INSTITUTE OF ENGINEERING
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FILTER DESIGN

LAB #2

DESIGN OF FILTER USING SCALING & FREQUENCY TRANSFORMATION

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DESIGN OF FILTER USING SCALING & FREQUENCY TRANSFORMATION

2. Objective

2.1. To be familiar with magnitude & frequency scaling
2.2. To be familiar with frequency transformation
2.3. To be familiar with the design of the required filter from the given normalized filter circuit

3. Requirement

3.1. Proteus Design Suite

Proteus is a simulation and design software tool developed by Labcenter Electronics for Electrical and Electronic circuit design. It is used to create a schematic of a circuit and Visualization of its operation.

4. Theory

4.1. Scaling

4.1.1. Impedance Scaling

Impedance scaling is the technique of scaling Impedances present in the circuit keeping the Frequency response constant. An impendence $|Z(j\omega)|$ is said to be magnitude scaled by a factor of K_m if it is multiplied by a
real positive constant K_m. Magnitude is scaled up if K_m>1 and scaled-down if K_m<1.

For Registor,Capacitor and Inductor Impedance scaling is performed below:

- **Resistor**
  Before scaling, $|Z_R|_{old}= R_{old}$
  After scaling by scale factor $\mathbf{K_m}$, we get
  $$K_m.\ |Z_R| = K_m.\ R_{old}$$
  $$\therefore\ R_{new} = K_m\ .\ R_{old}$$

- **Inductor**
  Before Scaling $|Z_L|= \omega.\ L_{old}$
  After scaling by scale factor $\mathbf{K_m}$, we get
  $$K_m.\ |Z_L| = K_m.\omega.\ L_{old}$$
  $$|Z_L|_{new} = \omega.(K_m\ .\ L_{old})$$
  $$\therefore L_{new} = K_m.L_{old}$$

- **Capacitor**
  Before scaling $|Z_c|_{old} = 1/\,(\omega.\ C_{old})$
  After scaling by scale factor $\mathbf{K_m}$, we get

$K_m.|Z_c| = K_m/(\omega.\ C_{old})$
$|Z_C|_{new} = 1/(\omega.\ C_{new})$

$\therefore\ C_{new} = 1/(\omega.\ |Z_C|_{new})$
$= C_{old}/K_m$

4.1.2. Frequency Scaling

Frequency scaling shifts the frequency response of the network in frequency axis keeping the impedance of the network constant. In frequency scaling only values of Capacitor and Inductor is scaled by factor K_f but keeping resistor same.

When old frequency ω is scaled by K_f new frequency $\Omega = \omega.K_f$

For Registor, Capacitor and Inductor Frequency scaling is performed below:

- **Resistor**

  $\therefore\ R_{new} = |Z_R|_{old} = R_{old}$

- **Inductor**

  $|Z_L|_{old} = \omega.\ L_{old} = |Z_L|_{new}$
  $= K_f.\omega.\ L_{old}/K_f$
  $= \Omega.\ L_{new}$
  $\therefore\ L_{new} = L_{old}/K_f$

- **Capacitor**

  $|Z_c|_{old} = \frac{1}{(\omega.Cold)}$
  $= \frac{1}{(\Omega.Cnew)}$
  $\therefore\ C_{new} = \frac{C}{Kf}$

Combining both Imepance and Frequency scaling, we get,

$R_{new} = K_m.\ R_{old}$
$L_{new} = (\frac{Km}{Kf}) * L_{old}$
$C_{new} = \frac{Cold}{(K_m.Kf)}$

4.2. Frequency Transformation

Frequency Transformation is a technique of transforming one filter to other. For example, transforming Low pass filter to the High pass filter.

4.2.1. Low Pass too High Pass

For a low pass filter $H_{LP}(s)$ having passband $0 < \omega < 1$

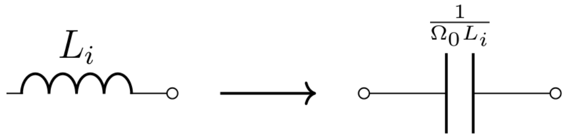
Using relation $\tilde{s} = (\frac{\Omega_0}{s})$ to convert to High pass

Now the high pass network is :

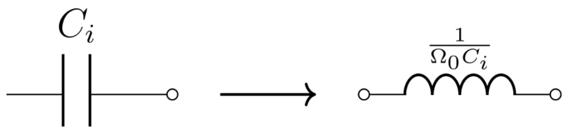
$$H_{HP}(s) = H_{LP}(\frac{\Omega_0}{s})$$

Transformation of Inductor and capacitor is given below:

$$|Z_L|_{HP} = \frac{1}{s\left(\frac{1}{\Omega_0 L_i}\right)}$$



$$|Z_C|_{HP} = s\left(\frac{1}{\Omega_0 C_i}\right)$$



4.2.2. Low Pass to Band Pass

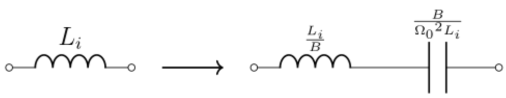
For a low pass filter $H_{LP}(s)$ having passband $0< \omega <1$

Using relation $\tilde{s}=\left(\frac{s^2+\Omega_0^2}{Bs}\right)$ to convert to Band pass.

