INSTITUTE OF ENGINEERING
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FILTER DESIGN

LAB #5

DESIGN OF HIGHER ORDER ACTIVE FILTER USING SALLEN & KEY BIQUAD CIRCUIT

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DESIGN OF HIGHER ORDER ACTIVE FILTER USING SALLEN & KEY BIQUAD CIRCUIT

2 Objective

- To be familiar with the design of higher order active filters using cascaded biquads
- To be familiar with design of filter using Sallen and Key biquad circuit
- To be familiar with RC-CR transformation

3 Requirement

3.1 Proteus Design Suite

Proteus is a simulation and design software tool developed by Labcenter Electronics for Electrical and Electronic circuit design.It is used to create schematic of a circuit and Visualization of its operation.

4 Exercises:

4.1 Question -1

What are the different techniques to design higher order active filters? Explain.

Simply utilizing the first and the second order filters, higher-order filters like 3rd, 4th, 5th and so on can be made. Some popular method to realize higher order filters are:

- Cascading Biquad filters
- Multiple-loop feedback circuit.
- Simulation of passive LC ladder network.

4.1.1 Cascading Biquad filters

In this method of Design higher order filters, the cascaded Biquad circuit along with bilinear circuit is used. Here, the requires Transfer function *Ts* is divided into Transfer functions for Biquad and Bilinear circuit.

$$T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_0}{s^n + b_{n-1} s^{n-1} + \ldots + b_0}$$

When n is even,

$$T(s) = \prod_{i=1}^{n/2} \frac{a_{2i} s^2 + a_{1i} s + a_{0i}}{s^2 + b_{1i} s + b_{0i}} = \prod_{i=1}^{n/2} T_i(s)$$

When n is odd,

$$T(s) = \frac{a_{11} s + a_{01}}{s + b_{01}} \prod_{i=2}^{(n-1)/2} \frac{a_{2i} s^2 + a_{1i} s + a_{0i}}{s^2 + b_{1i} s + b_{0i}} = T_1(s) \prod_{i=2}^{(n-1)/2} T_i(s)$$

The overall design of filter can be improved by allocating varying gain to the different sections of the filter. Another advantage of Cascading is Pole-zero paring where ripple is minimum in the system thus allowing factoring of Transfer function.

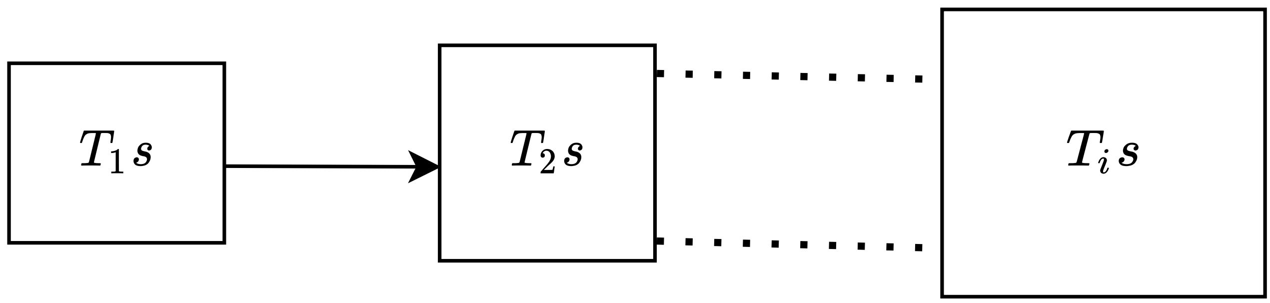


Figure 1: Higher order filter using Cascading Biquad circuit

4.2 Question -2

From the circuit given in figure 1 derive the transfer function $V_2(s)/V_1(s)$.

