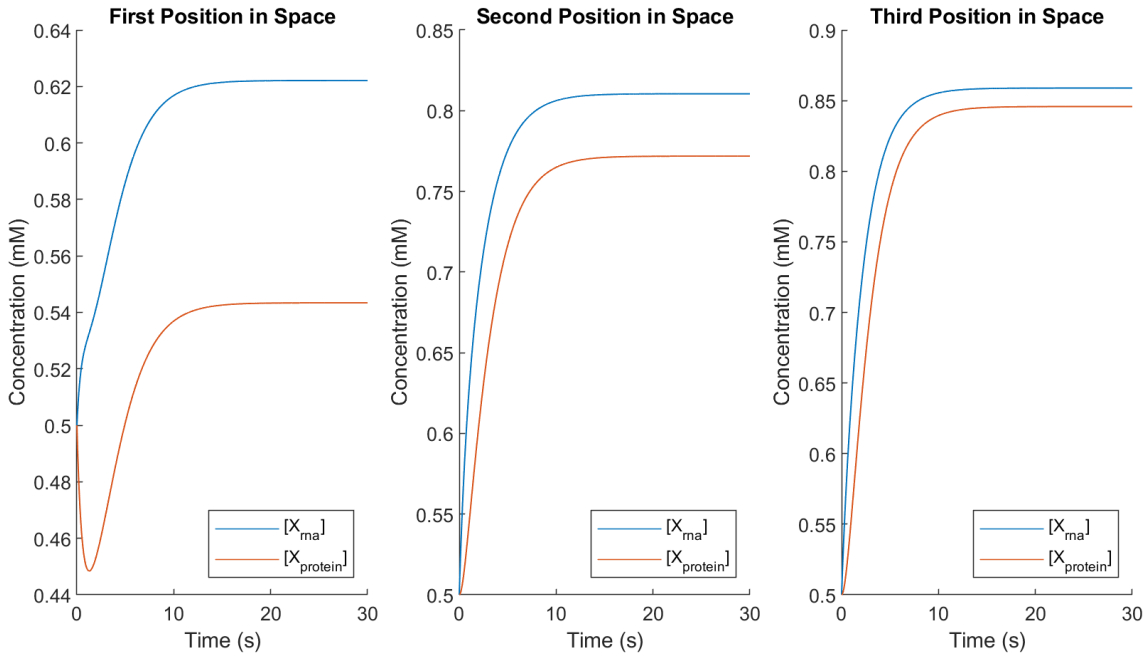
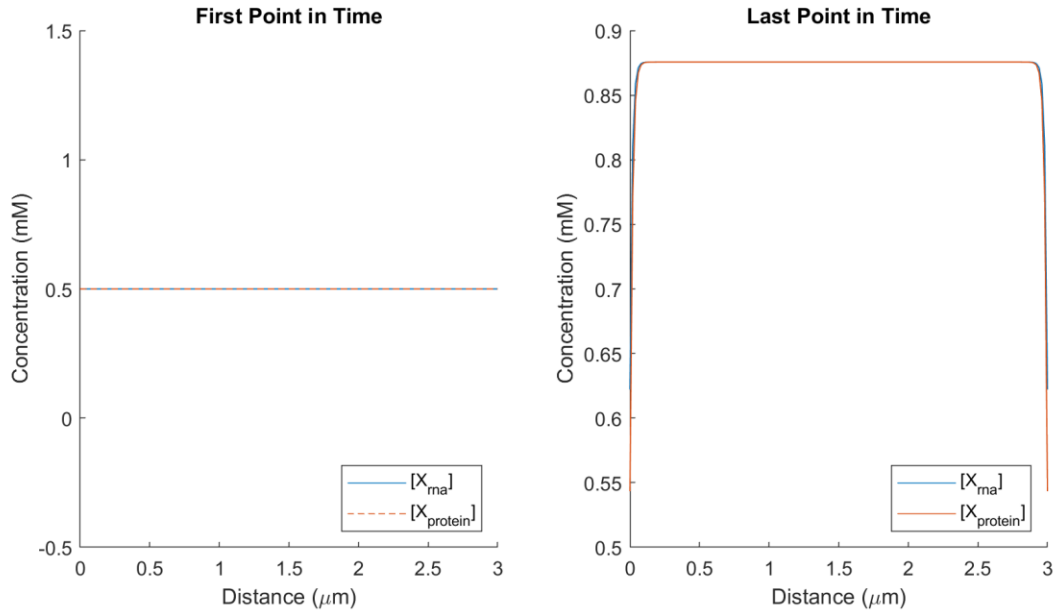


Figure A1b: Protein and RNA concentrations in mM were plotted as a function of space at the first and last points in time.



2. A system was simulated where the initial concentrations of X_{prot} and X_{rna} are 0.5mM only at the first position in space, and zero elsewhere. The total time was set to 30 seconds.

   Figure A2a: Protein and RNA concentrations in mM were plotted as a function of time at the first three positions in space.

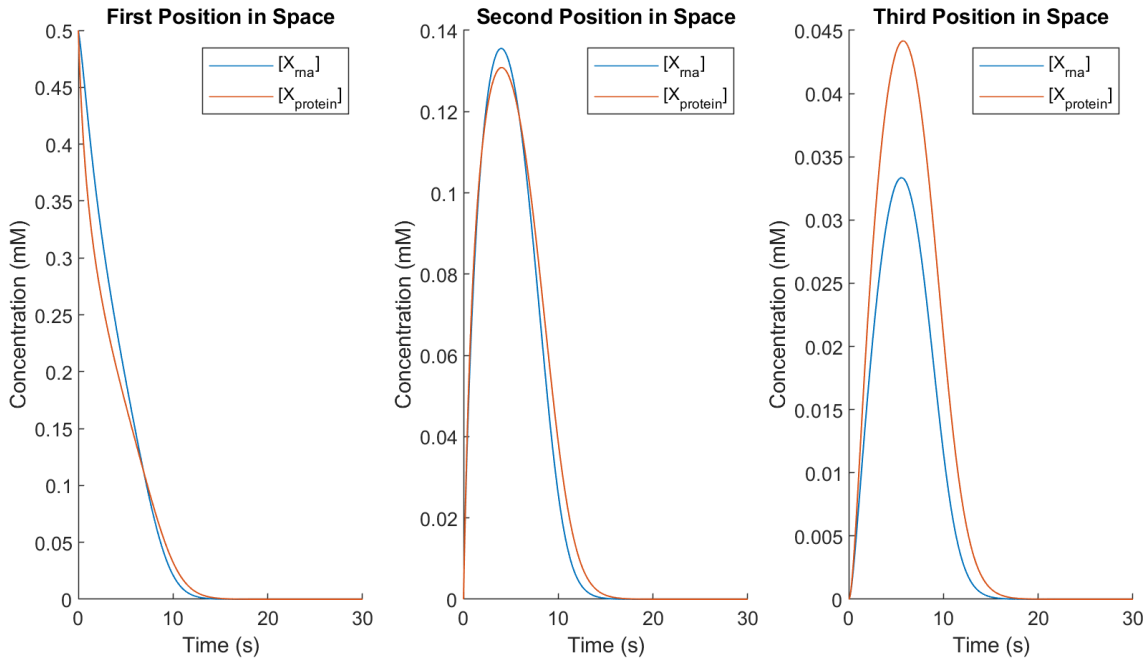
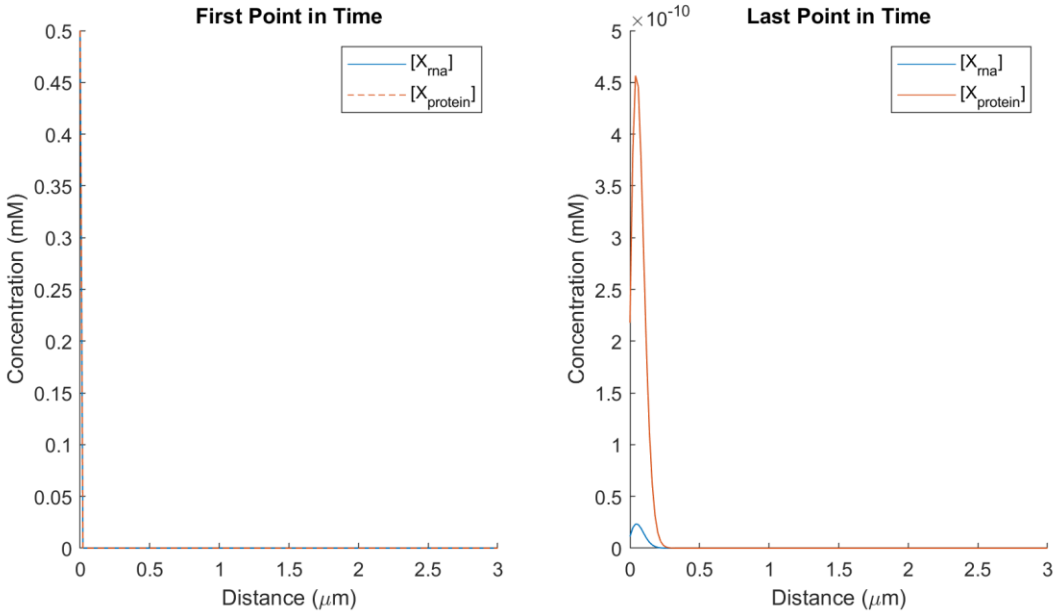


Figure A2b: Protein and RNA concentrations in mM were plotted as a function of space at the first and last points in time.



3. A system was simulated where the initial concentrations of X_{prot} and X_{rna} are 1.0mM only at the first position in space, and zero elsewhere. The total time was set to 30 seconds.

Figure A3a: Protein and RNA concentrations in mM were plotted as a function of time at the first three positions in space.

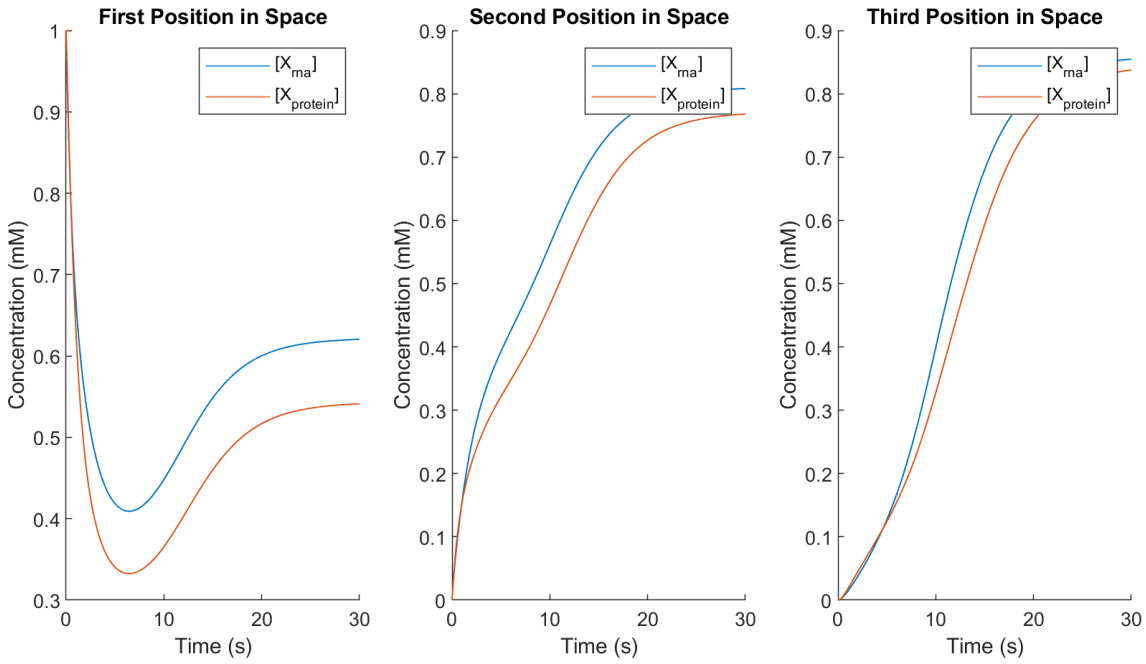
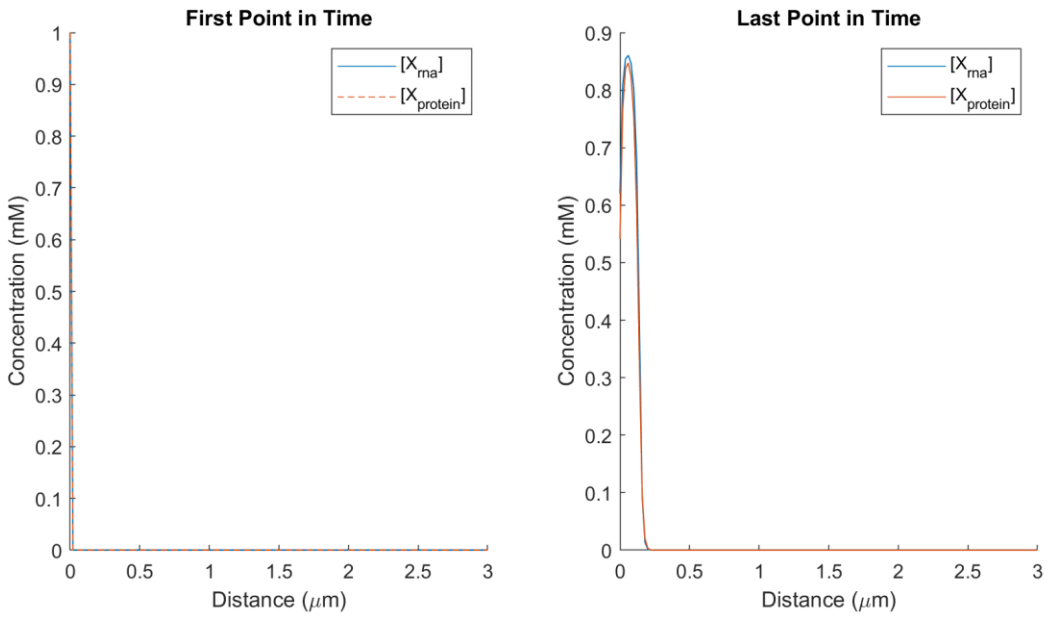


Figure A3b: Protein and RNA concentrations in mM were plotted as a function of space at the first and last points in time.



4. For the following 2 figures, the simulation from (3) was repeated with a total time of 50 seconds.

   Figure A4a-a: Protein and RNA concentrations in mM were plotted as a function of time at the first three positions in space.

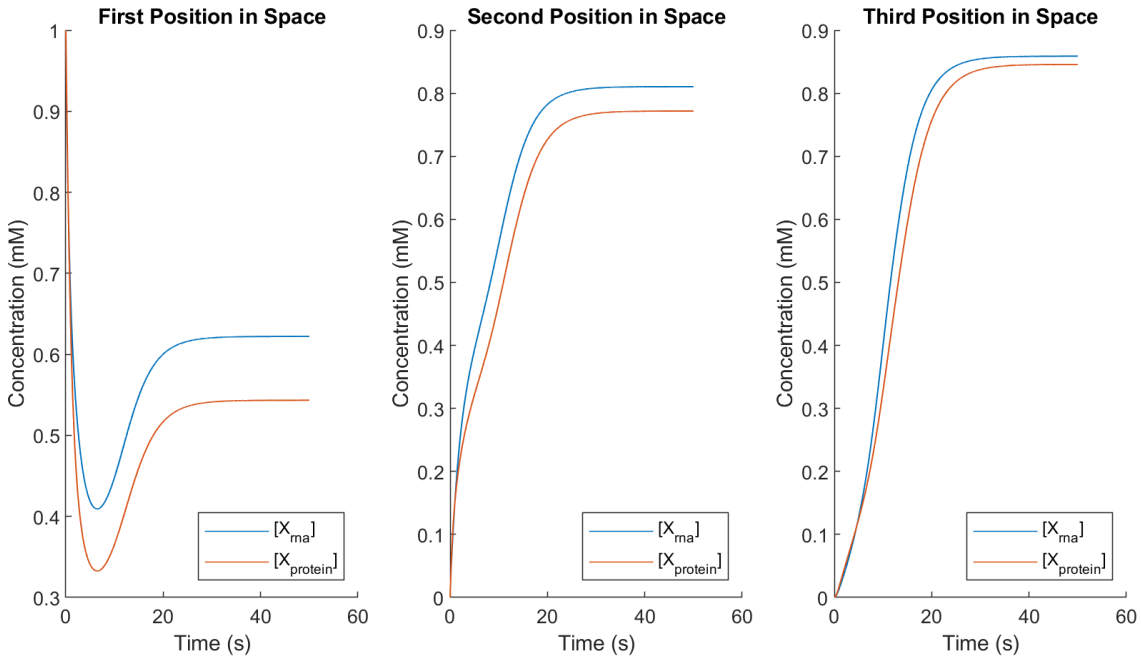
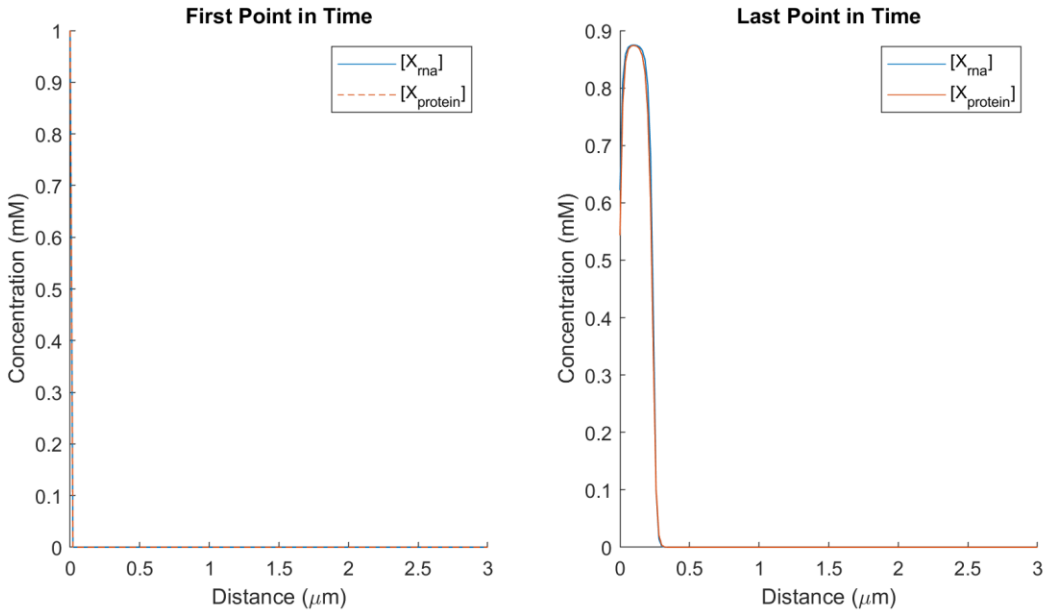


Figure A4a-b: Protein and RNA concentrations in mM were plotted as a function of space at the first and last points in time.



For the following 2 figures, the simulation from (3) was repeated with a total time of 100 seconds.

Figure A4b-a: Protein and RNA concentrations in mM were plotted as a function of time at the first three positions in space.

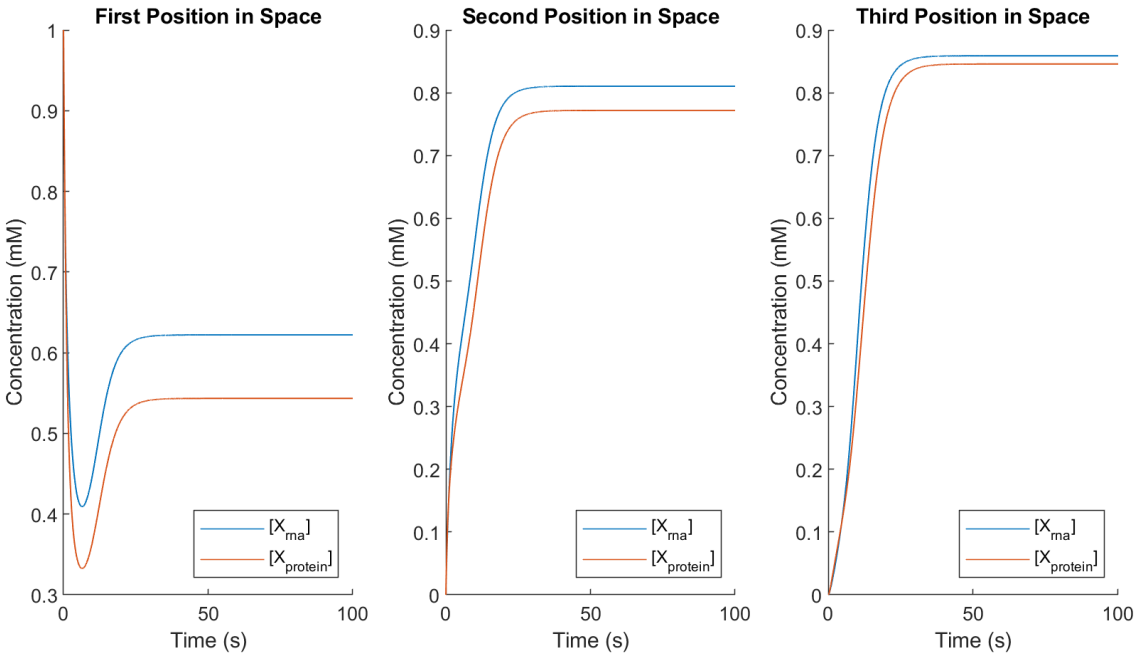
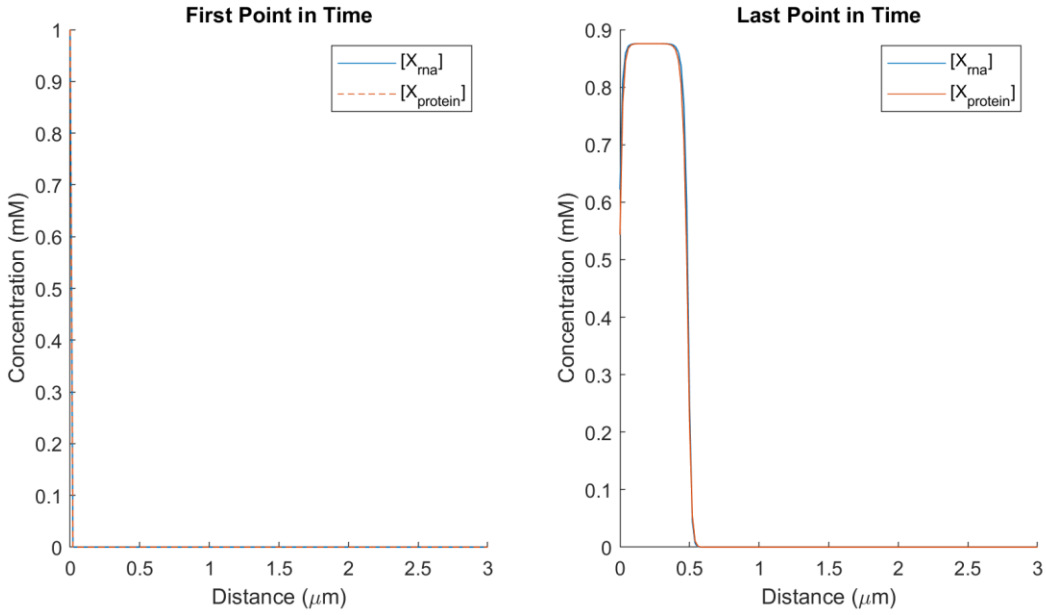


Figure A4b-b: Protein and RNA concentrations in mM were plotted as a function of space at the first and last points in time.



For the following 2 figures, the simulation from (3) was repeated with a total time of 200 seconds.

Figure A4c-a: Protein and RNA concentrations in mM were plotted as a function of time at the first three positions in space.

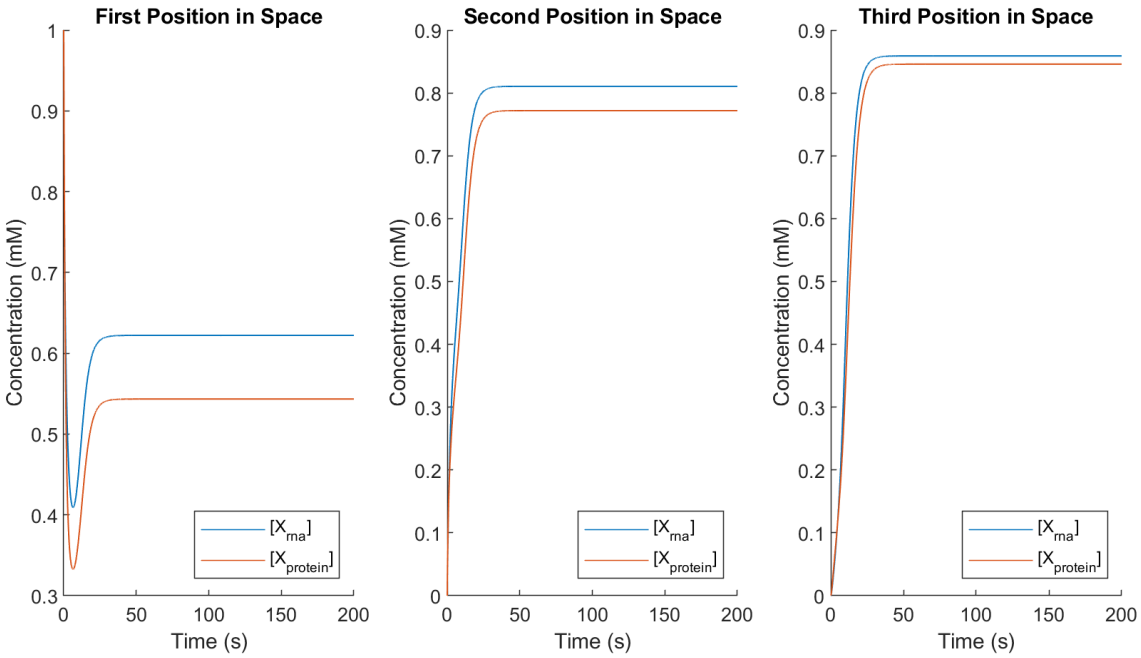
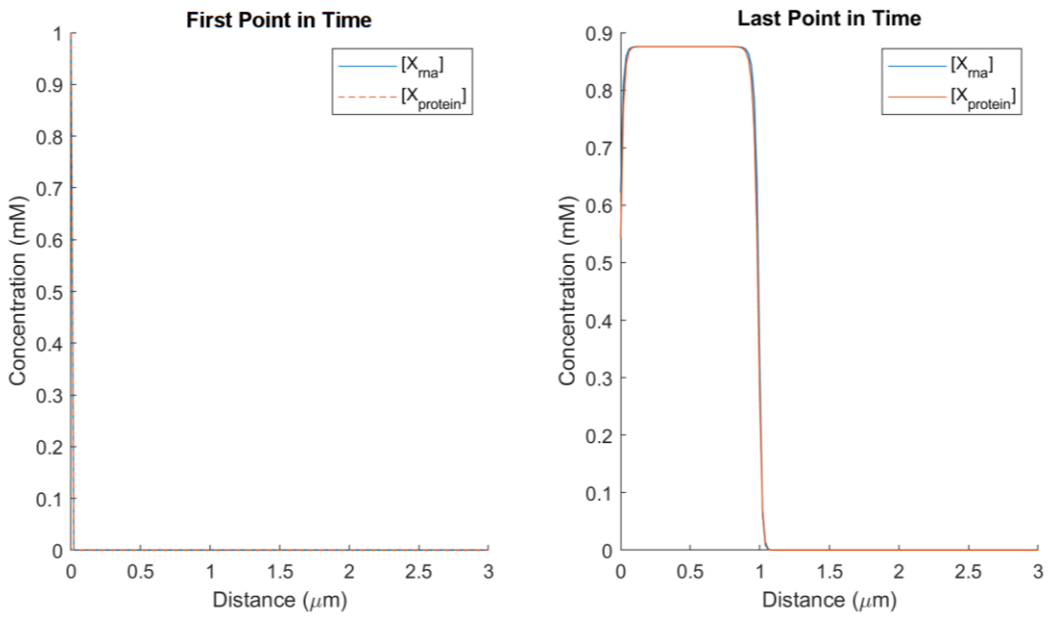


Figure A4c-b: Protein and RNA concentrations in mM were plotted as a function of space at the first and last points in time.