Now the band pass network is :

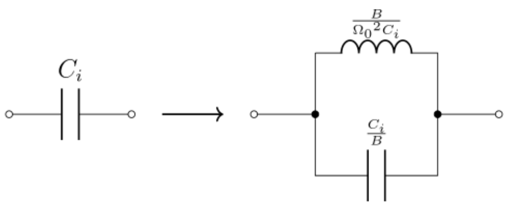
$$H_{BP}(s)= H_{LP}\left(\frac{s^2+\Omega_0^2}{Bs}\right)$$

Transformation of Inductor and capacitor is given below:



$$|Z_L|_{BP} = s\left(\frac{L_i}{B}\right) + \frac{1}{s\left(\frac{B}{\Omega_0^2 L_i}\right)}$$

$$|Y_C|_{BP} = s\left(\frac{C_i}{B}\right) + \frac{1}{s\left(\frac{B}{\Omega_0^2 C_i}\right)}$$



4.2.3. Low Pass to Band Stop

For a low pass filter $H_{LP}(s)$ having passband $0< \omega <1$

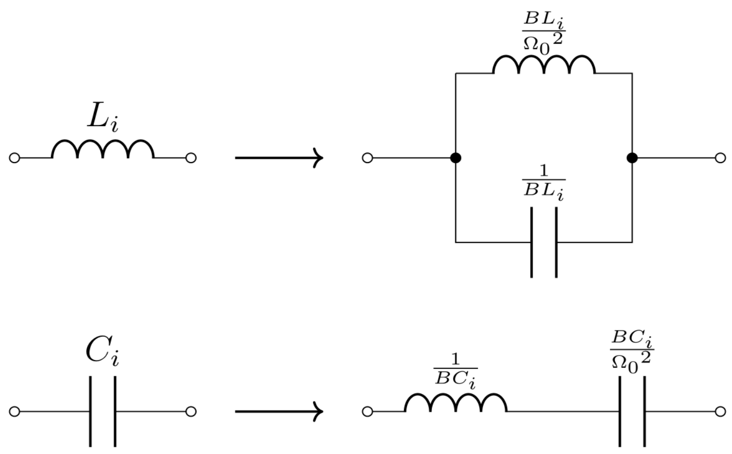
Using relation $\tilde{s}=\left(\frac{Bs}{s^2+\Omega_0^2}\right)$ to convert to Bandstop

Now the band stop network is :

$$H_{BS}(s)= H_{LP}\left(\frac{Bs}{s^2+\Omega_0^2}\right)$$

Transformation of Inductor and capacitor is given below:

$$|Z_C|_{BS} = s\left(\frac{1}{BC_i}\right) + \frac{1}{s\left(\frac{BC_i}{\Omega_0^2}\right)} \quad |Y_L|_{BS} = s\left(\frac{1}{BL_i}\right) + \frac{1}{s\left(\frac{BL_i}{\Omega_0^2}\right)}$$

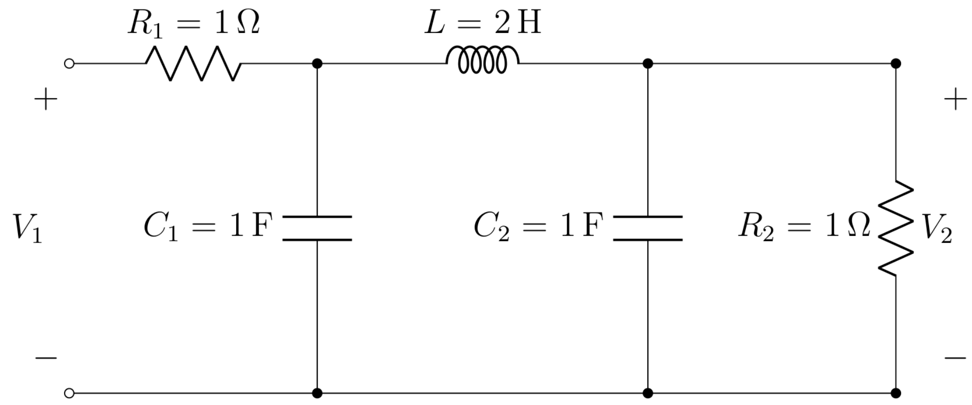


5. Exercise

5.1. Question 1

From the circuit given in figure 1, design a lowpass filter having half power frequency of 15915.4571 Hz. The inductance in your final circuit must be of 30 mH. Realize the circuit, observe and plot the magnitude response. Also show the highest gain in dB and half power frequency in your plot.

