INSTITUTE OF ENGINEERING
CENTRAL CAMPUS,PULCHOWK

FILTER DESIGN

LAB #4

DESIGN OF ACTIVE FILTER USING TOW THOMAS BIQUAD CIRCUIT

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July 31, 2021

Table of Contents

List of Figures

1 Title

DESIGN OF ACTIVE FILTER USING TOW THOMAS BIQUAD CIRCUIT

2 Objective

- To design a low pass filter using Tow Thomas biquad circuit from given specifications.
- To obtain bandpass, highpass, bandstop and allpass filter using Tow Thomas Biquad circuit.

3 Requirement

3.1 Proteus Design Suite

Proteus is a simulation and design software tool developed by Labcenter Electronics for Electrical and Electronic circuit design.It is used to create schematic of a circuit and Visualization of its operation.

4 Exercises:

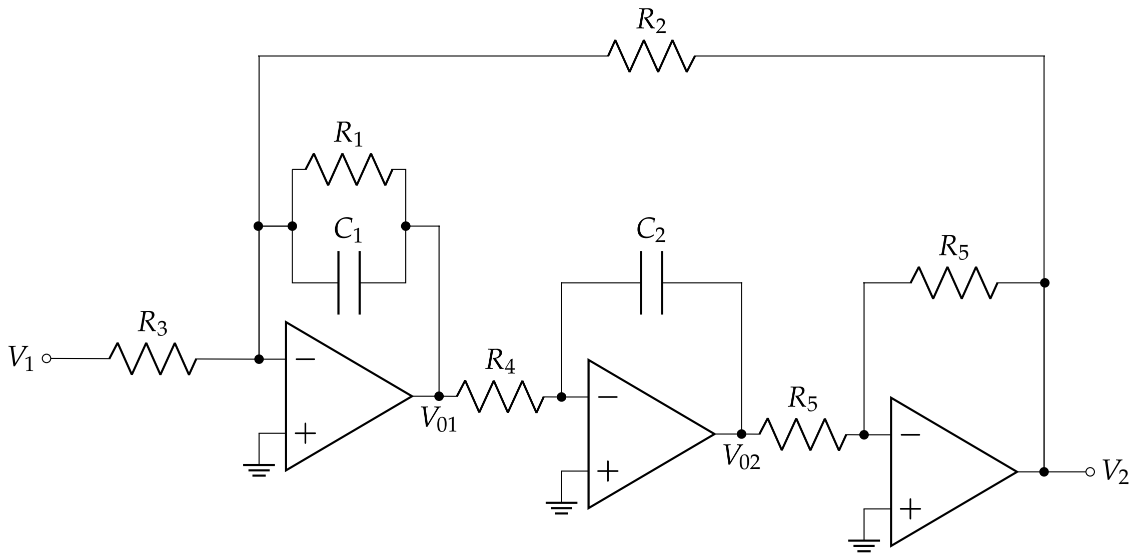


Figure 1: Tow Thomas biquad circuit

4.1 Question -1

From the circuit given in above figure:

1. **Derive the transfer function V_2/V_1 and determine the nature of the filter while taking output at V_2.**
2. **Also obtain the transfer function while observing output at V_{01} and input at V_1.**
3. **How can you obtain a band-stop, high-pass and all-pass filter using the Tow Thomas biquad circuit? Derive the transfer functions with necessary circuit diagrams.**

The whole circuit is divided into Lossy integrator (Summer), inverting integrator and unit gain inverter.

Lossy Integrator

If we assume Z is the total impedance obtained due to the parallel combination of R_1 and C_1, then,

$$\frac{1}{Z} = \frac{1}{R_1} + sC_1 = \frac{1 + sR_1C_1}{R_1}$$
$$\Rightarrow Z = \frac{R_1}{1 + sR_1C_1}$$

From Figure 2, we can write,

$$V_{01} = -\left(\frac{Z}{R_3}\right) V_1 - \left(\frac{Z}{R_2}\right) V_2 = -Z\left(\frac{V_1}{R_3} + \frac{V_2}{R_2}\right)$$
$$\Rightarrow V_{01} = -\left(\frac{R_1}{1 + sR_1C_1}\right)\left(\frac{V_1}{R_3} + \frac{V_2}{R_2}\right)$$

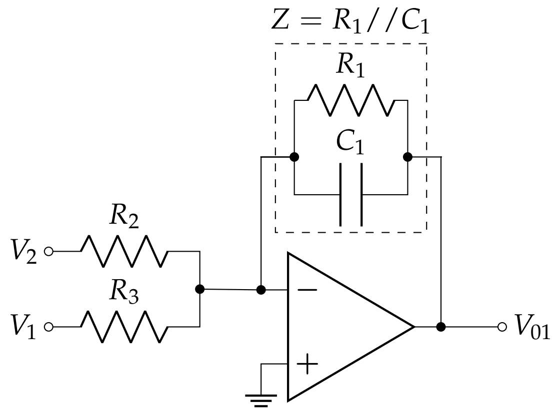


Figure 2: Lossy integrator (summer)

Inverting integrator

$$V_{02} = -\frac{V_{01}}{sR_4C_2}$$

Subsstuting value of V_{01} we get,

$$\Rightarrow V_{02} = \left(\frac{R_1}{1+sR_1C_1}\right)\left(\frac{1}{sR_4C_2}\right)\left(\frac{V_1}{R_3}+\frac{V_2}{R_2}\right)$$

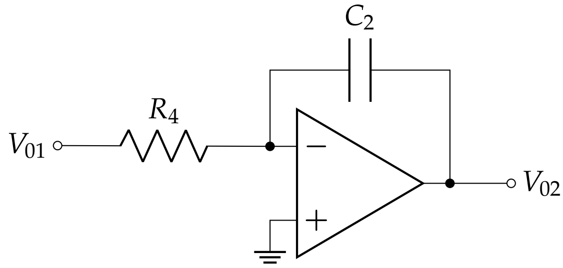


Figure 3: Inverting integrator

Unity gain inverter

From Figure 4, we can write,

$$V_2 = -\left(\frac{R_5}{R_5}\right)V_{02} = -V_{02}$$

Substituting value of V_{02}, we get,

$$V_2 = -\left(\frac{R_1}{1+sR_1C_1}\right)\left(\frac{1}{sR_4C_2}\right)\left(\frac{V_1}{R_3}+\frac{V_2}{R_2}\right)$$
$$V_2 = \frac{-V_1R_1}{sR_3R_4C_2(1+sR_1C_1)} - \frac{V_2R_1}{sR_2R_4C_2(1+sR_1C_1)}$$
$$V_2\left(1+\frac{R_1}{sR_2R_4C_2(1+sR_1C_1)}\right) = \frac{-V_1R_1}{sR_3R_4C_2(1+sR_1C_1)}$$
$$\frac{V_2}{V_1} = \frac{-R_1R_2}{(sR_2R_4C_2(1+sR_1C_1)+R_1)R_3}$$
$$\frac{V_2}{V_1} = \frac{-R_1R_2}{sR_2R_3R_4C_2(1+sR_1C_1)+R_1R_3}$$
$$\frac{V_2}{V_1} = \frac{-R_1R_2}{s^2R_1R_2R_3R_4C_1C_2+sR_2R_3R_4C_2+R_1R_3}$$

