INSTITUTE OF ENGINEERING
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DIGITAL SIGNAL PROCESSING

LAB #6

Design of FIR Digital Filters using Window method

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2 Objective

- To design FIR filter using Window method.

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4 Background

In previous experiment we discussed the most commonly used techniques for design of IIR filters based on transformations of continuous-time IIR systems into discrete time IIR systems. The major difficulty lies in the implementation of the non-iterative direct design method for IIR filters. However FIR filters are almost entirely restricted to discrete-time implementations. Thus the design techniques for FIR filters are based on directly approximating the desired frequency response of the discrete time system. Furthermore, most techniques for approximating the magnitude response of the FIR system assume a linear phase constraint; thereby avoiding the problem of spectrum factorization that complicates the direct design of IIR filters.

The simplest method of FIR design is called the window method. This method generally begins with an ideal desired frequency response Hd(w). The impulse response hd(n) of the filter exhibiting this desired frequency response can be obtained from inverse Fourier transform of Hd(w). However this impulse response exists for n= $-$ inf to $+$ inf and hence truncation is needed to make the finite duration impulse response. The truncation is similar to the multiplication of the hd(n) with the window function w(n). The multiplication in discrete time domain is equivalent to the convolution of the two in frequency domain, which actually gives the frequency response of the truncated FIR filter.

But depending on the tapering of the window to zero at each end, the nature of the window differs. For example, the rectangular window exhibits the most abrupt changes while approaching to zero at each end. For the specific value of length of the window, the rectangular window exhibits main lobe with the greatest width and the lowest side lobe attenuation, than the other windows like Bartlett, Hanning, Hamming, Blackman etc. There are also adjustable windows like Kaiser windows whose windowing function w(n) can be adjusted by changing the value of parameter β according to the stop band attenuation A as given below:

$$\beta = \begin{array}{ll} 0.1102(A-8.7), & if\ A > 50, \\ 0.5842(A-21)^{0.4} + 0.07886(A-21), & if\ 21 \le A \le 50, \\ 0.0, & if\ A < 21, \\ \text{where } A = -20log10\delta. & \end{array}$$

The length of the Kaiser window is given by;

$$M = \frac{(\delta - 7.95)}{(14.36 * \Delta w)} + 1, \quad \text{where } \frac{\Delta w = (w_s - w_p)}{2}$$

MATLAB Signal Processing Toolbox provides built-in functions to determine and plot the different types of windowing functions and their frequency responses. For this the functions like `Bartlett( )`, `Hamming( )`, `Hanning( )`, `Blackman( )` , `Kaiser( )` etc. are available. There is a function `fir1( )` that can be used to obtain the frequency response of the designed FIR filter. Use MATLAB `'Help'` for further information of the functions.

5 Lab Problems

5.1 Problem 1

Design an FIR linear phase digital filter approximating the ideal frequency response

$$H(w) = \begin{cases} 1 & |w| \leq \pi/6 \\ 0 & \pi/6 \leq |w| \leq \pi \end{cases}$$