For the following 2 figures, the simulation from (3) was repeated with a total time of 400 seconds.

Figure A4d-a: Protein and RNA concentrations in mM were plotted as a function of time at the first three positions in space.

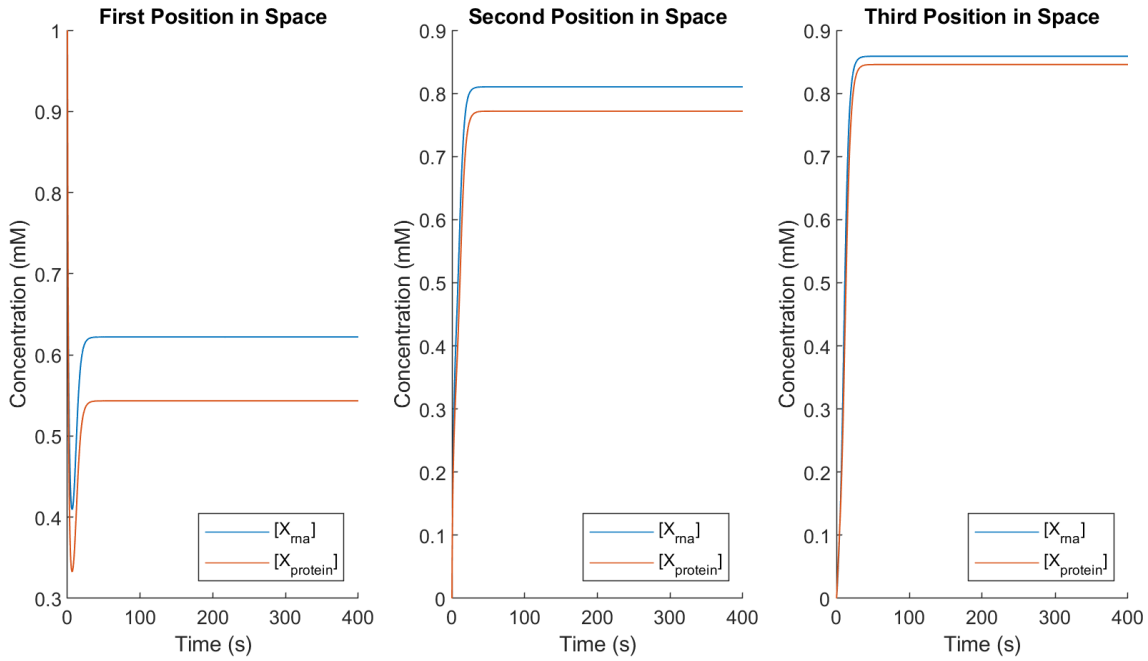
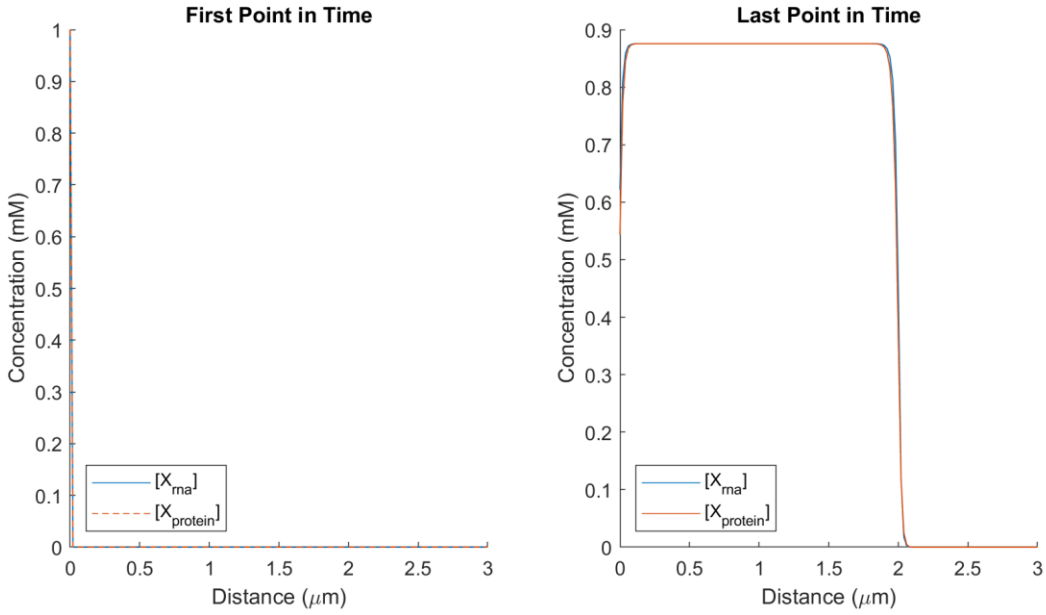


Figure A4d-b: Protein and RNA concentrations in mM were plotted as a function of space at the first and last points in time.



5. The diffusion term in the model and spatial dimension in the simulation allow one to understand the effect of spatial variation in a system. In Figures A1a and A1b, there is no spatial variation and the initial concentrations of RNA and protein are 0.5mM at all positions in space. Because of this, there is no spatial variation in the concentrations of protein and RNA at the first and last point in time (Figure A1b). At every position, the initial concentration is the same and diffusion occurs at the same rate, creating a state of equilibrium among the positions in space. There is only variation at the outermost positions (Figure A1a) caused by the absorbing boundary, which will be discussed later. This can be compared to Figures A2b and A3b, which have initial protein and RNA concentrations of 0.5mM and 1.0mM at the first position only, and zero elsewhere. In those systems, there is spatial variation in the concentrations of protein and RNA at the first point in time due to the spatially varied initial concentrations. There is also spatial variation in the concentrations of protein and RNA at the last point in time. This is because the molecules diffuse from the first position rightward toward the other positions where initial concentrations were zero. However, the system was only simulated for a total time of 30 seconds, so the molecules were only able to diffuse out to a distance of roughly 0.2μM.

   Like the ODE-based system in Project 1, two steady states exist in this system. One is where protein and RNA concentrations are both nearly zero, and one is where both are roughly 0.88mM. In Figures A2a and A2b, the initial concentration at the first position in space was 0.5mM. Protein and RNA concentrations approach the steady state at nearly zero. In Figures A3a and A3b, the initial concentration at the first position in space was 1.0mM. Protein and RNA concentrations approach the steady state at roughly 0.88mM.

It is important to note that the outermost positions in space that were subject to diffusion of protein and RNA do not approach the same steady states, but rather ones of smaller value. As the distance approaches these positions, the concentration steady states approach zero. In Figures A1a and A3a, the steady states at the first positions in space are less than those at the second positions, which are less than those at the third, which are less than those observed at the last point in time for the majority of positions in A1b and A3b, respectively. This is due to the absorbing (zero flux) boundary, which represents a hard boundary at the edges of the spatial area that does not allow for diffusion beyond the edges.

As seen in Figures A4a-b, A4b-b, A4c-b, and A4d-b, the distance covered by the non-zero steady state increased as the total time was increased. With a greater total time of the simulation, the protein and RNA had more time to diffuse a greater distance from the first position in space. As the total time was increased from 50s to 200s, the distance that the molecules diffused was increased from roughly 0.3μM to roughly 2.0μM.

In a biological context, the diffusion of the protein and RNA from one side of the spatial area to the next is similar to a moving wavefront of high X protein and RNA concentrations. As this wavefront passes a given cell, it could influence the expression of other genes in the cell or influence other biochemical processes.

Part B: A diffusing pair of mutually-inhibiting genes

2. An ODE model was implemented for a pair of mutually inhibitory genes. A reaction-diffusion model was used to account for spatial variation in the system.

   A system was simulated where the initial concentrations of X_{prot} and X_{rna} are 1.0mM at the first position in space and zero elsewhere, and the initial concentrations of Y_{prot} and Y_{rna} are 1.0mM at the last position in space and zero elsewhere. The total time was set to 5 seconds.

   Figure B2a: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of time at the first, middle, and last positions in space.

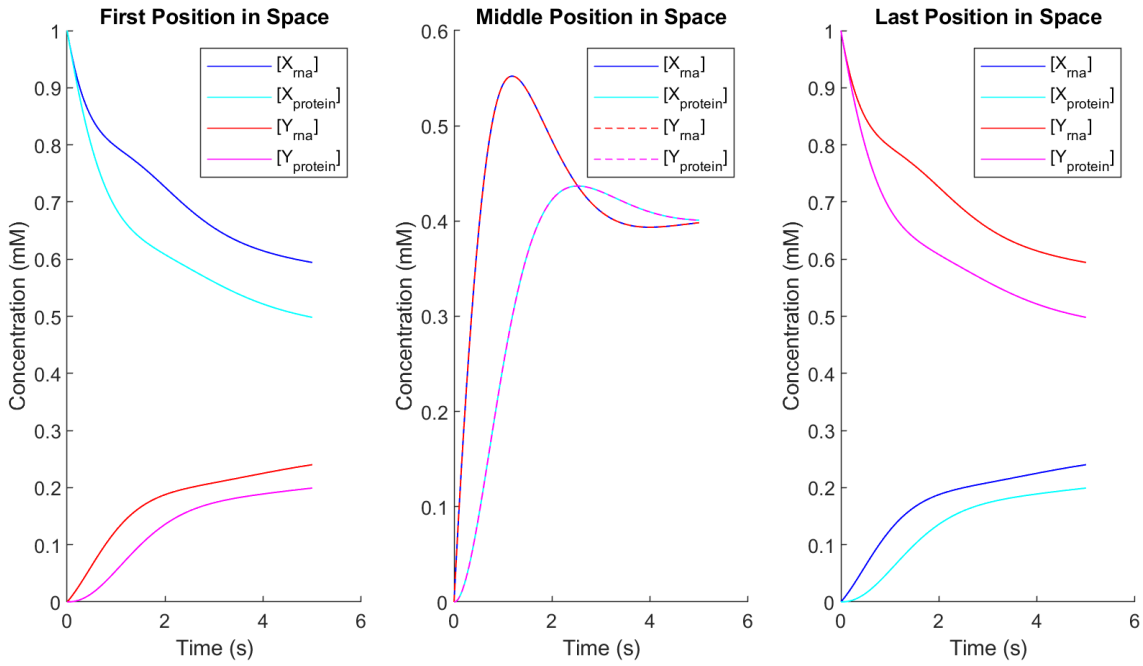
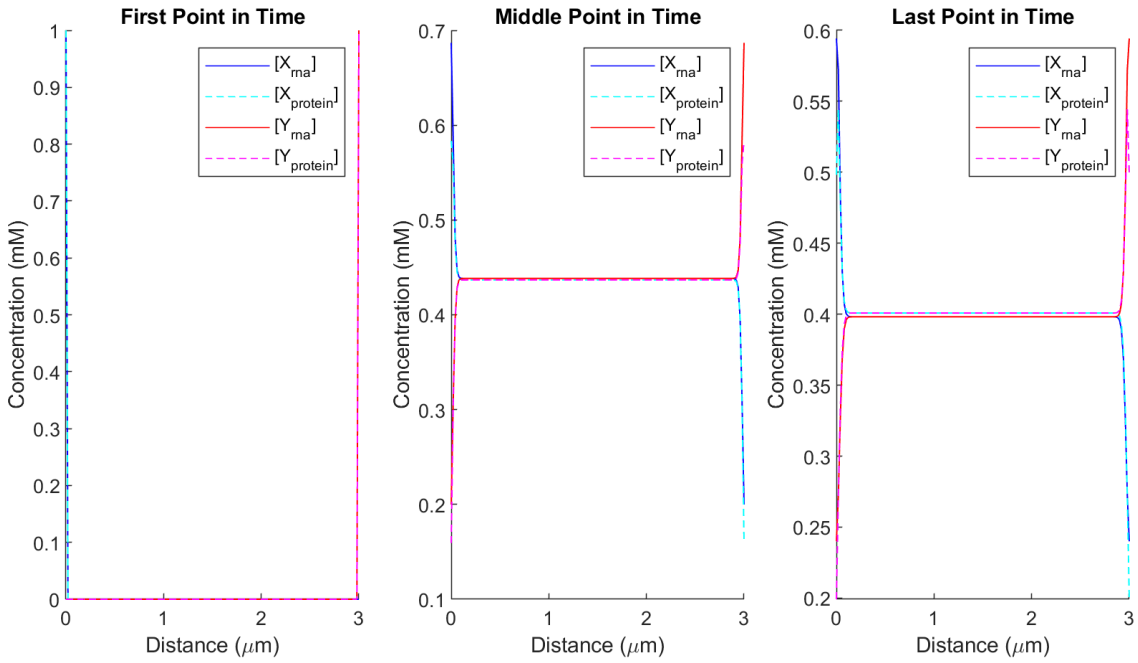


   Figure B2b: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of space at the first, middle, and last points in time.



3. For the following 2 figures, the simulation from (2) was repeated with a total time of 50 seconds.

   Figure B3a-a: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of time at the first, middle, and last positions in space.

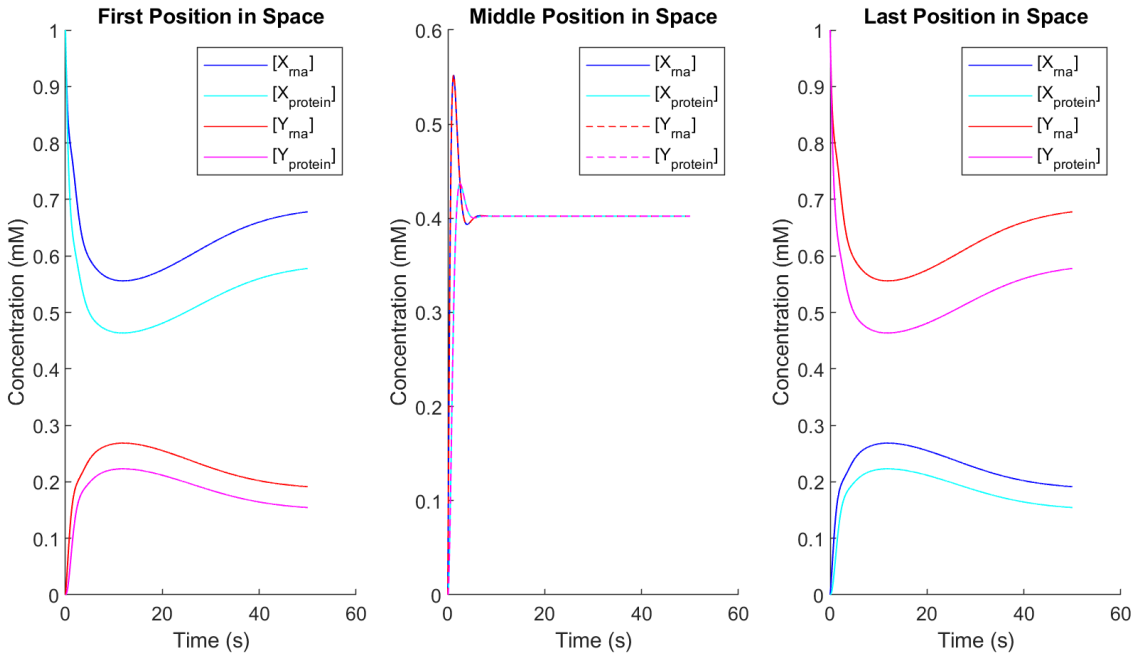
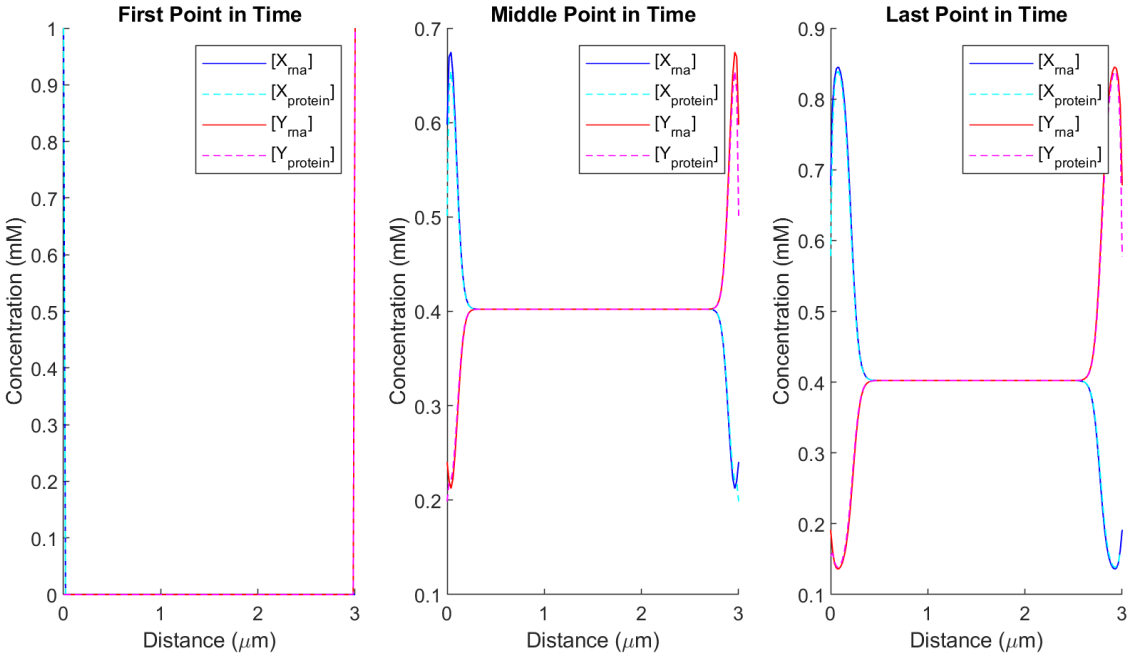


Figure B3a-b: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of space at the first, middle, and last points in time.



For the following 2 figures, the simulation from (2) was repeated with a total time of 100 seconds.

Figure B3b-a: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of time at the first, middle, and last positions in space.

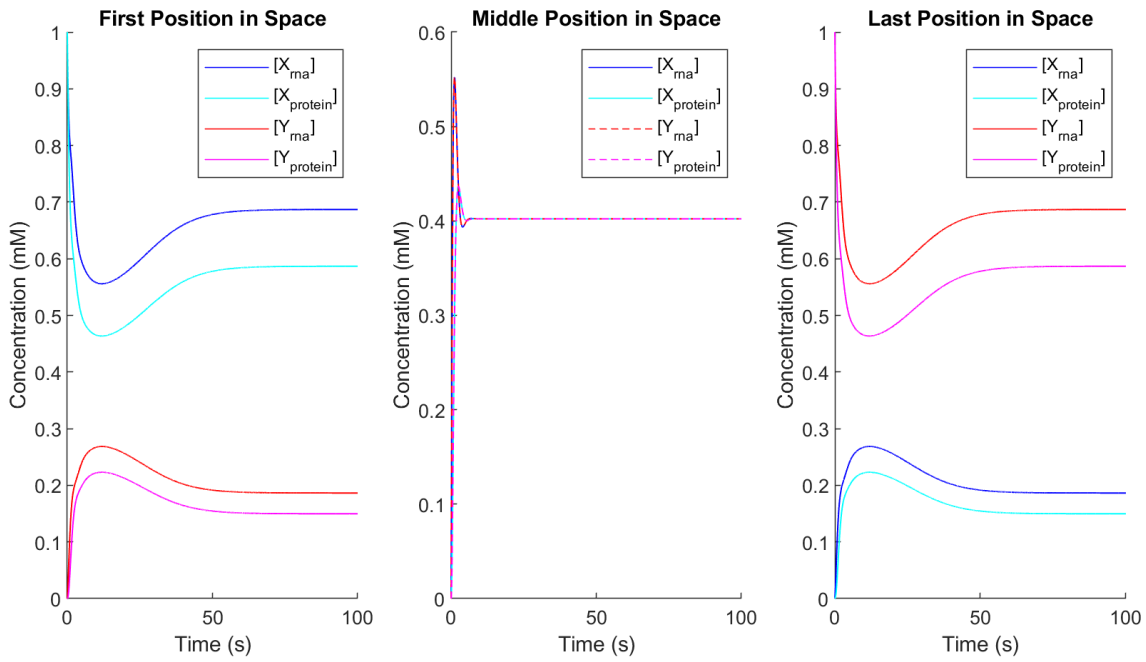
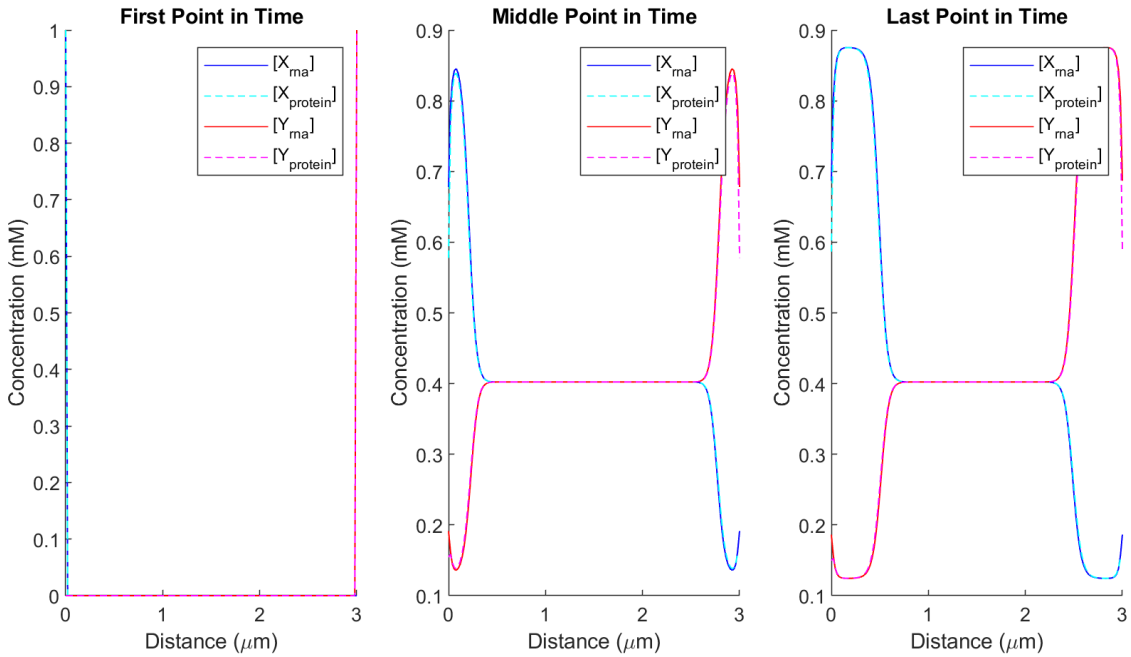


Figure B3b-b: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of space at the first, middle, and last points in time.



For the following 2 figures, the simulation from (2) was repeated with a total time of 200 seconds.

Figure B3c-a: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of time at the first, middle, and last positions in space.

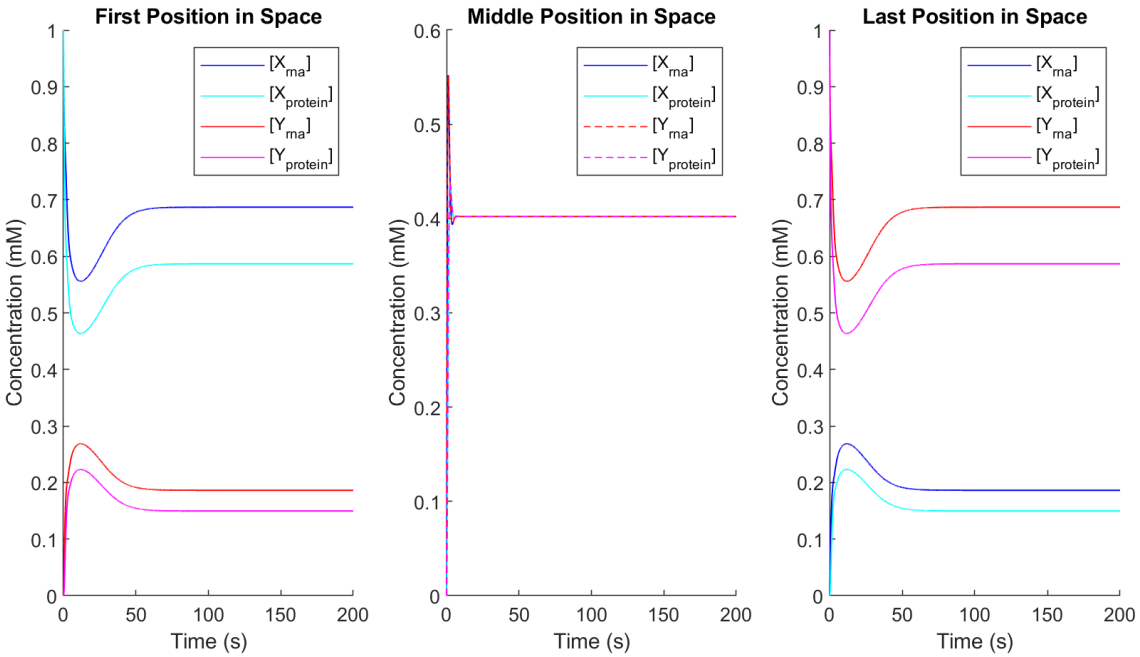
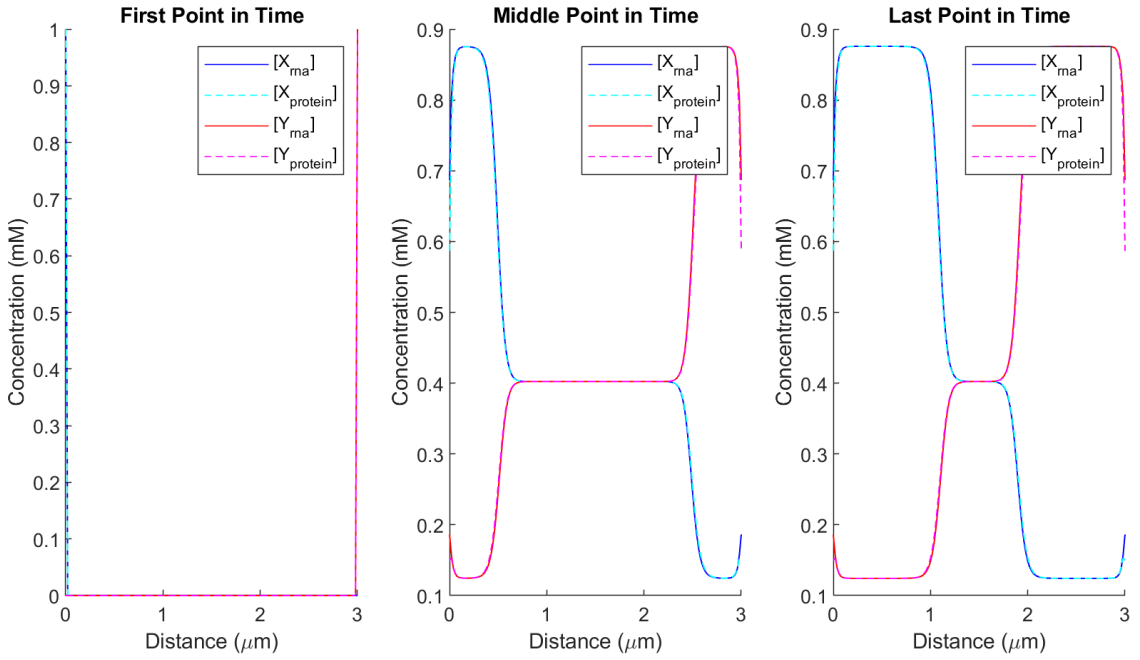


Figure B3c-b: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of space at the first, middle, and last points in time.