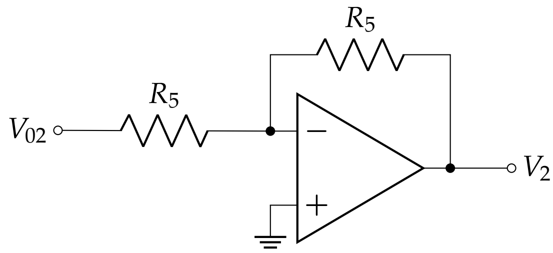


Figure 4: Unit gain inverter

$$\frac{V_2}{V_1} = \frac{-\left(\frac{R_1R_2}{R_1R_2R_3R_4C_1C_2}\right)}{s^2\left(\frac{R_1R_2R_3R_4C_1C_2}{R_1R_2R_3R_4C_1C_2}\right)+s\left(\frac{R_2R_3R_4C_2}{R_1R_2R_3R_4C_1C_2}\right)+\left(\frac{R_1R_3}{R_1R_2R_3R_4C_1C_2}\right)}$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{-\left(\frac{1}{R_3R_4C_1C_2}\right)}{s^2+s\left(\frac{1}{R_1C_1}\right)+\left(\frac{1}{R_2R_4C_1C_2}\right)}$$

Comparing to standard equation for the transfer function of a lowpass filter we can conclude that above equation is low pass filter ,

$$T_{LP}(s) = \frac{-H\omega_o^2}{s^2+s\left(\frac{\omega_o}{Q}\right)+\omega_o^2}$$

Transfer function while observing output V_{01} and input V_1

$$\frac{V_{01}}{V_1} = \left(\frac{V_{01}}{V_{02}}\right)\left(\frac{V_{02}}{V_2}\right)\left(\frac{V_2}{V_1}\right)$$

Substituting value we get,

$$\frac{V_{01}}{V_1} = (-sR_4C_2)(-1)\left(\frac{-\left(\frac{1}{R_3R_4C_1C_2}\right)}{s^2+s\left(\frac{1}{R_1C_1}\right)+\left(\frac{1}{R_2R_4C_1C_2}\right)}\right)$$

$$\frac{V_{01}}{V_1} = \frac{-s\left(\frac{R_4C_2}{R_3R_4C_1C_2}\right)}{s^2+s\left(\frac{1}{R_1C_1}\right)+\left(\frac{1}{R_2R_4C_1C_2}\right)}$$

$$\Rightarrow \frac{V_{01}}{V_1} = \frac{-\left(\frac{1}{R_3C_1}\right)s}{s^2+s\left(\frac{1}{R_1C_1}\right)+\left(\frac{1}{R_2R_4C_1C_2}\right)}$$

Compare with standard equation for the transfer function of a bandpass filter we can conclude that above equation $\frac{V_{01}}{V_1}$ is band pass filter ,

$$\Rightarrow T_{BP}(s) = \frac{H\left(\frac{\omega_o}{Q}\right)s}{s^2+s\left(\frac{\omega_o}{Q}\right)+\omega_o^2}$$

Bandstop using Tow Thomas biquad circuit

In addition to Figure 1, we need an inverting op-amp where V_1 and V_{01} are applied as input and V_2' as output voltage.

$$V_2' = -(V_1+V_{01})$$

$$\Rightarrow \frac{V_2'}{V_1} = -\left(1+\frac{V_{01}}{V_1}\right)$$

Substituting value of $\frac{V_{01}}{V_1}$, we get,

$$\text{or, } \frac{V_2'}{V_1} = -\left(1+\frac{-\left(\frac{1}{R_3C_1}\right)s}{s^2+s\left(\frac{1}{R_1C_1}\right)+\left(\frac{1}{R_2R_4C_1C_2}\right)}\right)$$

$$\text{or, } \frac{V_2'}{V_1} = -\left(\frac{s^2+s\left(\frac{1}{R_1C_1}\right)+\frac{1}{R_2R_4C_1C_2}-\left(\frac{1}{R_3C_1}\right)s}{s^2+s\left(\frac{1}{R_1C_1}\right)+\left(\frac{1}{R_2R_4C_1C_2}\right)}\right)$$

$$\Rightarrow \frac{V_2'}{V_1} = -\left(\frac{s^2 + s\left(\frac{1}{R_1C_1} - \frac{1}{R_3C_1}\right) + \frac{1}{R_2R_4C_1C_2}}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)} \right)$$

Comparing with standard equation for the transfer function of a bandstop filter and substituting values we get,

$$T_{BS}(s) = \frac{-H(s^2 + \omega_o^2)}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2} \Rightarrow T_{BS}(s) = \frac{V_2'}{V_1} = -\left(\frac{s^2 + \frac{1}{R_2R_4C_1C_2}}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)} \right)$$

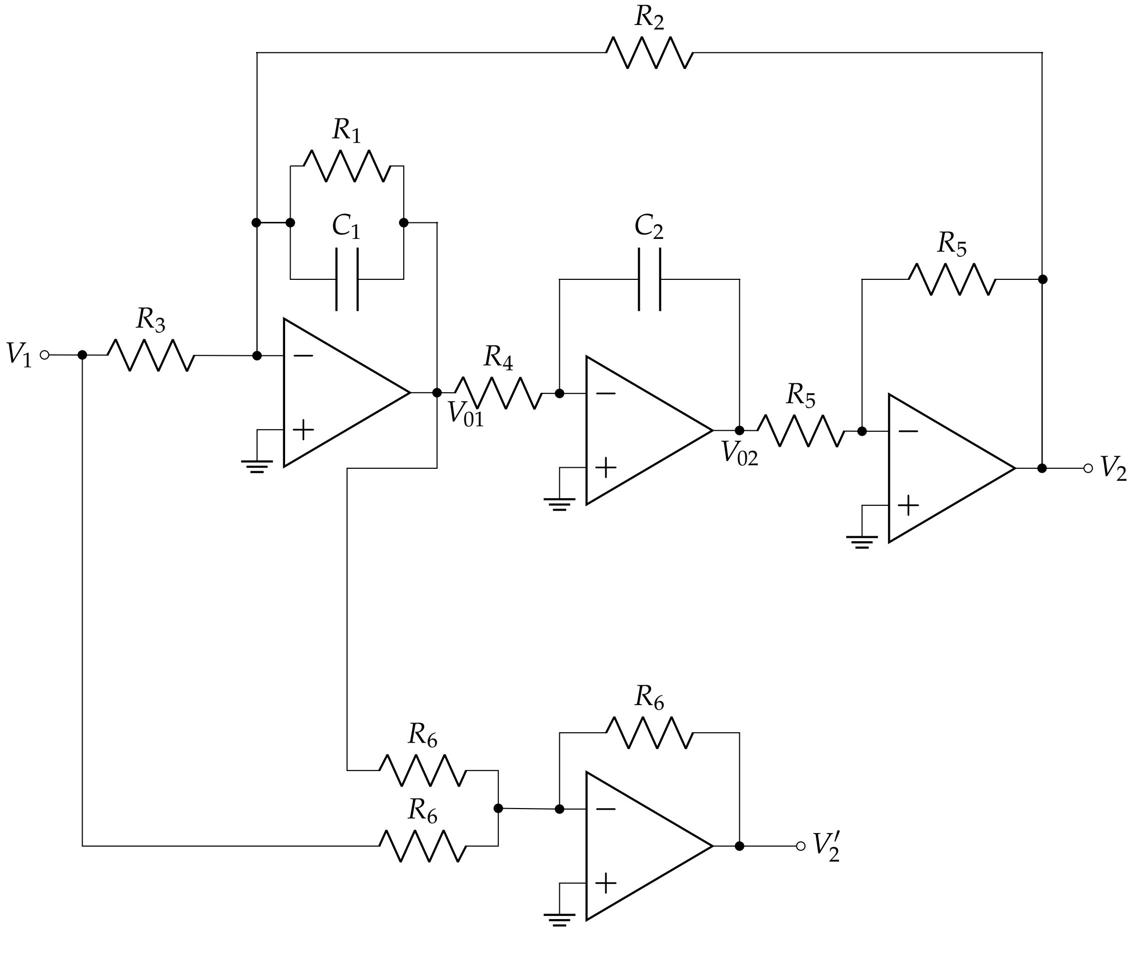


Figure 5: Biquad circuit for both Bandstop and Allpass Filters

Allpass using Tow Thomas biquad circuit

Comparing standard equation for the transfer function of an allpass filter with $\frac{V_2'}{V_1}$ we get two values for $\frac{\omega_o}{Q}$.