1. **Plot the window function for Hamming and its frequency response for length of $M = 31$.**
2. **Using the Hamming window plot the frequency response of the truncated FIR filter.**
3. **Repeat parts (a) and (b) for the Hanning, Blackman and Bartlett windows.**
4. **Repeat parts (a), (b) and (d) for filter length of M=61.**

```
f = [0 pi/6 pi/6 pi];
H = [1 1 0 0];
M = 31;
w = 0:pi/30:pi;
subplot(2,2,1);
h = hamming(M);
plot(w/pi,h);
title('31-point symmetric Hamming window');
subplot(2,2,2);
[H1,w] = freqz(h,1,1024);
plot(w/pi,abs(H1));
title('Frequency Response for Hamming window function');
subplot(2,2,3);
B = fir1(M-1,1/6,h);
[H2,w] = freqz(B,1,1024);
plot(w/pi,abs(H2));
ylabel('Amplitude');
xlabel('Radian Frequency');
title('Frequency Response of FIR filter using Hamming window');
gk = 20*log(abs(H2));
subplot(2,2,4);
plot(w/pi,gk);
ylabel('dB');
xlabel('Radian Frequency');



w = 0:pi/30:pi;
figure(2);
h = hanning(M);
subplot(2,2,1);
plot(w/pi,h);

title('31-point symmetric Hanning window');
subplot(2,2,2);
[H1,w] = freqz(h,1,1024);
plot(w/pi,abs(H1));
title('Frequency Response for Hanning window function');
subplot(2,2,3);
B = fir1(M-1,1/6,h);
[H2,w] = freqz(B,1,1024);
plot(w/pi,abs(H2));
ylabel('Amplitude');
xlabel('Radian Frequency');
title('Frequency Response of FIR filter using Hanning window');
```

```
gk = 20*log(abs(H2));
subplot(2,2,4);
plot(w/pi,gk);
ylabel('dB');
xlabel('Radian Frequency');


w = 0:pi/30:pi;
figure(3);
subplot(2,2,1);
h = blackman(M);
plot(w/pi,h);
title('31-point symmetric Blackman window');
subplot(2,2,2);
[H1,w] = freqz(h,1,1024);
plot(w/pi,abs(H1));
title('Frequency Response for Blackman window function');
subplot(2,2,3);
B = fir1(M-1,1/6,h);
[H2,w] = freqz(B,1,1024);
plot(w/pi,abs(H2));
ylabel('Amplitude');
xlabel('Radian Frequency');
title('Frequency Response of FIR filter using Blackman window');
gk = 20*log(abs(H2));
subplot(2,2,4);
plot(w/pi,gk);
ylabel('dB');
xlabel('Radian Frequency');



w = 0:pi/30:pi;
figure(4);
subplot(2,2,1);
h = bartlett(M);
plot(w/pi,h);
title('31-point Bartlett window');
subplot(2,2,2);
[H1,w] = freqz(h,1,1024);
plot(w/pi,abs(H1));
title('Frequency Response for Bartlett window function');
subplot(2,2,3);
B = fir1(M-1,1/6,h);
[H2,w] = freqz(B,1,1024);
plot(w/pi,abs(H2));
ylabel('Amplitude');
xlabel('Radian Frequency');
title('Frequency Response of FIR filter using Bartlett window');
gk = 20*log(abs(H2));
subplot(2,2,4);
plot(w/pi,gk);
ylabel('dB');
xlabel('Radian Frequency');



figure(5);
M = 61;
w = 0:pi/60:pi;
subplot(2,2,1);
h = hamming(M);
plot(w/pi,h);
title('61-point symmetric Hamming window');
subplot(2,2,2);
```

```
[H1,w] = freqz(h,1,1024);
plot(w/pi,abs(H1));
title('Frequency Response for Hamming window function');
subplot(2,2,3);
B = fir1(M-1,1/6,h);
[H2,w] = freqz(B,1,1024);
plot(w/pi,abs(H2));
ylabel('Amplitude');

xlabel('Radian Frequency');
title('Frequency Response of FIR filter using Hamming window');
gk = 20*log(abs(H2));
subplot(2,2,4);
plot(w/pi,gk);
ylabel('dB');
xlabel('Radian Frequency');
```

Code 1: Matlab code for FIR linear phase digital filter approximating the ideal frequency response

