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DIGITAL SIGNAL PROCESSING

LAB #5

Design of IIR Digital Filters

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2 Objective

- To design IIR filter using different methods.
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There are several methods that can be used to design digital filters having an infinite duration unit sample response. One of the popular methods is based on converting an analog filter into a digital filter. In this method we begin the design of digital filter in the analog domain and then convert the design into the digital domain. For this purpose, depending on the specifications of the required digital filter the various approximations like butterworth, chebyshev I, chebyshev II and elliptic filters are used.
Among the different approaches used in the design of digital IIR filters this lab experiment deals with impulse invariance method and bi-linear transformation.

4.1 Impulse Invariance Method

In impulse invariance method, the objective is to design an IIR filter having an unit sample response $h[n]$ that is the sampled version of the impulse response of the analog filter.

$$h[n] = h[nT] \qquad n = 0, 1, 2, \ldots\ldots, \text{where } T \text{ is the sampling interval}$$

4.2 Bi-Linear Transformation

In bi-linear transformation a conformal mapping from s-plane to z-plane is carried out with the relation given as,

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

For the design of IIR digital filters, there are built-in functions in MATLAB such as `Impinvar(..)`, `bilinear(..)`, `butter(..)`, `cheby1(..)`, `cheby2(..)`, `ellip(..)`, `buttord(..)`, `cheb1ord(..)`, `cheb2ord(..)`, `ellipord(..)` etc. For plotting the impulse responses for analog and digital filters refer to the functions `impulse(..)` and `dimpulse(..)`. For further information on the above functions, please refer to the MATLAB `'help'`.

5 Lab Problems

5.1 Problem 1

Convert the analog filter $H_a(s) = \dfrac{s+0.1}{(s+0.1)^2+9}$ into a digital IIR filter by means of the impulse invariance method. Plot the frequency response (magnitude) of the designed filter taking sampling interval (T) of 0.1, 0.5 seconds. Compare the response of the filter designed to that of the analog one. Comment on the effect of T on the response.

```
b = [1, 0.1];
a = [1, 0.2, 9.01];
Ts = [0.1 0.5];
fs = 1./Ts;
[Ha, wa] = freqs(b,a,512);
figure(1)
for i = 1:2
subplot(1,2,i)
plot(wa/pi, 20*log10(abs(Ha))); hold on;
[bz, az] = impinvar(b,a,fs(i));
[Hz, wz] = freqz(bz,az,512);
plot(wz/pi, 20*log10(abs(Hz)),'r')
xlabel('Normalized Frequency(* pi rad/sample)'),
ylabel('Magnitude(dB)');
title(['Magnitude response for Ts = ', num2str(Ts(i))]);
legend('Analog filter','Digital filter');
end
```

Code 1: Matlab code for converting Analog filter to Digital IIR and plotting the magnitude response