- **Observe the output in the oscilloscope when the square wave of 100 Hz is applied at input.**
- **Observe the output by increasing frequency upto 15 KHz. Comment on the result.**



Calculating the value of K_f

$$K_f = \frac{new\ frequency}{old\ frequency} = \frac{159.15.4571 \times 2\pi}{1} = 1.0*10^5$$

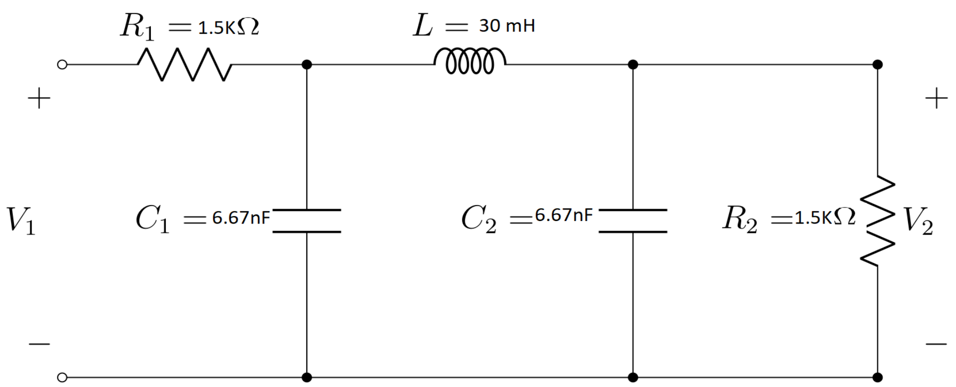
Here,Required $L_{new} = 30mH$ *and* $L_{old} = 2mH$

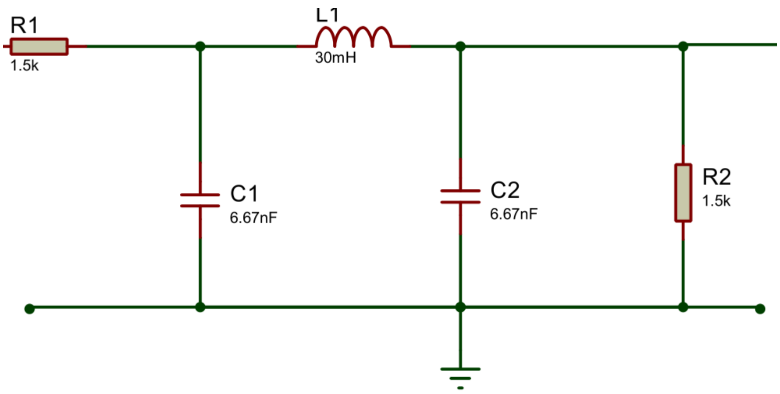
We know, $K_m = ((K_f.L_{new}) / (L_{old}) = \frac{100000 \times 0.03}{2}\ \ 1500$

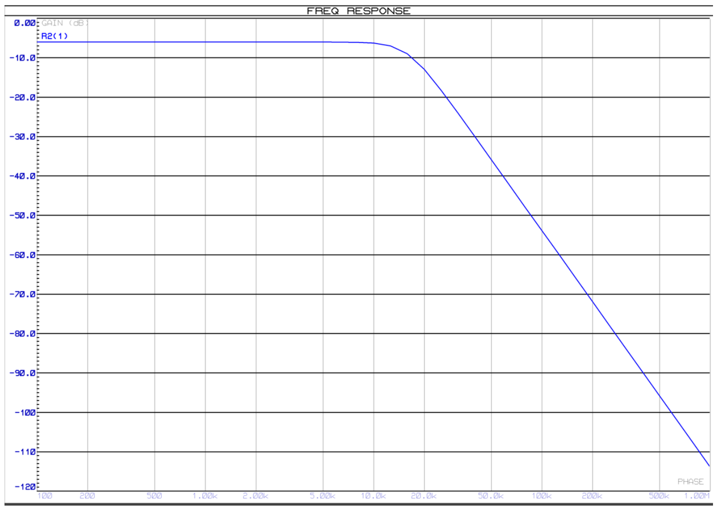
Hence, $R_{new} = K_m.R = 1.5\ K\Omega$

$$C_{new} = \frac{C_{old}}{K_m \times K_f} = \frac{1}{1500 \times 100000} = 6.67nF$$

Substituting new values of R, L & C we get







Highest gain = -6.02 dB
Half power frequency = 15.8 KHz

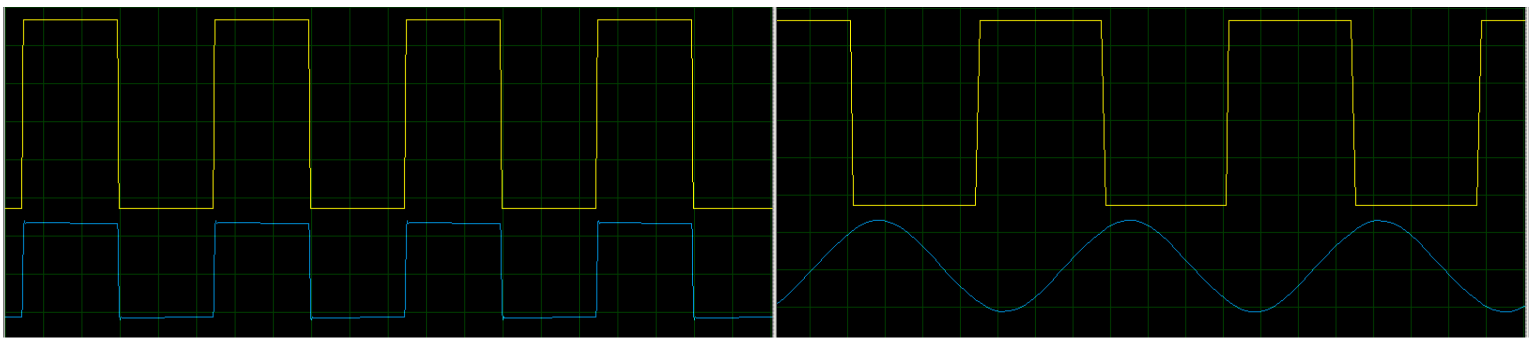


Fig: Output waveform when input is 100 Hz and 15 KHz respectively.