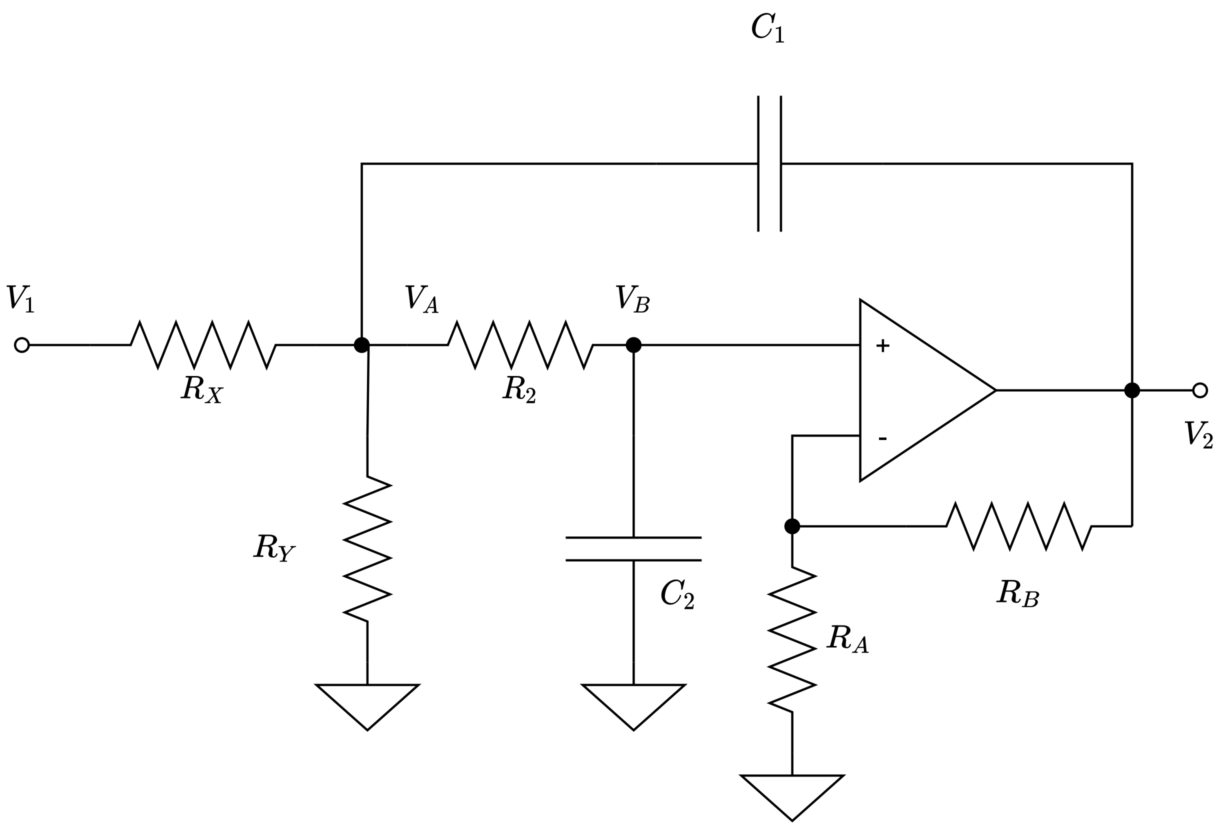


Figure 2: Sallen-Key Low pass filter

For an ideal op amp, we know,

$$V_B = \left(\frac{R_A}{R_A + R_B}\right) V_2$$

$$\Rightarrow V_2 = \left(1 + \frac{R_B}{R_A}\right) V_B$$

$$\Rightarrow V_2 = KV_B$$

$$\Rightarrow V_B = \frac{V_2}{K} \tag{1}$$

$$\therefore \frac{V_2}{V_A} = 1 + \frac{R_A}{R_B} = K \tag{2}$$

$$V_B = \frac{1/C_2 s}{R_2 + 1/C_2 s} V_A$$

$$\Rightarrow V_B = \frac{1}{1 + C_2 s R_2} V_A$$

$$\therefore V_A = \frac{1 + C_2 s R_2}{k} V_2 \tag{3}$$

Applying Nodal Analysis at V_A,

$$\frac{V_1}{R_1} + V_2(C_1 s) + \frac{V_B}{R_2} = V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + C_1 s\right)$$

Substituting value of V_A and V_B from above equations, we get,

$$\frac{V_1}{R_1} + V_2(C_1 s) + \frac{V_2}{kR_2} = \frac{V_2}{k}\left(\frac{(1 + C_2 s R_2)(R_1 + R_2 + C_1 s R_1 R_2)}{R_1 R_2}\right)$$

$$\text{or, } \frac{V_1}{R_1} = \frac{V_2}{kR_1R_2}\left(\frac{(1 + C_2 s R_2)(R_1 + R_2 + C_1 s R_1 R_2)}{R_1 R_2} - sKR_1R_2C_1 - R_1\right)$$

$$\text{or, } \frac{V_1}{R_1} = \frac{V_2C_1C_2R_2}{K}\left(s^2 + s(\frac{1}{R_2C_2} + \frac{1}{R_2C_1} + \frac{1}{R_1C_1} - \frac{k}{R_2C_2}) + \frac{1}{R_1R_2C_1C_2}\right)$$

Thus transfer function $V_2(s)/V_1(s)$ is

$$\frac{V_2}{V_1} = \frac{\frac{K}{R_1R_2C_1C_2}}{s^2 + s\left(\frac{1}{R_2C_2} + \frac{1}{R_2C_1} + \frac{1}{R_1C_1} - \frac{k}{R_2C_2}\right) + \frac{1}{R_1R_2C_1C_2}} \tag{4}$$

4.3 Question -3

Realize the fourth order Butterworth filter (refer table 1) using Sallen-Key circuit. Perform gain compensation if necessary.

n=2	n=3	n=4	n=5
-0.7071068 ± j 0.7071068	-0.50 ± j 0.86603	-0.3826834 ± j0.9238795	-0.809017 ± j 0.5877852
	-1.0	-0.9238795 ± j 0.3826834	-0.309017 ± j 0.9510565
			-1.0

Table 1: Pole location for Butterworth Response

From the table above, we can see that the pole locations for the fourth order Butterworth filter are $-0.3826834 \pm j0.9238795$ and $-0.9238795 \pm j0.3826834$.It has characteristic polynomial as

$$H(s) = (s^2 + 0.7653s + 1)(s^2 + 1.8478s + 1)$$

The Transfer function $T(s)$ is cascaded form of two biquad circuit as

$$T(s) = T_1(s)T_2(s) = \frac{1}{(s^2 + 0.7653s + 1)(s^2 + 1.8478s + 1)} \tag{5}$$

where,

$$T_1(s) = \frac{1}{s^2 + 0.7653s + 1} \tag{6}$$

$$T_2(s) = \frac{1}{s^2 + 1.8478s + 1} \tag{7}$$

We have to realise the transfer function of each circuit individually.