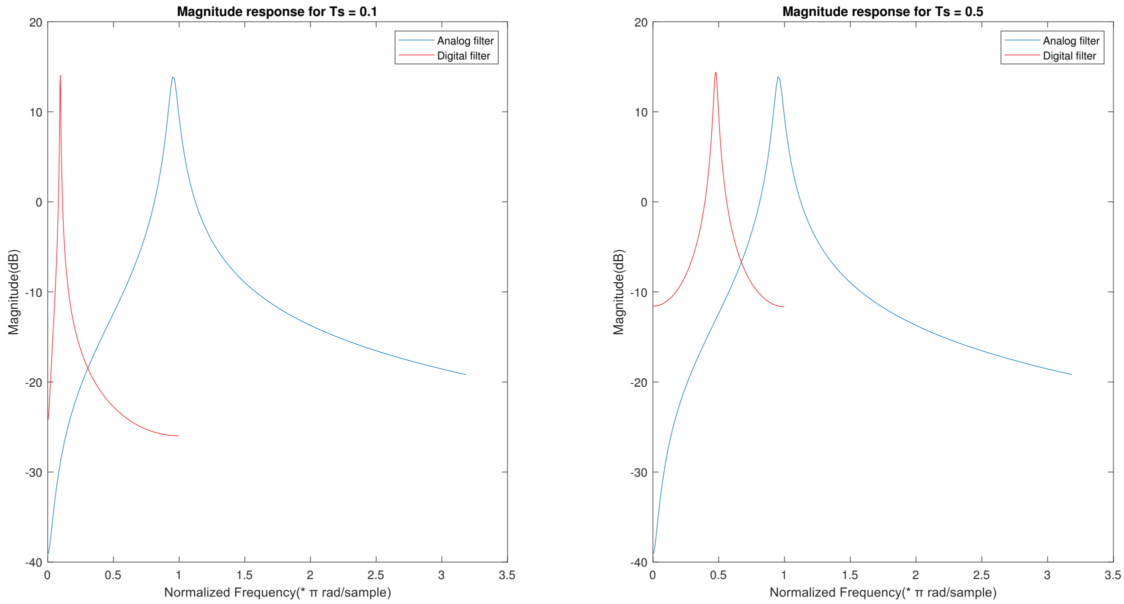

Figure 1: Plot for magnitude response of the designed filter

The response for $T_s = 0.1$ is slightly shifted to the left while the one for $T_s = 0.5$ is slightly shifted to the right.

5.2 Problem 2

Compare the unit sample response of the designed digital IIR filter with the impulse response of analog filter for T=0.1 and 0.5.

```
b = [1, 0.1];
a = [1, 0.2, 9.01];
Ts = [0.1 0.5];
fs = 1./Ts;
figure(2)
```

```
for i = 1:2
subplot(1,2,i)
impulse(b,a,'r');
%impulse response of analog filter
hold on;
[bz, az] = impinvar(b,a,fs(i));
dimpulse(bz,az);
%impulse response of digital filter
title(['Impulse response comparison for Ts = ', num2str(Ts(i))]);

legend('Analog filter','Digital filter');
end
```

Code 2: Matlab code for Comapring Unit sample response of digital IIR with Impulse response of analog filter

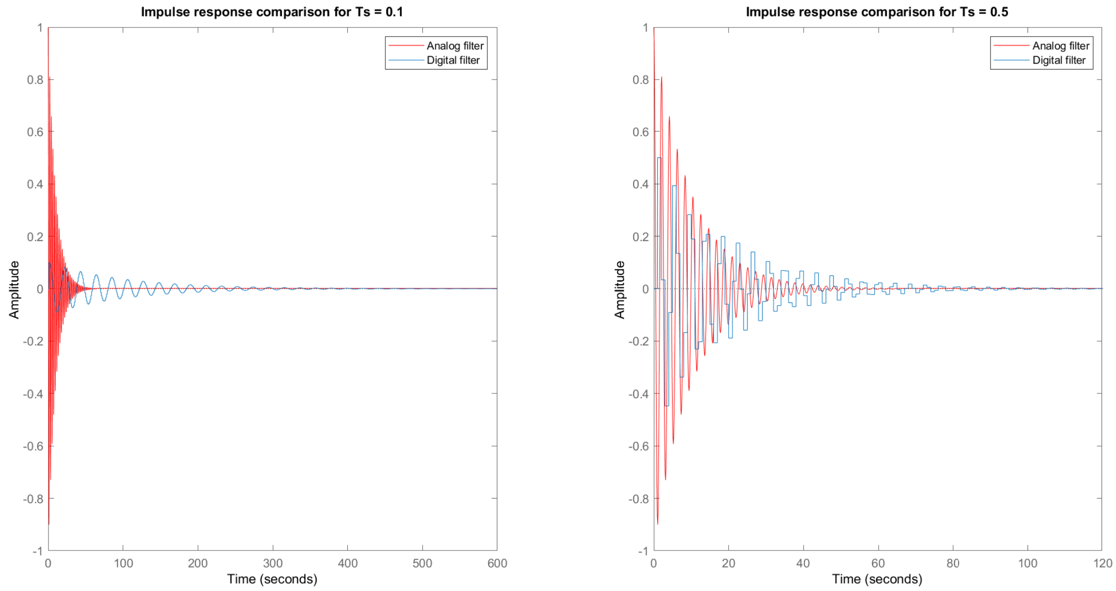

Figure 2: Plot of Unit sample response of digital IIR filter and Impulse response of analog filter