$$T_{AP}(s) = \frac{H\left(s^2 - \left(\frac{\omega_o}{Q}\right)s + \omega_o^2\right)}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}$$

$$-\left(\frac{1}{R_1C_1} - \frac{1}{R_3C_1}\right) = \frac{1}{R_1C_1} \Rightarrow \frac{2}{R_1C_1} = \frac{1}{R_3C_1} \Rightarrow R_3 = \frac{R_1}{2}$$

Thus $\frac{V_2'}{V_1}$ can be in form of standard transfer function of Allpass filter, if $R_3 = \frac{R_1}{2}$ satisfied and final transfer function is,

$$\Rightarrow T_{AP}(s) = \frac{V_2'}{V_1} = -\left(\frac{s^2 - \left(\frac{1}{R_1C_1}\right)s + \frac{1}{R_2R_4C_1C_2}}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}\right) \tag{1}$$

Highpass using Tow Thomas biquad circuit

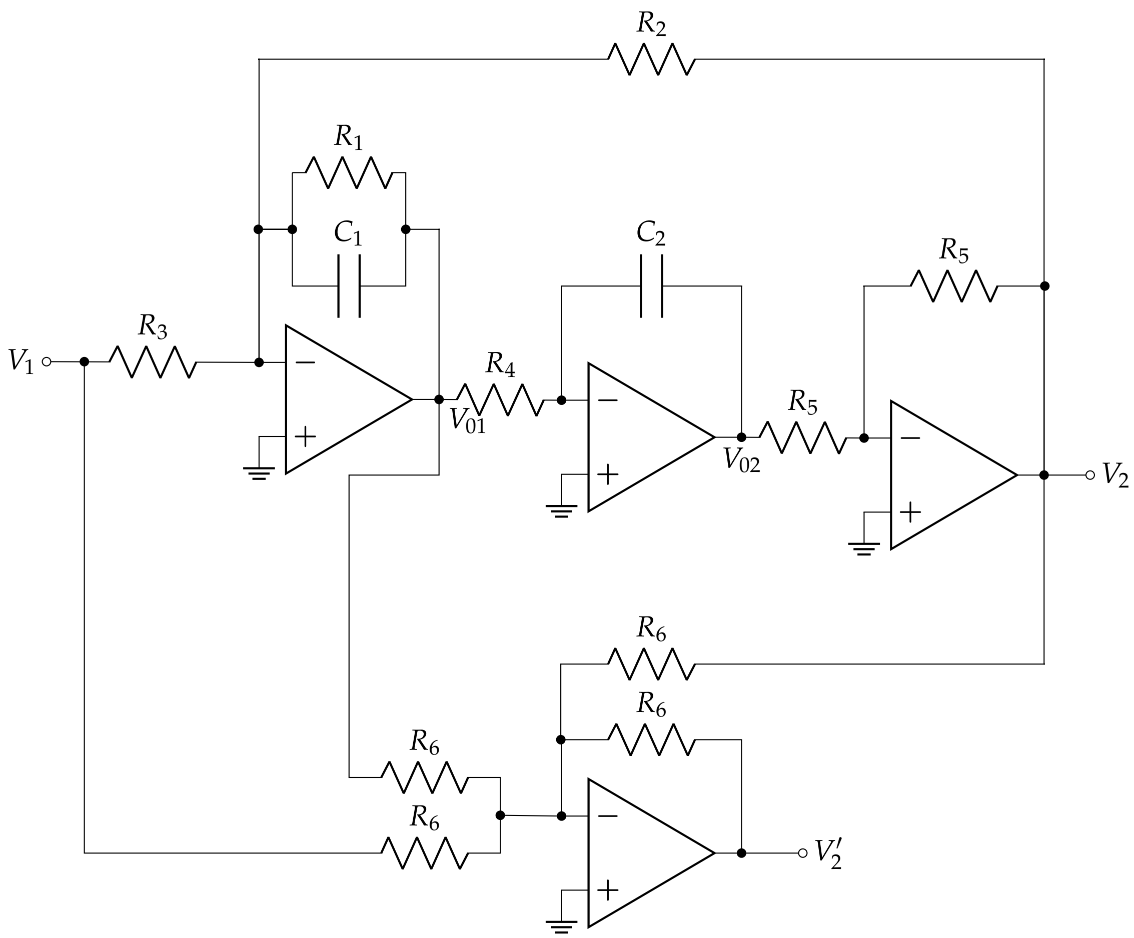


Figure 6: Four op-amp biquad circuit for highpass filter

In addition to Figure 1, we need an inverting op-amp where V_1, V_2 and V_{01} are applied as input and V_2' as output voltage.

$$V_2' = -(V_1 + V_2 + V_{01})$$
$$\Rightarrow \frac{V_2'}{V_1} = -\left(1 + \frac{V_2}{V_1} + \frac{V_{01}}{V_1}\right)$$

Substituting values of $\frac{V_2}{V_1}$ and $\frac{V_{01}}{V_1}$ we get,

$$\text{or, } \frac{V_2'}{V_1} = -\left(1 - \frac{\left(\frac{1}{R_3R_4C_1C_2}\right) + \left(\frac{1}{R_3C_1}\right)s}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}\right)$$

$$\text{or, } \frac{V_2'}{V_1} = -\left(\frac{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right) - \left(\frac{1}{R_3R_4C_1C_2}\right) - \left(\frac{1}{R_3C_1}\right)s}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}\right)$$

$$\Rightarrow \frac{V_2'}{V_1} = -\left(\frac{s^2 + \left(\frac{1}{R_1C_1} - \frac{1}{R_3C_1}\right)s + \left(\frac{1}{R_2R_4C_1C_2} - \frac{1}{R_3R_4C_1C_2}\right)}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}\right)$$

Thus $\frac{V_2'}{V_1}$ can be in form of standard transfer function of High passs filter, if $R_1 = R_2 = R_3$ satisfied and final transfer function is,

$$T_{HP}(s) = \frac{-Hs^2}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}$$

$$\Rightarrow T_{HP}(s) = \frac{V_2'}{V_1} = -\left(\frac{s^2}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}\right)$$

4.2 Question -2

Design a low-pass filter having poles at – 5000 ± j8660.25404 and a DC gain of 2 using Tow-Thomas biquad circuit. Your final circuit should consist of practically realizable elements. Realize the circuit and observe the magnitude response. And determine the characteristic features such as 3 dB frequency and DC gain.