We have denominator for 2^{nd} butterworth filter as

$$s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2 \tag{8}$$

And for $T_1(s)$ when compared to equation 6, we get

$$\omega_o = 1$$

$$\frac{\omega_o}{Q} = 0.7653 \Rightarrow Q = 1.3067$$

$$K = 1$$

For Sallen-key circuit with elemental values $C_1 = C_2 = 1$ F and $R_1 = R_2 = R$ we get,

$$R = \frac{1}{\omega_o} = 1\,\Omega$$
$$K = 3 - \frac{1}{Q} \Rightarrow K = 2.2347$$

Form gain relation

$$K = 1 + \frac{R_B}{R_A}$$
$$\frac{R_B}{R_A} = K - 1 = 1.2347$$

For $R_A = 1\,\Omega$ and $R_B = 1.2347\,\Omega$ we need to perform gain reduction compensation to make gain unity. For this purpose we need Sallen-key gain reduction circuit.

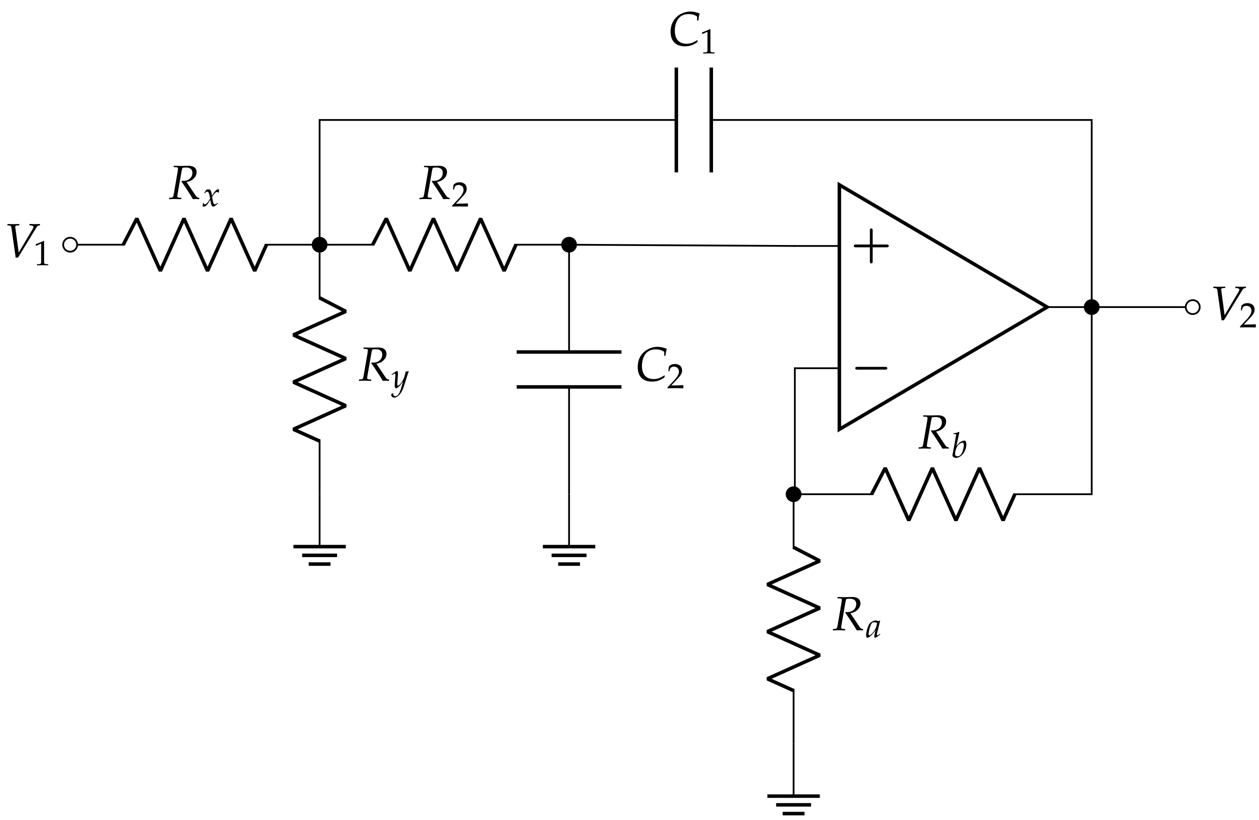


Figure 3: Gain reduction circuit for Sallen- Key

For desired gain 1,

$$\frac{R_y}{R_x + R_y} = \frac{H}{K} = \frac{1}{2.2347} = 0.4474$$
$$\Rightarrow R_y = 0.8096 R_x \tag{9}$$

From above figure,

$$R_1 = R_x || R_y = \frac{R_x R_y}{R_x + R_y}$$
$$\Rightarrow R_x R_y = R_x + R_y \tag{10}$$

Solving equation 9 an 10 we get, $R_x = 2.2351\,\Omega$ and $R_y = 1.8095\,\Omega$

Similarly for $T_2(s)$ when denominator of equation 7 compared to equation 8 we get,

$$\omega_o = 1\,\mathrm{rad\,s^{-1}}$$

$$\frac{\omega_o}{Q} = 1.8478 \Rightarrow Q = 0.5412$$

$$K = 1$$

For Sallen-key circuit with elemental values $C_1' = C_2' = 1$ F and $R_1' = R_2' = R'$we get,

$$R' = \frac{1}{\omega_o'} = 1\Omega$$

$$K' = 3 - \frac{1}{Q'} \Rightarrow K' = 1.1522$$

From gain relation,

$$K' = 1 + \frac{R_b'}{R_a'} \Rightarrow \frac{R_b'}{R_a'} = K' - 1 = 0.1522$$

We again have to perform gain reduction to make gain unity. For this purpose we need Sallen-key gain reduction circuit as above.