For the following 2 figures, the simulation from (2) was repeated with a total time of 400 seconds.

Figure B3d-a: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of time at the first, middle, and last positions in space.

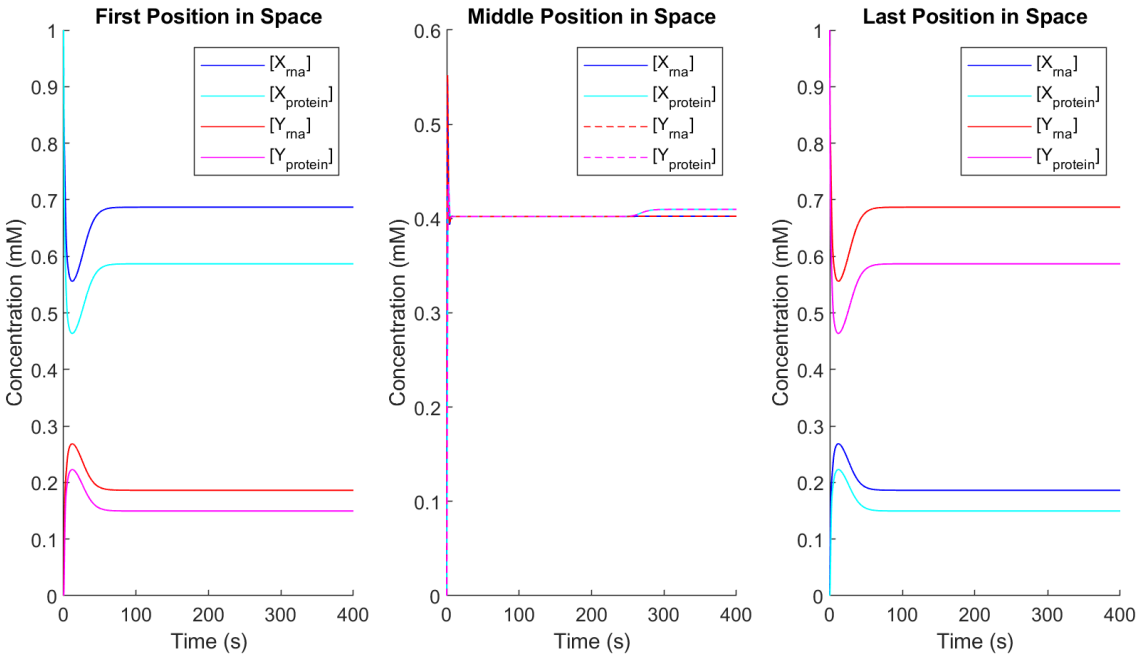
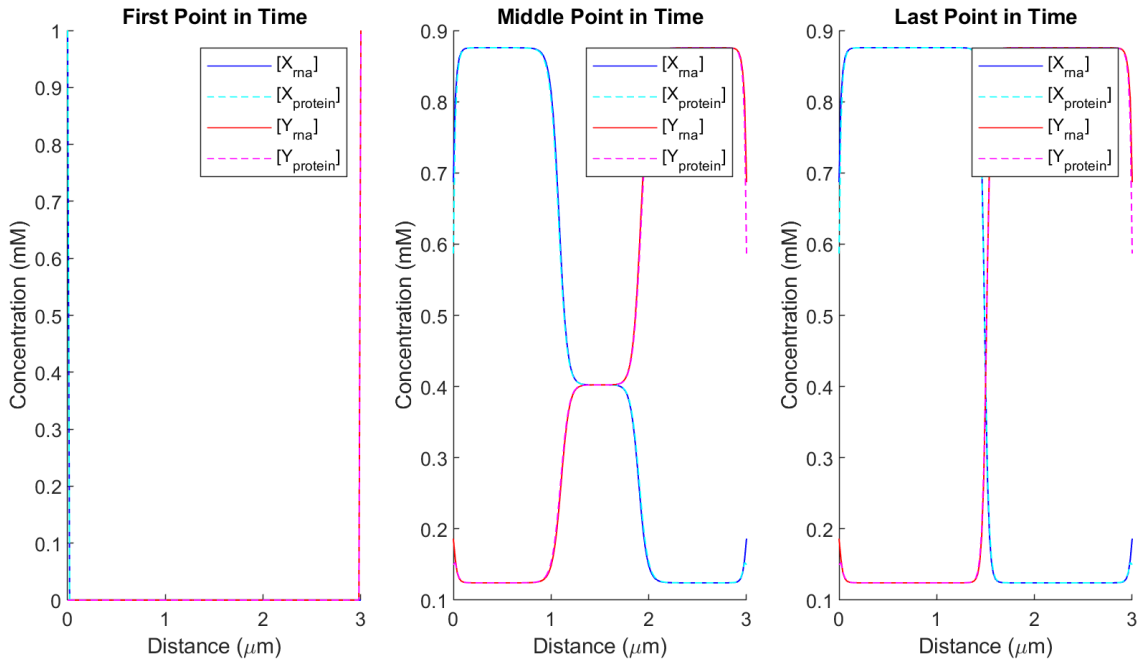


Figure B3d-b: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of space at the first, middle, and last points in time.



4. A system was simulated where the initial concentrations of X_{prot} and X_{rna} are 1.0mM at the first and last positions in space and zero elsewhere, and the initial concentrations of Y_{prot} and Y_{rna} are 1.0mM at the middle position in space and zero elsewhere.

   For the following 2 figures, the total time was set to 50 seconds.

   Figure B4a-a: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of time at the first, middle, and last positions in space.

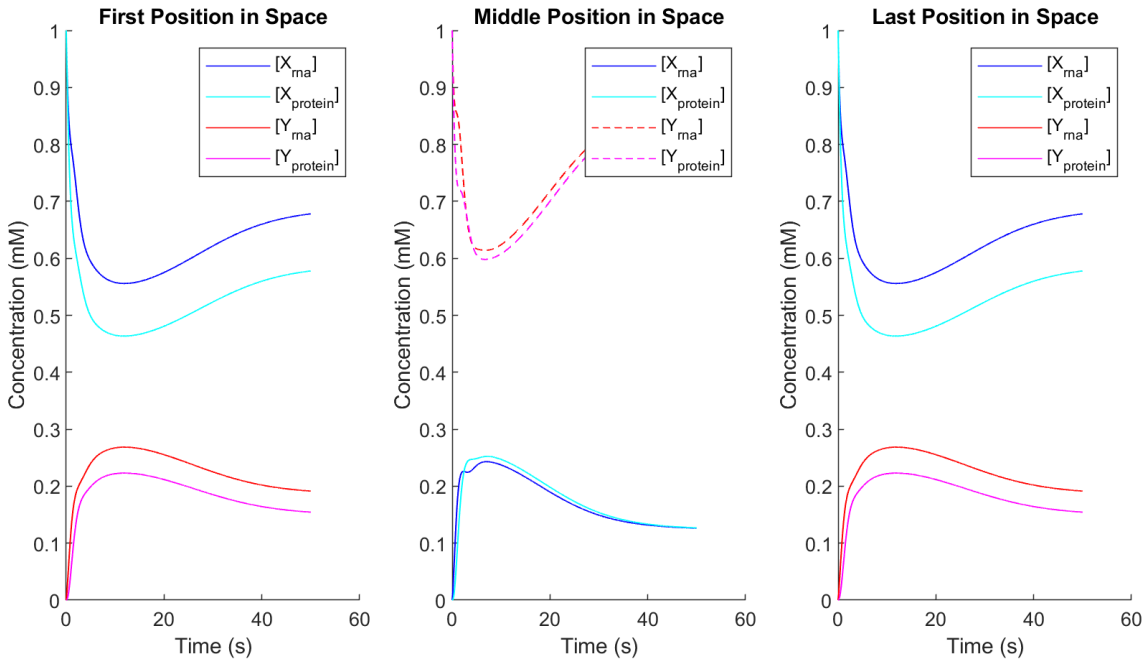
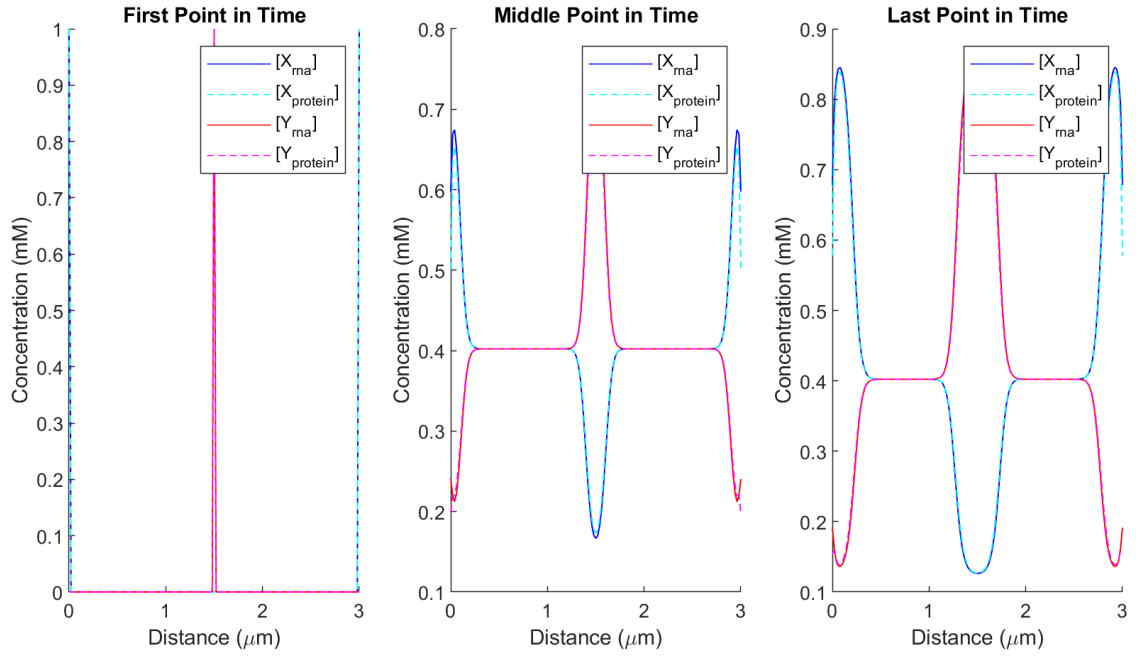


Figure B4a-b: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of space at the first, middle, and last points in time.



For the following 2 figures, the total time was set to 100 seconds.

Figure B4b-a: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of time at the first, middle, and last positions in space.

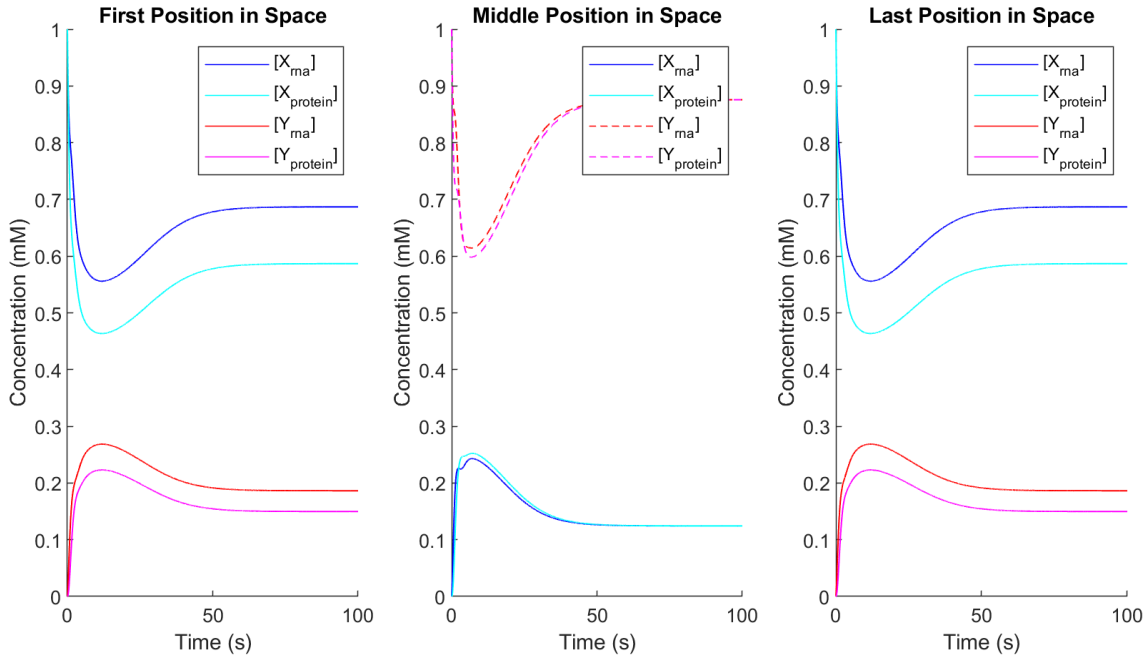
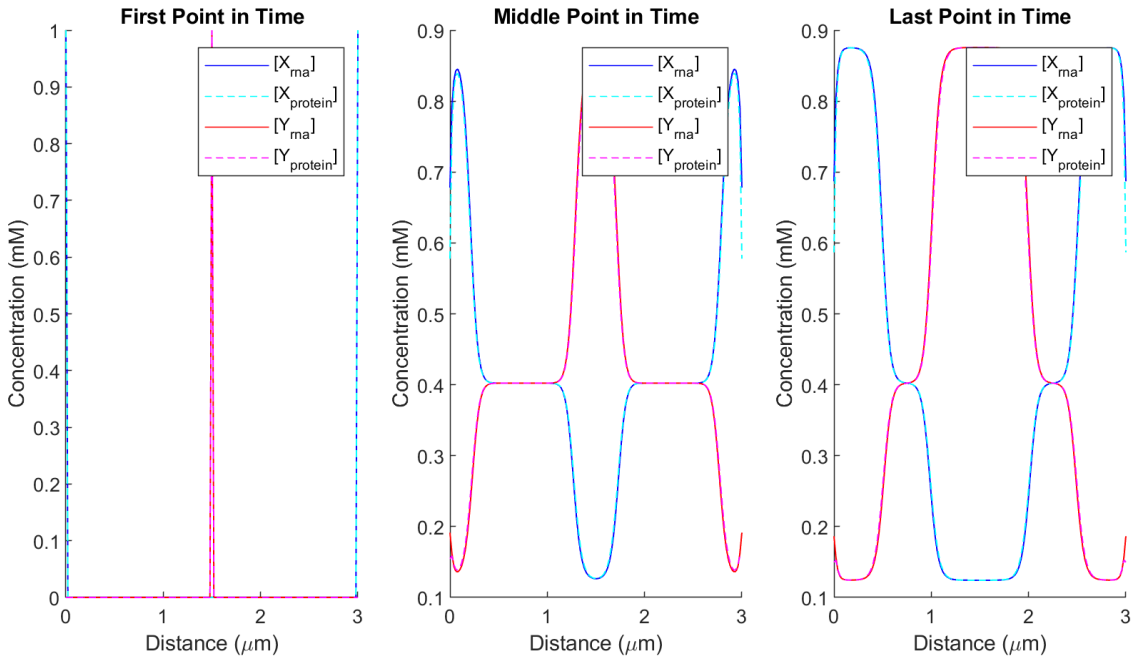


Figure B4b-b: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of space at the first, middle, and last points in time.



For the following 2 figures, the total time was set to 200 seconds.

Figure B4c-a: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of time at the first, middle, and last positions in space.

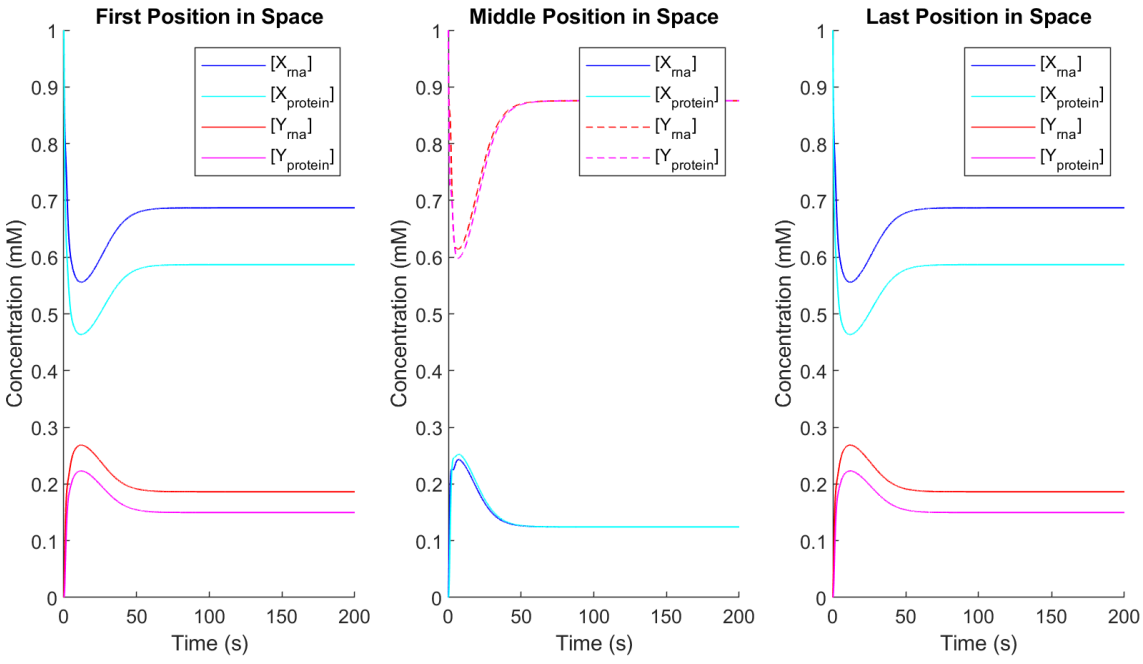
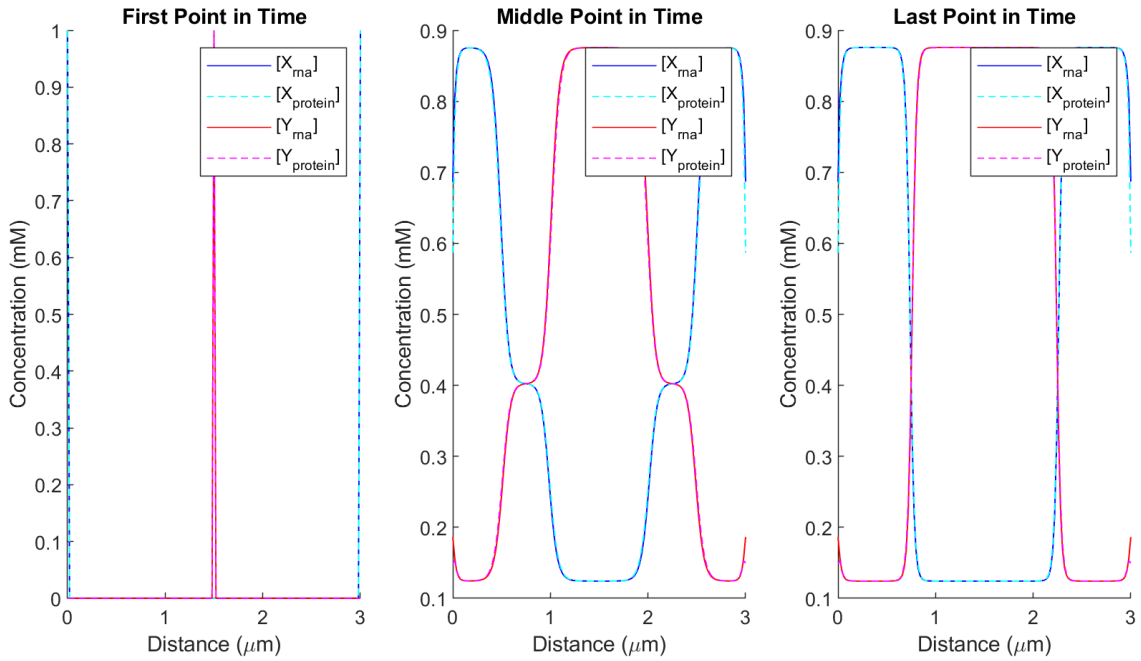


Figure B4c-b: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of space at the first, middle, and last points in time.



For the following 2 figures, the total time was set to 400 seconds.

Figure B4d-a: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of time at the first, middle, and last positions in space.

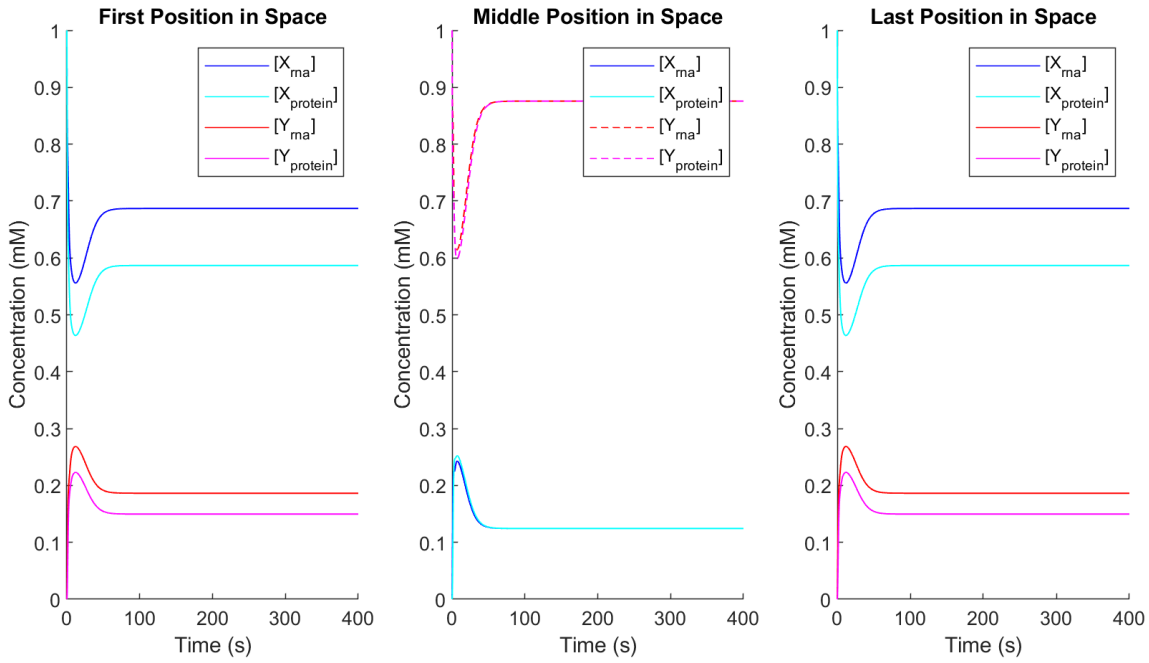
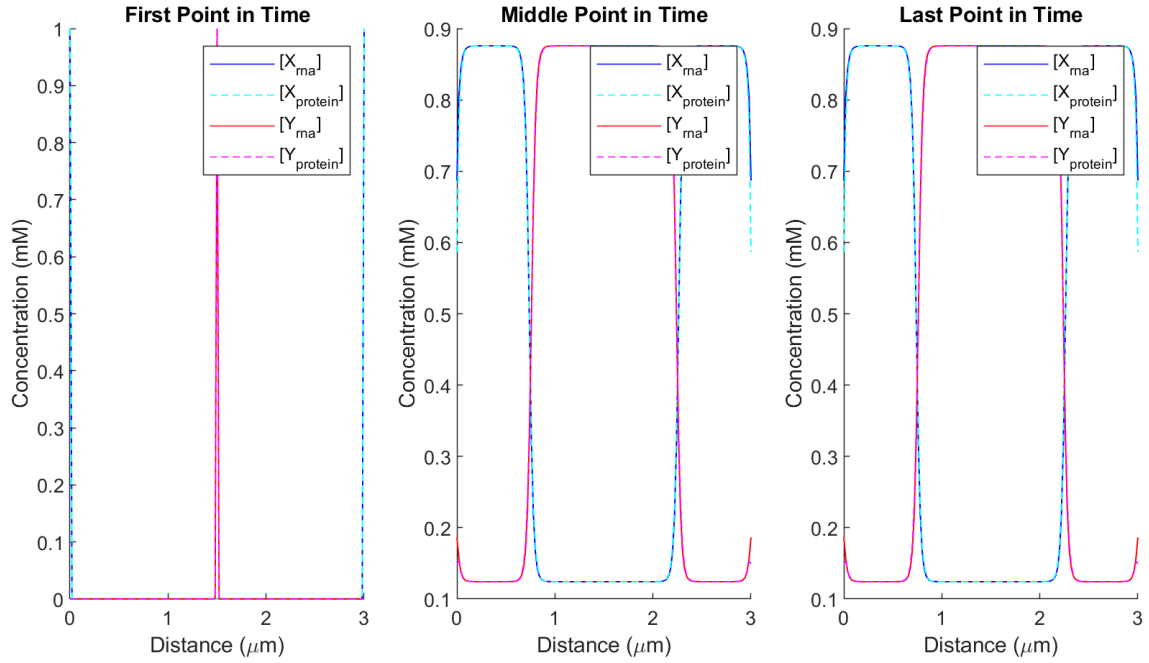


Figure B4d-b: For genes X and Y, protein and RNA concentrations in mM were plotted as a function of space at the first, middle, and last points in time.