1. **Determine the nature of the response by observing output at V_{01} with input V_1.**
2. **Observe the magnitude response by obtaining each of the following filter from your design and note down passband gain and half power frequencies:**
   (a) **High-pass filter**
   (b) **Band-stop filter and**
   (c) **All-pass filter**

Given:

$s = -\alpha \pm j(\beta) = -5000 \pm j8660.25404$

$$\alpha = 5000 = (\frac{w_o}{2Q})$$
$$\beta = 8660.25404$$

$$\omega_o = \sqrt[2]{(\alpha)^2 + (\beta)^2} = \sqrt[2]{10000} = 10000 rad/sec$$

$$\alpha = \frac{w_o}{2Q} \Rightarrow Q = \frac{\omega_o}{2\alpha}$$
$$Q = \frac{10000}{2 * 5000} = 1$$

$$Gain(H) = 2$$

Comparing below the two equation of low pass filter

$$\frac{V_2}{V_1} = \frac{-\left(\frac{1}{R_3R_4C_1C_2}\right)}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}$$

$$\text{and } T_{LP}(s) = \frac{-H\omega_o^2}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}$$

we get, $H = \frac{R_2}{R_3}$

$$\text{We assume,} \Omega_o = 1 rad/sec, R_4 = 1\Omega and C_1 = C_2 = 1F,$$
$$\text{or, } \Omega_o^2 = \frac{1}{R_2R_4C_1C_2}$$
$$\Rightarrow R_2 = \frac{1}{\Omega_o^2 R_4C_1C_2} = 1\Omega$$

$$Q = \sqrt{\frac{R_1^2C_1}{R_2R_4C_2}} = \sqrt{R_1^2} = R_1$$
$$\Rightarrow R_1 = Q = 1\Omega$$

$$H = \frac{R_2}{R_3}$$
$$\text{or, } 2 = \frac{1}{R_3}$$
$$\Rightarrow R_3 = 0.5\Omega$$

Since we have to design the filter at $\omega_o = 10000 rad/sec$ and $K_f = \frac{10000}{1} = 10000$

We assume, $K_m = 10000$(in order to obtain practically realizable values)

$$R_1 = 10K\Omega$$
$$R_2 = R_5 = 10K\Omega$$
$$R_3 = 5K\Omega$$
$$R_4 = 10K\Omega$$
$$R_5 = 10K\Omega$$
$$C_1 = C_2 = \frac{1}{K_m * K_f} = 0.1nF$$

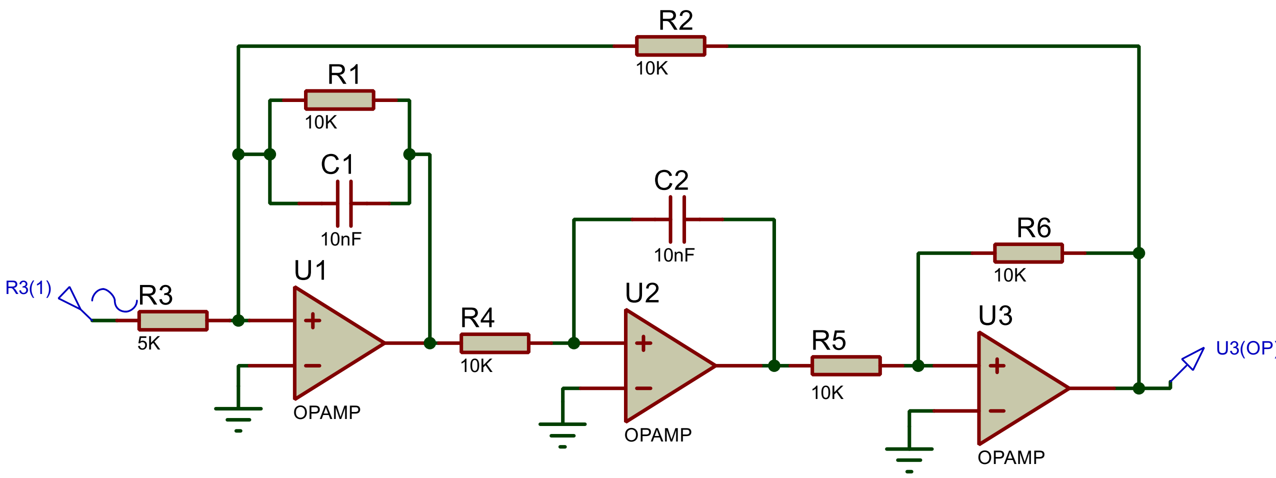
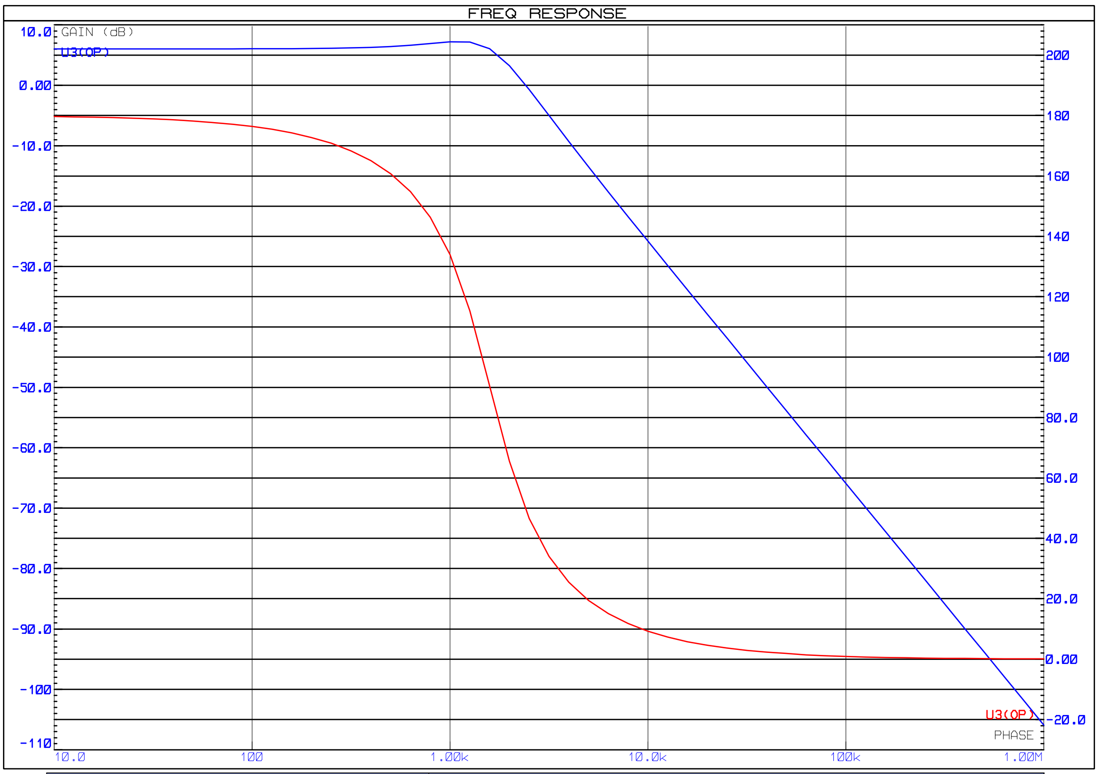