For desired gain 1,

$$\frac{R_y'}{R_x' + R_y'} = \frac{H}{K} = \frac{1}{1.1522} = 0.8679$$

$$\Rightarrow R_y' = 6.57R_x' \tag{11}$$

From above figure,

$$R_1' = R_x'||R_y' = \frac{R_x'R_y'}{R_x' + R_y'}$$

$$\Rightarrow R_x'R_y' = R_x' + R_y' \tag{12}$$

Solving equation 9 an 10 we get, $R_x' = 1.1522\,\Omega$ and $R_y' = 7.57\,\Omega$

Hence final cascaded filter is shown below.

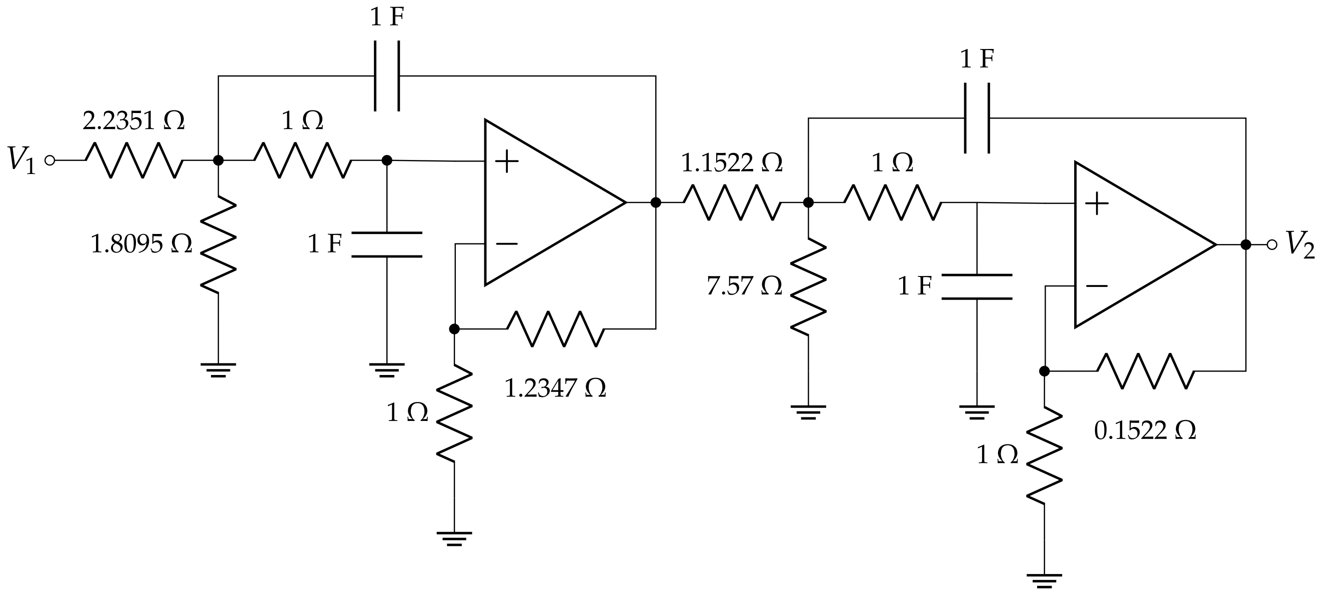


Figure 4: Fourth order butterworth lowpass filter using Sallen-Key circuit

4.4 Question -4

Obtain the final design of lowpass filter having half power frequency of 3.1831 KHz and practically realizable elements. Realize it in circuit and observe the magnitude response. Also note down the gain in passband and half power frequency.

For required half power frequency of 3.1831 KHz, Frequency Scaling factor K_f is given by,

$$K_f = \frac{\Omega}{\omega_o} = \frac{2\pi \times 3.1831 \times 10^3}{1} \approx 20 \times 10^3 \tag{13}$$

Furthermore, Impedance scaling $K_m = 1 * 10^3$ is also applied to obtain the practically realizable values.

Normalized value 1^{st} part	**Final Value 1^{st} part**	**Normalized value 2^{nd} part**	**Final Value 2^{nd} part**
1 Ω	R_a = 1 $K\Omega$	1 Ω	R'_a= 1 $k\Omega$
1.2347 Ω	R_b = 1.2347 $K\Omega$	0.1522 Ω	R'_b= 152.2 Ω
2.2351 Ω	R_x = 1 $K\Omega$	1.1522 Ω	R'_x= 1 $K\Omega$
1.8095 Ω	R_y = 1 $K\Omega$	7.57 Ω	R'_y= 1 $K\Omega$
1 Ω	R_2 = 1$K\Omega$	1 Ω	R'_2= 1 $K\Omega$
1 nF	C_1 = 50 nF	1 nF	C'_1= 50 nF
1 nF	C_2 = 50 nF	1 nF	C'_2= 50 nF