For 100 Hz the output is still square wave but for 15KHz the output is sinusoidal.

5.2. Question 2

From the circuit given in figure 1, design a high pass filter having ω0 of 50000 rad/sec. The inductance in your final circuit must be of 60 mH. Realize the circuit and plot the magnitude response. Also show the highest gain in dB and half power frequency in your plot.

- **Observe the output in an oscilloscope when the square wave of 100 Hz is applied at input.**
- **Observe the output by increasing frequency upto 10 KHz. Comment on the result.**

Here Inductor L is transformed into a capacitor with a capacitance of $\frac{1}{\Omega_0 L}$ and similarly, capacitor C_1 and C_2 are transformed into inductors L_1 and L_2 with inductances of $\frac{1}{\Omega_0 C_1}$ and $\frac{1}{\Omega_0 C_2}$ respectively

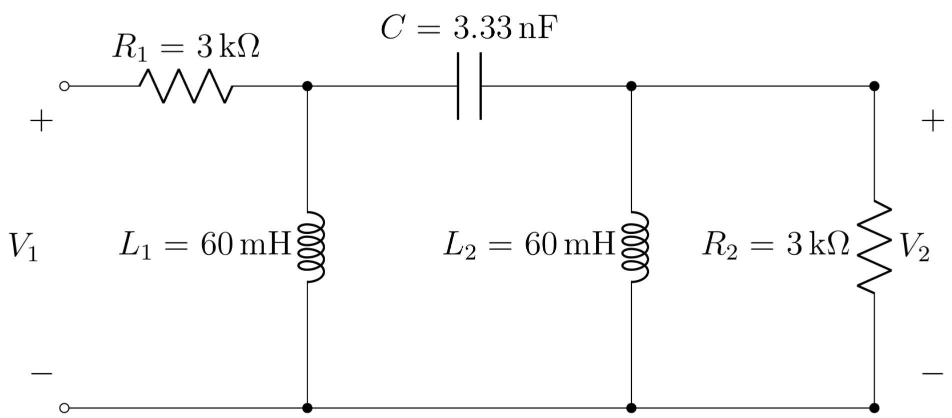
Frequency scaling factor $\Omega_0 = \frac{\omega}{\omega_n} = \frac{50000}{1} = 50000$

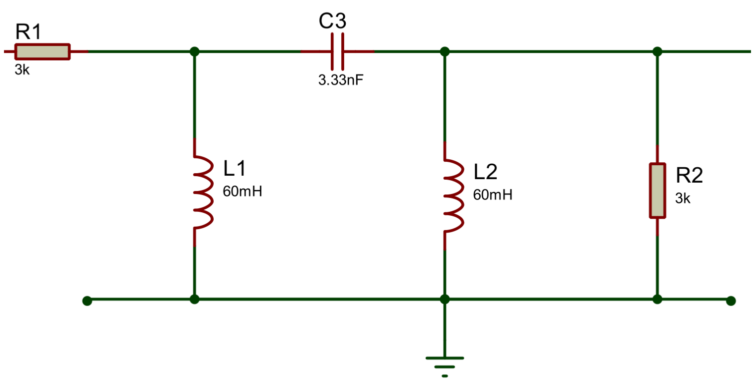
So, Here, $L_{new} = 60mH$

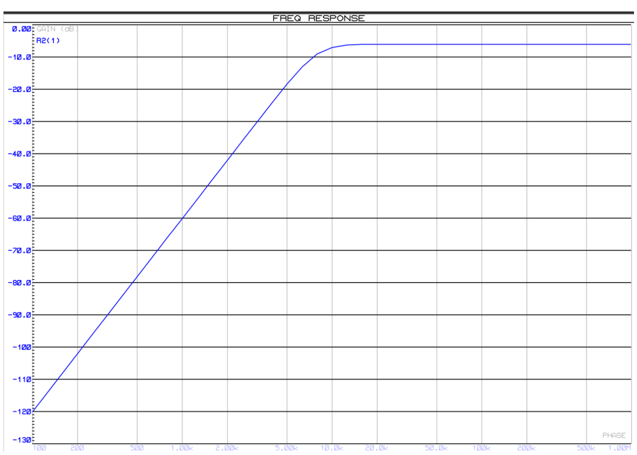
We know, $K_m = (L_{new} * \Omega_0 * C_1) = 0.06 * 50000 * 1 = 300$