Figure 7: Proteus Circuit Figure output at V2



Gain	Half power frequency
6.02 dB	(2.02) KHz

Figure 8: Proteus Observation for output at V2

Response when output is at V_{01} and input is V_1

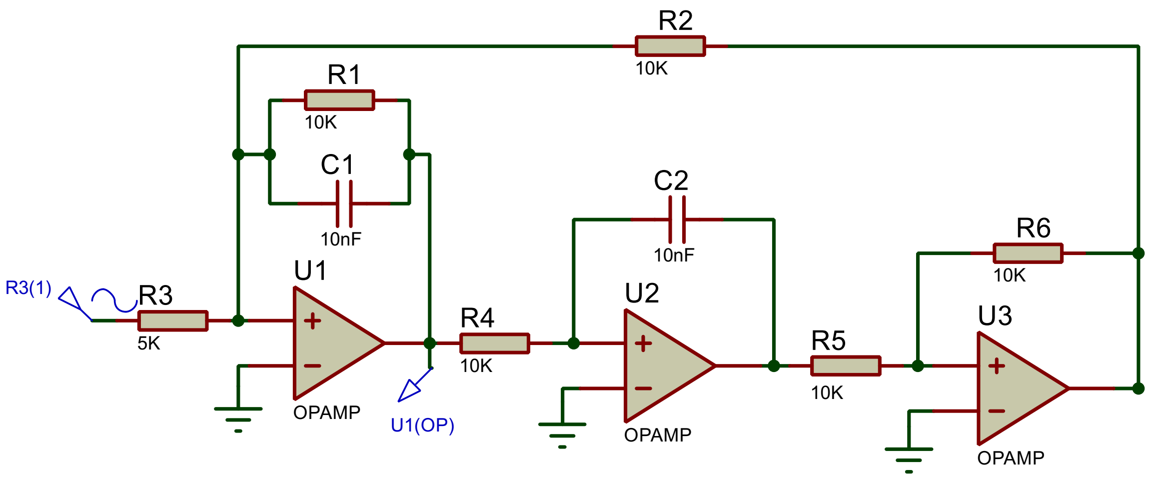
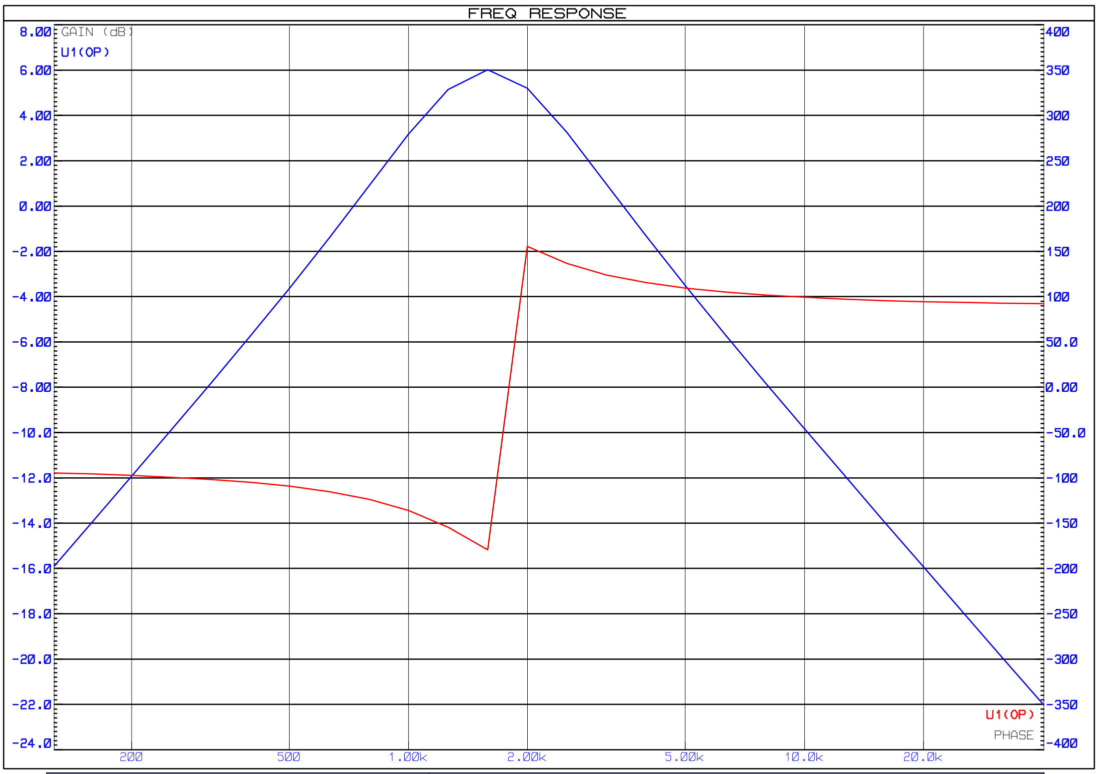


Figure 9: Proteus Circuit Figure output at V01



Gain	Half power frequency
6.02 dB	(0.98 and 2.55) KHz

Figure 10: Proteus Observation for output at V01

Thus the bandwidth is : 1.57 KHz.

High-pass Filter

Here, $R_1 = R_2 = R_3 = 10K\Omega$ and fourth op-amp is needed as explained earlier. Additionally R_6 is also needed having value $10K\Omega$.

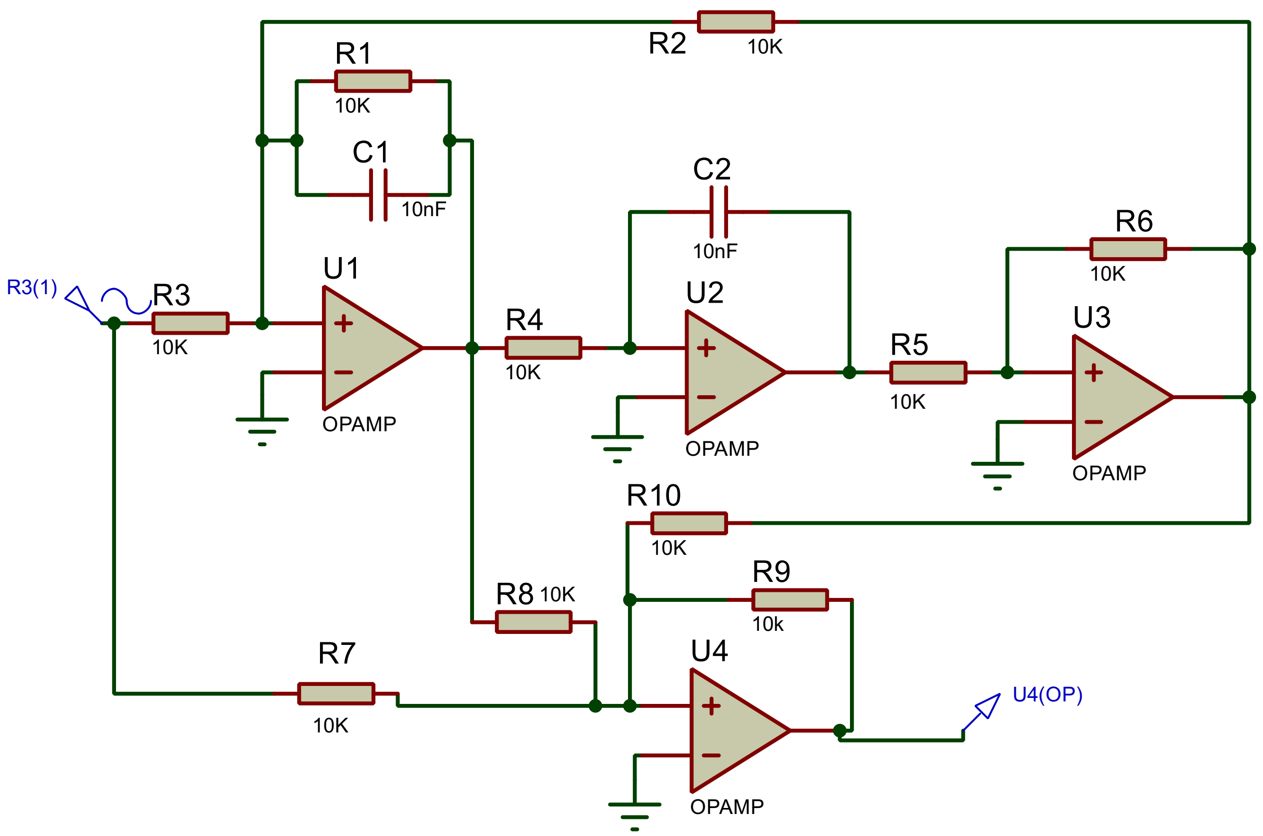
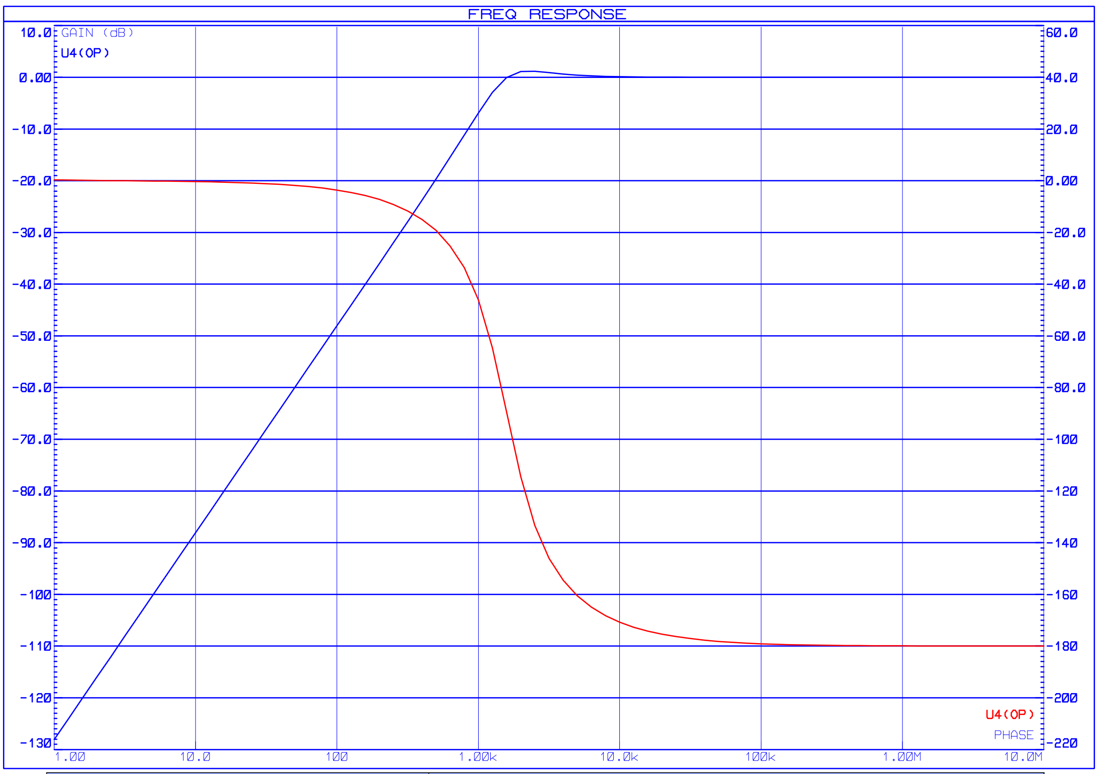


