Bounded Reachability Problems are Decidable in FIFO Machines

Benedikt Bollig Alain Finkel **Amrita Suresh**

LSV – CNRS & ENS Paris-Saclay, Université Paris-Saclay, France

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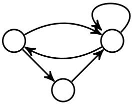
FIFO Machines

Distributed processes such that

- Each process P_i is a finite state machine

P_i :



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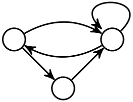

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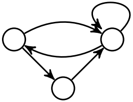

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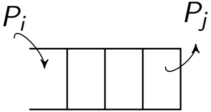
$P_1, \ldots, P_n$

- They communicate using FIFO queues



FIFO Machines

- Studied since the 1980s. Widely used in distributed settings.
- FIFO machines simulate TM, hence underapproximations.

[1] Gouda et al., *On deadlock detection in systems of communicating finite state machines*, 1987.
[2] Esparza et al., *Perfect Model for Bounded Verification*, 2012
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- (Input-)Bounded FIFO machines strictly contain these subclasses.

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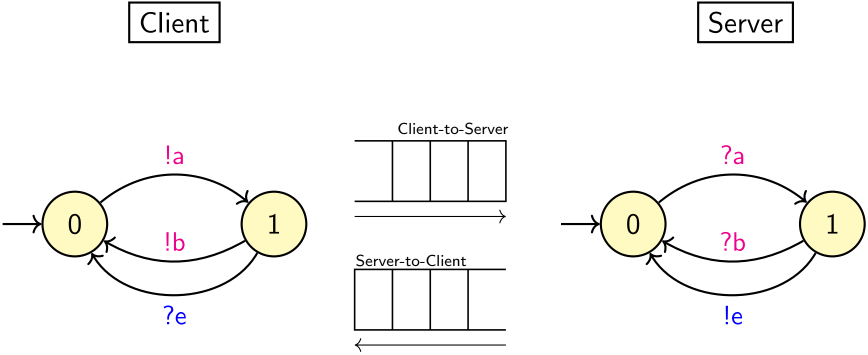
Example (Connection-Deconnection Protocol) [4]



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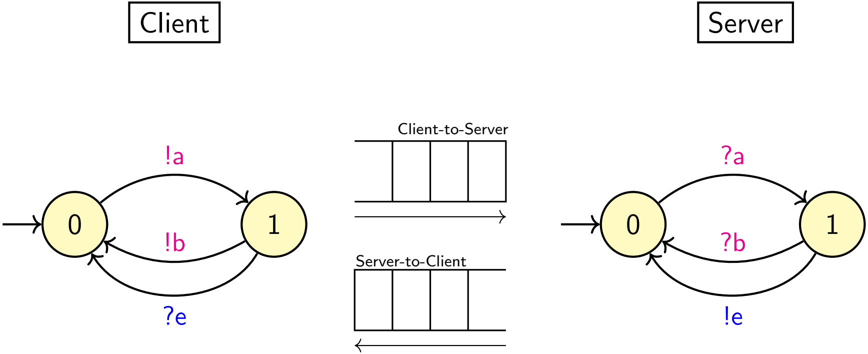

Initial configuration $(0, 0; \varepsilon, \varepsilon)$

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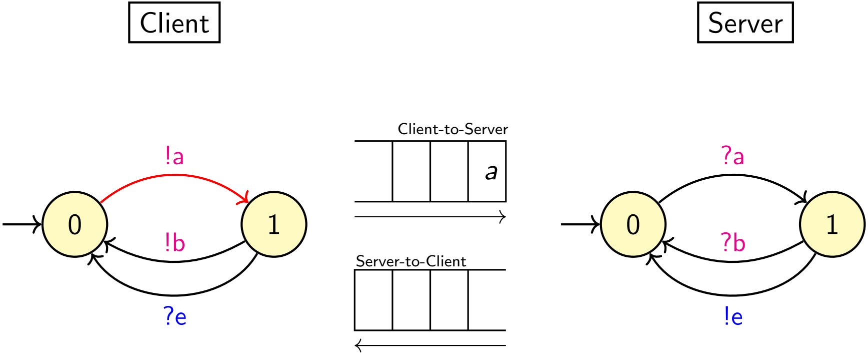