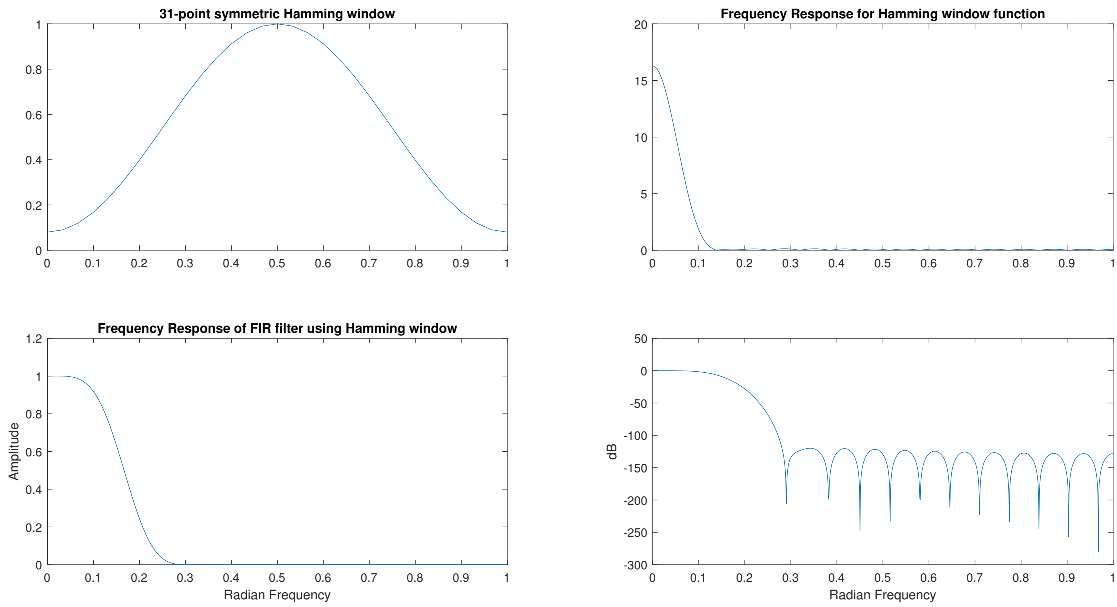


Figure 1: Plot window function for Hamming window and its frequency response

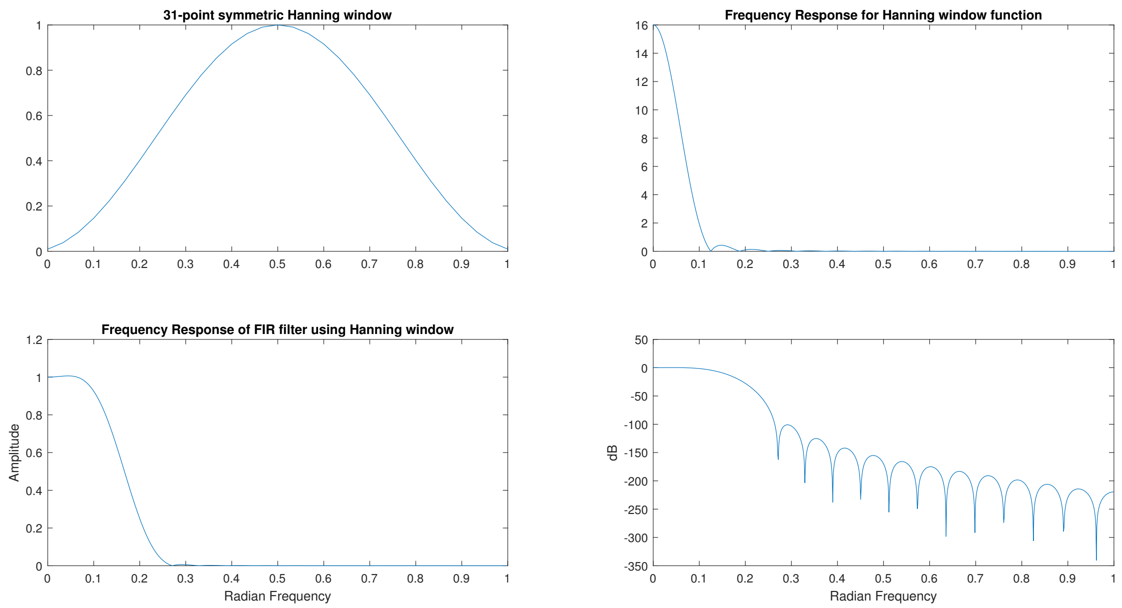


Figure 2: Plot window function for Hanning window and its frequency response

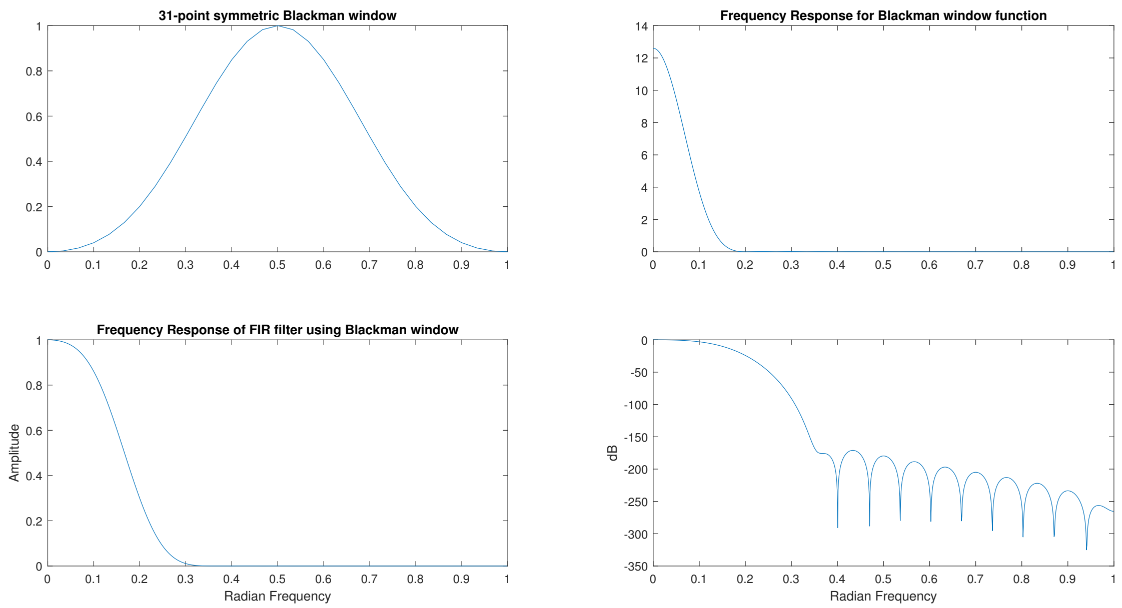


Figure 3: Plot window function for Blackman window and its frequency response