5.3 Problem 3

Convert the above analog filter in to a digital IIR filter by means of bilinear transformation and repeat all the procedures as specified in Problem 1

```
b = [1, 0.1];
a = [1, 0.2, 9.01];
Ts = [0.1 0.5];
fs = 1./Ts;
[Ha, wa] = freqs(b,a,512);
figure(1)
for i = 1:2
subplot(1,2,i)
plot(wa/pi, 20*log10(abs(Ha)));
hold on;
[bz, az] = bilinear(b,a,fs(i));
[Hz, wz] = freqz(bz,az,512);
plot(wz/pi, 20*log10(abs(Hz)),'r')
xlabel('Normalized Frequency(* pi rad/sample)'),
```

```
ylabel('Magnitude(dB)');
title(['Magnitude response for Ts = ', num2str(Ts(i))]);
legend('Analog filter','Digital filter');
end
figure(2)
for i = 1:2
subplot(1,2,i)
impulse(b,a,'r--');
%impulse response of analog filter
hold on;
[bz, az] = bilinear(b,a,fs(i));
dimpulse(bz,az);
%impulse response of digital filter
title(['Impulse response comparison for Ts = ', num2str(Ts(i))]);
legend('Analog filter','Digital filter');
end
```

Code 3: Matlab code for converting Analog filter to Digital IIR by using bilinear transformation and plotting and comparing different responses