Run - $(0, 0; \varepsilon, \varepsilon) \xrightarrow{!a} (1, 0; a, \varepsilon)$

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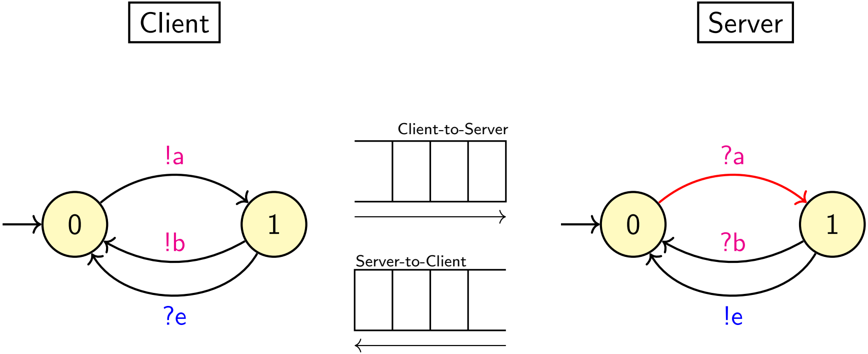

$$(0, 0; \varepsilon, \varepsilon) \xrightarrow{!a} (1, 0; a, \varepsilon) \xrightarrow{?a} (1, 1; \varepsilon, \varepsilon)$$

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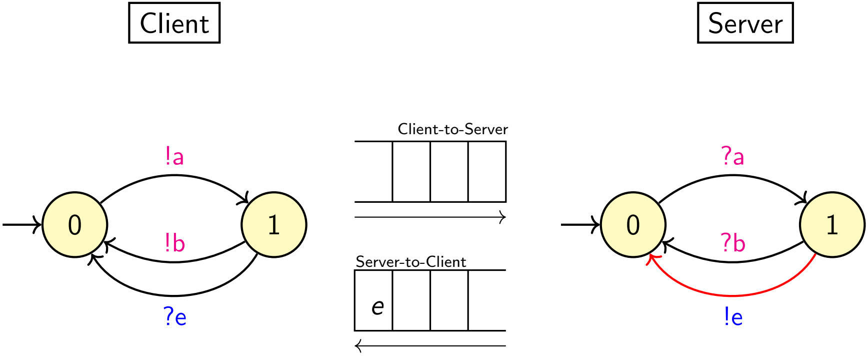

$(0, 0; \varepsilon, \varepsilon) \xrightarrow{!a} (1, 0; a, \varepsilon) \xrightarrow{?a} (1, 1; \varepsilon, \varepsilon) \xrightarrow{!e} (1, 0; \varepsilon, e)$

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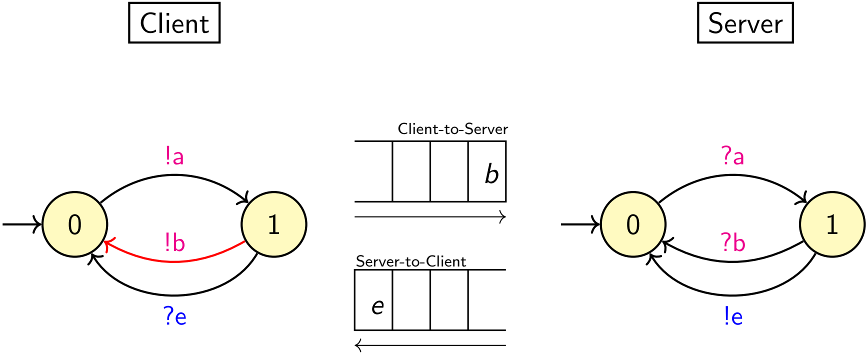

$$(0,0;\varepsilon,\varepsilon) \xrightarrow{!a} (1,0;a,\varepsilon) \xrightarrow{?a} (1,1;\varepsilon,\varepsilon) \xrightarrow{!e} (1,0;\varepsilon,e) \xrightarrow{!b} (0,0;b,e)$$

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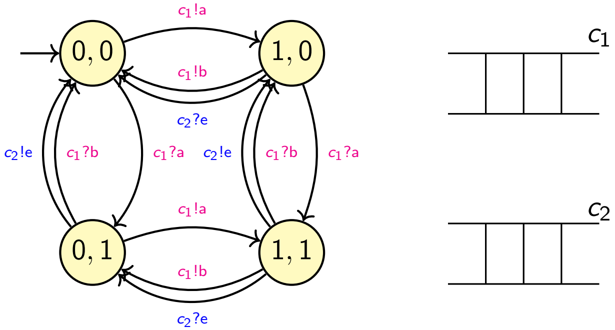
Formal model

A FIFO machine is a tuple $M = (Q, Ch, \Sigma, T, q_0)$ where



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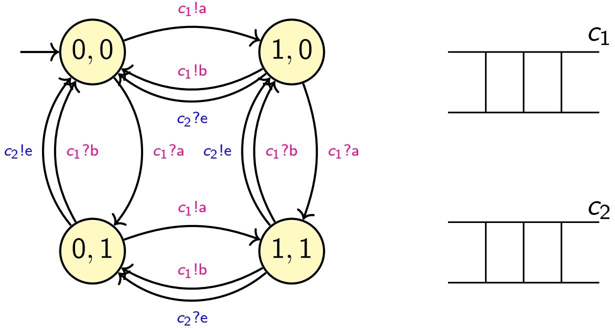

Q is a finite set of control-states.