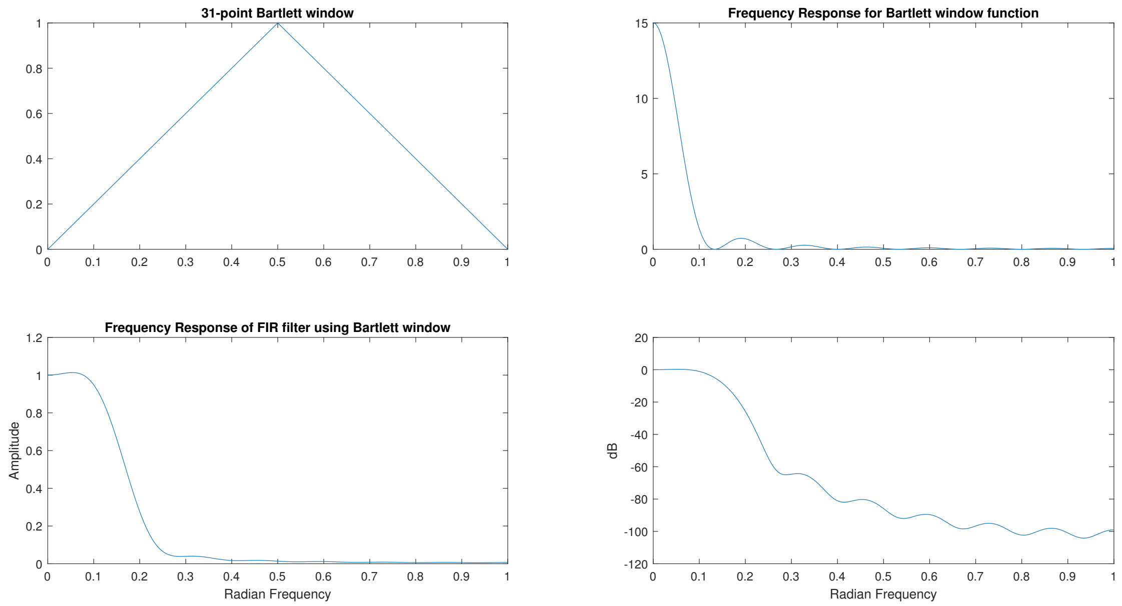


Figure 4: Plot window function for Bartlett window and its frequency response

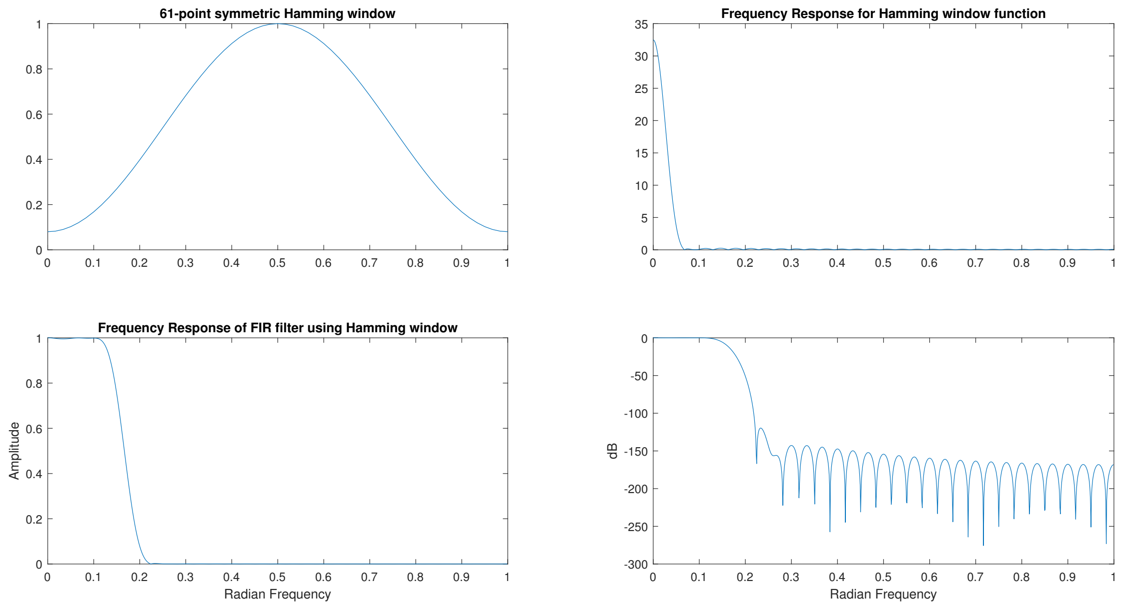


Figure 5: Plot window function for Hamming window and its frequency response for length of $M = 61$

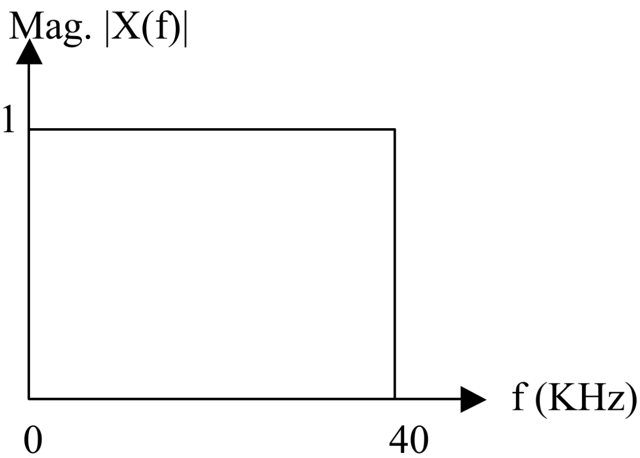
5.2 Problem 2

Discuss the effects of different types of the windowing functions on the frequency response of the FIR filter. Carry out the comparative study on the basis of the peak side-lobe level, the approximate transition width of the main lobe etc. Also discuss on the effects of increasing value of M.