Figure 11: Proteus Circuit Figure High pass



Gain	Half power frequency
0 dB	(1.26) KHz

Figure 12: Proteus Observation for High pass

Band-Stop Filter

Here, $R_1 = R_3 = 10K\Omega$ and fourth op-amp is needed as explained earlier. Additionally R_6 is also needed having value $10K\Omega$.

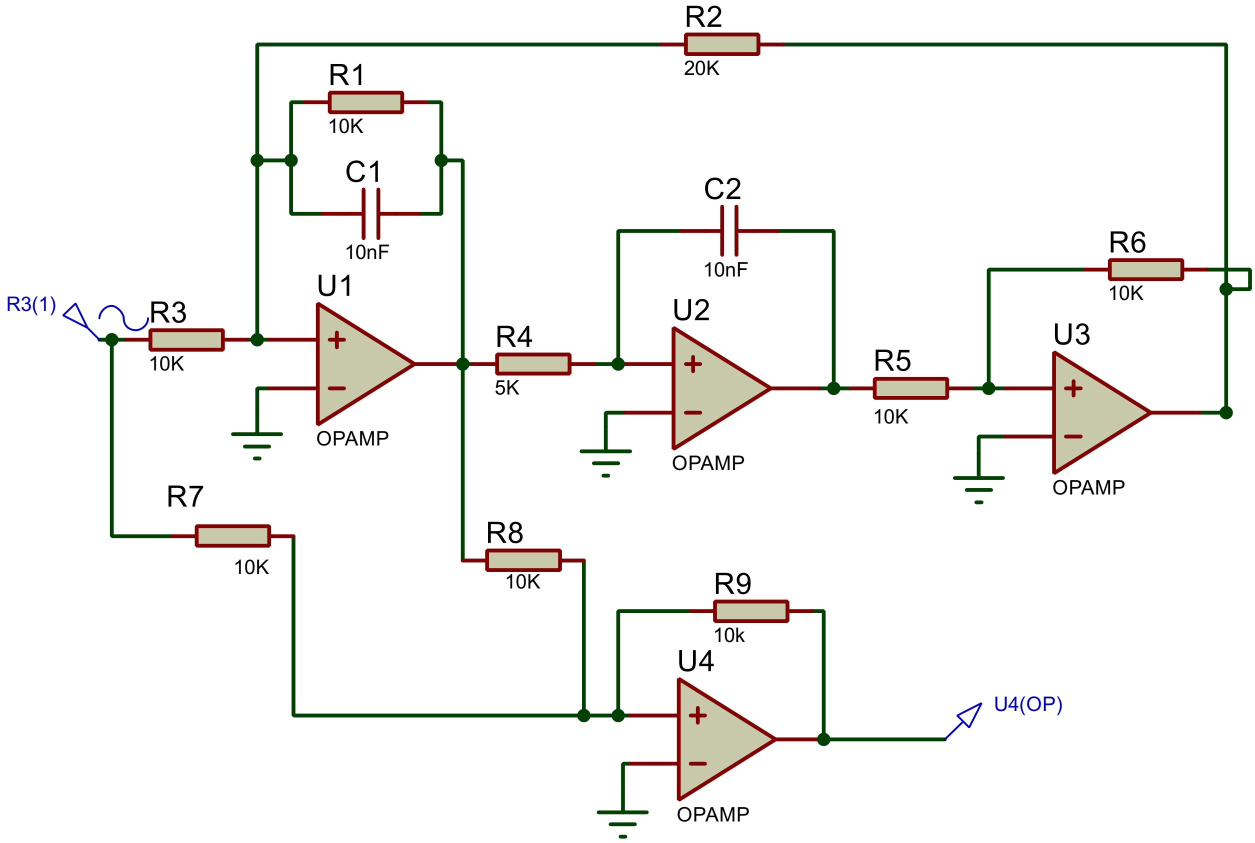
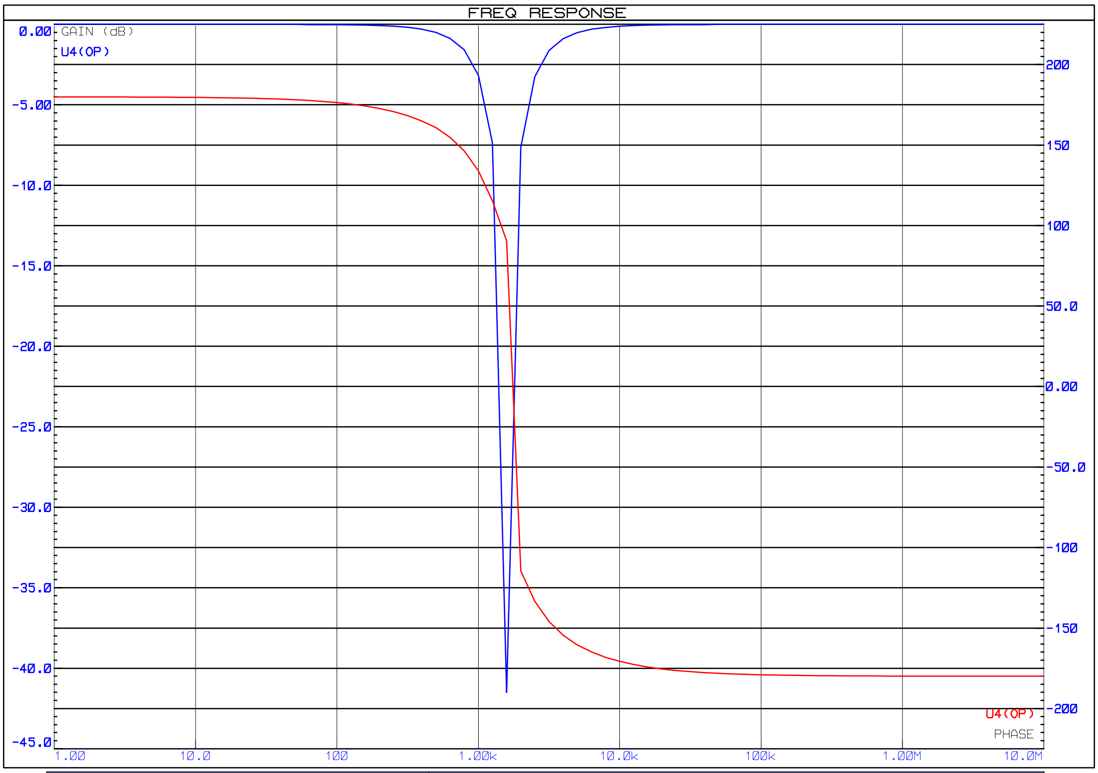