Hence, $R_{new} = K_m.R = 300\Omega$

And $C_{new} = \frac{1}{K_m \omega_0 L} = \frac{1}{300*50000*2} = 33.33nF$







Highest gain = -6.02 dB
Half power frequency = 7.93 KHz

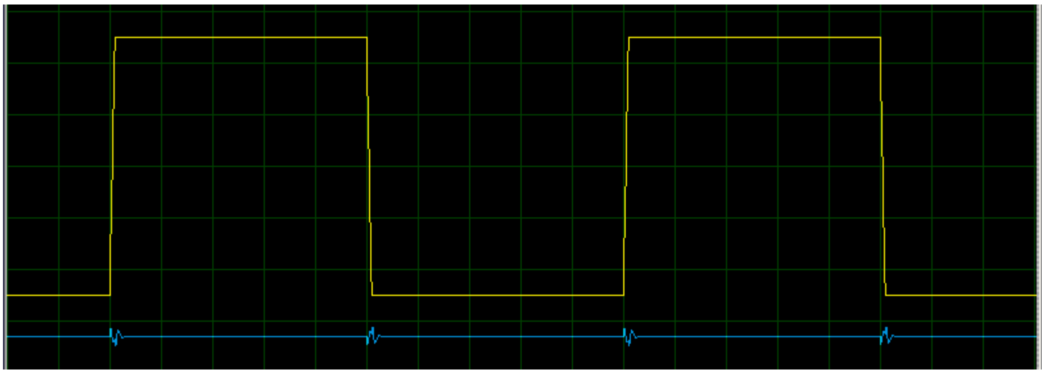


Fig: Output waveform when input is 100 Hz

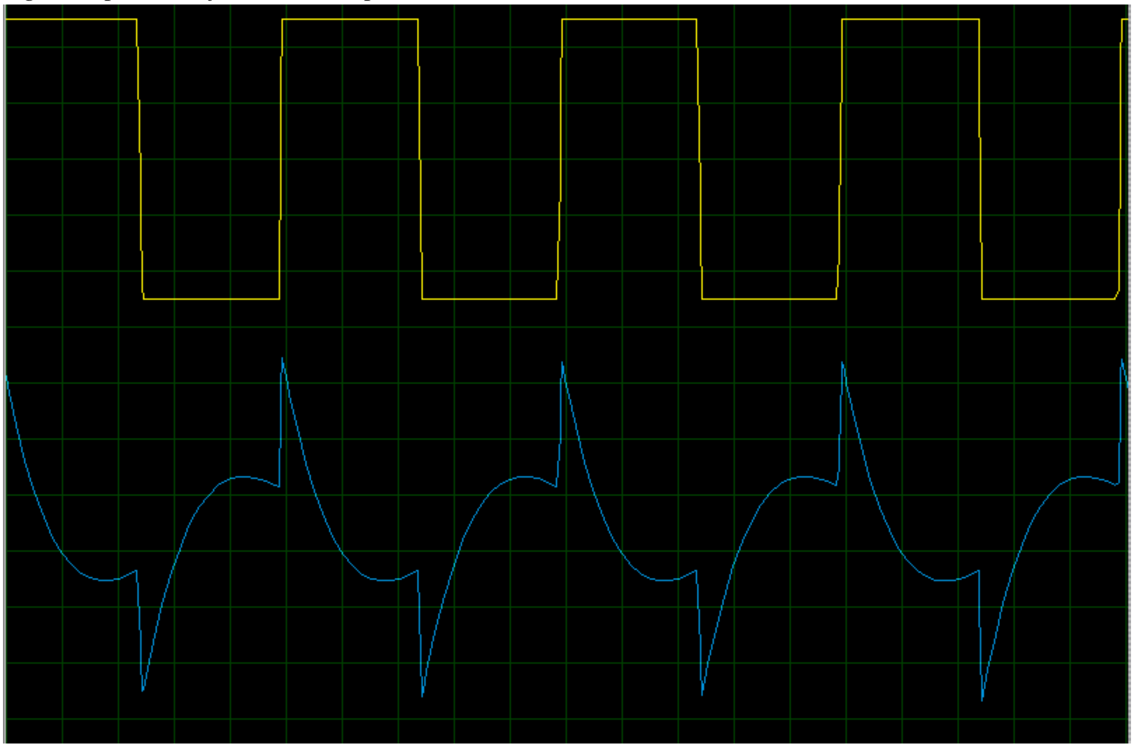


Fig: Output waveform when input is 15 kHz

Due to high-frequency, spikes are seen in both 100Hz and 15 kHz but in 15 kHz the spikes are more sharp and distinct.

5.3. Question 3

From the circuit given in figure 1, design a bandpass filter having ω0 of 40000 rad/sec and Bandwidth of 4000 rad/sec. In your final design the largest inductance should be of 1H and all other elements should be practically realizable. Realize the circuit and plot

the magnitude response. Also show the highest gain in dB, gain at 40000 rad/sec and half power frequencies in your plot.