From the observations in the above graphs, the effect of using a different kinds of windowing functions are seen on the main lobe transition band and the side lobes in the frequency response of the FIR filter. Here, it can be noted that there is a tradeoff between the width of the transition band and the attenuation of the side lobes when the windowing function is varied. In the case of Hamming window, the transition band is slightly increased but has reduced side lobes in comparison to the Hanning window. The Blackman window has very small side lobes but has an increased width of the main lobe in comparison to other windows. Bartlett window has significant side lobes but a lower transition band. The effect of increasing the size of the window results in an increment of the number of side lobes and a decrement in the width of the transition band.

5.3 Problem 3

An analog signal $x(t)$ consists of the sum of two components $x_1(t)$ and $x_2(t)$. The spectral characteristics of $x(t)$ is shown in the figure. The signal $x(t)$ is band limited to 40 kHz and is sampled at the rate of 100 kHz to yield the sequence $x[n]$. It is desired to suppress the



signal $x_2(t)$ by passing the sequence $x[n]$ through a digital low pass filter. The allowable distortion on $|X_1(f)|$ is $\pm 2\%$ ($\delta_1 = 0.02$) over the range $0 \leq |f| \leq$ 15 kHz. Above 20 kHz, the filter must have an attenuation of at least 40 DB ($\delta_2 = 0.01$).

- **a** **For obtaining the filter with above specifications, use the Kaiser window and determine the length of the required window. Plot the frequency response of the filter and its impulse response also.**
- **b** **If the same filter is to be designed using Hamming window what would be the length of the required window. Plot the frequency and impulse response of the filter.**

```
wsample = 100;
wp = 15;
ws = 20;
wpn = 2*wp/wsample;
wsn = 2*ws/wsample;
wn = (wpn + wsn)/2;
Rs = 40;
beta = 0.5842.*(Rs-21)^0.4+0.07886.*(Rs-21);
dw = (wsn - wpn)/2;
```

```
M = ((Rs-7.95)./(14.36*dw)) + 1;
N = round(M);
Kaiser_window_length = N
wk = kaiser(N,beta);
bk = fir1(N-1,wn,wk);
[H,w] = freqz(bk,1,512);
subplot(2,1,1);
plot(w/pi,abs(H));
title('Frequency Response of FIR filter using Kaiser Window');
subplot(2,1,2);
plot(w/pi,20*log10(abs(H)));
xlabel('radian frequency');
ylabel('dB');

figure('Name','impulse response');
nk = 0:N-1;
dimpulse(nk,bk);
title('Impulse Response of the FIR filter');
```

Code 2: Matlab code for obtaining filter with Kaiser window and its frequency response