$Q = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$.

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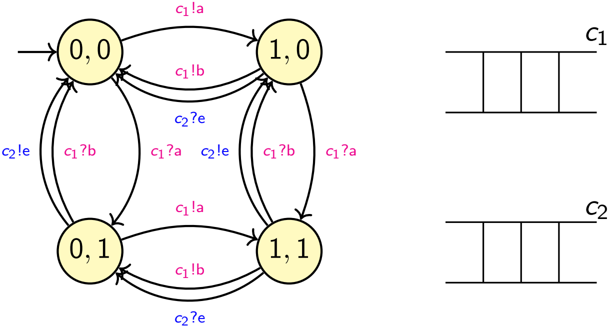

Ch is the number of channels.

$Ch = \{c_1, c_2\}$.

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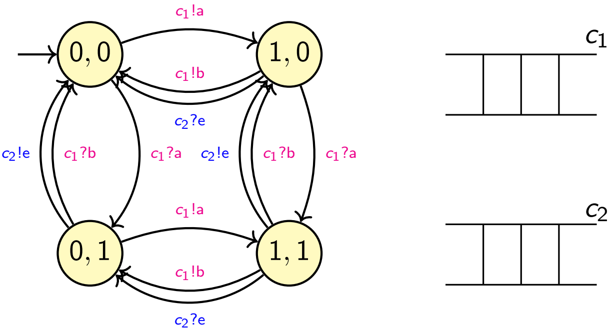

$\Sigma = \uplus_{c \in Ch} \Sigma_c$ is the alphabet.

$\Sigma = \{a, b, e\}$.

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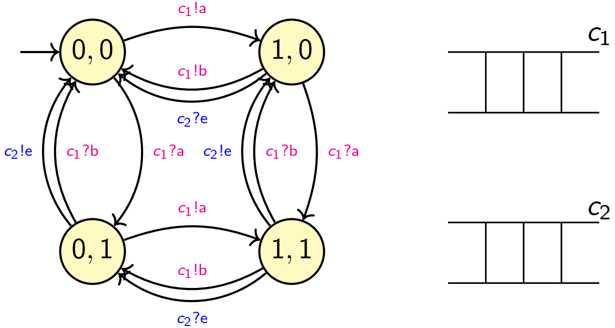

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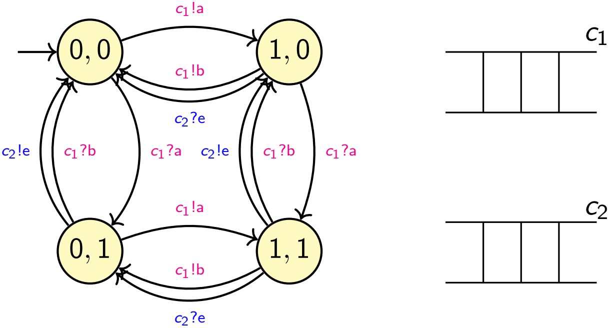

$T \subseteq Q \times A_M \times Q$ is the transition relation where
$A_M = \{c!a \mid a \in \Sigma \text{ and } c \in Ch\} \cup \{c?a \mid a \in \Sigma \text{ and } c \in Ch\}$

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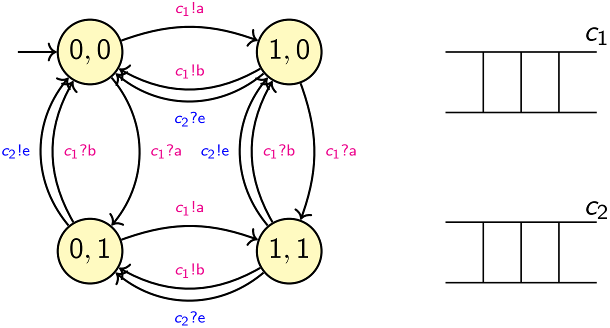

q_0 is the initial state.

$q_0 = (0, 0)$.

Configurations and Reachability

- A configuration is $(q, \mathbf{w})$ where q is the control-state and $\mathbf{w}$ is a tuple of the channel contents. The set of configurations is S_M.