Figure 13: Proteus Circuit Figure Band stop



Gain	Half power frequency
0 dB	(0.979 and 2.521) KHz

Figure 14: Proteus Observation for Band stop

Thus the bandwidth is : 1.542 KHz.

All-pass Filter

Here, $R_3 = R_1/2 = 5K\Omega$ and fourth op-amp is needed as explained earlier. Additionally R_6 is also needed having value $10K\Omega$.

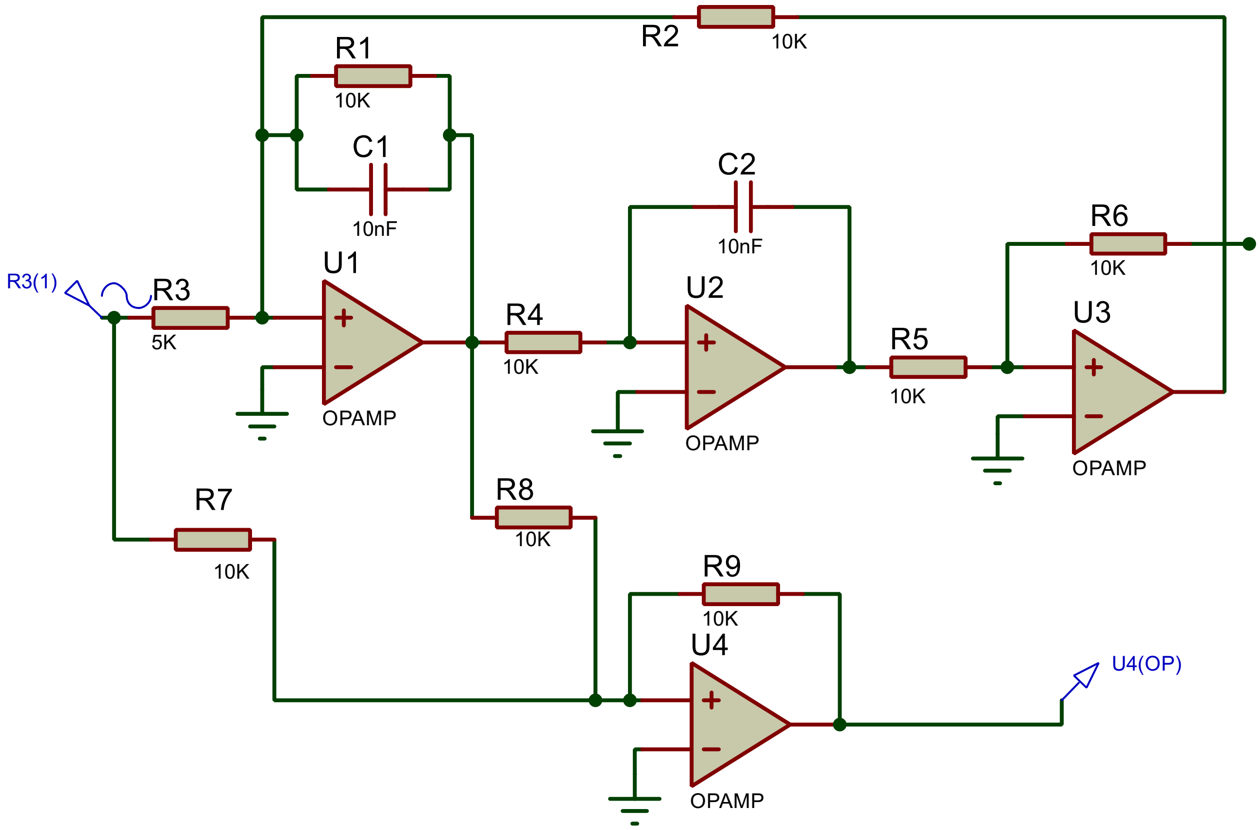
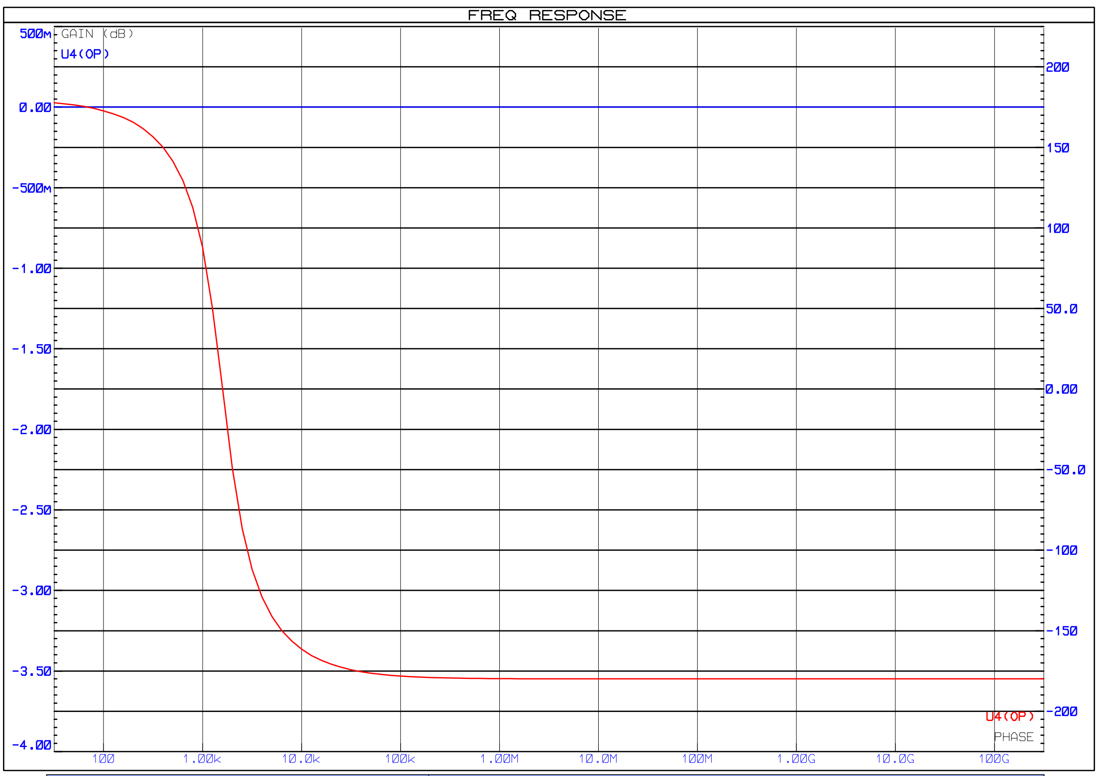


Figure 15: Proteus Circuit Figure All pass



Gain	Half power frequency
26.1 micro dB	(0) KHz

Figure 16: Proteus Observation for All pass

4.3 Question -3

Realize the second order Butterworth lowpass filter having half power frequency of 3.5 kHz using the Tow Thomas biquad circuit and observe the response. By plotting the response show the half power frequency and DC gain.