Table 2: Component values at half power frequency $k_f = 3.18 Khz$

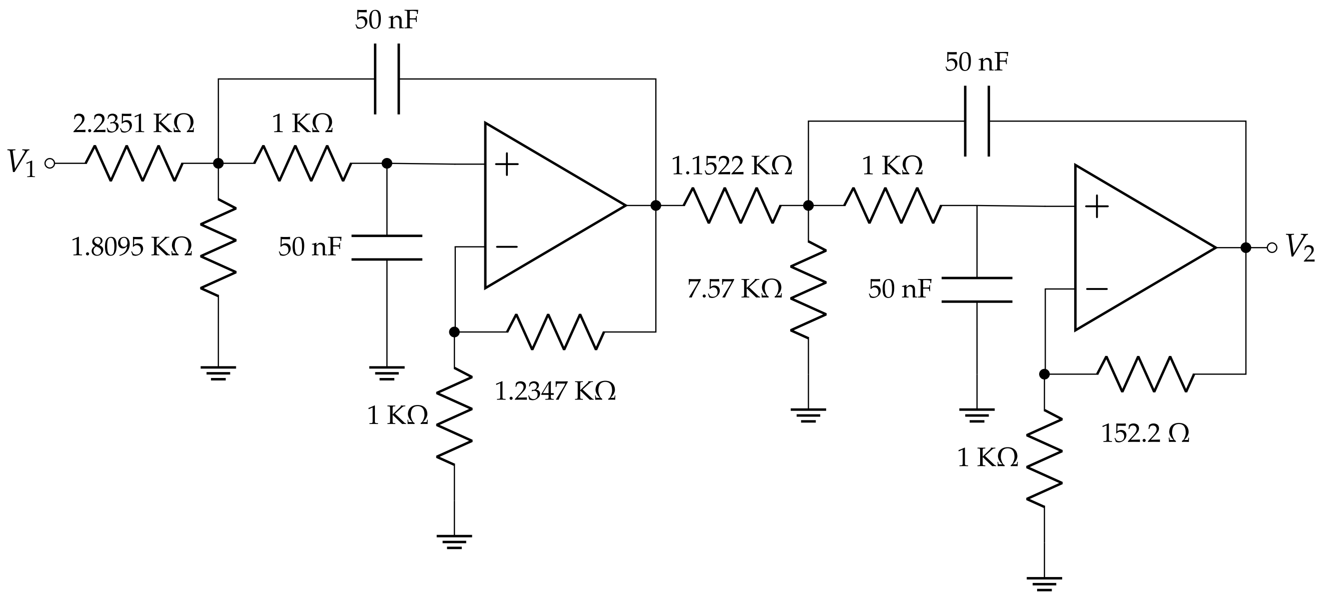


Figure 5: Fourth order butterworth lowpass filter with half power frequency 3.1831 KHz

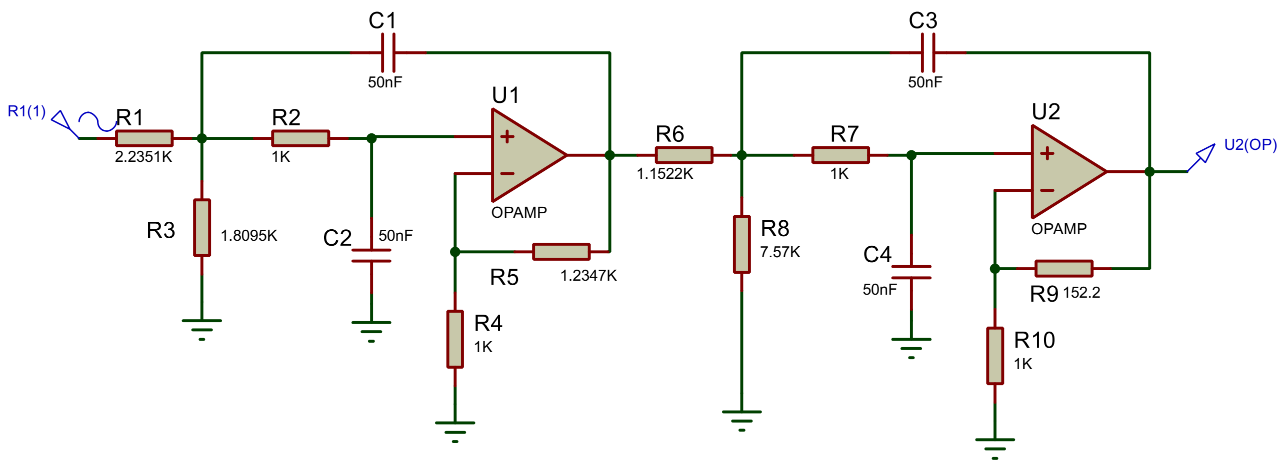
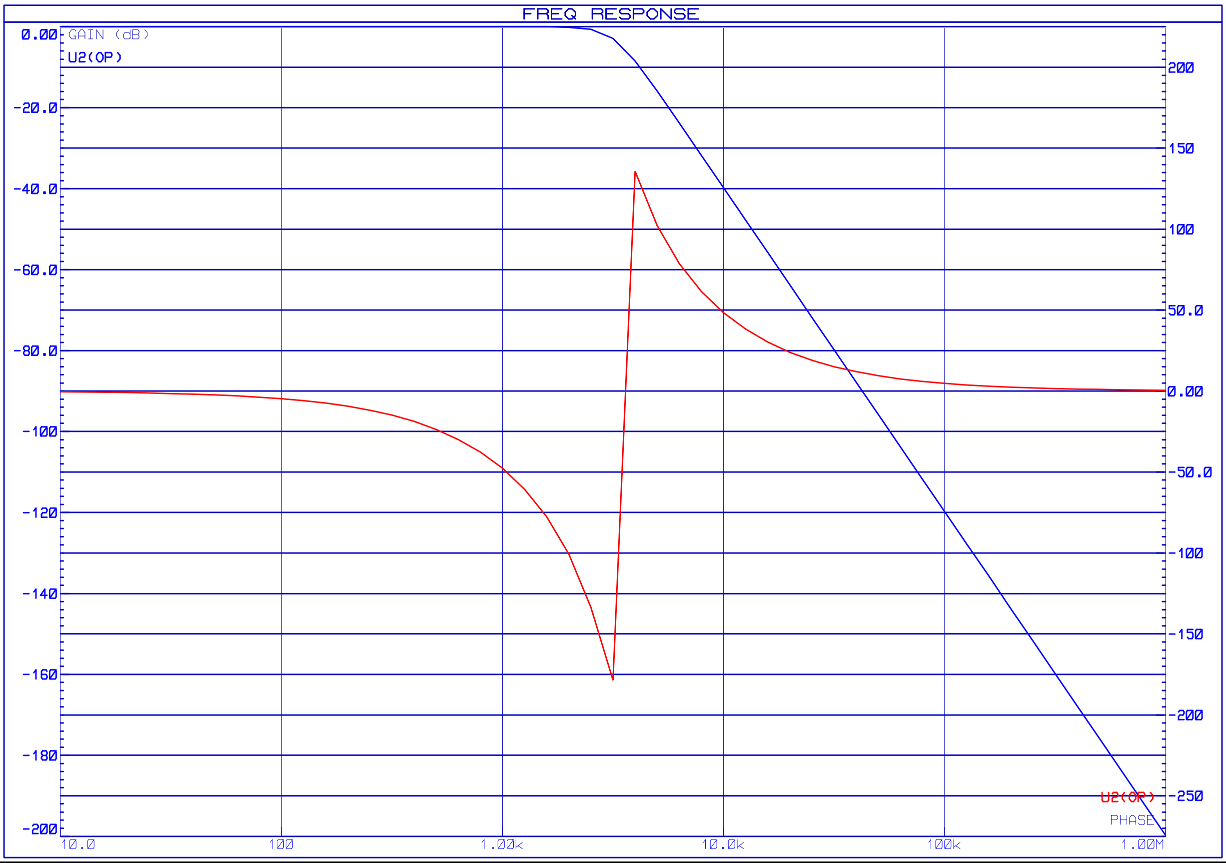