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Theorem

Testing the reachability of a configuration in a general FIFO system is undecidable. [a]

[a] Brand and Zafiropulo, *On Communicating Finite-State Machines*, 1983.

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- $Reach_M(L) = \bigcup_{\sigma \in L} Reach_M(\sigma)$.

Configurations and Reachability

We define the *send projection over c* $proj_{c!} : A_M^* \rightarrow \Sigma^*$

Example: $proj_{c!}(c!x.d!y.c?x.c!z.c!z) = xzz$

Bounded language

Let $w_1, ..., w_n \in \Sigma^+$ be non-empty words where $n \geq 1$.
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$((ab)^*(cd)^*)^*$ is not a bounded language.

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Let $L = (L_c)_{c \in Ch}$ be non-empty regular bounded languages over Σ.
$L_! = \{w \in A_M^* \mid proj_{c!}(w) \in L_c \text{ for all } c \in Ch\}$.

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(We define $L_?$ similarly.)

Rational relations

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We say that $\mathcal{R}$ is rational if there is a regular word language $R \in \Theta^*$ such that

$$\mathcal{R} = \{(\mathbf{a}_c^1 \cdot \ldots \cdot \mathbf{a}_c^n)_{c \in Ch} \mid \mathbf{a}^1 \ldots \mathbf{a}^n \in R \text{ with } n \in \mathbb{N} \text{ and } \mathbf{a}^i = (\mathbf{a}_c^i)_{c \in Ch} \in \Theta \text{ for } i \in \{1, \ldots, n\}\}.$$

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Example: $\mathcal{R} = \{(a^m, b^n \mid m \geq n)\}$ is a rational relation, witnessed by $R = ((a, b) + (a, \epsilon))^*$.

Input-Bounded Rational Reachability Problem

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Question: Do we have $(q, \mathbf{w}) \in Reach_M(L_!)$ for some $\mathbf{w} \in \mathcal{R}$?

Input-Bounded Rational Reachability Problem

Theorem

The Input-Bounded Rational Reachability Problem is decidable.

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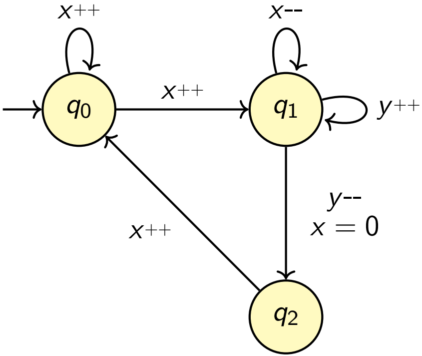
Proof using counter machines...

Counter machines

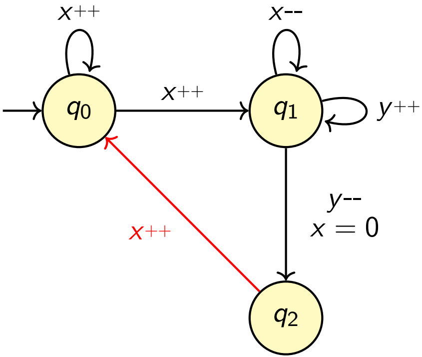
A counter machine (with zero tests) is a tuple $C = (Q, Cnt, T, q_0)$



Counter machines with RESTRICTED zero tests

Once a counter has been tested for zero, it cannot be incremented or decremented anymore.

Counter machines with RESTRICTED zero tests



Counter machines with RESTRICTED zero tests

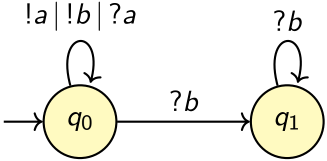
Theorem

The following problem is decidable: Given a counter machine $C = (Q, Cnt, T, q_0)$, a regular language $L \subseteq A_C^*$, a control state $q \in Q$, and counter valuation v, do we have $(q, v) \in Reach_C(L_{zero} \cap L)$?

Translation

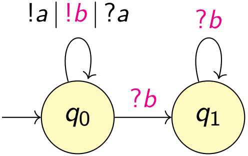
Intuition: Given a bounded language L, which is bounded over $(w_1, \ldots, w_n)$, we construct a counter x_i for each w_i.