Comparing below the two equation of low pass filter with the second order Butterworth equation for low pass filter $T_{LP}(s) = -\frac{1}{s^2 + 1.414 * s + 1}$

$$\frac{V_2}{V_1} = \frac{-\left(\frac{1}{R_3R_4C_1C_2}\right)}{s^2 + s\left(\frac{1}{R_1C_1}\right) + \left(\frac{1}{R_2R_4C_1C_2}\right)}$$

$$\text{and } T_{LP}(s) = \frac{-H\omega_o^2}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}$$

We get $\omega_0 = 1rad/sec$ and $H = \frac{R_2}{R_3}$ we assume $R_4 = 1\Omega$ and $C_1 = C_2 = 1F$,

$$w_o^2 = \frac{1}{R_2R_4C_1C_2}$$
$$\Rightarrow R_2 = \frac{1}{w_o^2R_4C_1C_2} = 1\Omega$$

$$R_1 = 1/1.414 = 0.71\Omega$$
$$R_3 = 1\Omega$$
$$For, \omega_0 = 3.5KhzK_f = 22000rad/sec$$
$$let, K_m = 10000$$

$$R_1 = 710\Omega$$
$$R_2 = R_5 = 10K\Omega$$
$$R_3 = 10K\Omega$$
$$R_4 = 10K\Omega$$
$$C_1 = C_2 = \frac{1}{K_m * K_f} = 4.54nF$$

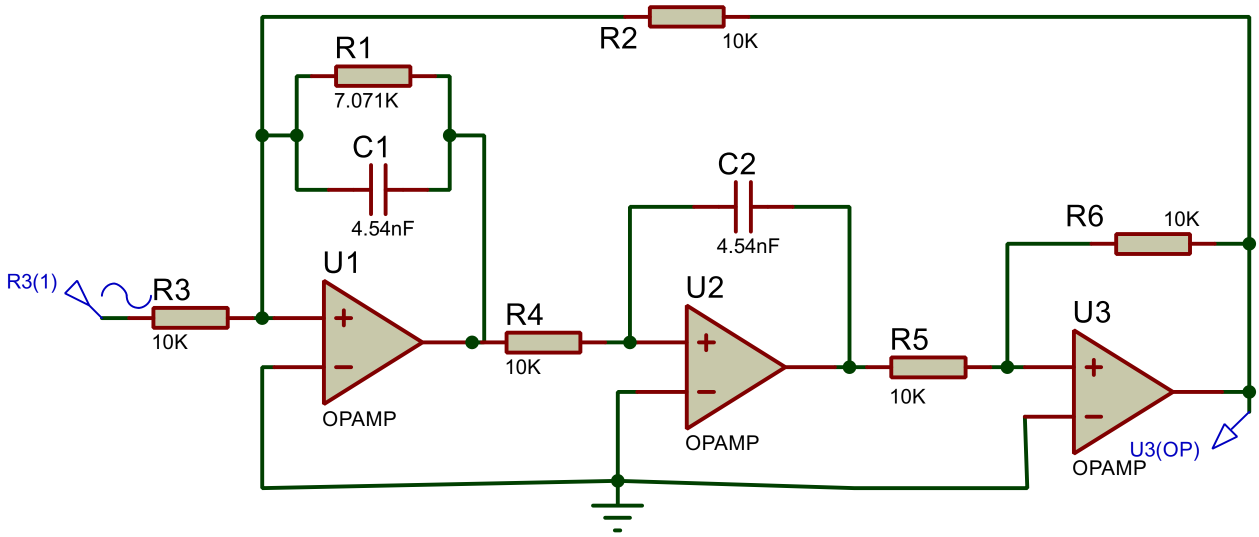
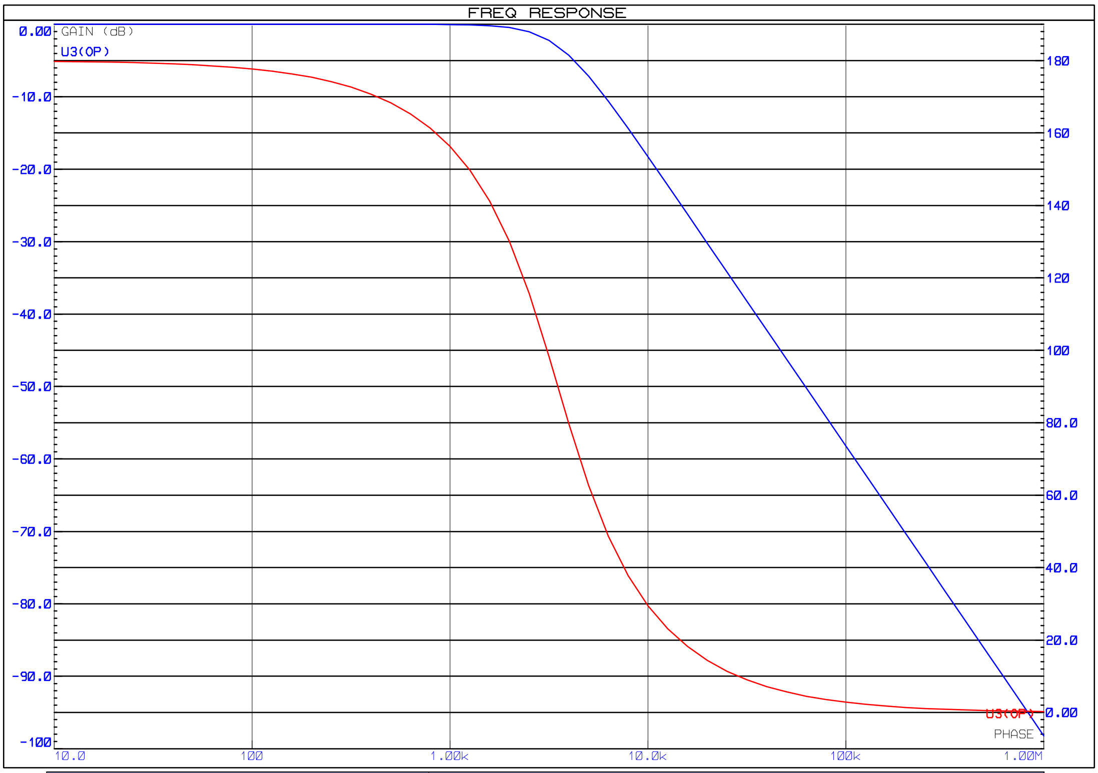


Figure 17: Proteus Circuit Figure Butterworth



Gain	Half power frequency
0 dB	(3.49) KHz

Figure 18: Proteus Observation for Butterworth

5 Discussion & Conclusion

In this Lab we design active filter using Tow Thomas Biquad Circuit. We used Proteus design suite to design and plot frequency response for the Low pass, High pass , all pass andBand stop filter. We also derive transfer function for these filters hence fulfilling our objectives of lab.