5. In these simulations, the molecules of interest include protein and RNA for two genes: X and Y. In Figures B2a and B2b, X protein and RNA were initially only present at the first position in space, while Y protein and RNA were initially only present at the last position in space. Figure A2a shows the preliminary result of these starting conditions in the first 5 seconds of simulation. Over time, the gene X products diffused rightward from the first position while the gene Y products diffused leftward from the last position. The molecules approach each other at the middle position, then they appear to approach a common concentration steady state. At the last point in time, the products of both genes are present and neither significantly dominates over the other (Figure A2b).
   Once again, it is visible that the distance covered by diffusion of protein and RNA increased as the total time was increased in (3). With a greater total time of the simulation, the protein and RNA for genes X and Y had more time to diffuse a greater distance from their starting positions in space, in greater amounts. Comparing Figures B3a-a, B3b-a, B3c-a, and B3d-a shows that the products of genes X and Y diffuse toward the middle position and eventually reach a steady state at the middle position where the RNA concentrations are equal and the protein concentrations are equal. However, since genes X and Y are mutually inhibitory, there are two additional steady states reached, where one of the genes dominates. At the first position in space, which initially had only gene X products present, a steady state of high gene X product concentrations and low gene Y product concentrations is reached. The opposite steady state is reached at the last position in space, which initially only had gene Y products present. The three steady states are most clearly observable in Figure B3d-a. Comparing Figures B3a-b, B3b-b, B3c-b, and B3d-b shows that the latter two steady states will eventually dominate as time

progresses. From B3a-b to B3d-b, the amount of space in the middle position that is at the 'neutral' steady state (where products of both genes are present at relatively equal and significant concentrations) shrinks. At the last point in time in Figure B3d-b, the neutral steady state is no longer present. The left half of the spatial area is at the gene X-dominant steady state and the right half is at the gene Y-dominant steady state. In the biological context of cro/cI genes, this indicates that one half of the cells in the spatial area would undergo lysis while the other undergoes lysogeny, and this would be determined by the cell's position in space.

The simulations in (4) are initiated with gene X products present at the first and last positions only, and gene Y products present at the middle position only. Similar to the previous simulation, the molecules at the first and last positions (in this case gene X products) diffuse toward the middle position. The molecules at the middle position (gene Y products) diffuse outward toward the first and last positions (Figure B4a-a). According to Figures B4a-a, B4b-a, B4c-a, and B4d-a, the concentrations reach gene X-dominant or gene Y-dominant steady states. Figures B4a-b, B4b-b, and B4c-b show that there is also a 'neutral' steady state where neither gene dominates and both are present at significant concentrations while the molecules are first diffusing. However, Figures B4c-b and B4d-b show that this neutral steady state is eventually lost, just as it was in the previous simulations. At the last point in time in Figure B4d-b, the spatial area becomes divided into three segments: gene X-dominant, gene Y-dominant, and gene X-dominant. In the biological context of cro/cI genes, this indicates that the cells in the spatial area would undergo lysis or lysogeny in such a striped pattern based on their spatial positioning. Other similar gene pairs may be involved in skin patterning in various animals. Their

mutual inhibition would create a striped or spotted pattern. In a hypothetical example, one gene could encode for black pigment, while the other encodes for lack of pigment, or white. Early in development, the animal's pattern may perhaps be less apparent due to the presence of the 'neutral' steady state. This could manifest as an area of grey coloring. As time progresses, the 'neutral' steady state is lost, and the single-gene-dominant steady states become more prevalent. The skin is segmented into defined areas of black or white, creating a black and white striped or spotted pattern.

Part A: A diffusing autoregulatory gene

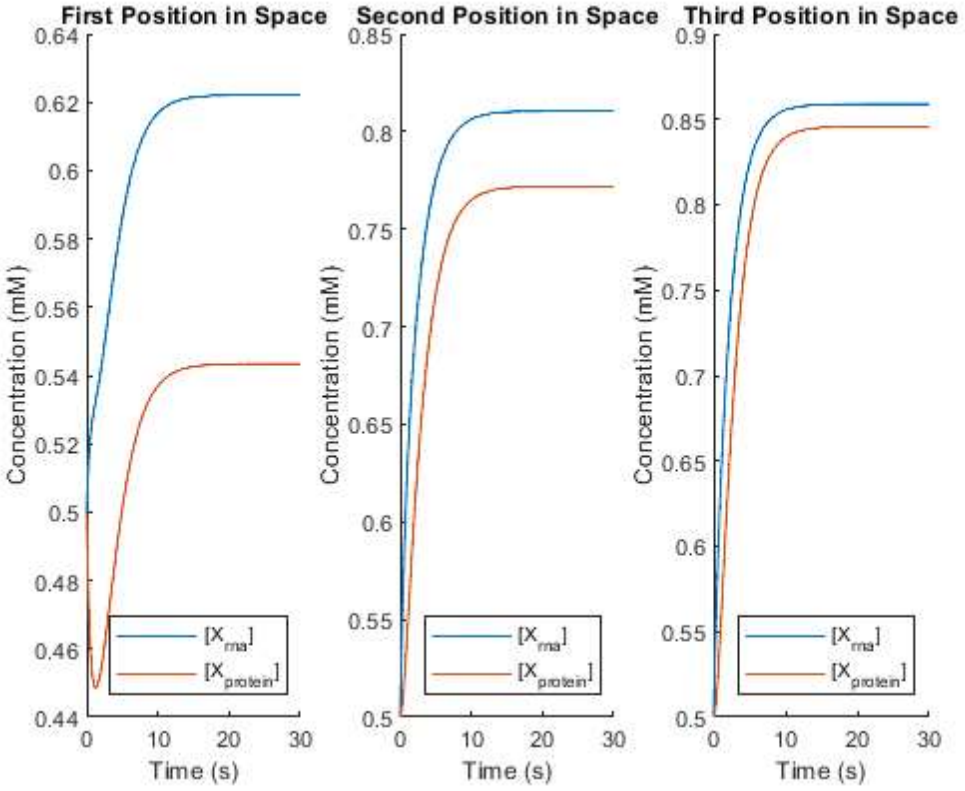
Contents

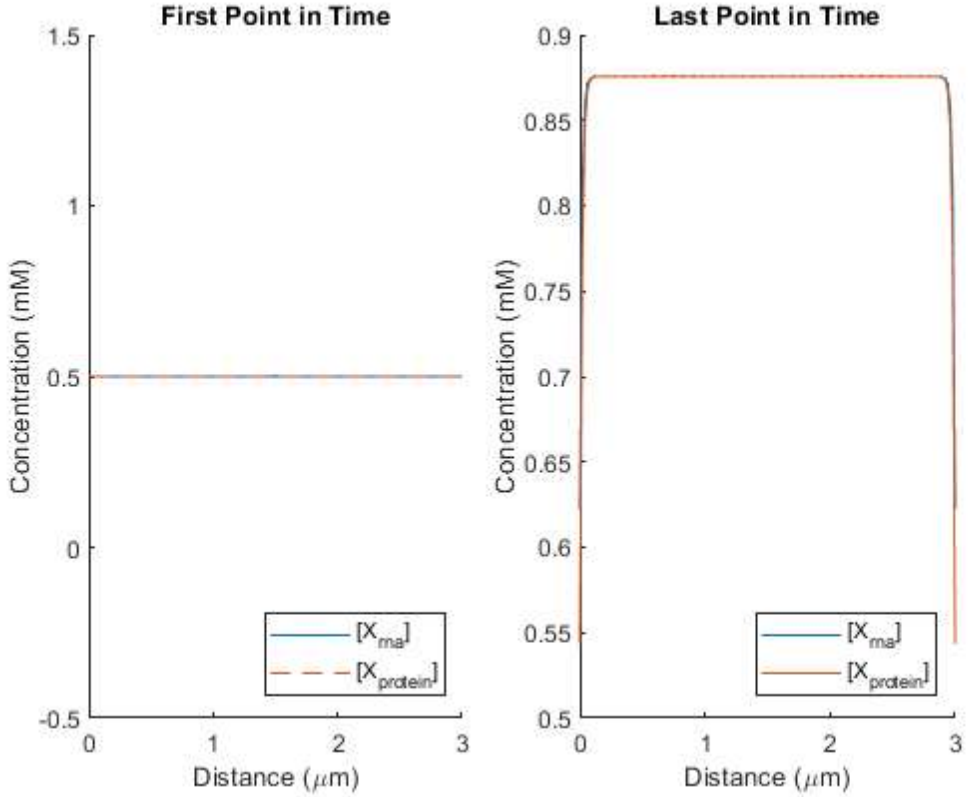
1. initial RNA & protein = 0.5mM everywhere

```
clear
close all

% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 30; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = 0.5.*(ones(spacesteps, timesteps));
Xrna = 0.5.*(ones(spacesteps, timesteps));

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) ); % rna synthesis
        v2 = Xr*Xrna(x,t); % rna degradation
        v3 = w*Xrna(x,t); % protein synthesis
        v4 = Xp*Xprot(x,t); % protein degradation

        % RNA Laplacian
        del2rna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2rna = del2rna + Xrna(x+1,t) + 0;
```

```
        elseif (x==spacesteps)
            del2rna = del2rna + 0 + Xrna(x-1, t);
        else
            del2rna = del2rna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2rna = del2rna/(deltaX^2); % Laplacian

        % RNA derivative
        drna_dt = v1 - v2 + Dr*del2rna;

        % RNA update
        Xrna(x,t+1) = Xrna(x,t) + (drna_dt*deltaT);

        % protein Laplacian
        del2prot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2prot = del2prot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2prot = del2prot + 0 + Xprot(x-1, t);
        else
            del2prot = del2prot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2prot = del2prot/(deltaX^2); % Laplacian

        % protein derivative
        dprot_dt = v3 - v4 + Dp*del2prot;

        % protein update
        Xprot(x,t+1) = Xprot(x,t) + (dprot_dt*deltaT);
    end
end

time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;

fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)))
plot(time, (Xprot(1,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location', 'southeast')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna(2,:)))
plot(time, (Xprot(2,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Second Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location', 'southeast')
hold off

subplot(1,3,3)
hold on
```

```
plot(time, (Xrna(3,:)))
plot(time, (Xprot(3,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Third Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location', 'southeast')
hold off

fig2 = figure(2);
subplot(1,2,1)
hold on
plot(space, (Xrna(:,1)))
plot(space, (Xprot(:,1)), '--')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]', 'Location', 'southeast')
hold off

subplot(1,2,2)
hold on
plot(space, (Xrna(:,timesteps)))
plot(space, (Xprot(:,timesteps)))
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]', 'Location', 'southeast')
hold off
```





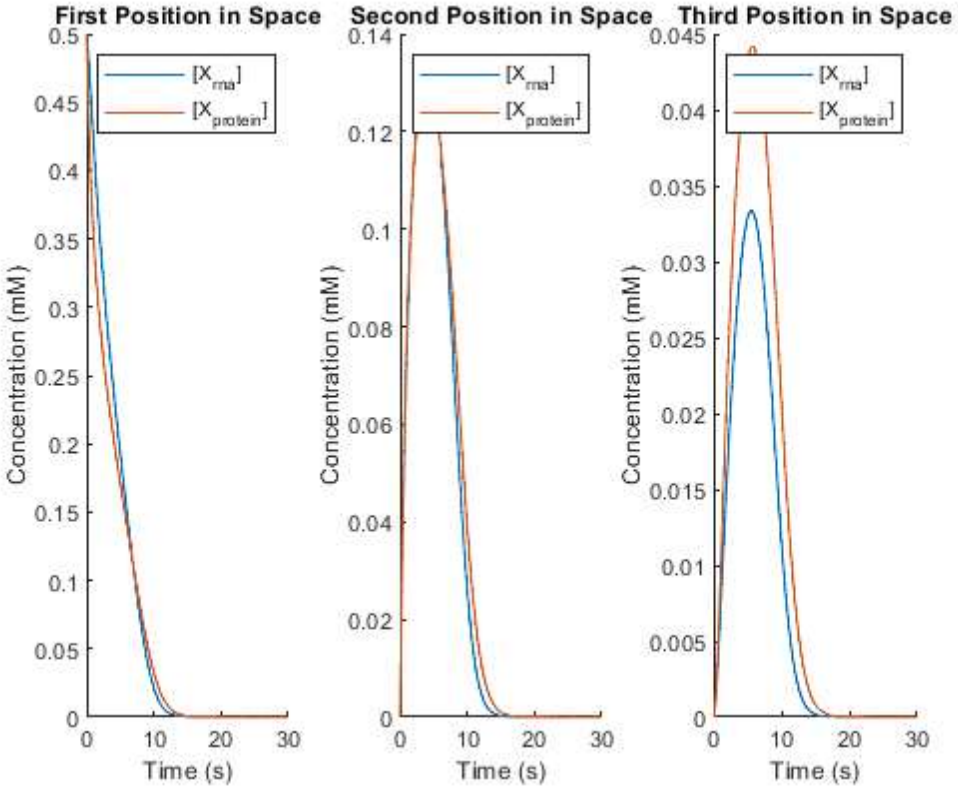
2. initial RNA & protein = 0.5mM at 1st position, 0 elsewhere

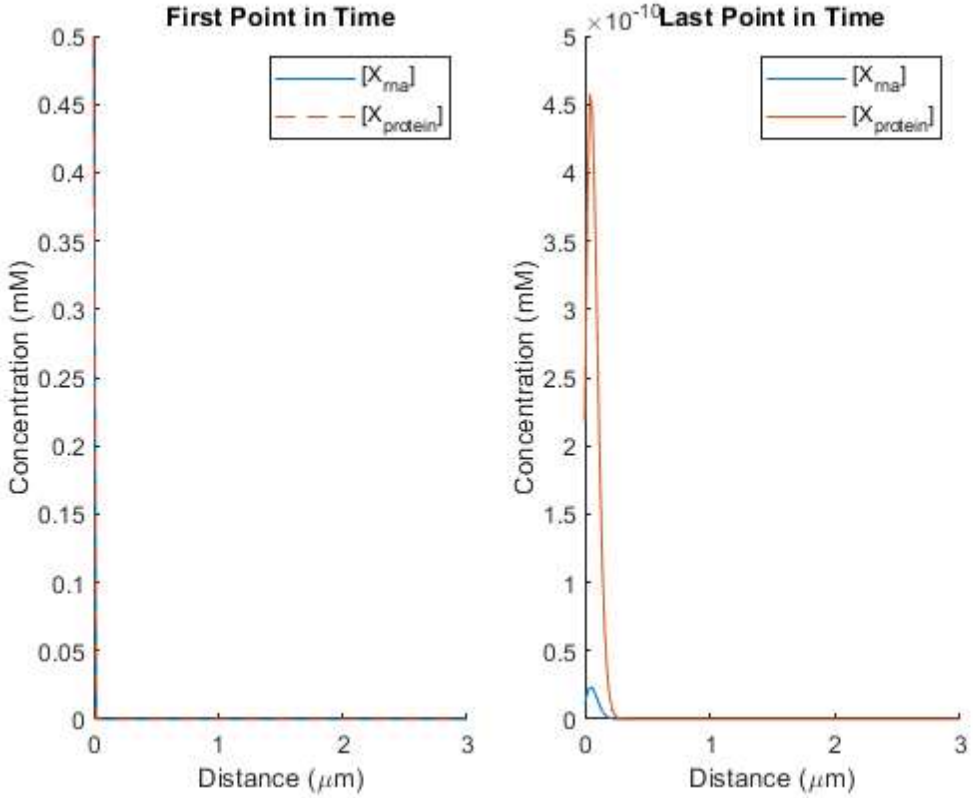
```
clear
close all
```

```
% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 30; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 0.5;
Xrna(1,1) = 0.5;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) ); % rna synthesis
        v2 = Xr*Xrna(x,t); % rna degradation
        v3 = w*Xrna(x,t); % protein synthesis
        v4 = Xp*Xprot(x,t); % protein degradation

        % RNA Laplacian
        del2rna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2rna = del2rna + Xrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2rna = del2rna + 0 + Xrna(x-1, t);
        else
            del2rna = del2rna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2rna = del2rna/(deltaX^2); % Laplacian

        % RNA derivative
        drna_dt = v1 - v2 + Dr*del2rna;

        % RNA update
        Xrna(x,t+1) = Xrna(x,t) + (drna_dt*deltaT);

        % protein Laplacian
        del2prot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2prot = del2prot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2prot = del2prot + 0 + Xprot(x-1, t);
        else
```

```
            del2prot = del2prot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2prot = del2prot/(deltaX^2); % Laplacian

        % protein derivative
        dprot_dt = v3 - v4 + Dp*del2prot;

        % protein update
        Xprot(x,t+1) = Xprot(x,t) + (dprot_dt*deltaT);
    end
end

time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;

fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)))
plot(time, (Xprot(1,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna(2,:)))
plot(time, (Xprot(2,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Second Position in Space')
legend('[X_{rna}]', '[X_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(3,:)))
plot(time, (Xprot(3,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Third Position in Space')
legend('[X_{rna}]', '[X_{protein}]')
hold off

fig2 = figure(2);
subplot(1,2,1)
hold on
plot(space, (Xrna(:,1)))
plot(space, (Xprot(:,1)), '--')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]')
hold off

subplot(1,2,2)
```

```
hold on
plot(space, (Xrna(:,timesteps)))
plot(space, (Xprot(:,timesteps)))
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]')
hold off
```





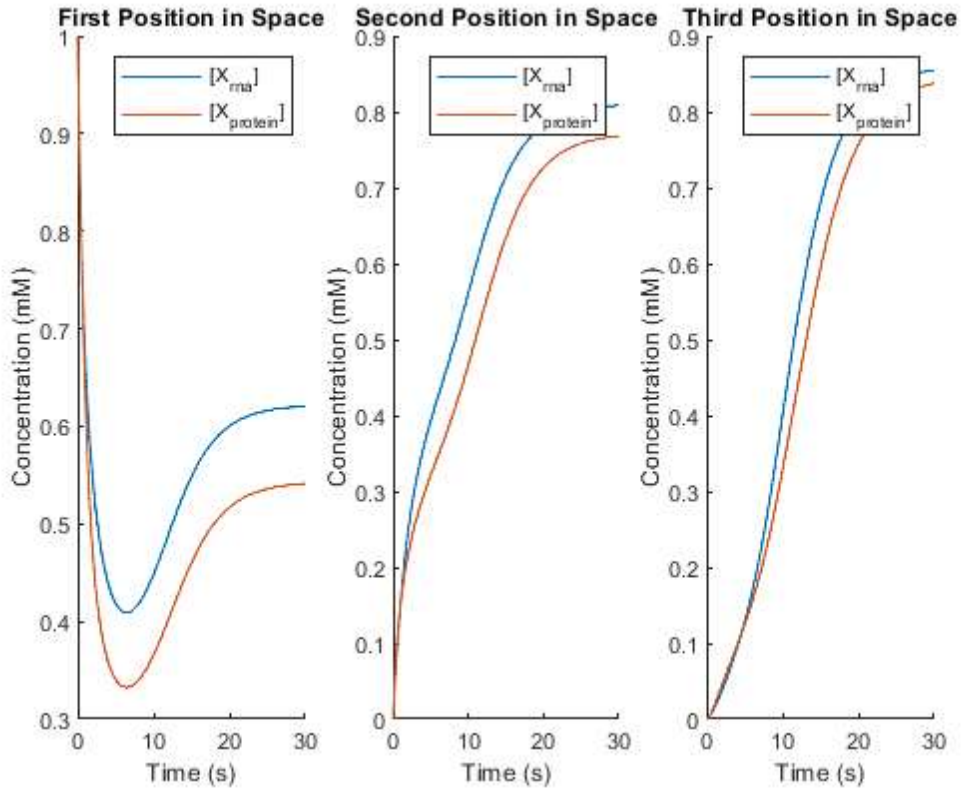
3. initial RNA & protein = 1.0mM at 1st position, 0 elsewhere

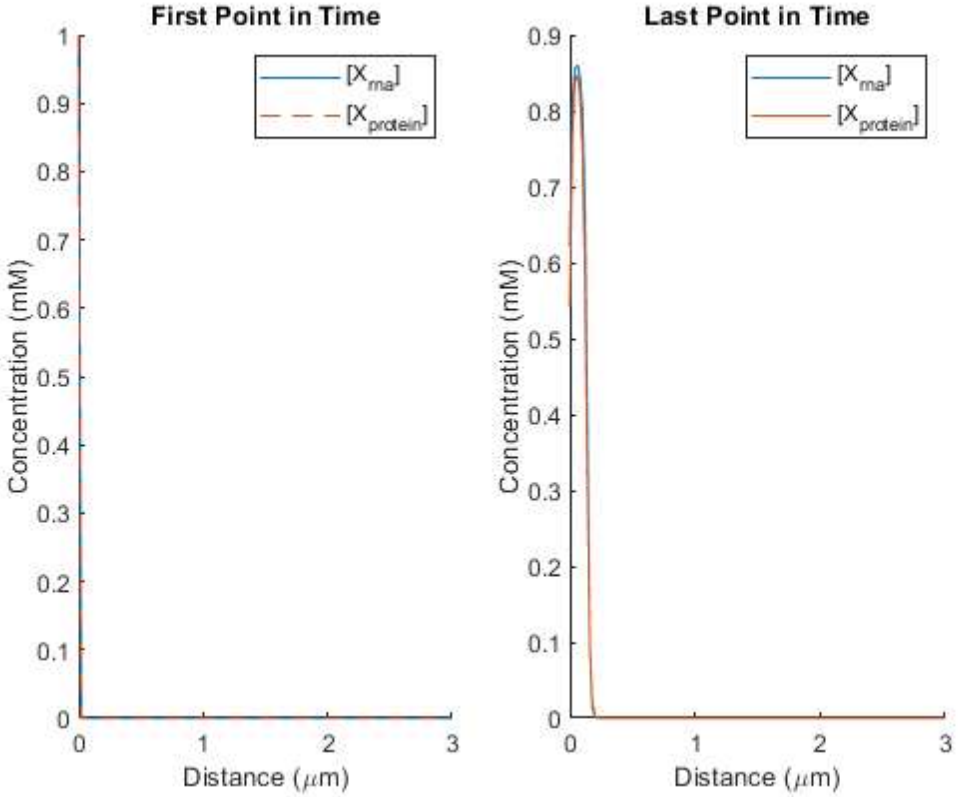
```
clear
close all
```

```
% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 30; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 1.0;
Xrna(1,1) = 1.0;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) ); % rna synthesis
        v2 = Xr*Xrna(x,t); % rna degradation
        v3 = w*Xrna(x,t); % protein synthesis
        v4 = Xp*Xprot(x,t); % protein degradation

        % RNA Laplacian
        del2rna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2rna = del2rna + Xrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2rna = del2rna + 0 + Xrna(x-1, t);
        else
            del2rna = del2rna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2rna = del2rna/(deltaX^2); % Laplacian

        % RNA derivative
        drna_dt = v1 - v2 + Dr*del2rna;

        % RNA update
        Xrna(x,t+1) = Xrna(x,t) + (drna_dt*deltaT);

        % protein Laplacian
        del2prot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2prot = del2prot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2prot = del2prot + 0 + Xprot(x-1, t);
        else
```

```
            del2prot = del2prot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2prot = del2prot/(deltaX^2); % Laplacian

        % protein derivative
        dprot_dt = v3 - v4 + Dp*del2prot;

        % protein update
        Xprot(x,t+1) = Xprot(x,t) + (dprot_dt*deltaT);
    end
end

time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;

fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)))
plot(time, (Xprot(1,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna(2,:)))
plot(time, (Xprot(2,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Second Position in Space')
legend('[X_{rna}]', '[X_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(3,:)))
plot(time, (Xprot(3,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Third Position in Space')
legend('[X_{rna}]', '[X_{protein}]')
hold off

fig2 = figure(2);
subplot(1,2,1)
hold on
plot(space, (Xrna(:,1)))
plot(space, (Xprot(:,1)), '--')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]')
hold off

subplot(1,2,2)
```

```
hold on
plot(space, (Xrna(:,timesteps)))
plot(space, (Xprot(:,timesteps)))
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]')
hold off
```





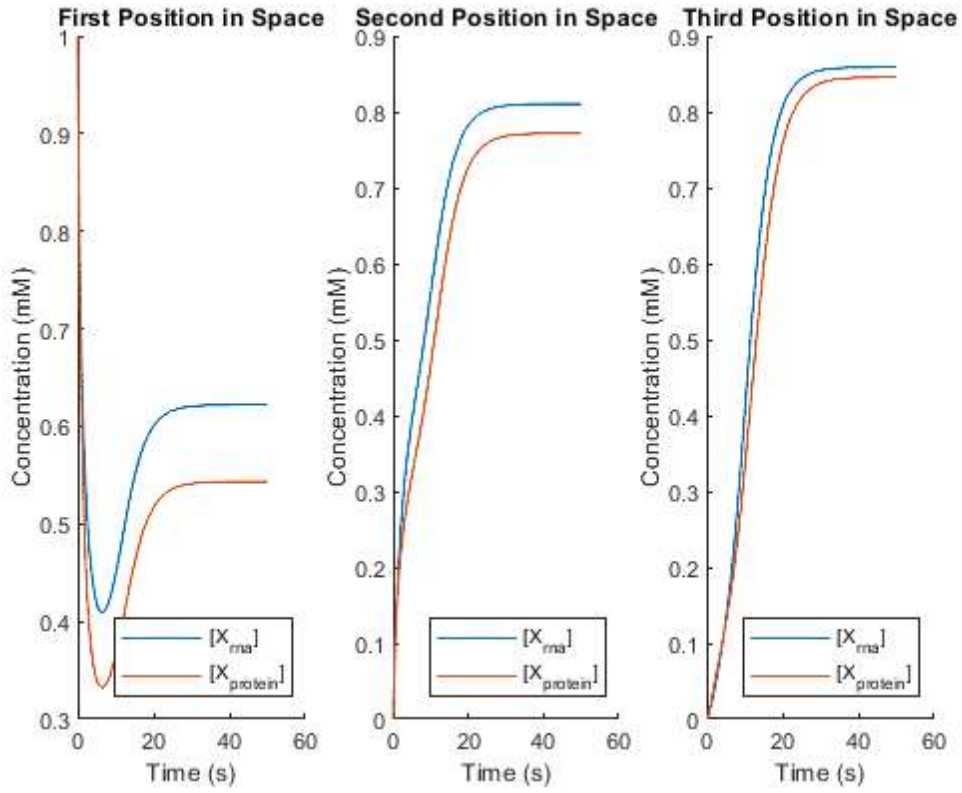
4. (a) total time = 50 s

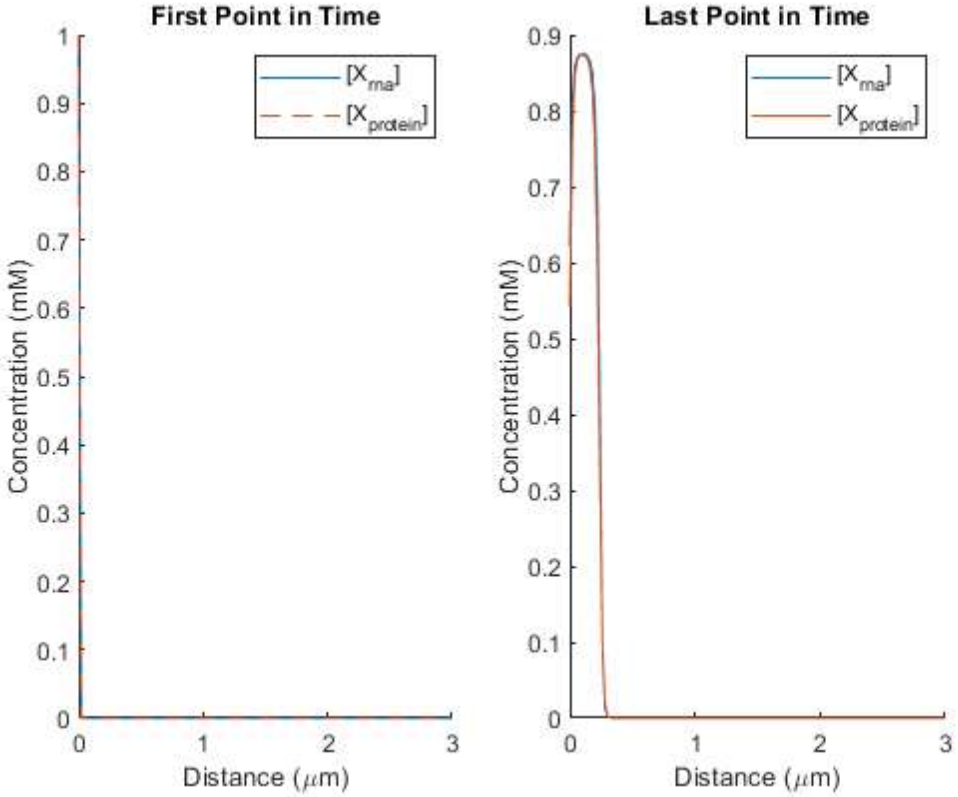
```
clear
close all
```

```
% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 50; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 1.0;
Xrna(1,1) = 1.0;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) ); % rna synthesis
        v2 = Xr*Xrna(x,t); % rna degradation
        v3 = w*Xrna(x,t); % protein synthesis
        v4 = Xp*Xprot(x,t); % protein degradation

        % RNA Laplacian
        del2rna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2rna = del2rna + Xrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2rna = del2rna + 0 + Xrna(x-1, t);
        else
            del2rna = del2rna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2rna = del2rna/(deltaX^2); % Laplacian

        % RNA derivative
        drna_dt = v1 - v2 + Dr*del2rna;

        % RNA update
        Xrna(x,t+1) = Xrna(x,t) + (drna_dt*deltaT);

        % protein Laplacian
        del2prot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2prot = del2prot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2prot = del2prot + 0 + Xprot(x-1, t);
        else
```

```
            del2prot = del2prot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2prot = del2prot/(deltaX^2); % Laplacian

        % protein derivative
        dprot_dt = v3 - v4 + Dp*del2prot;

        % protein update
        Xprot(x,t+1) = Xprot(x,t) + (dprot_dt*deltaT);
    end
end

time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;

fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)))
plot(time, (Xprot(1,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location','southeast')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna(2,:)))
plot(time, (Xprot(2,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Second Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location','southeast')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(3,:)))
plot(time, (Xprot(3,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Third Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location','southeast')
hold off

fig2 = figure(2);
subplot(1,2,1)
hold on
plot(space, (Xrna(:,1)))
plot(space, (Xprot(:,1)), '--')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]')
hold off

subplot(1,2,2)
```

```
hold on
plot(space, (Xrna(:,timesteps)))
plot(space, (Xprot(:,timesteps)))
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]')
hold off
```