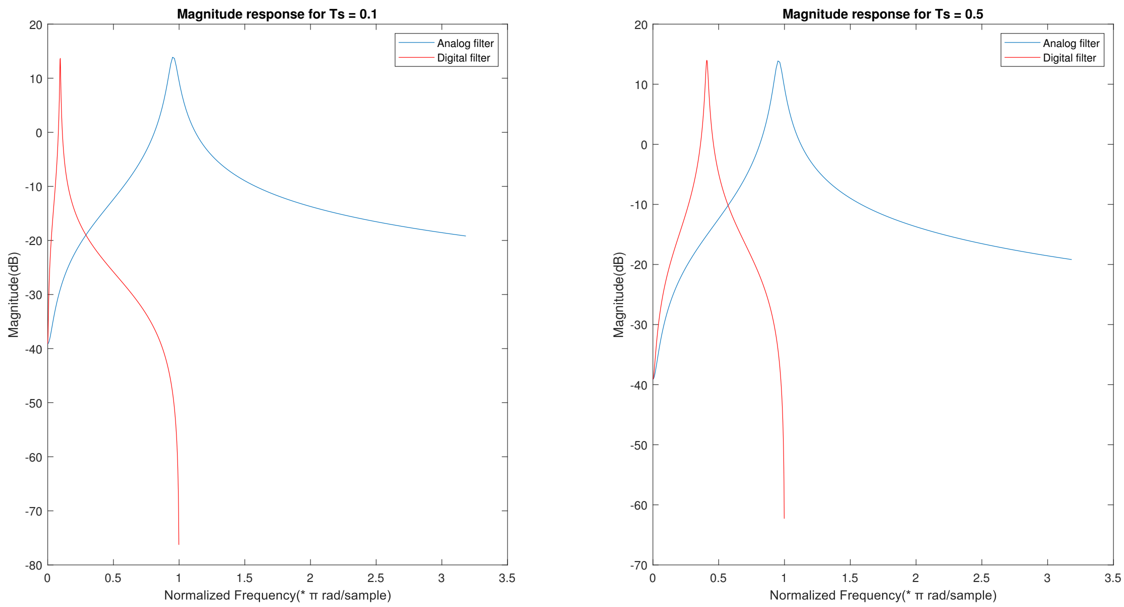

Figure 3: Plot for magnitude response of the designed filter

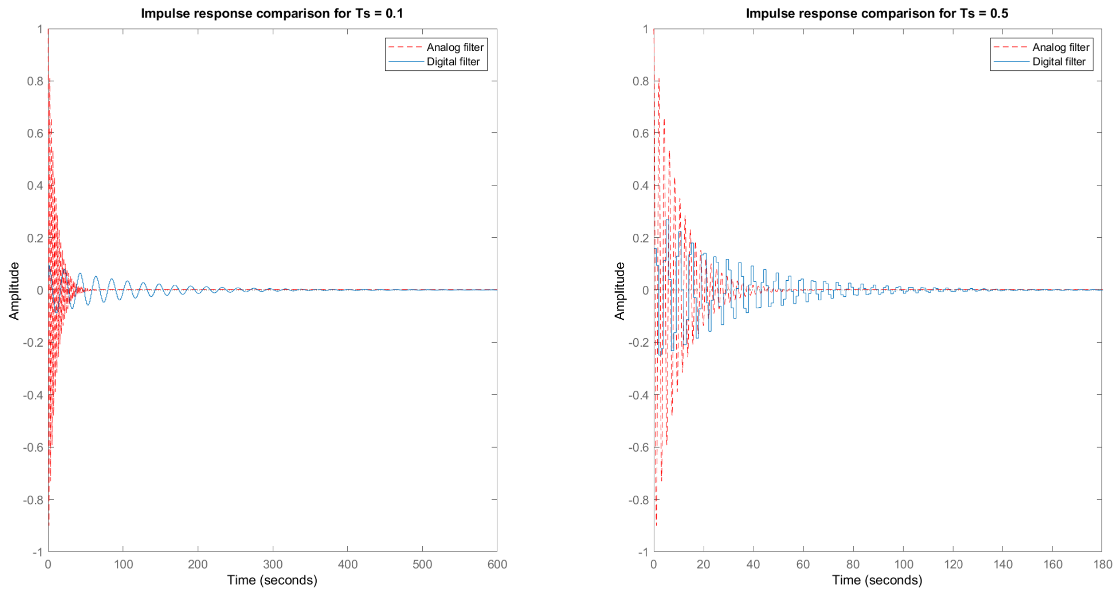

Figure 4: Plot for Unit sample response of digital IIR filter and Impulse response of analog filter

5.4 Problem 4

An IIR digital low pass filter is required to meet the following specifications:

- **Pass band ripple (or peak to peak ripple):$\leq$ 0.5 dB**
- **Passband edge: 1.2 kHz**
- **Stopband attenuation:$\geq$ 40 dB**
- **Stopband edge: 2.0 kHz**
- **Sample rate: 8.0 kHz**

Use the MATLAB Signal Processing functions to determine

- **The required filter order,**
- **The cutoff frequency,**
- **The numerator and the denominator coefficients**

for the digital Butterwoth, digital Chebyshev and digital Elliptic filters. Also plot their frequency responses. Describe the nature of each response.

```
fs = 8000;
fn = fs/2;

wp = 1200/fn;
ws = 2000/fn;
rp = 0.5;
rs = 40;
[N,Wc] = buttord(wp,ws,rp,rs);

[b1,a1] = butter(N,Wc);
freqz(b1,a1);
subplot(2,1,1);
title("Frequency response of the designed Butterworth Filter")
```

```
fprintf("Order of Butterworth filter: %d\n", N);
disp("Cutoff frequencies:");
disp(Wc);
disp("Numerator coefficients of Butterworth Filter: ");
disp(b1);
disp("Denominator coefficients of Butterworth Filter: : ");
disp(a1);


figure(2);
[N,Wc] = cheb1ord(wp,ws,rp,rs);

[b1,a1] = cheby1(N,rp,wp);
freqz(b1,a1);
subplot(2,1,1);

title("Frequency response of the designed Chebyshev-1 Filter")

fprintf("Order of Chebyshev 1 filter: %d\n", N);
disp("Cutoff frequencies:");
disp(Wc);
disp("Numerator coefficients of Chebyshev-1 Filter: ");
disp(b1);
disp("Denominator coefficients of Chebyshev-1 Filter: ");
disp(a1);

figure(3);
[N,Wc] = cheb2ord(wp,ws,rp,rs);

[b1,a1] = cheby2(N,rs,ws);
freqz(b1,a1);
subplot(2,1,1);

title("Frequency response of the designed Chebyshev-2 Filter")

fprintf("Order of Chebyshev 2 filter: %d\n", N);
disp("Cutoff frequencies:");
disp(Wc);
disp("Numerator coefficients of Chebyshev-2 Filter: ");
disp(b1);
disp("Denominator coefficients of Chebyshev-2 Filter: ");
disp(a1);

figure(4);
[N,Wc] = ellipord(wp,ws,rp,rs);

[b1,a1] = ellip(N,rp,rs,wp);
freqz(b1,a1);
subplot(2,1,1);

title("Frequency response of the designed Elliptic Filter")

fprintf("Order of Elliptic Filter: %d\n", N);
disp("Cutoff frequencies:");
disp(Wc);
disp("Numerator coefficients of Elliptic Filter: ");
disp(b1);
disp("Denominator coefficients of Elliptic Filter: ");
disp(a1);
```