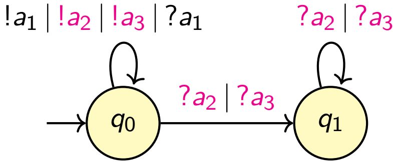
Translation



$$\hat{L} = (ab)^* bb^*$$

Step 1: Distinct letter property





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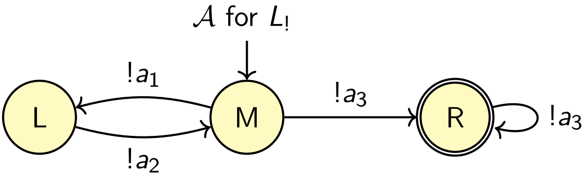
$L = (a_1 a_2)^* a_3 a_3{}^*$

Step 2: Trace property



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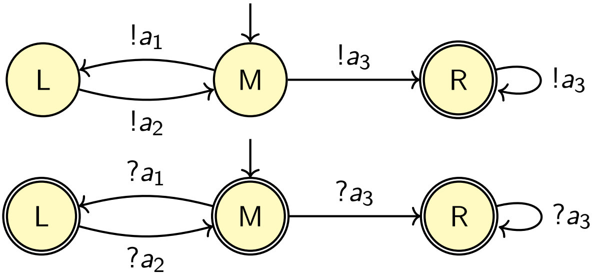
$\mathcal{A}$ for $L_! \cap Pref(L_?)$



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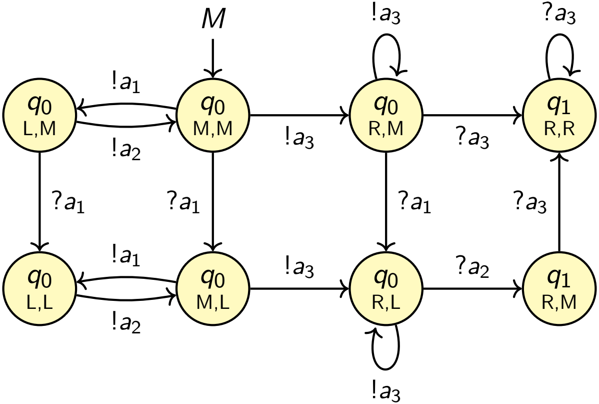
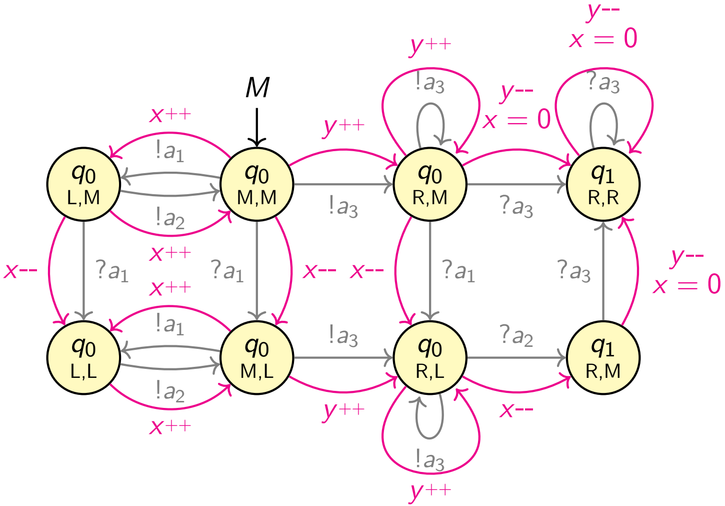
Step 3: Normal form



Step 4: Conversion to counter machine



Equivalence between configurations

Given the normal form and counter automata, is there a 1-1 equivalence between the configurations?

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NO!
Given a counter configuration $(q; 3, 0)$ for some q, where $L = (ab)^*(c)^*$, what is the corresponding FIFO machine configuration?

Equivalence between configurations

Given the normal form and counter automata, is there a 1-1 equivalence between the configurations?

But we can keep track of the last message sent.

Equivalence between configurations

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$L_a^{\text{last}} \subseteq A_M^*$ be the set of words where a describes the **last sent** messages.

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We can now conclude that runs in the FIFO machine are faithfully simulated by runs in the counter machine.

Other bounded problems

Table: Summary of key results. (D stands for Decidable)

	Letter-bounded	Bounded $\lvert Ch \rvert = 1$	Bounded $\lvert Ch \rvert > 1$
UNBOUND	D	D	D [5]
TERM	D	**EXPTIME**	**D**
REACH	D [6]	**EXPTIME**	**D, not ELEM**
CS-REACH	D	**EXPTIME**	D

[5] Jéron and Jard, *Testing for unboundedness of FIFO channels*, 1993.

[6] Gouda et al., *On deadlock detection in systems of communicating finite state machines*, 1987.

Future work

- Precise complexity for termination and boundedness
- Upper bounds for single channel case
- Output bounded reachability problems