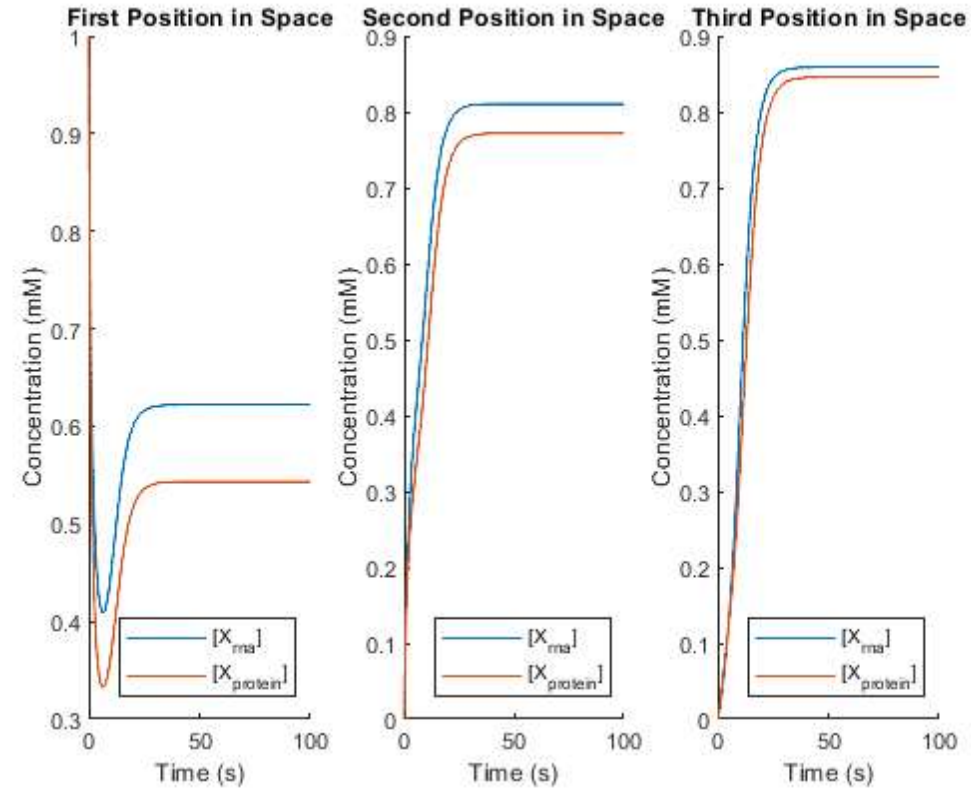
4. (b) total time = 100 s

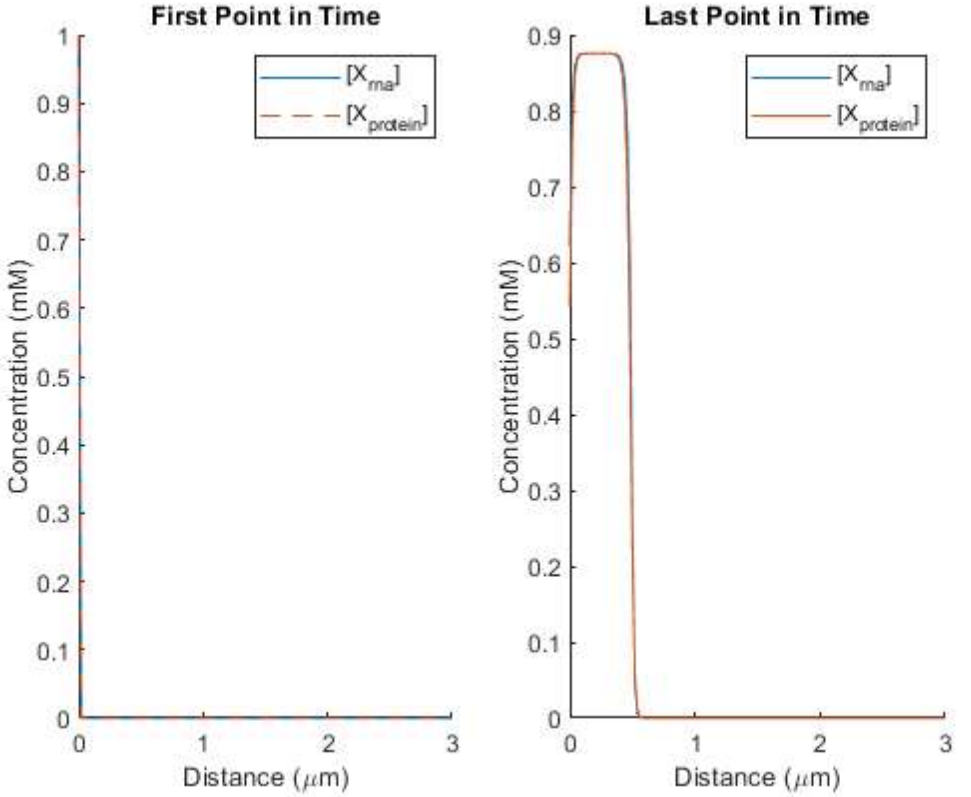
```
clear
close all
```

```
% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 100; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 1.0;
Xrna(1,1) = 1.0;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) ); % rna synthesis
        v2 = Xr*Xrna(x,t); % rna degradation
        v3 = w*Xrna(x,t); % protein synthesis
        v4 = Xp*Xprot(x,t); % protein degradation

        % RNA Laplacian
        del2rna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2rna = del2rna + Xrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2rna = del2rna + 0 + Xrna(x-1, t);
        else
            del2rna = del2rna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2rna = del2rna/(deltaX^2); % Laplacian

        % RNA derivative
        drna_dt = v1 - v2 + Dr*del2rna;

        % RNA update
        Xrna(x,t+1) = Xrna(x,t) + (drna_dt*deltaT);

        % protein Laplacian
        del2prot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2prot = del2prot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2prot = del2prot + 0 + Xprot(x-1, t);
        else
```

```
            del2prot = del2prot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2prot = del2prot/(deltaX^2); % Laplacian

        % protein derivative
        dprot_dt = v3 - v4 + Dp*del2prot;

        % protein update
        Xprot(x,t+1) = Xprot(x,t) + (dprot_dt*deltaT);
    end
end

time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;

fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)))
plot(time, (Xprot(1,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location','southeast')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna(2,:)))
plot(time, (Xprot(2,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Second Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location','southeast')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(3,:)))
plot(time, (Xprot(3,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Third Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location','southeast')
hold off

fig2 = figure(2);
subplot(1,2,1)
hold on
plot(space, (Xrna(:,1)))
plot(space, (Xprot(:,1)), '--')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]')
hold off

subplot(1,2,2)
```

```
hold on
plot(space, (Xrna(:,timesteps)))
plot(space, (Xprot(:,timesteps)))
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]')
hold off
```





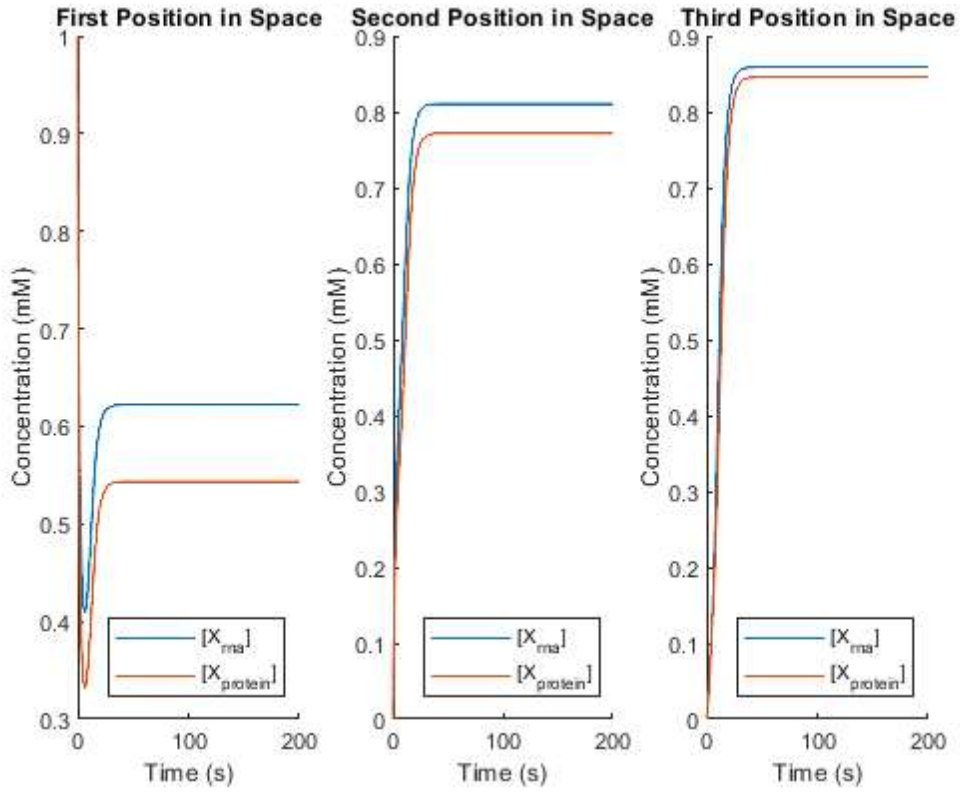
4. (c) total time = 200 s

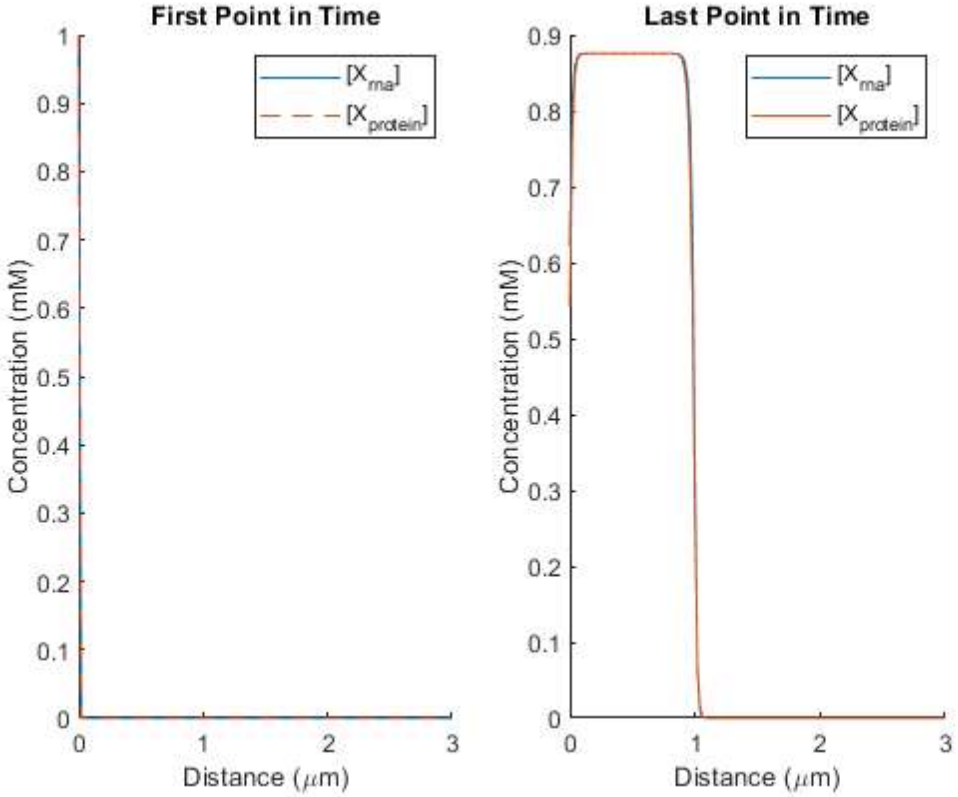
```
clear
close all
```

```
% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 200; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 1.0;
Xrna(1,1) = 1.0;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) ); % rna synthesis
        v2 = Xr*Xrna(x,t); % rna degradation
        v3 = w*Xrna(x,t); % protein synthesis
        v4 = Xp*Xprot(x,t); % protein degradation

        % RNA Laplacian
        del2rna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2rna = del2rna + Xrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2rna = del2rna + 0 + Xrna(x-1, t);
        else
            del2rna = del2rna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2rna = del2rna/(deltaX^2); % Laplacian

        % RNA derivative
        drna_dt = v1 - v2 + Dr*del2rna;

        % RNA update
        Xrna(x,t+1) = Xrna(x,t) + (drna_dt*deltaT);

        % protein Laplacian
        del2prot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2prot = del2prot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2prot = del2prot + 0 + Xprot(x-1, t);
        else
```

```
            del2prot = del2prot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2prot = del2prot/(deltaX^2); % Laplacian

        % protein derivative
        dprot_dt = v3 - v4 + Dp*del2prot;

        % protein update
        Xprot(x,t+1) = Xprot(x,t) + (dprot_dt*deltaT);
    end
end

time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;

fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)))
plot(time, (Xprot(1,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location','southeast')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna(2,:)))
plot(time, (Xprot(2,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Second Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location','southeast')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(3,:)))
plot(time, (Xprot(3,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Third Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location','southeast')
hold off

fig2 = figure(2);
subplot(1,2,1)
hold on
plot(space, (Xrna(:,1)))
plot(space, (Xprot(:,1)), '--')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]')
hold off

subplot(1,2,2)
```

```
hold on
plot(space, (Xrna(:,timesteps)))
plot(space, (Xprot(:,timesteps)))
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]')
hold off
```





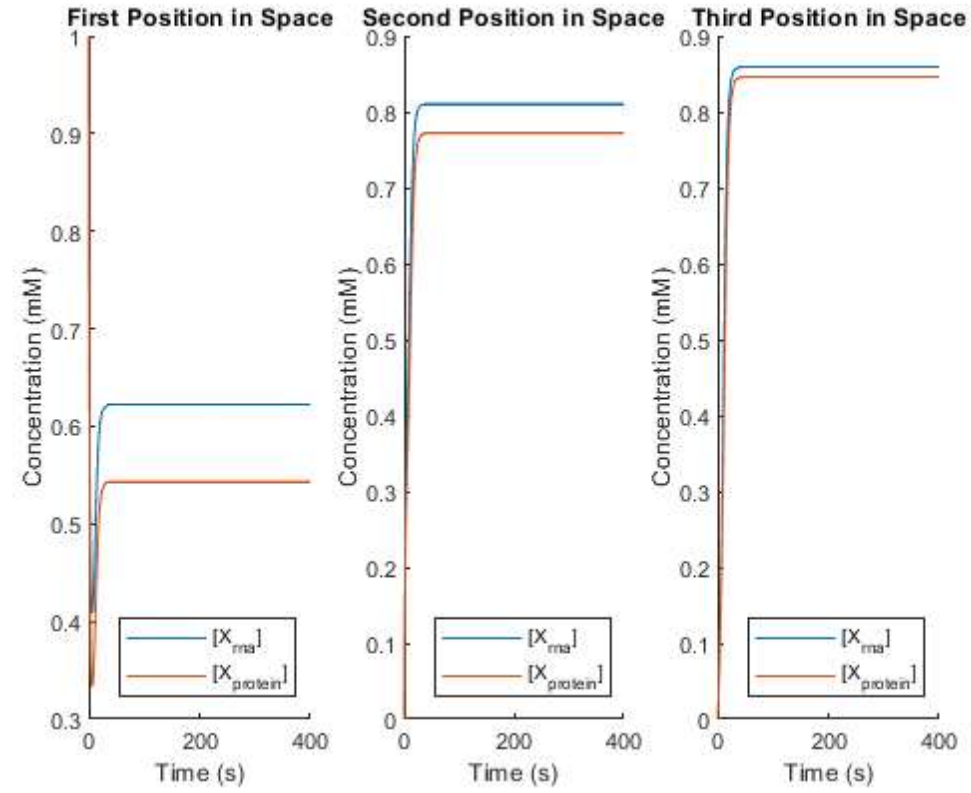
4. (d) total time = 400 s

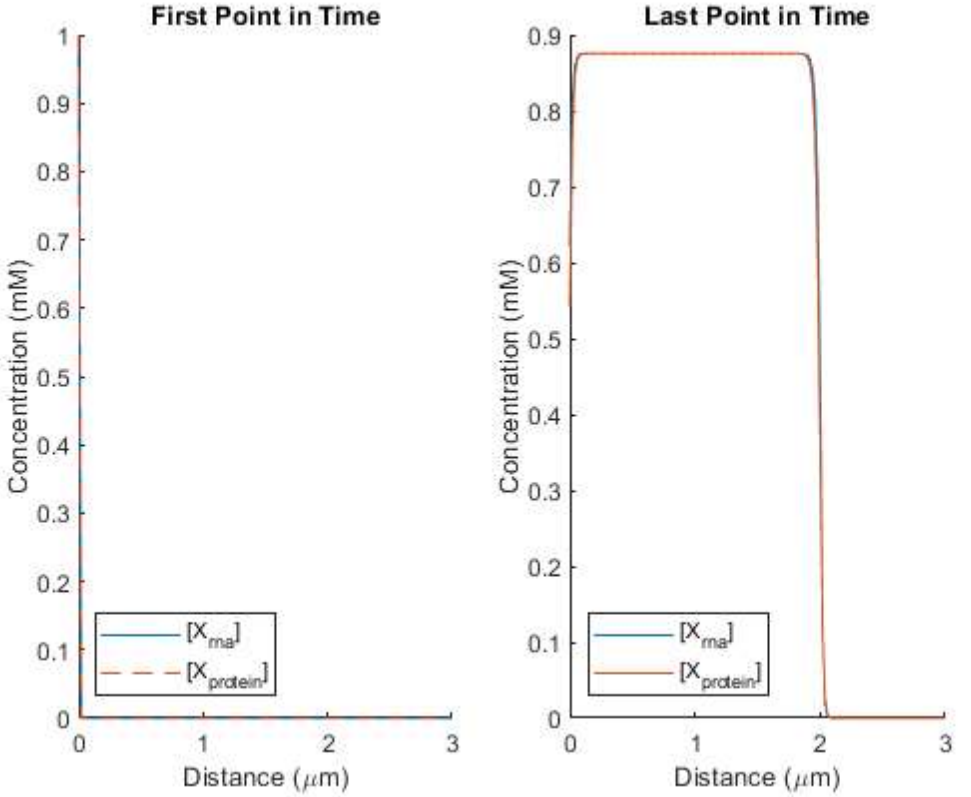
```
clear
close all
```

```
% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 400; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 1.0;
Xrna(1,1) = 1.0;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) ); % rna synthesis
        v2 = Xr*Xrna(x,t); % rna degradation
        v3 = w*Xrna(x,t); % protein synthesis
        v4 = Xp*Xprot(x,t); % protein degradation

        % RNA Laplacian
        del2rna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2rna = del2rna + Xrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2rna = del2rna + 0 + Xrna(x-1, t);
        else
            del2rna = del2rna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2rna = del2rna/(deltaX^2); % Laplacian

        % RNA derivative
        drna_dt = v1 - v2 + Dr*del2rna;

        % RNA update
        Xrna(x,t+1) = Xrna(x,t) + (drna_dt*deltaT);

        % protein Laplacian
        del2prot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2prot = del2prot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2prot = del2prot + 0 + Xprot(x-1, t);
        else
```

```
            del2prot = del2prot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2prot = del2prot/(deltaX^2); % Laplacian

        % protein derivative
        dprot_dt = v3 - v4 + Dp*del2prot;

        % protein update
        Xprot(x,t+1) = Xprot(x,t) + (dprot_dt*deltaT);
    end
end

time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;

fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)))
plot(time, (Xprot(1,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location','southeast')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna(2,:)))
plot(time, (Xprot(2,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Second Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location','southeast')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(3,:)))
plot(time, (Xprot(3,:)))
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Third Position in Space')
legend('[X_{rna}]', '[X_{protein}]', 'Location','southeast')
hold off

fig2 = figure(2);
subplot(1,2,1)
hold on
plot(space, (Xrna(:,1)))
plot(space, (Xprot(:,1)), '--')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]', 'Location','southwest')
hold off

subplot(1,2,2)
```

```
hold on
plot(space, (Xrna(:,timesteps)))
plot(space, (Xprot(:,timesteps)))
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]', 'Location','southwest')
hold off
```





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Part B: A diffusing pair of mutually-inhibiting genes

Contents

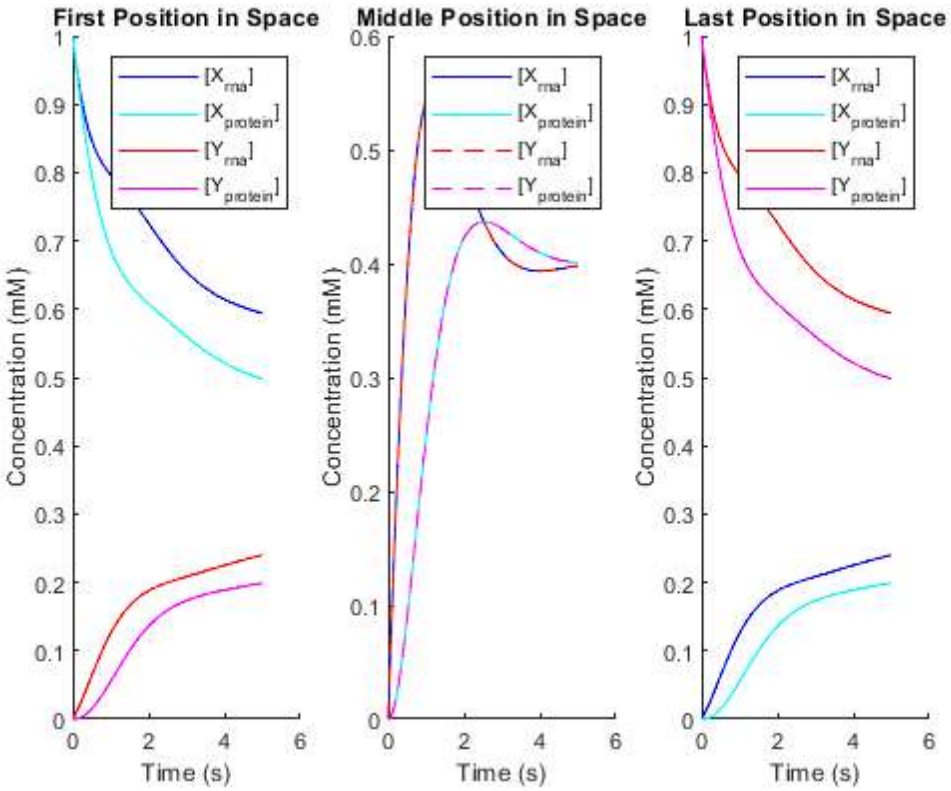
1-2. first position: initial X = 1mM. last position: initial Y = 1mM