Code 4: Matlab code for determining filter order, cutoff frequency, numerator and denominator and plotting the frequency response

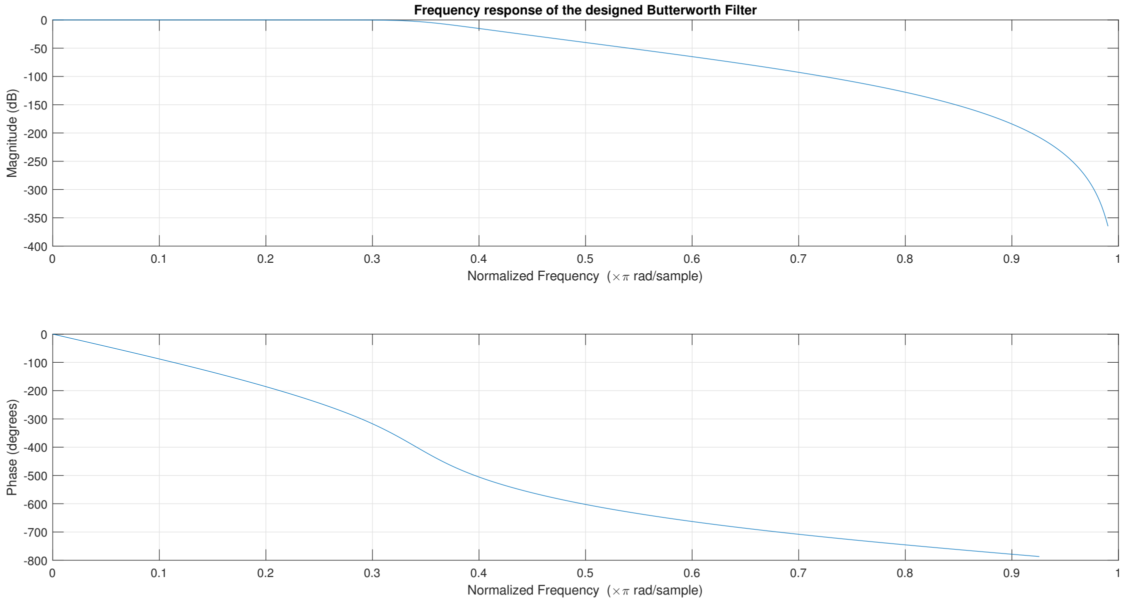

Figure 5: Plot of frequency response of Butterworth filter

```
Order of Butterworth filter: 9
Cutoff frequencies:
    0.3438

Numerator coefficients of Butterworth Filter:
    0.0004    0.0032    0.0129    0.0302    0.0453    0.0453    0.0302    0.0129    0.0032

Denominator coefficients of Butterworth Filter: :
    1.0000   -2.7996    4.4582   -4.5412    3.2404   -1.6330    0.5780   -0.1370    0.0197
```