Required center frequency for band pass filter (Ω_0) = 40000 rad/s
Since, half power frequency (ω) = 1 rad/s
The required bandwidth for band pass filter (B) = 4000 rad/s

To transform low pass to high pass filter we have to calculate the new value of Inductor and capacitance

A. Series

a. Inductor L is transformed into capacitor C_1 of capacitance $\frac{B}{\Omega_0^2 . L}$

$C_1 = \frac{B}{\Omega_0^2 . L} = \frac{4000}{4000^2 * 2} = 1.25\ \mu F$

b. Inductor L_1 of inductance $\frac{L}{B}$

$L_1 = \frac{L}{B} = \frac{2}{4000} = 0.5\ mH$

B. Parallel

a. Capacitor C is transformed into inductor L of inductance $\frac{B}{\Omega_0^2 . C}$

$L_2 = \frac{B}{\Omega_0^2 . C} = \frac{4000}{4000^2 * 1} = 2.5\ \mu H$

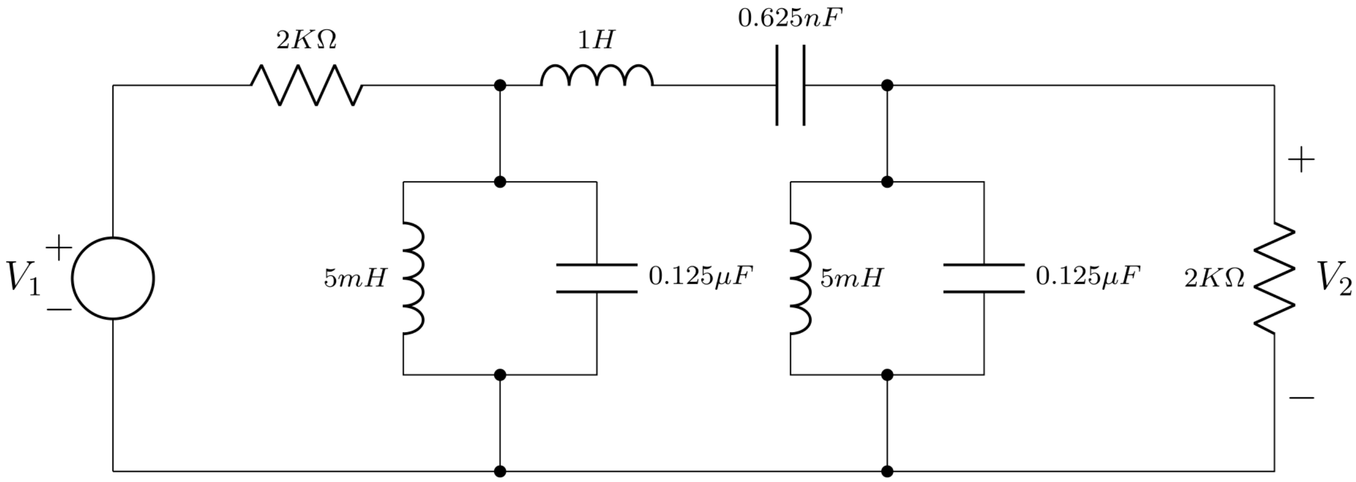
b. capacitor C of capacitance $\frac{C}{B}$