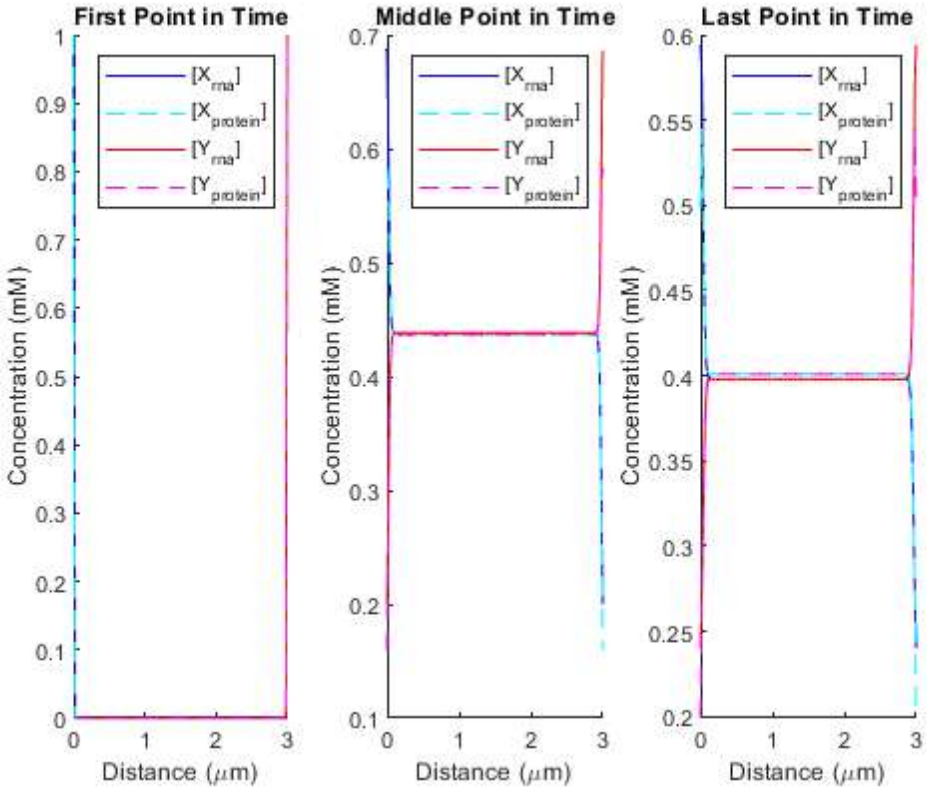
```
clear
close all

% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 5; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Yprot = (zeros(spacesteps, timesteps));
Yrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 1.0;
Xrna(1,1) = 1.0;
Yprot(spacesteps,1) = 1.0;
Yrna(spacesteps,1) = 1.0;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*(1-( (Yprot(x,t)^2) / ((K^2)+(Yprot(x,t)^2)) )); % X rna synthesis
```

```
v2 = Xr*Xrna(x,t); % X rna degradation
v3 = w*Xrna(x,t); % X protein synthesis
v4 = Xp*Xprot(x,t); % X protein degradation
v5 = u*(1-( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) )); % Y rna synthesis
v6 = Xr*Yrna(x,t); % Y rna degradation %document shows Xr*Xrna - typo?
v7 = w*Yrna(x,t); % Y protein synthesis
v8 = Xp*Yprot(x,t); % Y protein degradation

% X RNA Laplacian
del2Xrna = -2*Xrna(x,t); %reset value
if (x==1)
    del2Xrna = del2Xrna + Xrna(x+1,t) + 0;
elseif (x==spacesteps)
    del2Xrna = del2Xrna + 0 + Xrna(x-1, t);
else
    del2Xrna = del2Xrna + Xrna(x+1,t) + Xrna(x-1,t);
end
del2Xrna = del2Xrna/(deltaX^2); % Laplacian

% X RNA derivative
dXrna_dt = v1 - v2 + Dr*del2Xrna;

% X RNA update
Xrna(x,t+1) = Xrna(x,t) + (dXrna_dt*deltaT);

% X protein Laplacian
del2Xprot = -2*Xprot(x,t); %reset value
if (x==1)
    del2Xprot = del2Xprot + Xprot(x+1,t) + 0;
elseif (x==spacesteps)
    del2Xprot = del2Xprot + 0 + Xprot(x-1, t);
else
    del2Xprot = del2Xprot + Xprot(x+1,t) + Xprot(x-1,t);
end
del2Xprot = del2Xprot/(deltaX^2); % Laplacian

% X protein derivative
dXprot_dt = v3 - v4 + Dp*del2Xprot;

% X protein update
Xprot(x,t+1) = Xprot(x,t) + (dXprot_dt*deltaT);



% Y RNA Laplacian
del2Yrna = -2*Yrna(x,t); %reset value
if (x==1)
    del2Yrna = del2Yrna + Yrna(x+1,t) + 0;
elseif (x==spacesteps)
    del2Yrna = del2Yrna + 0 + Yrna(x-1, t);
else
    del2Yrna = del2Yrna + Yrna(x+1,t) + Yrna(x-1,t);
end
del2Yrna = del2Yrna/(deltaX^2); % Laplacian

% Y RNA derivative
dYrna_dt = v5 - v6 + Dr*del2Yrna;
```

```
        % Y RNA update
        Yrna(x,t+1) = Yrna(x,t) + (dYrna_dt*deltaT);

        % Y protein Laplacian
        del2Yprot = -2*Yprot(x,t); %reset value
        if (x==1)
            del2Yprot = del2Yprot + Yprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yprot = del2Yprot + 0 + Yprot(x-1, t);
        else
            del2Yprot = del2Yprot + Yprot(x+1,t) + Yprot(x-1,t);
        end
        del2Yprot = del2Yprot/(deltaX^2); % Laplacian

        % Y protein derivative
        dYprot_dt = v7 - v8 + Dp*del2Yprot;

        % Y protein update
        Yprot(x,t+1) = Yprot(x,t) + (dYprot_dt*deltaT);
    end
end

time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;

fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)), 'b')
plot(time, (Xprot(1,:)), 'c')
plot(time, (Yrna(1,:)), 'r')
plot(time, (Yprot(1,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna((ceil(spacesteps/2)),:)), 'b')
plot(time, (Xprot((ceil(spacesteps/2)),:)), 'c')
plot(time, (Yrna((ceil(spacesteps/2)),:)), '--r')
plot(time, (Yprot((ceil(spacesteps/2)),:)), '--m')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Middle Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(spacesteps,:)), 'b')
plot(time, (Xprot(spacesteps,:)), 'c')
plot(time, (Yrna(spacesteps,:)), 'r')
plot(time, (Yprot(spacesteps,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
```

```
title('Last Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

fig2 = figure(2);
subplot(1,3,1)
hold on
plot(space, (Xrna(:,1)),'b')
plot(space, (Xprot(:,1)), '--c')
plot(space, (Yrna(:,1)), 'r')
plot(space, (Yprot(:,1)), '--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(space, (Xrna(:,((timesteps/2)+1))), 'b')
plot(space, (Xprot(:,((timesteps/2)+1))),'--c')
plot(space, (Yrna(:,((timesteps/2)+1))),'r')
plot(space, (Yprot(:,((timesteps/2)+1))),'--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Middle Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(space, (Xrna(:,timesteps)),'b')
plot(space, (Xprot(:,timesteps)),'--c')
plot(space, (Yrna(:,timesteps)),'r')
plot(space, (Yprot(:,timesteps)),'--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off
```





3. (a) total time = 50 s

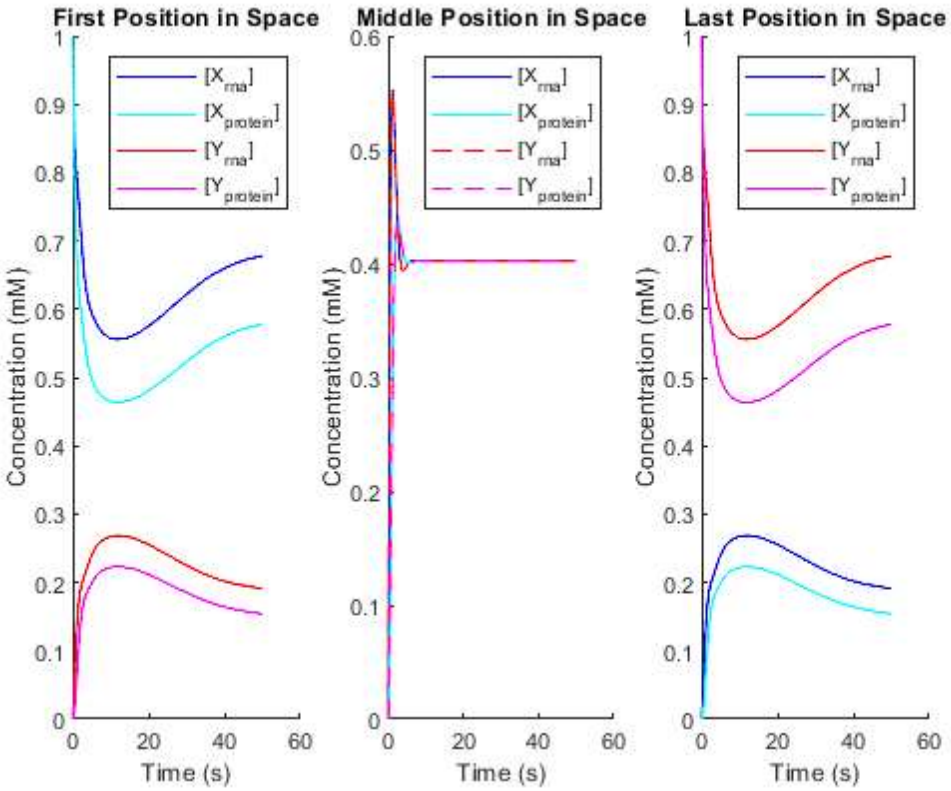
```
clear
close all
```

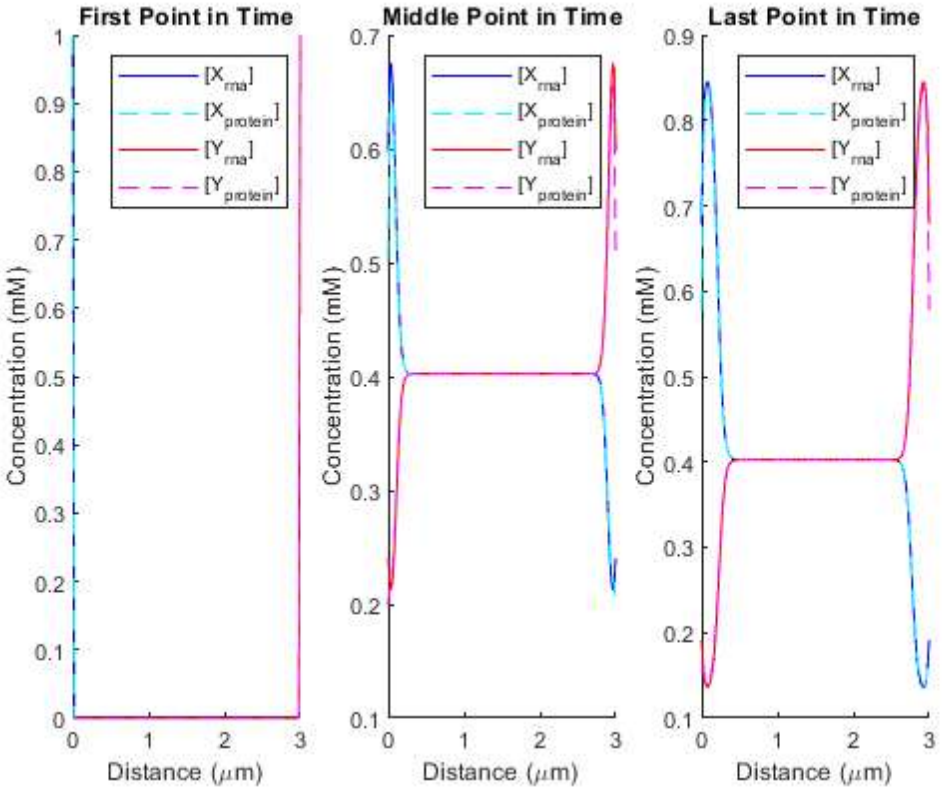
```
% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 50; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Yprot = (zeros(spacesteps, timesteps));
Yrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 1.0;
Xrna(1,1) = 1.0;
Yprot(spacesteps,1) = 1.0;
Yrna(spacesteps,1) = 1.0;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*(1-( (Yprot(x,t)^2) / ((K^2)+(Yprot(x,t)^2)) )); % X rna synthesis
        v2 = Xr*Xrna(x,t); % X rna degradation
        v3 = w*Xrna(x,t); % X protein synthesis
        v4 = Xp*Xprot(x,t); % X protein degradation
        v5 = u*(1-( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) )); % Y rna synthesis
        v6 = Xr*Yrna(x,t); % Y rna degradation %document shows Xr*Xrna - typo?
        v7 = w*Yrna(x,t); % Y protein synthesis
        v8 = Xp*Yprot(x,t); % Y protein degradation

        % X RNA Laplacian
        del2Xrna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2Xrna = del2Xrna + Xrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xrna = del2Xrna + 0 + Xrna(x-1, t);
        else
            del2Xrna = del2Xrna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2Xrna = del2Xrna/(deltaX^2); % Laplacian

        % X RNA derivative
        dXrna_dt = v1 - v2 + Dr*del2Xrna;

        % X RNA update
        Xrna(x,t+1) = Xrna(x,t) + (dXrna_dt*deltaT);
```

```
        % X protein Laplacian
        del2Xprot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2Xprot = del2Xprot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xprot = del2Xprot + 0 + Xprot(x-1, t);
        else
            del2Xprot = del2Xprot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2Xprot = del2Xprot/(deltaX^2); % Laplacian

        % X protein derivative
        dXprot_dt = v3 - v4 + Dp*del2Xprot;

        % X protein update
        Xprot(x,t+1) = Xprot(x,t) + (dXprot_dt*deltaT);



        % Y RNA Laplacian
        del2Yrna = -2*Yrna(x,t); %reset value
        if (x==1)
            del2Yrna = del2Yrna + Yrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yrna = del2Yrna + 0 + Yrna(x-1, t);
        else
            del2Yrna = del2Yrna + Yrna(x+1,t) + Yrna(x-1,t);
        end
        del2Yrna = del2Yrna/(deltaX^2); % Laplacian

        % Y RNA derivative
        dYrna_dt = v5 - v6 + Dr*del2Yrna;

        % Y RNA update
        Yrna(x,t+1) = Yrna(x,t) + (dYrna_dt*deltaT);

        % Y protein Laplacian
        del2Yprot = -2*Yprot(x,t); %reset value
        if (x==1)
            del2Yprot = del2Yprot + Yprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yprot = del2Yprot + 0 + Yprot(x-1, t);
        else
            del2Yprot = del2Yprot + Yprot(x+1,t) + Yprot(x-1,t);
        end
        del2Yprot = del2Yprot/(deltaX^2); % Laplacian

        % Y protein derivative
        dYprot_dt = v7 - v8 + Dp*del2Yprot;

        % Y protein update
        Yprot(x,t+1) = Yprot(x,t) + (dYprot_dt*deltaT);
    end
end

time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;
```

```
fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)), 'b')
plot(time, (Xprot(1,:)), 'c')
plot(time, (Yrna(1,:)), 'r')
plot(time, (Yprot(1,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna((ceil(spacesteps/2)),:)), 'b')
plot(time, (Xprot((ceil(spacesteps/2)),:)), 'c')
plot(time, (Yrna((ceil(spacesteps/2)),:)), '--r')
plot(time, (Yprot((ceil(spacesteps/2)),:)), '--m')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Middle Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(spacesteps,:)), 'b')
plot(time, (Xprot(spacesteps,:)), 'c')
plot(time, (Yrna(spacesteps,:)), 'r')
plot(time, (Yprot(spacesteps,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Last Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

fig2 = figure(2);
subplot(1,3,1)
hold on
plot(space, (Xrna(:,1)),'b')
plot(space, (Xprot(:,1)), '--c')
plot(space, (Yrna(:,1)), 'r')
plot(space, (Yprot(:,1)), '--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(space, (Xrna(:,((timesteps/2)+1))), 'b')
plot(space, (Xprot(:,((timesteps/2)+1))),'--c')
plot(space, (Yrna(:,((timesteps/2)+1))),'r')
plot(space, (Yprot(:,((timesteps/2)+1))),'--m')
xlabel('Distance (\mum)')
```

```
ylabel('Concentration (mM)')
title('Middle Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(space, (Xrna(:,timesteps)),'b')
plot(space, (Xprot(:,timesteps)),'--c')
plot(space, (Yrna(:,timesteps)),'r')
plot(space, (Yprot(:,timesteps)),'--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off
```





3. (b) total time = 100 s

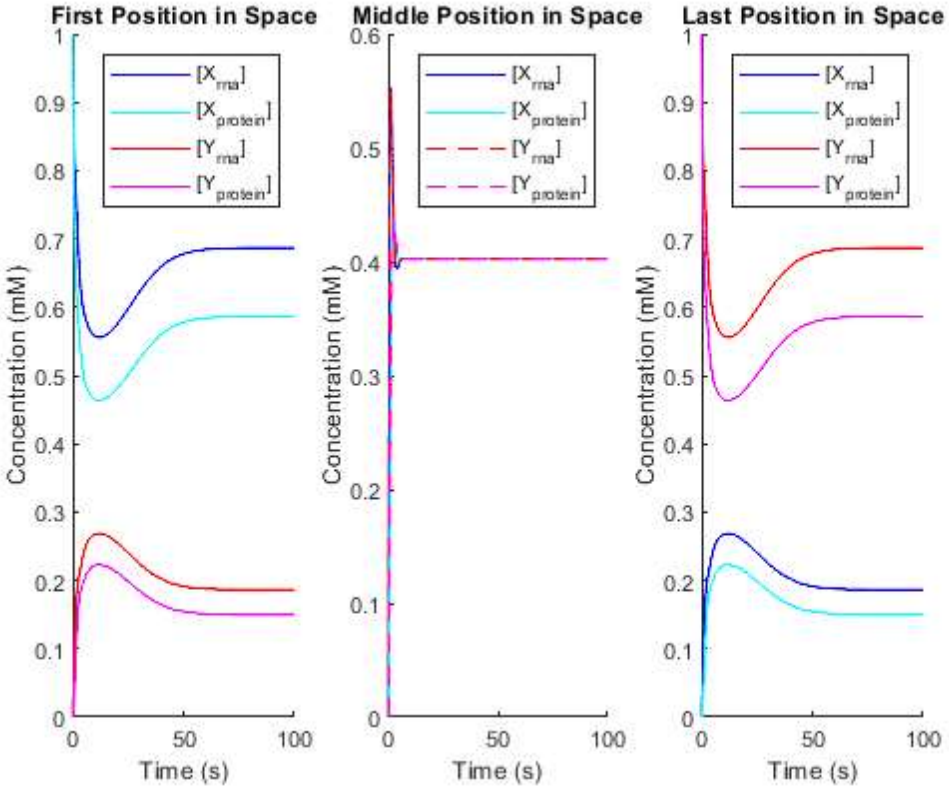
```
clear
close all
```

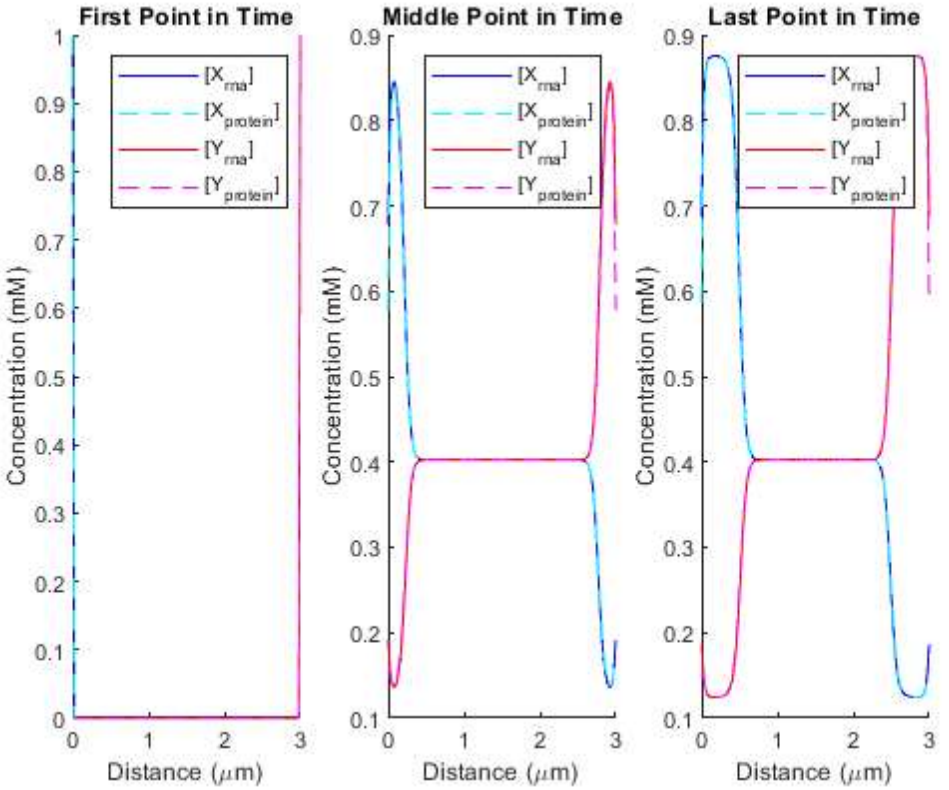
```
% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 100; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Yprot = (zeros(spacesteps, timesteps));
Yrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 1.0;
Xrna(1,1) = 1.0;
Yprot(spacesteps,1) = 1.0;
Yrna(spacesteps,1) = 1.0;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*(1-( (Yprot(x,t)^2) / ((K^2)+(Yprot(x,t)^2)) )); % X rna synthesis
        v2 = Xr*Xrna(x,t); % X rna degradation
        v3 = w*Xrna(x,t); % X protein synthesis
        v4 = Xp*Xprot(x,t); % X protein degradation
        v5 = u*(1-( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) )); % Y rna synthesis
        v6 = Xr*Yrna(x,t); % Y rna degradation %document shows Xr*Xrna - typo?
        v7 = w*Yrna(x,t); % Y protein synthesis
        v8 = Xp*Yprot(x,t); % Y protein degradation

        % X RNA Laplacian
        del2Xrna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2Xrna = del2Xrna + Xrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xrna = del2Xrna + 0 + Xrna(x-1, t);
        else
            del2Xrna = del2Xrna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2Xrna = del2Xrna/(deltaX^2); % Laplacian

        % X RNA derivative
        dXrna_dt = v1 - v2 + Dr*del2Xrna;

        % X RNA update
        Xrna(x,t+1) = Xrna(x,t) + (dXrna_dt*deltaT);
```

```
        % X protein Laplacian
        del2Xprot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2Xprot = del2Xprot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xprot = del2Xprot + 0 + Xprot(x-1, t);
        else
            del2Xprot = del2Xprot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2Xprot = del2Xprot/(deltaX^2); % Laplacian

        % X protein derivative
        dXprot_dt = v3 - v4 + Dp*del2Xprot;

        % X protein update
        Xprot(x,t+1) = Xprot(x,t) + (dXprot_dt*deltaT);



        % Y RNA Laplacian
        del2Yrna = -2*Yrna(x,t); %reset value
        if (x==1)
            del2Yrna = del2Yrna + Yrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yrna = del2Yrna + 0 + Yrna(x-1, t);
        else
            del2Yrna = del2Yrna + Yrna(x+1,t) + Yrna(x-1,t);
        end
        del2Yrna = del2Yrna/(deltaX^2); % Laplacian

        % Y RNA derivative
        dYrna_dt = v5 - v6 + Dr*del2Yrna;

        % Y RNA update
        Yrna(x,t+1) = Yrna(x,t) + (dYrna_dt*deltaT);

        % Y protein Laplacian
        del2Yprot = -2*Yprot(x,t); %reset value
        if (x==1)
            del2Yprot = del2Yprot + Yprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yprot = del2Yprot + 0 + Yprot(x-1, t);
        else
            del2Yprot = del2Yprot + Yprot(x+1,t) + Yprot(x-1,t);
        end
        del2Yprot = del2Yprot/(deltaX^2); % Laplacian

        % Y protein derivative
        dYprot_dt = v7 - v8 + Dp*del2Yprot;

        % Y protein update
        Yprot(x,t+1) = Yprot(x,t) + (dYprot_dt*deltaT);
    end
end

time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;
```

```
fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)), 'b')
plot(time, (Xprot(1,:)), 'c')
plot(time, (Yrna(1,:)), 'r')
plot(time, (Yprot(1,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna((ceil(spacesteps/2)),:)), 'b')
plot(time, (Xprot((ceil(spacesteps/2)),:)), 'c')
plot(time, (Yrna((ceil(spacesteps/2)),:)), '--r')
plot(time, (Yprot((ceil(spacesteps/2)),:)), '--m')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Middle Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(spacesteps,:)), 'b')
plot(time, (Xprot(spacesteps,:)), 'c')
plot(time, (Yrna(spacesteps,:)), 'r')
plot(time, (Yprot(spacesteps,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Last Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

fig2 = figure(2);
subplot(1,3,1)
hold on
plot(space, (Xrna(:,1)),'b')
plot(space, (Xprot(:,1)), '--c')
plot(space, (Yrna(:,1)), 'r')
plot(space, (Yprot(:,1)), '--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(space, (Xrna(:,((timesteps/2)+1))), 'b')
plot(space, (Xprot(:,((timesteps/2)+1))),'--c')
plot(space, (Yrna(:,((timesteps/2)+1))),'r')
plot(space, (Yprot(:,((timesteps/2)+1))),'--m')
xlabel('Distance (\mum)')
```

```
ylabel('Concentration (mM)')
title('Middle Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(space, (Xrna(:,timesteps)),'b')
plot(space, (Xprot(:,timesteps)),'--c')
plot(space, (Yrna(:,timesteps)),'r')
plot(space, (Yprot(:,timesteps)),'--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off
```