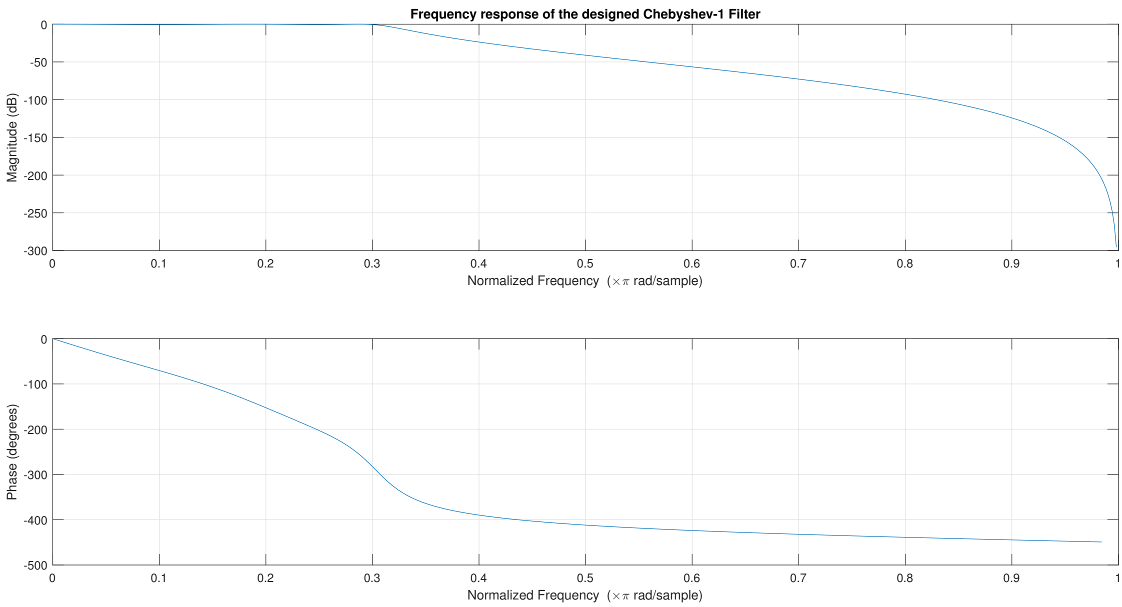

Figure 6: Plot of frequency response of Chebyshev-I filter

```
Order of Chebyshev 1 filter: 5
Cutoff frequencies:
    0.3000

Numerator coefficients of Chebyshev-1 Filter:
    0.0026    0.0132    0.0264    0.0264    0.0132    0.0026

Denominator coefficients of Chebyshev-1 Filter:
    1.0000   -2.9775    4.2932   -3.5124    1.6145   -0.3334
```