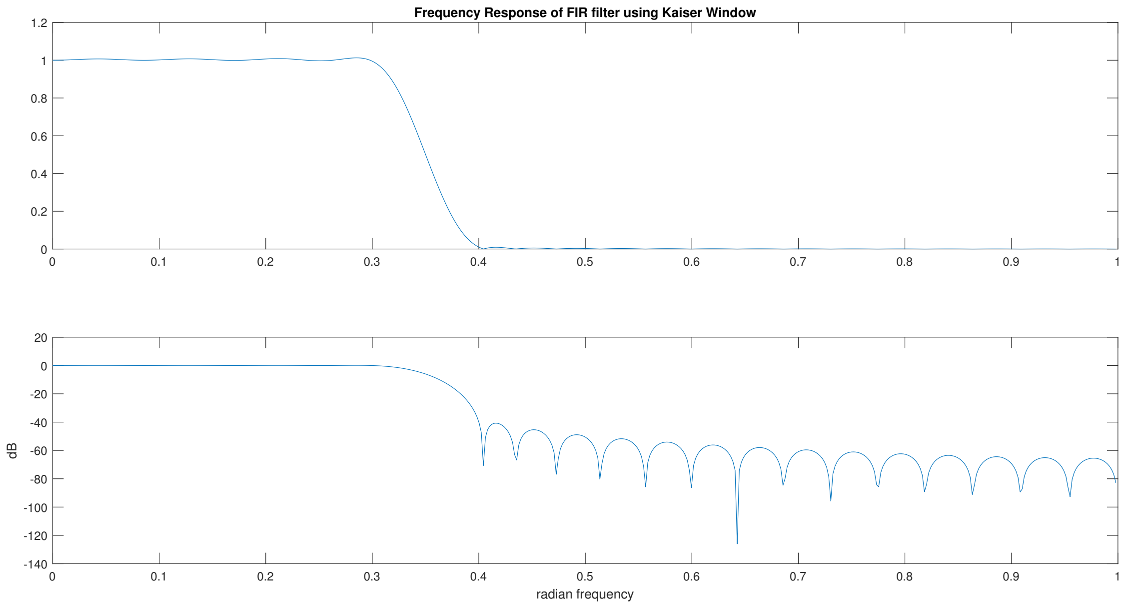


Figure 6: Plot for Frequency Response of the FIR filter using Kaiser window

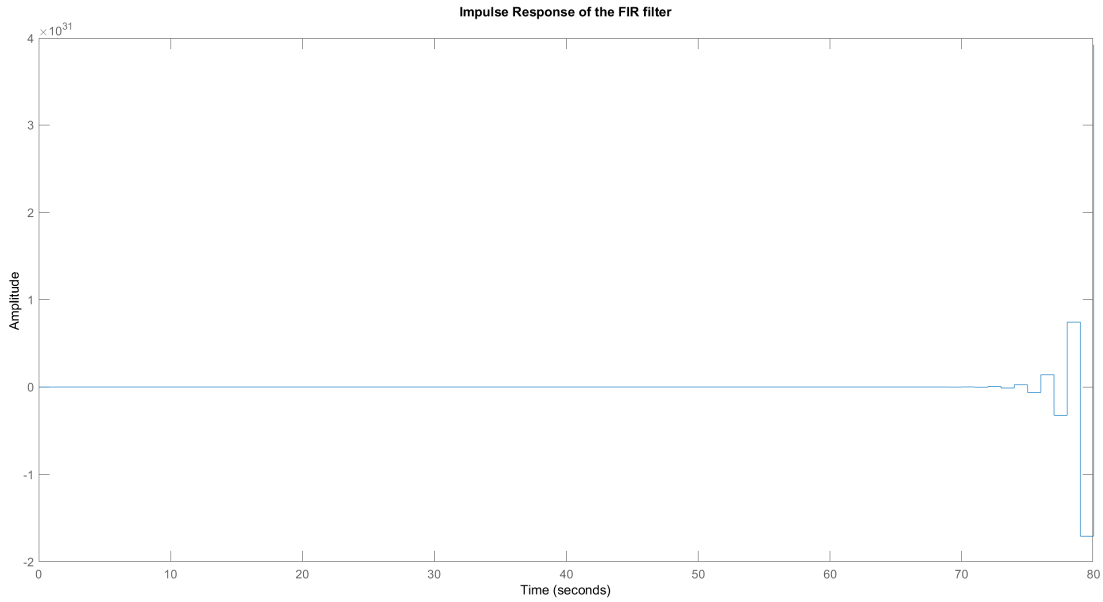


Figure 7: Plot for Impulse Response of the FIR filter

6 Discussion and Conclusion

In this lab, we have discussed the different types of windowing functions and their effects on the frequency response of the FIR filter.We explore these properties using various MATLAB functions for the signal processing. We have also discussed the effects of different types of the windowing functions on the frequency response of the FIR filter. We have also discussed the effects of increasing value of M. The effects of using different kinds of windowing functions are seen on the main lobe transition band and the side lobes in the frequency response of the FIR filter. Here, it can be noted that there is a tradeoff between the width of the transition band and the attenuation of the side lobes when the windowing function is varied. Finally we explore Kaiser window and its frequency response.