Figure 6: Proteus Circuit for fourth order low pass



Gain	**Half power frequency**
0 dB	(3.177) KHz

Figure 7: Proteus Observation for fourth order low pass

4.5 Question -5

Apply RC-CR transformation in your filter circuit obtained in problem C, to obtain the highpass filter.

To convert low pass filter to high pass filter, we need to apply RC-CR transformation. Here each resistor with value R is replaced by a capacitor with value $\frac{1}{R}$ and each capacitor with value C is replaced by a resistor with value $\frac{1}{C}$. Additionally we have to perform gain reduction(compensation).

First Part

$$Z_x||Z_y = \frac{1}{s \times 1}$$

$$\Rightarrow \frac{1}{s} = \frac{\left(\frac{1}{sC_x}\right)\left(\frac{1}{sC_y}\right)}{\frac{1}{sC_x} + \frac{1}{sC_y}}$$

$$\therefore C_x + C_y = 1 \tag{14}$$

From gain relation,

$$\frac{Z_y}{Z_x + Z_y} = \frac{1}{2.2347} = 0.4474$$

$$\Rightarrow 0.5526Z_y = 0.4474Z_x$$

$$\Rightarrow Z_y = 0.8096Z_x$$

$$\Rightarrow \frac{1}{sC_y} = 0.8096\frac{1}{sC_x}$$

$$\therefore C_x = 0.8096C_y \tag{15}$$

Solving equation 14 and 15 we get, $C_x = 0.4474$ F and $C_y = 0.5526$ F.

Second Part

$$\frac{1}{s \times 1} = Z'_x||Z'_y$$

$$\therefore C'_x + C'_y = 1 \tag{16}$$

From gain relation,

$$\frac{Z'_y}{Z'_x + Z'_y} = \frac{1}{1.1522} = 0.8679$$

$$\Rightarrow 0.1321Z'_y = 0.8679Z'_x$$

$$\Rightarrow \frac{1}{sC'_y} = 6.57\frac{1}{sC'_x}$$

$$\therefore C'_x = 6.57C'_y \tag{17}$$

Solving Equation 16 and 17, we get, $C'_x = 0.8679$ F and $C_y = 0.1321$ F.

Hence the high pass filter transformed using RC-CR method is shown below.