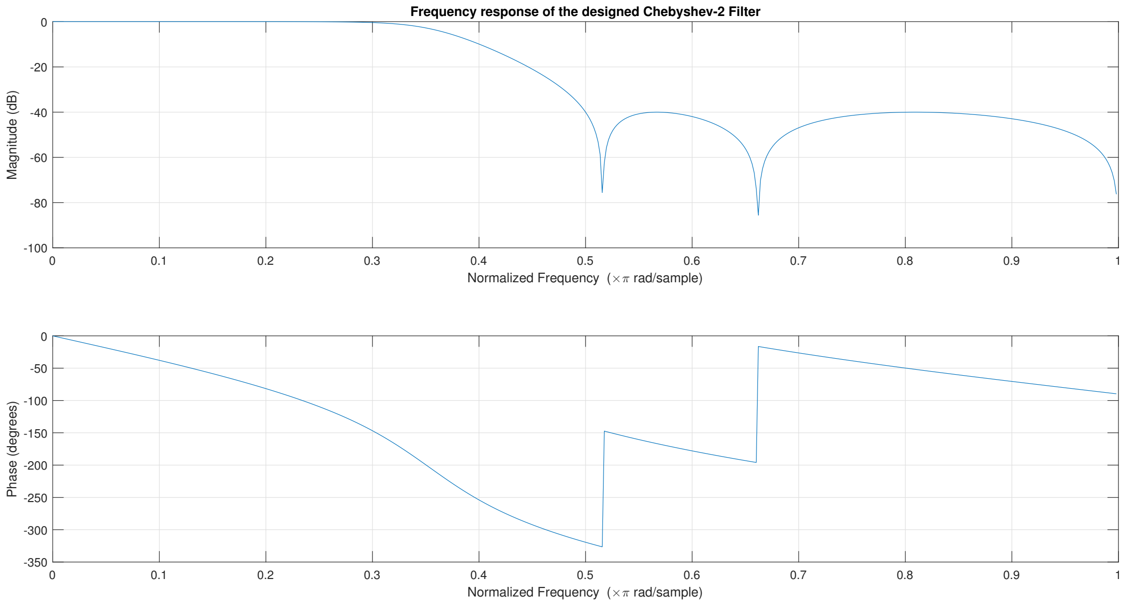

Figure 7: Plot of frequancy response of Chebyshev-II filter

```
    Order of Chebyshev 2 filter: 5
Cutoff frequencies:
    0.5000

Numerator coefficients of Chebyshev-2 Filter:
    0.0524    0.1086    0.1661    0.1661    0.1086    0.0524

Denominator coefficients of Chebyshev-2 Filter:
    1.0000   -1.2056    1.2136   -0.4984    0.1583   -0.0136
```

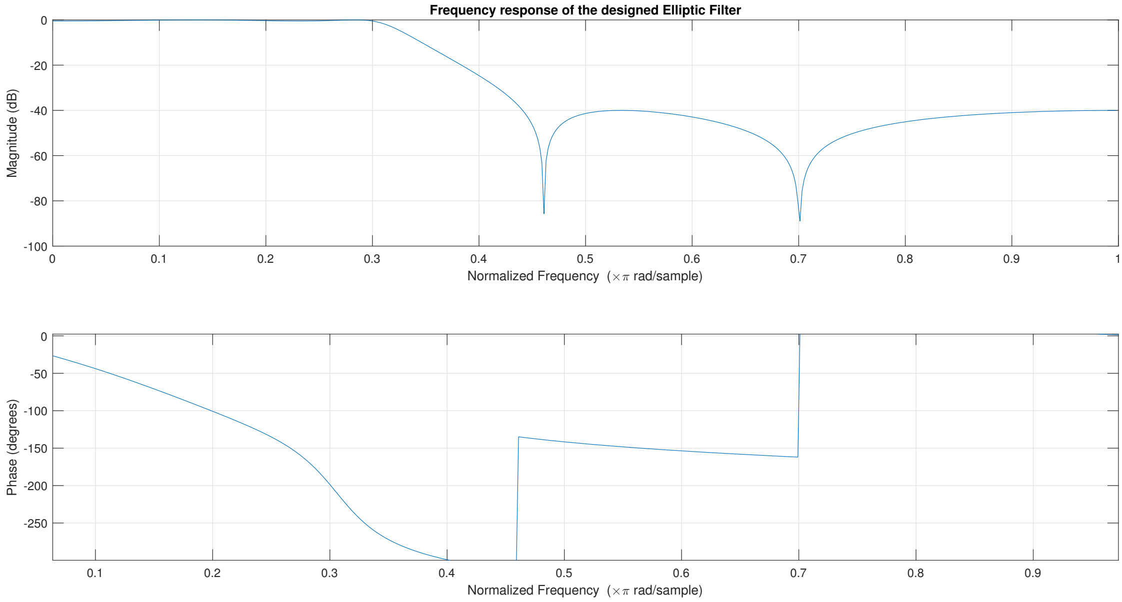

Figure 8: Plot of frequency response of Elliptic filter

```
Order of Elliptic Filter: 4
Cutoff frequencies:
    0.3000

Numerator coefficients of Elliptic Filter:
    0.0389    0.0363    0.0665    0.0363    0.0389

Denominator coefficients of Elliptic Filter:
    1.0000   -2.1444    2.3658   -1.3250    0.3332
```

6 Discussion and Conclusion

In this lab we familiarize ourself with Matlab and its signal processing toolbox and some useful functions like `Impinvar(..)`,`bilinear(..)`, `butter(..)`, `cheby1(..)`, `cheby2(..)`, `ellip(..)`, `buttord(..)`, `cheb2ord(..)`, `ellipord(..)`. We design Digital IIR filter from anaolg filter using impulse varience method and bilinear transformation. We compare the response of the designed digital IIR filter with the response of the analog filter. We also compare the unit sample response of the designed digital IIR filter with the impulse response of analog filter and comared with different values of the sampling rate Ts.