$C_2 = \frac{C}{B} = 0.25\ mF$

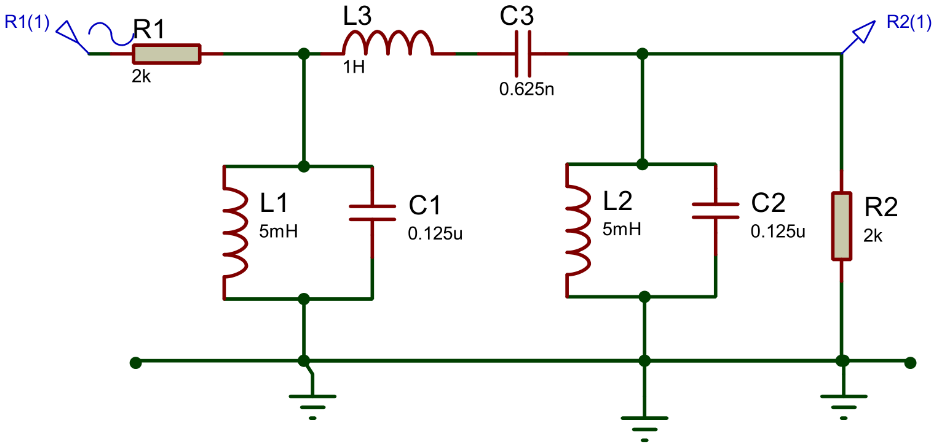
Required inductance L_{new}= 1H

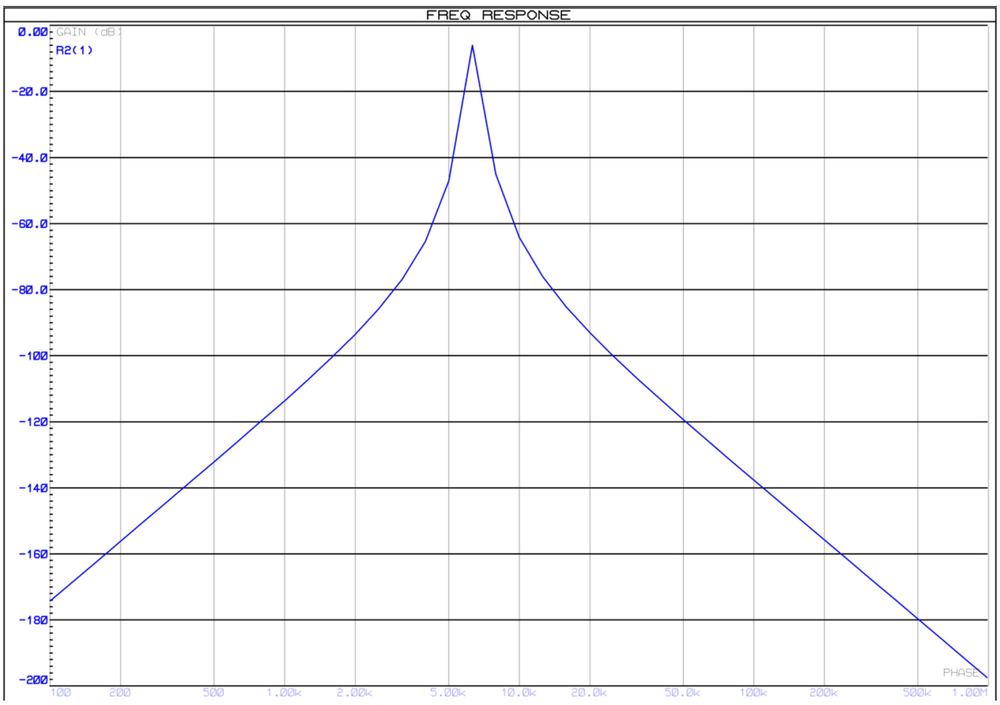
$K_m = \frac{L^{new}}{L\ old} = \frac{1}{5*10^{-4}} = 2000$

Now using value ok K_m we get ,
R_{new}= 2 KΩ
$L^{Parallel}_{new}$=5 mH
C^{series}_{new}=0.625 nF



$C^{parallel}_{new}$=0.125 μF





Highest gain = -6.02 dB

Half power frequency = 6.215 KHz and 6.435 KHz
Gain at 40000 rad/sec =-7.37 dB

5.4. Question 4

From the circuit given in figure 1, design a bandstop filter having ω0 of 40000 rad/sec and Bandwidth of 4000 rad/sec. In your final design the largest inductance should be of 1H and all other elements should be practically realizable. Realize the circuit and plot the magnitude response. Also show the highest gain in dB, gain at 40000 rad/sec and half power frequencies in your plot.

Required center frequency for band stop filter (Ω_0) = 40000 rad/s
Since, half power frequency (ω) = 1 rad/s
The required bandwidth for band stop filter (B) = 4000 rad/s

To transform low pass to high pass filter we have to calculate the new value of Inductor and capacitance

C. Parallel

  a. Inductor L is transformed into Inductor L_1 of Inductance $\frac{BL}{\Omega_0^{2}}$.