3. (c) total time = 200 s

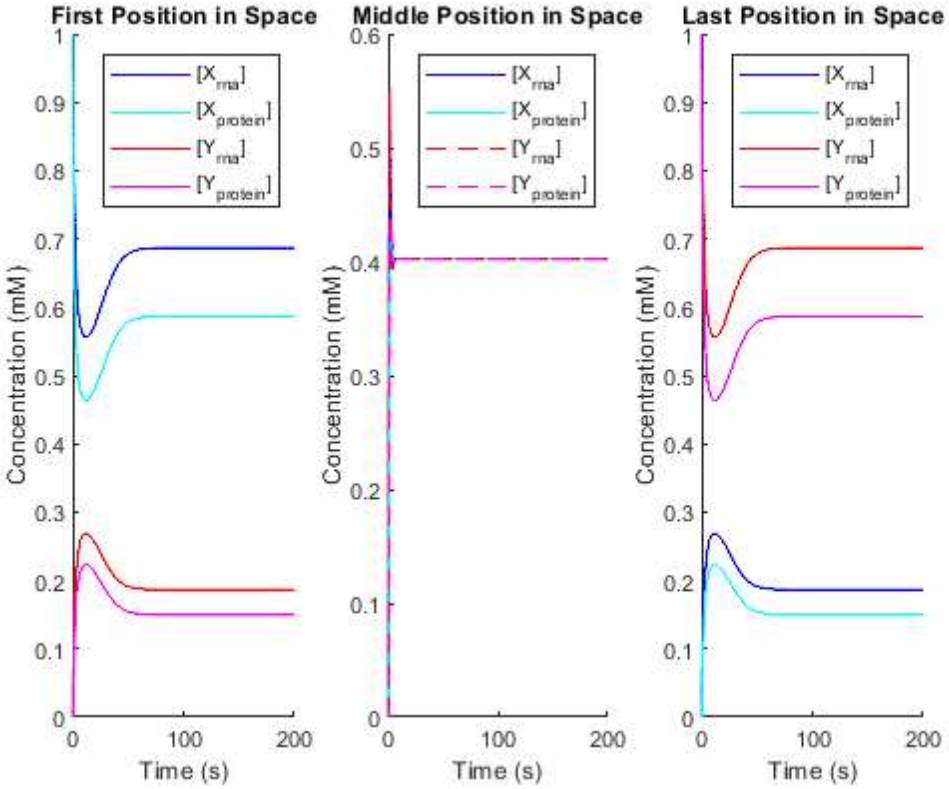
```
clear
close all
```

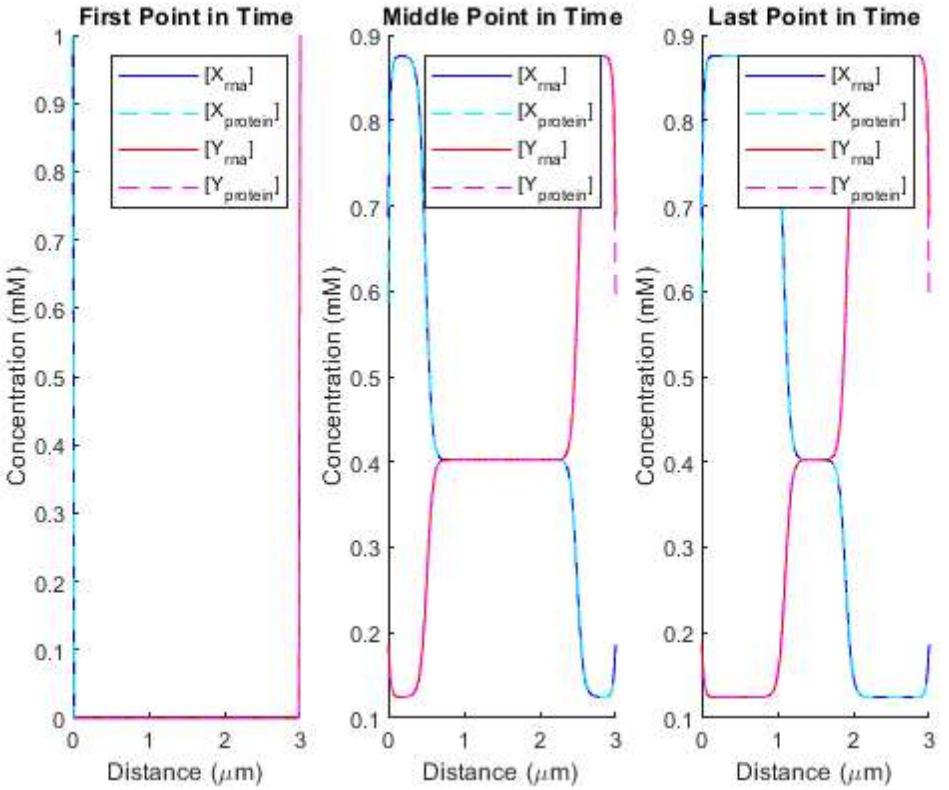
```
% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 200; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Yprot = (zeros(spacesteps, timesteps));
Yrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 1.0;
Xrna(1,1) = 1.0;
Yprot(spacesteps,1) = 1.0;
Yrna(spacesteps,1) = 1.0;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*(1-( (Yprot(x,t)^2) / ((K^2)+(Yprot(x,t)^2)) )); % X rna synthesis
        v2 = Xr*Xrna(x,t); % X rna degradation
        v3 = w*Xrna(x,t); % X protein synthesis
        v4 = Xp*Xprot(x,t); % X protein degradation
        v5 = u*(1-( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) )); % Y rna synthesis
        v6 = Xr*Yrna(x,t); % Y rna degradation %document shows Xr*Xrna - typo?
        v7 = w*Yrna(x,t); % Y protein synthesis
        v8 = Xp*Yprot(x,t); % Y protein degradation

        % X RNA Laplacian
        del2Xrna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2Xrna = del2Xrna + Xrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xrna = del2Xrna + 0 + Xrna(x-1, t);
        else
            del2Xrna = del2Xrna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2Xrna = del2Xrna/(deltaX^2); % Laplacian

        % X RNA derivative
        dXrna_dt = v1 - v2 + Dr*del2Xrna;

        % X RNA update
        Xrna(x,t+1) = Xrna(x,t) + (dXrna_dt*deltaT);
```

```
        % X protein Laplacian
        del2Xprot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2Xprot = del2Xprot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xprot = del2Xprot + 0 + Xprot(x-1, t);
        else
            del2Xprot = del2Xprot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2Xprot = del2Xprot/(deltaX^2); % Laplacian

        % X protein derivative
        dXprot_dt = v3 - v4 + Dp*del2Xprot;

        % X protein update
        Xprot(x,t+1) = Xprot(x,t) + (dXprot_dt*deltaT);



        % Y RNA Laplacian
        del2Yrna = -2*Yrna(x,t); %reset value
        if (x==1)
            del2Yrna = del2Yrna + Yrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yrna = del2Yrna + 0 + Yrna(x-1, t);
        else
            del2Yrna = del2Yrna + Yrna(x+1,t) + Yrna(x-1,t);
        end
        del2Yrna = del2Yrna/(deltaX^2); % Laplacian

        % Y RNA derivative
        dYrna_dt = v5 - v6 + Dr*del2Yrna;

        % Y RNA update
        Yrna(x,t+1) = Yrna(x,t) + (dYrna_dt*deltaT);

        % Y protein Laplacian
        del2Yprot = -2*Yprot(x,t); %reset value
        if (x==1)
            del2Yprot = del2Yprot + Yprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yprot = del2Yprot + 0 + Yprot(x-1, t);
        else
            del2Yprot = del2Yprot + Yprot(x+1,t) + Yprot(x-1,t);
        end
        del2Yprot = del2Yprot/(deltaX^2); % Laplacian

        % Y protein derivative
        dYprot_dt = v7 - v8 + Dp*del2Yprot;

        % Y protein update
        Yprot(x,t+1) = Yprot(x,t) + (dYprot_dt*deltaT);
    end
end

time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;
```

```
fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)), 'b')
plot(time, (Xprot(1,:)), 'c')
plot(time, (Yrna(1,:)), 'r')
plot(time, (Yprot(1,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna((ceil(spacesteps/2)),:)), 'b')
plot(time, (Xprot((ceil(spacesteps/2)),:)), 'c')
plot(time, (Yrna((ceil(spacesteps/2)),:)), '--r')
plot(time, (Yprot((ceil(spacesteps/2)),:)), '--m')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Middle Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(spacesteps,:)), 'b')
plot(time, (Xprot(spacesteps,:)), 'c')
plot(time, (Yrna(spacesteps,:)), 'r')
plot(time, (Yprot(spacesteps,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Last Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

fig2 = figure(2);
subplot(1,3,1)
hold on
plot(space, (Xrna(:,1)),'b')
plot(space, (Xprot(:,1)), '--c')
plot(space, (Yrna(:,1)), 'r')
plot(space, (Yprot(:,1)), '--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(space, (Xrna(:,((timesteps/2)+1))), 'b')
plot(space, (Xprot(:,((timesteps/2)+1))),'--c')
plot(space, (Yrna(:,((timesteps/2)+1))),'r')
plot(space, (Yprot(:,((timesteps/2)+1))),'--m')
xlabel('Distance (\mum)')
```

```
ylabel('Concentration (mM)')
title('Middle Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(space, (Xrna(:,timesteps)),'b')
plot(space, (Xprot(:,timesteps)),'--c')
plot(space, (Yrna(:,timesteps)),'r')
plot(space, (Yprot(:,timesteps)),'--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off
```





3. (d) total time = 400 s

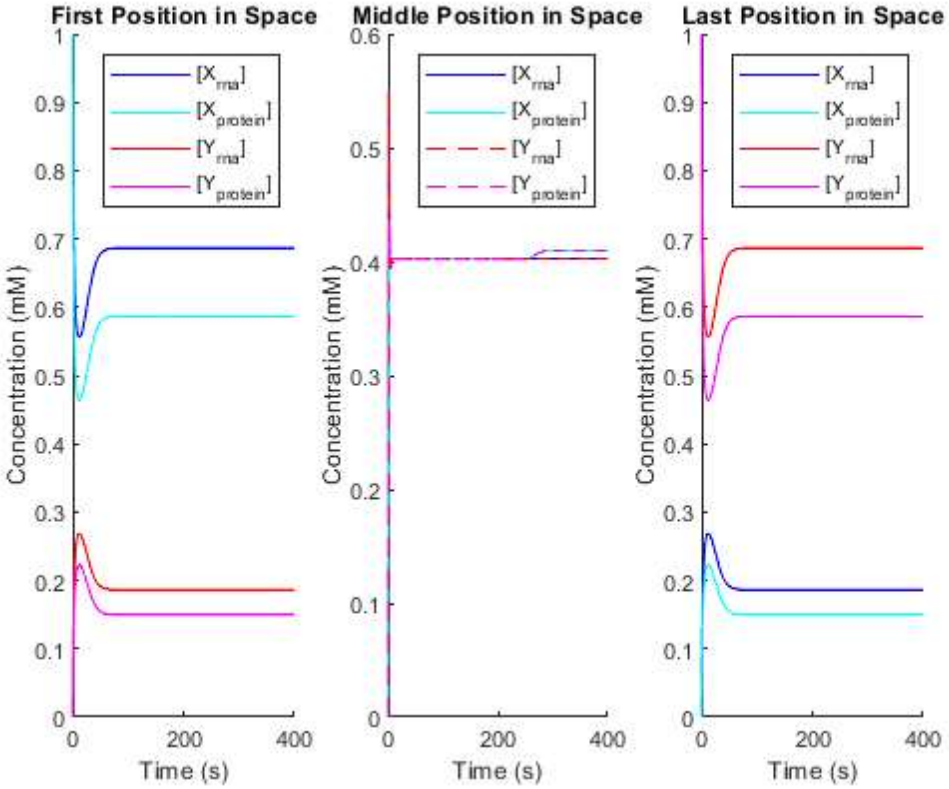
```
clear
close all
```

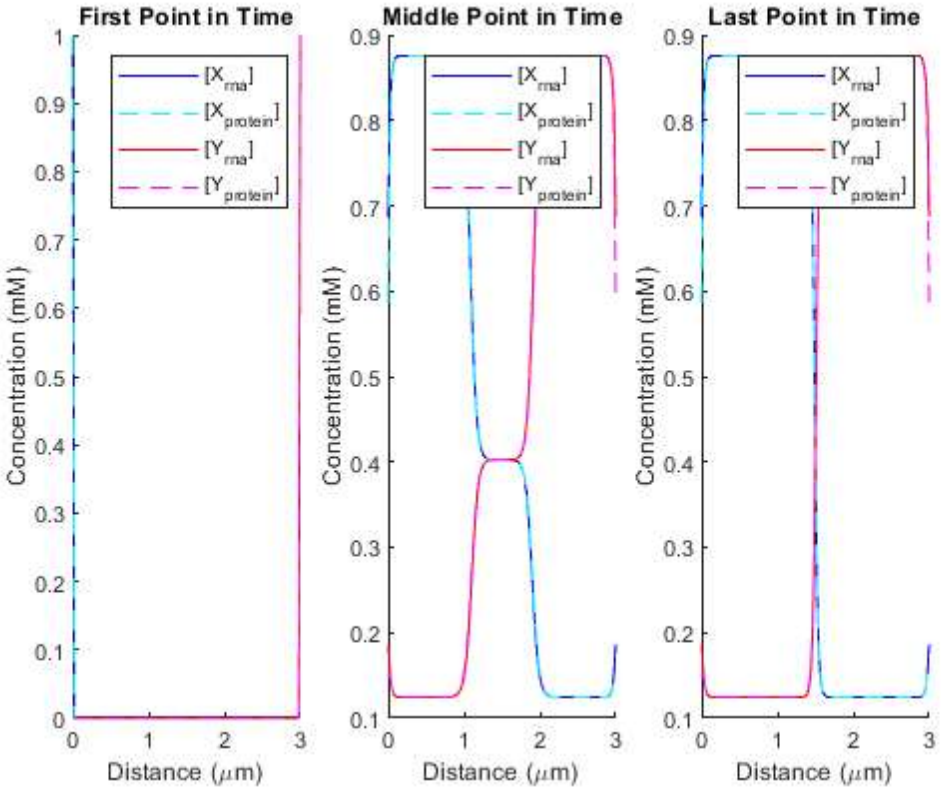
```
% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 400; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Yprot = (zeros(spacesteps, timesteps));
Yrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 1.0;
Xrna(1,1) = 1.0;
Yprot(spacesteps,1) = 1.0;
Yrna(spacesteps,1) = 1.0;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*(1-( (Yprot(x,t)^2) / ((K^2)+(Yprot(x,t)^2)) )); % X rna synthesis
        v2 = Xr*Xrna(x,t); % X rna degradation
        v3 = w*Xrna(x,t); % X protein synthesis
        v4 = Xp*Xprot(x,t); % X protein degradation
        v5 = u*(1-( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) )); % Y rna synthesis
        v6 = Xr*Yrna(x,t); % Y rna degradation %document shows Xr*Xrna - typo?
        v7 = w*Yrna(x,t); % Y protein synthesis
        v8 = Xp*Yprot(x,t); % Y protein degradation

        % X RNA Laplacian
        del2Xrna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2Xrna = del2Xrna + Xrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xrna = del2Xrna + 0 + Xrna(x-1, t);
        else
            del2Xrna = del2Xrna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2Xrna = del2Xrna/(deltaX^2); % Laplacian

        % X RNA derivative
        dXrna_dt = v1 - v2 + Dr*del2Xrna;

        % X RNA update
        Xrna(x,t+1) = Xrna(x,t) + (dXrna_dt*deltaT);
```

```
        % X protein Laplacian
        del2Xprot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2Xprot = del2Xprot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xprot = del2Xprot + 0 + Xprot(x-1, t);
        else
            del2Xprot = del2Xprot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2Xprot = del2Xprot/(deltaX^2); % Laplacian

        % X protein derivative
        dXprot_dt = v3 - v4 + Dp*del2Xprot;

        % X protein update
        Xprot(x,t+1) = Xprot(x,t) + (dXprot_dt*deltaT);



        % Y RNA Laplacian
        del2Yrna = -2*Yrna(x,t); %reset value
        if (x==1)
            del2Yrna = del2Yrna + Yrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yrna = del2Yrna + 0 + Yrna(x-1, t);
        else
            del2Yrna = del2Yrna + Yrna(x+1,t) + Yrna(x-1,t);
        end
        del2Yrna = del2Yrna/(deltaX^2); % Laplacian

        % Y RNA derivative
        dYrna_dt = v5 - v6 + Dr*del2Yrna;

        % Y RNA update
        Yrna(x,t+1) = Yrna(x,t) + (dYrna_dt*deltaT);

        % Y protein Laplacian
        del2Yprot = -2*Yprot(x,t); %reset value
        if (x==1)
            del2Yprot = del2Yprot + Yprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yprot = del2Yprot + 0 + Yprot(x-1, t);
        else
            del2Yprot = del2Yprot + Yprot(x+1,t) + Yprot(x-1,t);
        end
        del2Yprot = del2Yprot/(deltaX^2); % Laplacian

        % Y protein derivative
        dYprot_dt = v7 - v8 + Dp*del2Yprot;

        % Y protein update
        Yprot(x,t+1) = Yprot(x,t) + (dYprot_dt*deltaT);
    end
end

time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;
```

```
fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)), 'b')
plot(time, (Xprot(1,:)), 'c')
plot(time, (Yrna(1,:)), 'r')
plot(time, (Yprot(1,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna((ceil(spacesteps/2)),:)), 'b')
plot(time, (Xprot((ceil(spacesteps/2)),:)), 'c')
plot(time, (Yrna((ceil(spacesteps/2)),:)), '--r')
plot(time, (Yprot((ceil(spacesteps/2)),:)), '--m')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Middle Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(spacesteps,:)), 'b')
plot(time, (Xprot(spacesteps,:)), 'c')
plot(time, (Yrna(spacesteps,:)), 'r')
plot(time, (Yprot(spacesteps,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Last Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

fig2 = figure(2);
subplot(1,3,1)
hold on
plot(space, (Xrna(:,1)),'b')
plot(space, (Xprot(:,1)), '--c')
plot(space, (Yrna(:,1)), 'r')
plot(space, (Yprot(:,1)), '--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(space, (Xrna(:,((timesteps/2)+1))), 'b')
plot(space, (Xprot(:,((timesteps/2)+1))),'--c')
plot(space, (Yrna(:,((timesteps/2)+1))),'r')
plot(space, (Yprot(:,((timesteps/2)+1))),'--m')
xlabel('Distance (\mum)')
```

```
ylabel('Concentration (mM)')
title('Middle Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(space, (Xrna(:,timesteps)),'b')
plot(space, (Xprot(:,timesteps)),'--c')
plot(space, (Yrna(:,timesteps)),'r')
plot(space, (Yprot(:,timesteps)),'--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off
```





4. (a) total time = 50 s

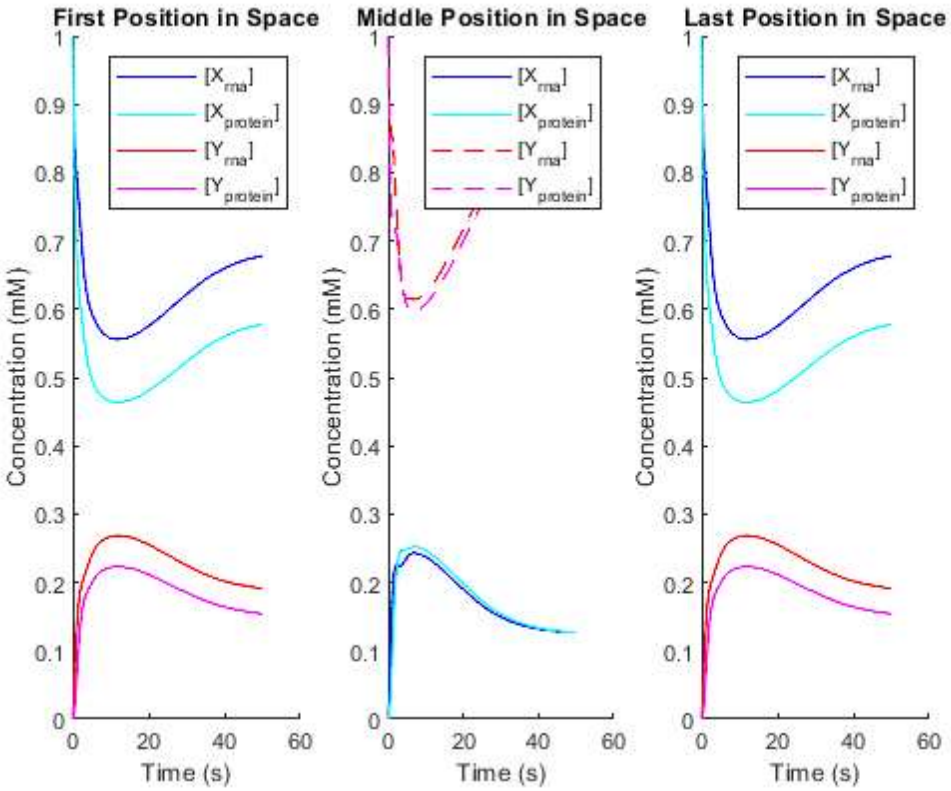
```
clear
close all
```

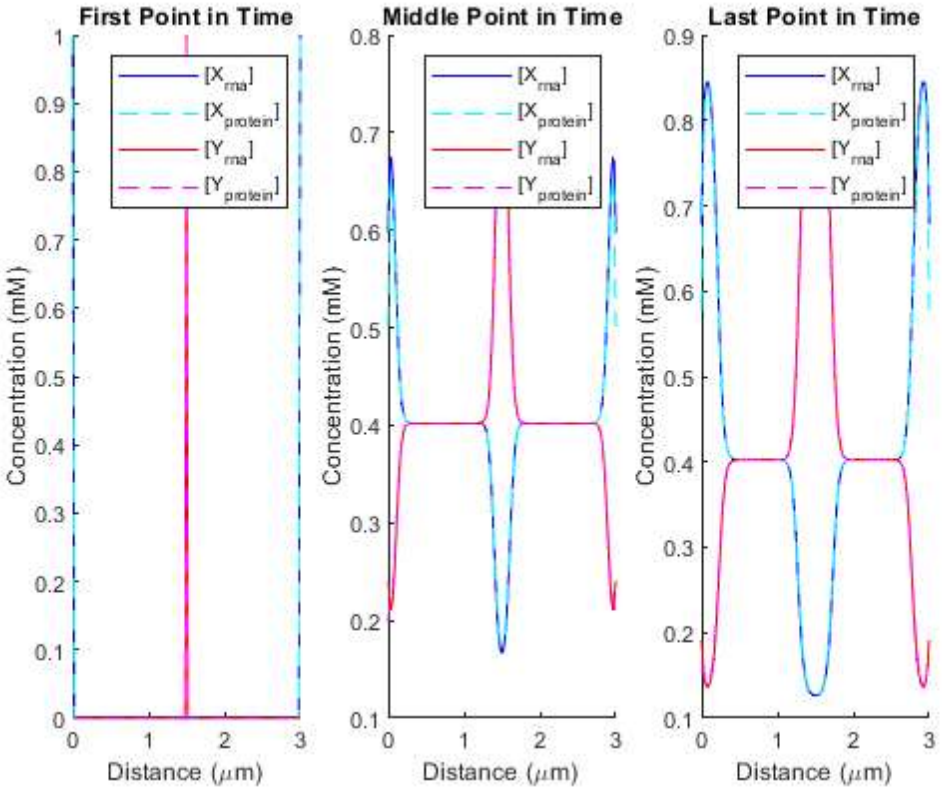
```
% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 50; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Yprot = (zeros(spacesteps, timesteps));
Yrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 1.0;
Xrna(1,1) = 1.0;
Xprot(spacesteps,1) = 1.0;
Xrna(spacesteps,1) = 1.0;
Yprot((ceil(spacesteps/2)),1) = 1.0;
Yrna((ceil(spacesteps/2)),1) = 1.0;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*(1-( (Yprot(x,t)^2) / ((K^2)+(Yprot(x,t)^2)) )); % X rna synthesis
        v2 = Xr*Xrna(x,t); % X rna degradation
        v3 = w*Xrna(x,t); % X protein synthesis
        v4 = Xp*Xprot(x,t); % X protein degradation
        v5 = u*(1-( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) )); % Y rna synthesis
        v6 = Xr*Yrna(x,t); % Y rna degradation %document shows Xr*Xrna - typo?
        v7 = w*Yrna(x,t); % Y protein synthesis
        v8 = Xp*Yprot(x,t); % Y protein degradation

        % X RNA Laplacian
        del2Xrna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2Xrna = del2Xrna + Xrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xrna = del2Xrna + 0 + Xrna(x-1, t);
        else
            del2Xrna = del2Xrna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2Xrna = del2Xrna/(deltaX^2); % Laplacian

        % X RNA derivative
        dXrna_dt = v1 - v2 + Dr*del2Xrna;
```

```
        % X RNA update
        Xrna(x,t+1) = Xrna(x,t) + (dXrna_dt*deltaT);

        % X protein Laplacian
        del2Xprot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2Xprot = del2Xprot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xprot = del2Xprot + 0 + Xprot(x-1, t);
        else
            del2Xprot = del2Xprot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2Xprot = del2Xprot/(deltaX^2); % Laplacian

        % X protein derivative
        dXprot_dt = v3 - v4 + Dp*del2Xprot;

        % X protein update
        Xprot(x,t+1) = Xprot(x,t) + (dXprot_dt*deltaT);



        % Y RNA Laplacian
        del2Yrna = -2*Yrna(x,t); %reset value
        if (x==1)
            del2Yrna = del2Yrna + Yrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yrna = del2Yrna + 0 + Yrna(x-1, t);
        else
            del2Yrna = del2Yrna + Yrna(x+1,t) + Yrna(x-1,t);
        end
        del2Yrna = del2Yrna/(deltaX^2); % Laplacian

        % Y RNA derivative
        dYrna_dt = v5 - v6 + Dr*del2Yrna;

        % Y RNA update
        Yrna(x,t+1) = Yrna(x,t) + (dYrna_dt*deltaT);

        % Y protein Laplacian
        del2Yprot = -2*Yprot(x,t); %reset value
        if (x==1)
            del2Yprot = del2Yprot + Yprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yprot = del2Yprot + 0 + Yprot(x-1, t);
        else
            del2Yprot = del2Yprot + Yprot(x+1,t) + Yprot(x-1,t);
        end
        del2Yprot = del2Yprot/(deltaX^2); % Laplacian

        % Y protein derivative
        dYprot_dt = v7 - v8 + Dp*del2Yprot;

        % Y protein update
        Yprot(x,t+1) = Yprot(x,t) + (dYprot_dt*deltaT);
    end
end
```

```
time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;

fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)), 'b')
plot(time, (Xprot(1,:)), 'c')
plot(time, (Yrna(1,:)), 'r')
plot(time, (Yprot(1,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna((ceil(spacesteps/2)),:)), 'b')
plot(time, (Xprot((ceil(spacesteps/2)),:)), 'c')
plot(time, (Yrna((ceil(spacesteps/2)),:)), '--r')
plot(time, (Yprot((ceil(spacesteps/2)),:)), '--m')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Middle Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(spacesteps,:)), 'b')
plot(time, (Xprot(spacesteps,:)), 'c')
plot(time, (Yrna(spacesteps,:)), 'r')
plot(time, (Yprot(spacesteps,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Last Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

fig2 = figure(2);
subplot(1,3,1)
hold on
plot(space, (Xrna(:,1)),'b')
plot(space, (Xprot(:,1)), '--c')
plot(space, (Yrna(:,1)), 'r')
plot(space, (Yprot(:,1)), '--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(space, (Xrna(:,((timesteps/2)+1))), 'b')
plot(space, (Xprot(:,((timesteps/2)+1))),'--c')
plot(space, (Yrna(:,((timesteps/2)+1))),'r')
```

```
plot(space, (Yprot(:,((timesteps/2)+1))),'--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Middle Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(space, (Xrna(:,timesteps)),'b')
plot(space, (Xprot(:,timesteps)),'--c')
plot(space, (Yrna(:,timesteps)),'r')
plot(space, (Yprot(:,timesteps)),'--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off
```