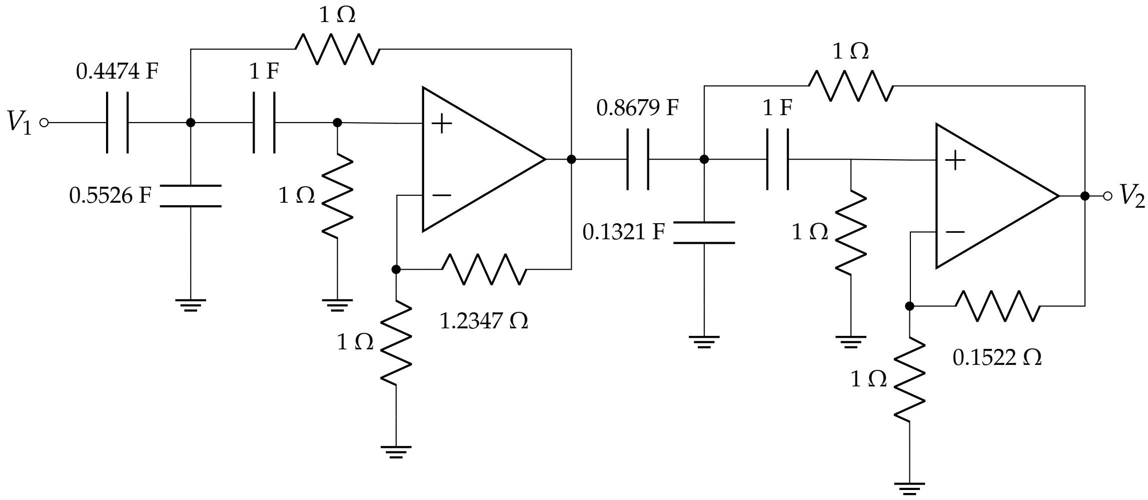


Figure 8: Fourth order butterworth highpass filter using RC-CR transformation

4.6 Question -6

Finally obtain a highpass filter (from the circuit obtained in problem E) having half power frequency of 4.775 kHz with practically realizable elements. Realize the network and observe the response and note down the gain in passband and half power frequency.

For required half power frequency $k_f = 4.775Khz$, Frequency Scaling factor K_f is given by,

$$K_f = \frac{\Omega}{\omega_o} = \frac{2\pi \times 4.775 \times 10^3}{1} \approx 30 \times 10^3 \tag{18}$$

Furthermore, Impedance scaling $K_m = 1 * 10^3$ is also applied to obtain the practically realizable values.

Normalized value 1^{st} part	**Final Value 1^{st} part**	**Normalized value 2^{nd} part**	**Final Value 2^{nd} part**
1 Ω	R_a = 1 $K\Omega$	1 Ω	R'_a= 1 $k\Omega$
1.2347 Ω	R_b = 1.2347 $K\Omega$	0.1522 Ω	R'_b= 152.2 Ω
1 Ω	R_1 = 1 $K\Omega$	1 Ω	R'_1= 1 $K\Omega$
1 Ω	R_2 = 1$K\Omega$	1 Ω	R'_2= 1 $K\Omega$
0.4474 nF	C_x = 14.913 nF	0.8679 nF	C'_x= 28.93 nF
0.5526 nF	C_y = 18.42 nF	0.1321 nF	C'_y= 4.4 nF
1 nF	C_2 = 33.33 nF	1 nF	C'_2= 33.33 nF

Table 3: Component values at half power frequency $k_f = 4.775Khz$

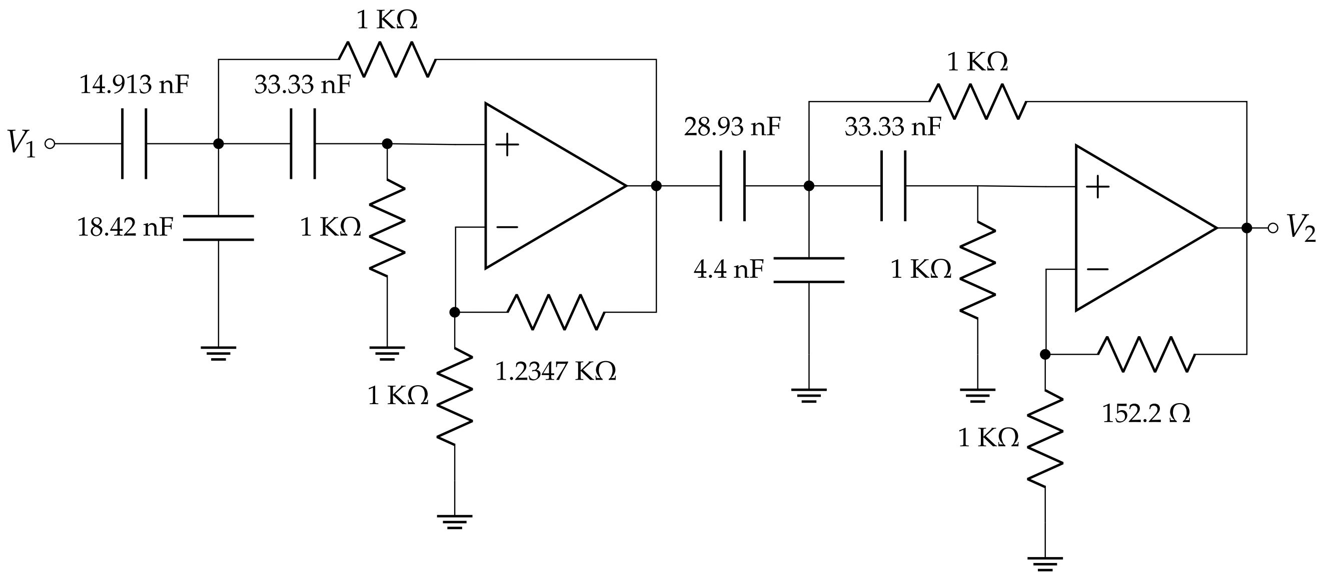


Figure 9: Fourth order butterworth Highpass filter with half power frequency 4.775 KHz