  $L_1 = \frac{BL}{\Omega_0^{2}}. = \frac{4000*2}{4000^{2}}$=5 µF

  b. Capacitor C_1 of Capacitance $\frac{1}{BL}$

  $C_1 = \frac{1}{BL} = \frac{1}{4000*2}$ =0.125 mH

D. Series

  a. Capacitor C is transformed into inductor L_2 of inductance $\frac{1}{B\,C}$

  $L_2 = \frac{1}{B\,C} = \frac{1}{4000*1}$=0.25 mH

  b. Capacitor C of capacitance $\frac{BC}{\Omega_0^{2}}$

  $C_2 = \frac{BC}{\Omega_0^{2}}$=2.5 µF

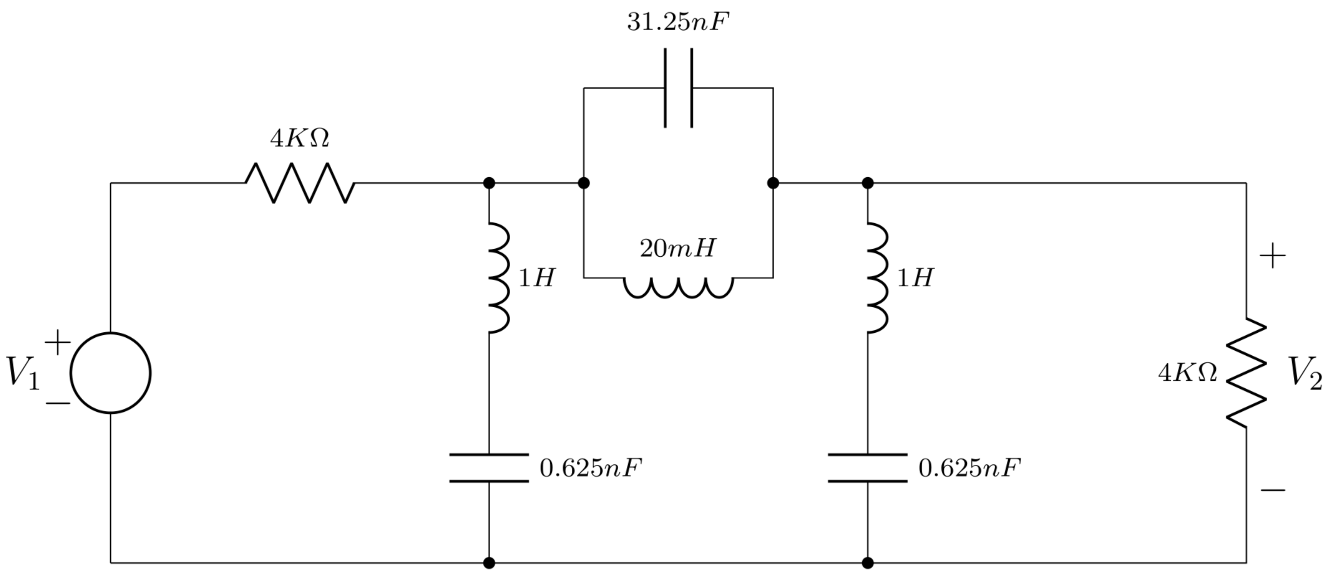
Required inductance L_{new}= 1H

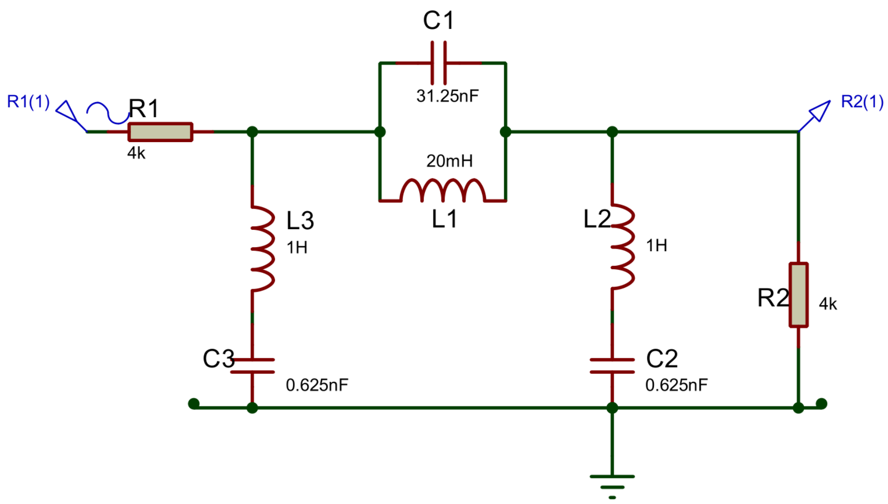
$K_m = \frac{L^{new}}{L\ old} = \frac{1}{2.5*10^{-4}}$=4000

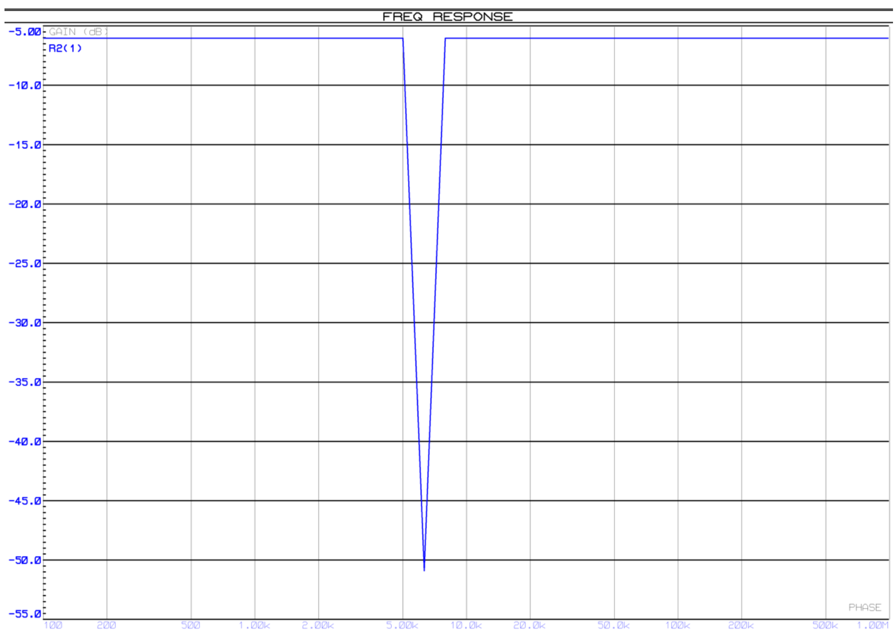
Now using value ok K_m we get ,
R_{new}= 4 KΩ
$L^{Parallel}{}_{new}$=1 mH

C^{series}_{new}=31.25 nF

$C^{parallel}_{new}$=0.625 nF







Highest gain = -6.02 dB
Half power frequency = 5.1 KHz and 7.8 KHz
Gain at 40000 rad/sec =-7.37 dB

6. Conclusion

This Lab is mainly focused on Impedance scaling and frequency scaling. Proteus simulation and its tools helped us to realize and analyze the circuit. Here we transformed the Low Pass filter of the given Half power frequency to High Pass, Band Pass and Band Stop Filters as per the condition supplied.