4. (b) total time = 100 s

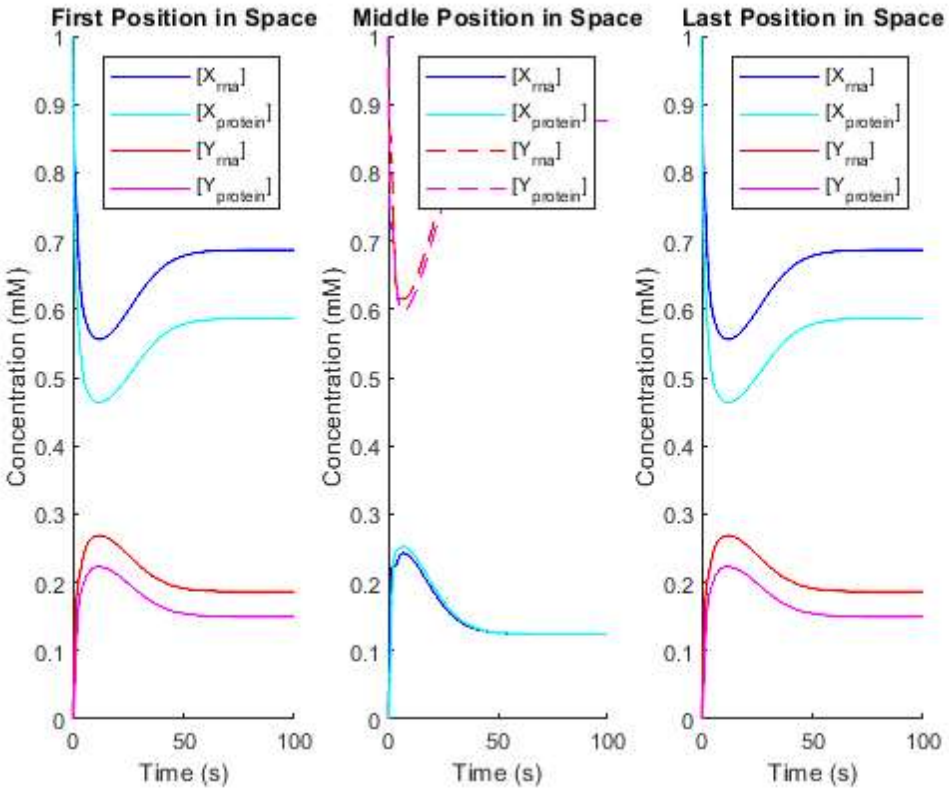
```
clear
close all
```

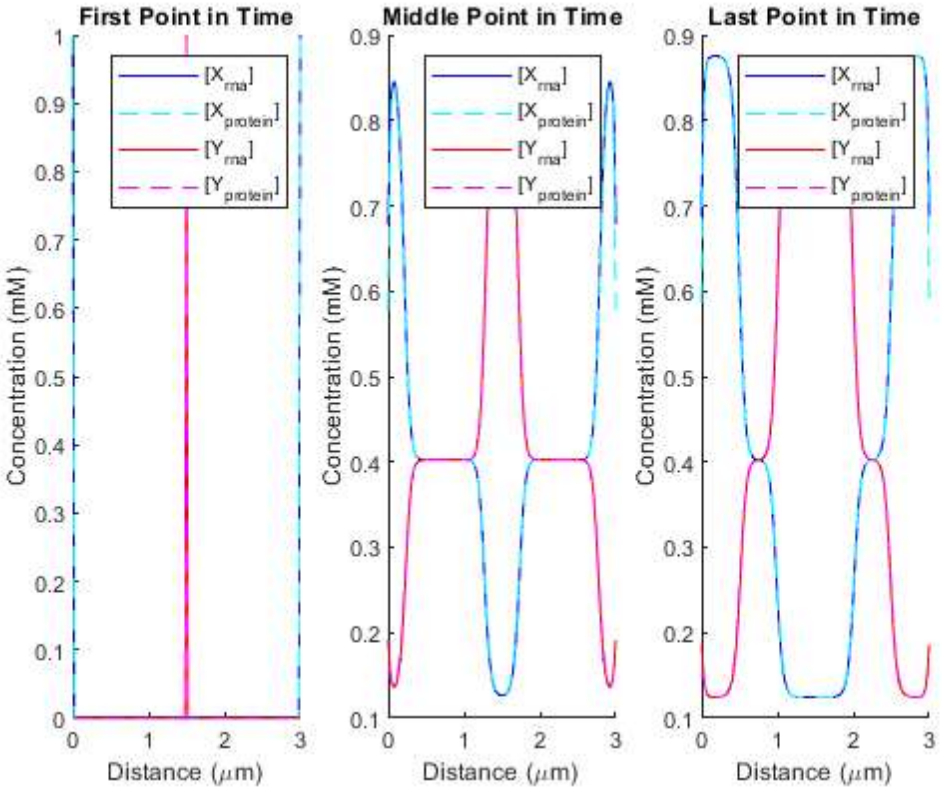
```
% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 100; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Yprot = (zeros(spacesteps, timesteps));
Yrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 1.0;
Xrna(1,1) = 1.0;
Xprot(spacesteps,1) = 1.0;
Xrna(spacesteps,1) = 1.0;
Yprot((ceil(spacesteps/2)),1) = 1.0;
Yrna((ceil(spacesteps/2)),1) = 1.0;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*(1-( (Yprot(x,t)^2) / ((K^2)+(Yprot(x,t)^2)) )); % X rna synthesis
        v2 = Xr*Xrna(x,t); % X rna degradation
        v3 = w*Xrna(x,t); % X protein synthesis
        v4 = Xp*Xprot(x,t); % X protein degradation
        v5 = u*(1-( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) )); % Y rna synthesis
        v6 = Xr*Yrna(x,t); % Y rna degradation %document shows Xr*Xrna - typo?
        v7 = w*Yrna(x,t); % Y protein synthesis
        v8 = Xp*Yprot(x,t); % Y protein degradation

        % X RNA Laplacian
        del2Xrna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2Xrna = del2Xrna + Xrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xrna = del2Xrna + 0 + Xrna(x-1, t);
        else
            del2Xrna = del2Xrna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2Xrna = del2Xrna/(deltaX^2); % Laplacian

        % X RNA derivative
        dXrna_dt = v1 - v2 + Dr*del2Xrna;
```

```
        % X RNA update
        Xrna(x,t+1) = Xrna(x,t) + (dXrna_dt*deltaT);

        % X protein Laplacian
        del2Xprot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2Xprot = del2Xprot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xprot = del2Xprot + 0 + Xprot(x-1, t);
        else
            del2Xprot = del2Xprot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2Xprot = del2Xprot/(deltaX^2); % Laplacian

        % X protein derivative
        dXprot_dt = v3 - v4 + Dp*del2Xprot;

        % X protein update
        Xprot(x,t+1) = Xprot(x,t) + (dXprot_dt*deltaT);



        % Y RNA Laplacian
        del2Yrna = -2*Yrna(x,t); %reset value
        if (x==1)
            del2Yrna = del2Yrna + Yrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yrna = del2Yrna + 0 + Yrna(x-1, t);
        else
            del2Yrna = del2Yrna + Yrna(x+1,t) + Yrna(x-1,t);
        end
        del2Yrna = del2Yrna/(deltaX^2); % Laplacian

        % Y RNA derivative
        dYrna_dt = v5 - v6 + Dr*del2Yrna;

        % Y RNA update
        Yrna(x,t+1) = Yrna(x,t) + (dYrna_dt*deltaT);

        % Y protein Laplacian
        del2Yprot = -2*Yprot(x,t); %reset value
        if (x==1)
            del2Yprot = del2Yprot + Yprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yprot = del2Yprot + 0 + Yprot(x-1, t);
        else
            del2Yprot = del2Yprot + Yprot(x+1,t) + Yprot(x-1,t);
        end
        del2Yprot = del2Yprot/(deltaX^2); % Laplacian

        % Y protein derivative
        dYprot_dt = v7 - v8 + Dp*del2Yprot;

        % Y protein update
        Yprot(x,t+1) = Yprot(x,t) + (dYprot_dt*deltaT);
    end
end
```

```
time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;

fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)), 'b')
plot(time, (Xprot(1,:)), 'c')
plot(time, (Yrna(1,:)), 'r')
plot(time, (Yprot(1,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna((ceil(spacesteps/2)),:)), 'b')
plot(time, (Xprot((ceil(spacesteps/2)),:)), 'c')
plot(time, (Yrna((ceil(spacesteps/2)),:)), '--r')
plot(time, (Yprot((ceil(spacesteps/2)),:)), '--m')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Middle Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(spacesteps,:)), 'b')
plot(time, (Xprot(spacesteps,:)), 'c')
plot(time, (Yrna(spacesteps,:)), 'r')
plot(time, (Yprot(spacesteps,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Last Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

fig2 = figure(2);
subplot(1,3,1)
hold on
plot(space, (Xrna(:,1)),'b')
plot(space, (Xprot(:,1)), '--c')
plot(space, (Yrna(:,1)), 'r')
plot(space, (Yprot(:,1)), '--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(space, (Xrna(:,((timesteps/2)+1))), 'b')
plot(space, (Xprot(:,((timesteps/2)+1))),'--c')
plot(space, (Yrna(:,((timesteps/2)+1))),'r')
```

```
plot(space, (Yprot(:,((timesteps/2)+1))),'--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Middle Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(space, (Xrna(:,timesteps)),'b')
plot(space, (Xprot(:,timesteps)),'--c')
plot(space, (Yrna(:,timesteps)),'r')
plot(space, (Yprot(:,timesteps)),'--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off
```





4. (c) total time = 200 s

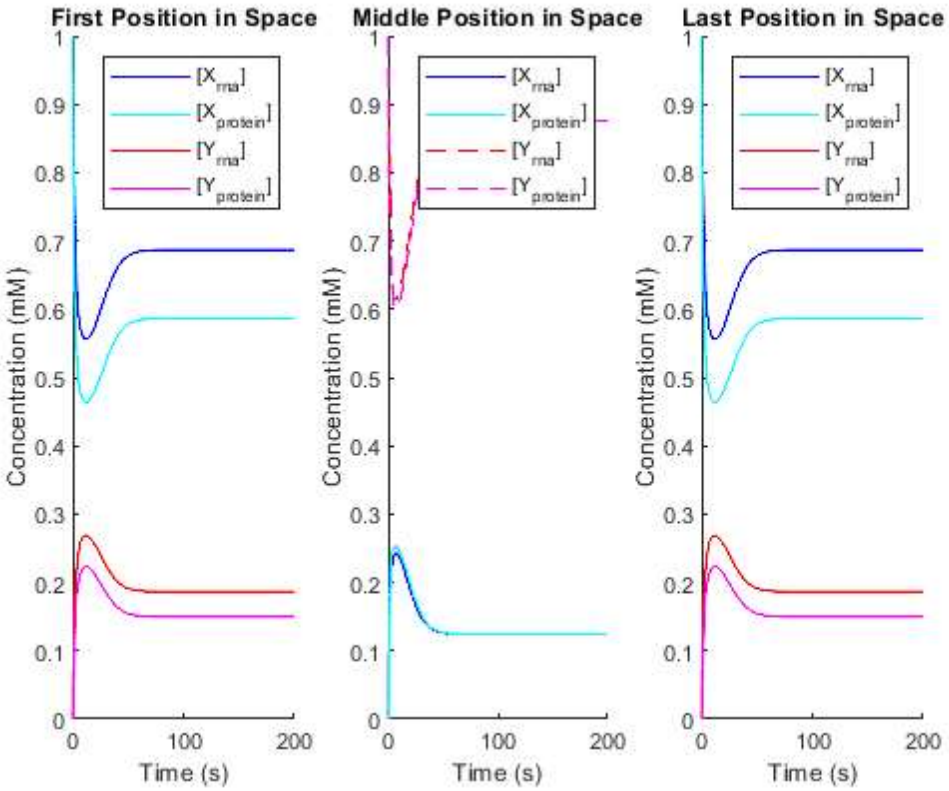
```
clear
close all
```

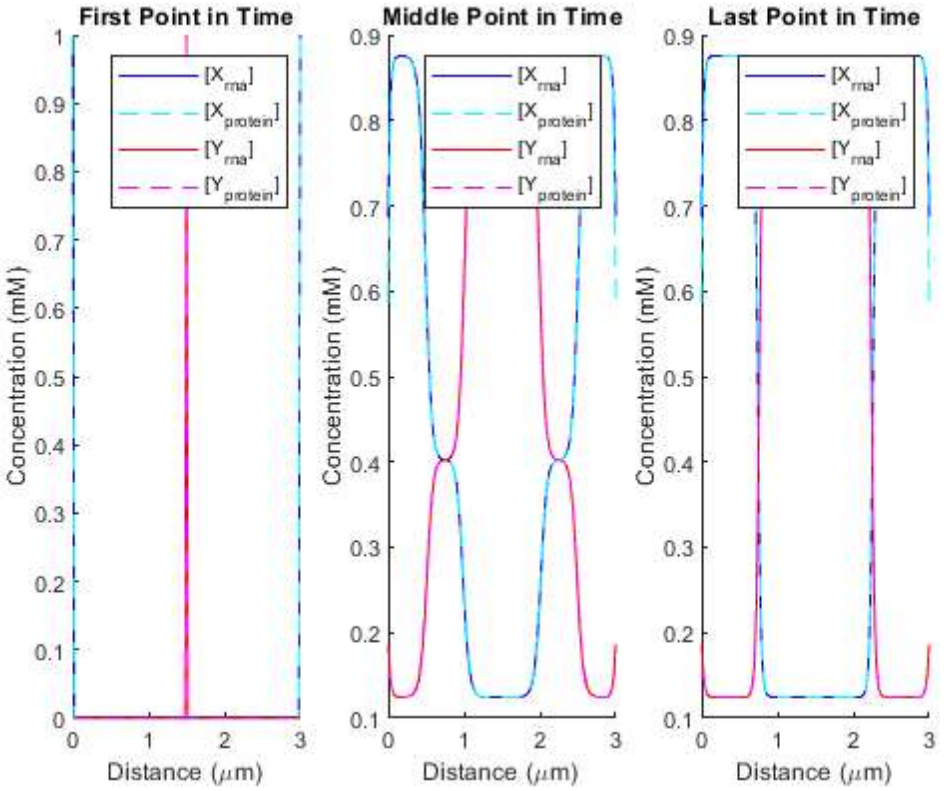
```
% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 200; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Yprot = (zeros(spacesteps, timesteps));
Yrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 1.0;
Xrna(1,1) = 1.0;
Xprot(spacesteps,1) = 1.0;
Xrna(spacesteps,1) = 1.0;
Yprot((ceil(spacesteps/2)),1) = 1.0;
Yrna((ceil(spacesteps/2)),1) = 1.0;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*(1-( (Yprot(x,t)^2) / ((K^2)+(Yprot(x,t)^2)) )); % X rna synthesis
        v2 = Xr*Xrna(x,t); % X rna degradation
        v3 = w*Xrna(x,t); % X protein synthesis
        v4 = Xp*Xprot(x,t); % X protein degradation
        v5 = u*(1-( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) )); % Y rna synthesis
        v6 = Xr*Yrna(x,t); % Y rna degradation %document shows Xr*Xrna - typo?
        v7 = w*Yrna(x,t); % Y protein synthesis
        v8 = Xp*Yprot(x,t); % Y protein degradation

        % X RNA Laplacian
        del2Xrna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2Xrna = del2Xrna + Xrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xrna = del2Xrna + 0 + Xrna(x-1, t);
        else
            del2Xrna = del2Xrna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2Xrna = del2Xrna/(deltaX^2); % Laplacian

        % X RNA derivative
        dXrna_dt = v1 - v2 + Dr*del2Xrna;
```

```
        % X RNA update
        Xrna(x,t+1) = Xrna(x,t) + (dXrna_dt*deltaT);

        % X protein Laplacian
        del2Xprot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2Xprot = del2Xprot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xprot = del2Xprot + 0 + Xprot(x-1, t);
        else
            del2Xprot = del2Xprot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2Xprot = del2Xprot/(deltaX^2); % Laplacian

        % X protein derivative
        dXprot_dt = v3 - v4 + Dp*del2Xprot;

        % X protein update
        Xprot(x,t+1) = Xprot(x,t) + (dXprot_dt*deltaT);



        % Y RNA Laplacian
        del2Yrna = -2*Yrna(x,t); %reset value
        if (x==1)
            del2Yrna = del2Yrna + Yrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yrna = del2Yrna + 0 + Yrna(x-1, t);
        else
            del2Yrna = del2Yrna + Yrna(x+1,t) + Yrna(x-1,t);
        end
        del2Yrna = del2Yrna/(deltaX^2); % Laplacian

        % Y RNA derivative
        dYrna_dt = v5 - v6 + Dr*del2Yrna;

        % Y RNA update
        Yrna(x,t+1) = Yrna(x,t) + (dYrna_dt*deltaT);

        % Y protein Laplacian
        del2Yprot = -2*Yprot(x,t); %reset value
        if (x==1)
            del2Yprot = del2Yprot + Yprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yprot = del2Yprot + 0 + Yprot(x-1, t);
        else
            del2Yprot = del2Yprot + Yprot(x+1,t) + Yprot(x-1,t);
        end
        del2Yprot = del2Yprot/(deltaX^2); % Laplacian

        % Y protein derivative
        dYprot_dt = v7 - v8 + Dp*del2Yprot;

        % Y protein update
        Yprot(x,t+1) = Yprot(x,t) + (dYprot_dt*deltaT);
    end
end
```

```
time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;

fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)), 'b')
plot(time, (Xprot(1,:)), 'c')
plot(time, (Yrna(1,:)), 'r')
plot(time, (Yprot(1,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna((ceil(spacesteps/2)),:)), 'b')
plot(time, (Xprot((ceil(spacesteps/2)),:)), 'c')
plot(time, (Yrna((ceil(spacesteps/2)),:)), '--r')
plot(time, (Yprot((ceil(spacesteps/2)),:)), '--m')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Middle Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(spacesteps,:)), 'b')
plot(time, (Xprot(spacesteps,:)), 'c')
plot(time, (Yrna(spacesteps,:)), 'r')
plot(time, (Yprot(spacesteps,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Last Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

fig2 = figure(2);
subplot(1,3,1)
hold on
plot(space, (Xrna(:,1)),'b')
plot(space, (Xprot(:,1)), '--c')
plot(space, (Yrna(:,1)), 'r')
plot(space, (Yprot(:,1)), '--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(space, (Xrna(:,((timesteps/2)+1))), 'b')
plot(space, (Xprot(:,((timesteps/2)+1))),'--c')
plot(space, (Yrna(:,((timesteps/2)+1))),'r')
```

```
plot(space, (Yprot(:,((timesteps/2)+1))),'--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Middle Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(space, (Xrna(:,timesteps)),'b')
plot(space, (Xprot(:,timesteps)),'--c')
plot(space, (Yrna(:,timesteps)),'r')
plot(space, (Yprot(:,timesteps)),'--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off
```





4. (d) total time = 400 s

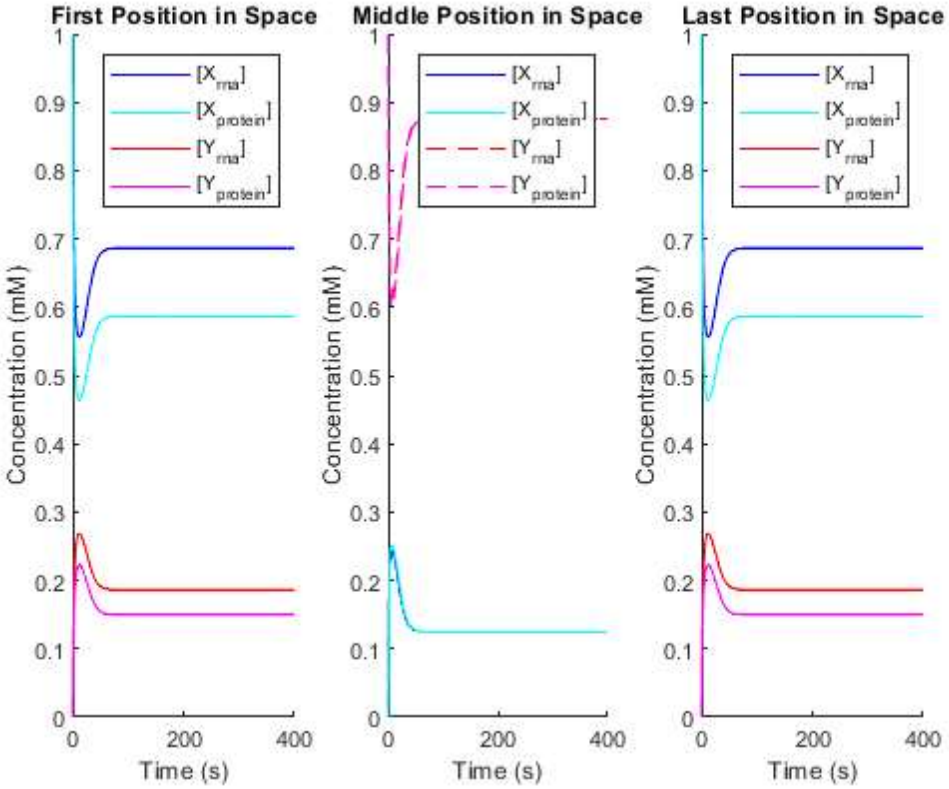
```
clear
close all
```

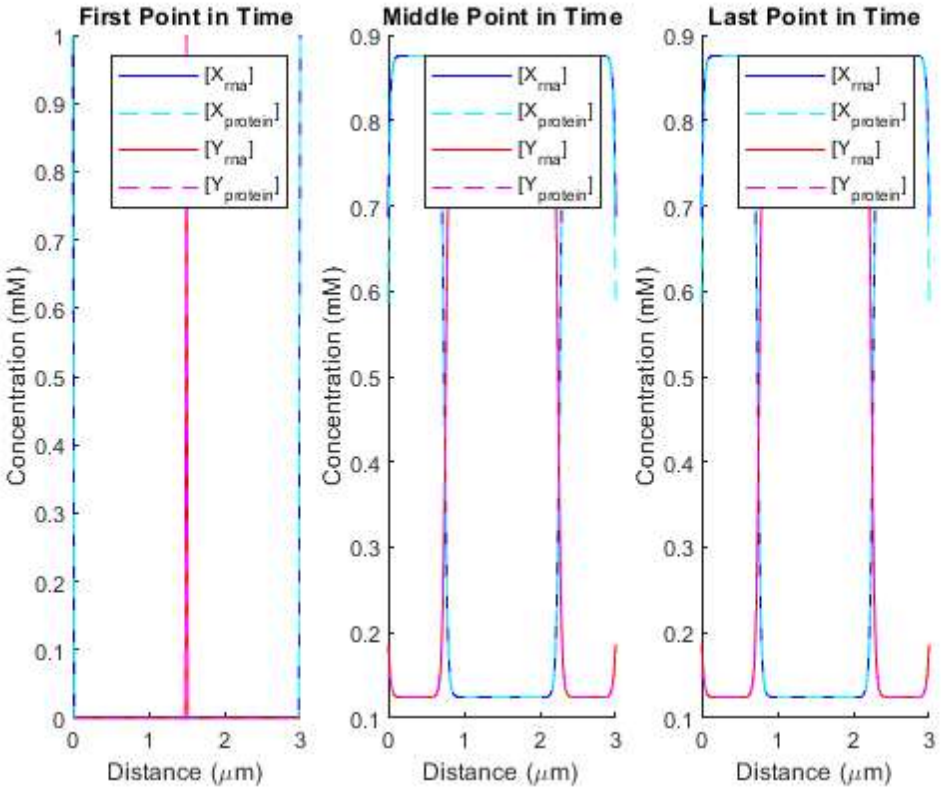
```
% constants
u = 1; % 1/s
w = 1; % 1/s
Xp = 1; % 1/s
Xr = 1; % 1/s
K = 0.33; % mM
Dp = 1E-4; % um/s
Dr = 1E-4; % um/s

% system parameters
totalT = 400; % s
deltaT = 0.01; % s
deltaX = 0.02; % um
Xmin = 0; % um
Xmax = 3.0; % um

% numbers of points
spacesteps = ceil( (Xmax-Xmin) / deltaX) + 1;
timesteps = ceil(totalT/deltaT);

% set arrays & initial conditions
Xprot = (zeros(spacesteps, timesteps));
Xrna = (zeros(spacesteps, timesteps));
Yprot = (zeros(spacesteps, timesteps));
Yrna = (zeros(spacesteps, timesteps));
Xprot(1,1) = 1.0;
Xrna(1,1) = 1.0;
Xprot(spacesteps,1) = 1.0;
Xrna(spacesteps,1) = 1.0;
Yprot((ceil(spacesteps/2)),1) = 1.0;
Yrna((ceil(spacesteps/2)),1) = 1.0;

% loop over time and x
for t = 1:timesteps
    for x = 1:spacesteps
        v1 = u*(1-( (Yprot(x,t)^2) / ((K^2)+(Yprot(x,t)^2)) )); % X rna synthesis
        v2 = Xr*Xrna(x,t); % X rna degradation
        v3 = w*Xrna(x,t); % X protein synthesis
        v4 = Xp*Xprot(x,t); % X protein degradation
        v5 = u*(1-( (Xprot(x,t)^2) / ((K^2)+(Xprot(x,t)^2)) )); % Y rna synthesis
        v6 = Xr*Yrna(x,t); % Y rna degradation %document shows Xr*Xrna - typo?
        v7 = w*Yrna(x,t); % Y protein synthesis
        v8 = Xp*Yprot(x,t); % Y protein degradation

        % X RNA Laplacian
        del2Xrna = -2*Xrna(x,t); %reset value
        if (x==1)
            del2Xrna = del2Xrna + Xrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xrna = del2Xrna + 0 + Xrna(x-1, t);
        else
            del2Xrna = del2Xrna + Xrna(x+1,t) + Xrna(x-1,t);
        end
        del2Xrna = del2Xrna/(deltaX^2); % Laplacian

        % X RNA derivative
        dXrna_dt = v1 - v2 + Dr*del2Xrna;
```

```
        % X RNA update
        Xrna(x,t+1) = Xrna(x,t) + (dXrna_dt*deltaT);

        % X protein Laplacian
        del2Xprot = -2*Xprot(x,t); %reset value
        if (x==1)
            del2Xprot = del2Xprot + Xprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Xprot = del2Xprot + 0 + Xprot(x-1, t);
        else
            del2Xprot = del2Xprot + Xprot(x+1,t) + Xprot(x-1,t);
        end
        del2Xprot = del2Xprot/(deltaX^2); % Laplacian

        % X protein derivative
        dXprot_dt = v3 - v4 + Dp*del2Xprot;

        % X protein update
        Xprot(x,t+1) = Xprot(x,t) + (dXprot_dt*deltaT);



        % Y RNA Laplacian
        del2Yrna = -2*Yrna(x,t); %reset value
        if (x==1)
            del2Yrna = del2Yrna + Yrna(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yrna = del2Yrna + 0 + Yrna(x-1, t);
        else
            del2Yrna = del2Yrna + Yrna(x+1,t) + Yrna(x-1,t);
        end
        del2Yrna = del2Yrna/(deltaX^2); % Laplacian

        % Y RNA derivative
        dYrna_dt = v5 - v6 + Dr*del2Yrna;

        % Y RNA update
        Yrna(x,t+1) = Yrna(x,t) + (dYrna_dt*deltaT);

        % Y protein Laplacian
        del2Yprot = -2*Yprot(x,t); %reset value
        if (x==1)
            del2Yprot = del2Yprot + Yprot(x+1,t) + 0;
        elseif (x==spacesteps)
            del2Yprot = del2Yprot + 0 + Yprot(x-1, t);
        else
            del2Yprot = del2Yprot + Yprot(x+1,t) + Yprot(x-1,t);
        end
        del2Yprot = del2Yprot/(deltaX^2); % Laplacian

        % Y protein derivative
        dYprot_dt = v7 - v8 + Dp*del2Yprot;

        % Y protein update
        Yprot(x,t+1) = Yprot(x,t) + (dYprot_dt*deltaT);
    end
end
```

```
time = 0:deltaT:totalT;
space = 0:deltaX:Xmax;

fig1 = figure(1);
subplot(1,3,1)
hold on
plot(time, (Xrna(1,:)), 'b')
plot(time, (Xprot(1,:)), 'c')
plot(time, (Yrna(1,:)), 'r')
plot(time, (Yprot(1,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('First Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(time, (Xrna((ceil(spacesteps/2)),:)), 'b')
plot(time, (Xprot((ceil(spacesteps/2)),:)), 'c')
plot(time, (Yrna((ceil(spacesteps/2)),:)), '--r')
plot(time, (Yprot((ceil(spacesteps/2)),:)), '--m')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Middle Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(time, (Xrna(spacesteps,:)), 'b')
plot(time, (Xprot(spacesteps,:)), 'c')
plot(time, (Yrna(spacesteps,:)), 'r')
plot(time, (Yprot(spacesteps,:)), 'm')
xlabel('Time (s)')
ylabel('Concentration (mM)')
title('Last Position in Space')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

fig2 = figure(2);
subplot(1,3,1)
hold on
plot(space, (Xrna(:,1)),'b')
plot(space, (Xprot(:,1)), '--c')
plot(space, (Yrna(:,1)), 'r')
plot(space, (Yprot(:,1)), '--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('First Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,2)
hold on
plot(space, (Xrna(:,((timesteps/2)+1))), 'b')
plot(space, (Xprot(:,((timesteps/2)+1))),'--c')
plot(space, (Yrna(:,((timesteps/2)+1))),'r')
```

```
plot(space, (Yprot(:,((timesteps/2)+1))),'--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Middle Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off

subplot(1,3,3)
hold on
plot(space, (Xrna(:,timesteps)),'b')
plot(space, (Xprot(:,timesteps)),'--c')
plot(space, (Yrna(:,timesteps)),'r')
plot(space, (Yprot(:,timesteps)),'--m')
xlabel('Distance (\mum)')
ylabel('Concentration (mM)')
title('Last Point in Time')
legend('[X_{rna}]', '[X_{protein}]', '[Y_{rna}]', '[Y_{protein}]')
hold off
```





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