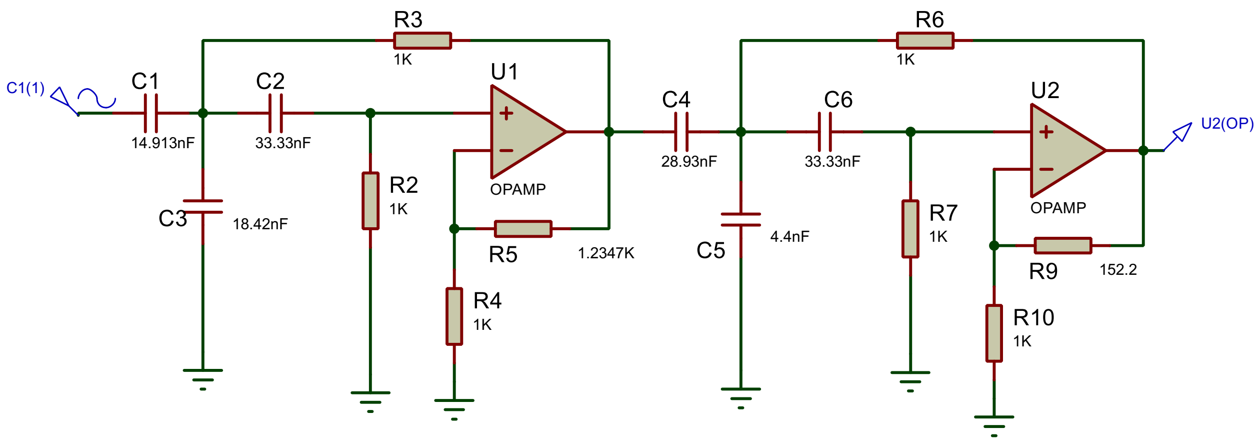
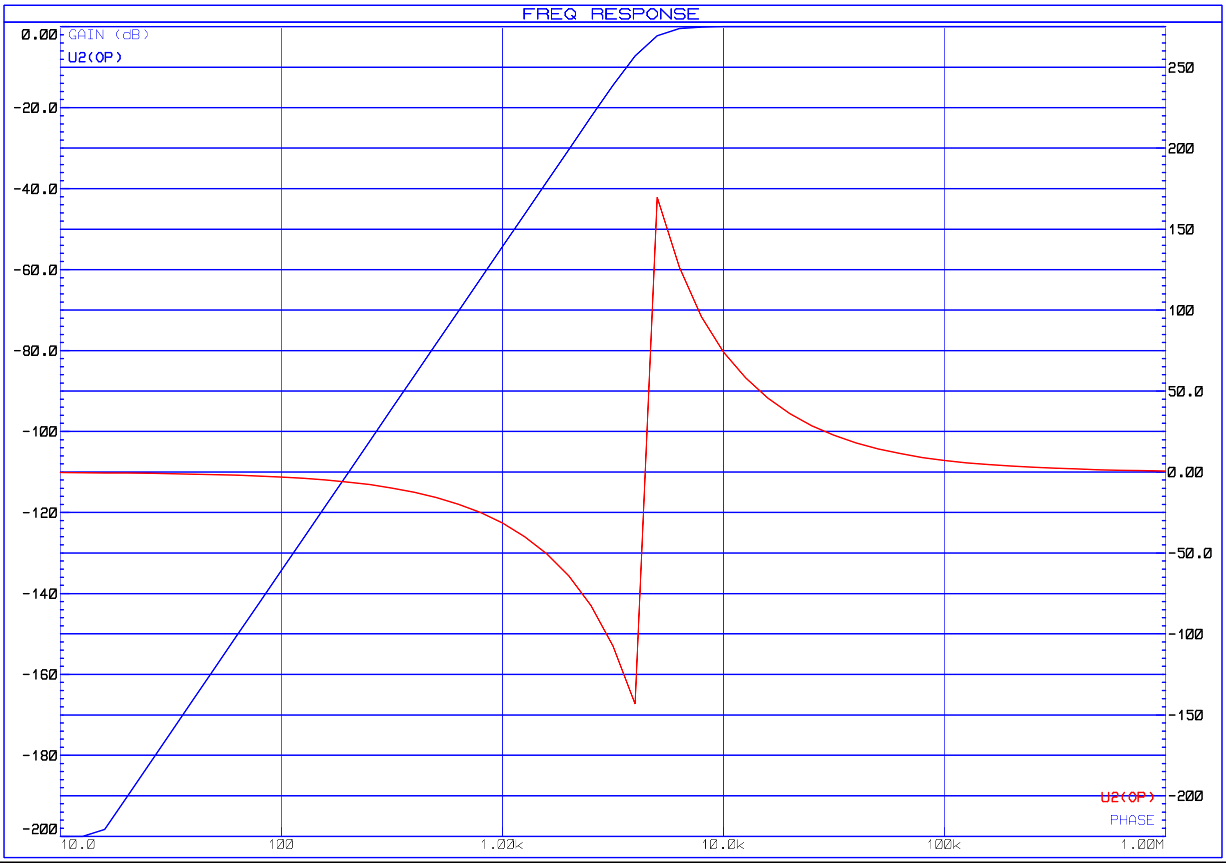


Figure 10: Proteus Circuit for fourth order high pass



Gain	**Half power frequency**
0 dB	(4.86) KHz

Figure 11: Proteus Observation for fourth order high pass

5 Discussion and Conclusion

In this lab we have derived the Transfer function of Sallen-Key circuits and used it further to design a Fourth order cascaded filter using two second order biquad circuit. We also designed a low pass filter using Sallen-Key method and use RC-CR transformation to obtain high pass of required Half power frequency hence fulfilling our Lab objective.