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Team 18

Investigating Latent Factors in Currencies

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Abstract

We explore latent factor construction in the FX market. To this end, we utilize the decomposition technique Instrumented Principal Component Analysis (IPCA) which is an extension of Principal Component Analysis (PCA). The IPCA factors are contrasted against the extant PCA factors and Fama-French (FFR) style factors of carry, value, and momentum. We find evidence that IPCA and PCA factors are better at explaining the cross-section of returns. However, the IPCA factors implied Stochastic Discount Factor (SDF) has a Sharpe ratio that is marginally lesser than the FFR factors for comparable risk, leverage and turnover. On balance, IPCA seems to be able to construct factors that explain the cross-section better than PCA or FFR, and can be combined to create high Sharpe ratio portfolios comparable to FFR and better than PCA.

Contents

1 Introduction

Foreign Exchange (FX) is one of the largest asset classes by trading volumes. According to the Bank of International Settlements, the average daily turnover in the FX market was $5.1 Trillion in April 2016 (see BIS 2019). It is an important market that facilitates international trade and capital flows, hedging and speculation. Understanding the drivers of movements in exchange rates and asset prices, in general, has been one of the central problems in Finance.

Factor modeling is one approach used to explain returns of an underlying asset. This framework involves devising systematic factors that co-vary with the underlying asset and computing factor loadings that measure the degree of co-variation. Factors can be constructed in the following three main ways:

- **Macro factors**: Describe the risk premium earned by an asset due to shocks in macroeconomic variables such as economic growth, inflation, and volatility.
- **Fundamental dynamic factors**: Incorporate a prior understanding of what drives differences in expected returns of assets and use this to create factors using portfolio sorts on the corresponding attributes. This method was pioneered by Fama and French and we will refer to it as the FFR approach. The typical method used is summarized in Appendix 1. The FFR approach is frequently applied due to its simplicity and connection to a prior understanding of the assets.
- **Latent style factors**: Treat the factors as latent (i.e. not directly observable) and utilize decomposition techniques such as Principal Component Analysis (PCA) to learn the factors and respective loadings using the data.

Each method described above is not without shortcomings. Macro factors are explanatory to different degrees but are typically not directly tradable (see Ang 2014, pages 214-225).

Fundamental dynamic factors rely on a prior understanding that is partial at best. Moreover, there is no evidence to suggest that the method of portfolio construction used for dynamic factors is the appropriate form of the factors[1]. Latent style factor construction makes the structure of factors an explicit part of the econometric problem. However, trading on these latent factors can be risky due to the lack of interpretability and heavy dependency on past data. In addition, financial data is riddled with shifting regimes. This makes the form of latent style factors liable to change in ways that are not easily explained.

Can we find a technique that allows us to explicitly learn latent factors that drive asset returns while incorporating established domain knowledge in currency markets? Instrumented PCA (IPCA) is one such technique that attempts to do so (see Kelly, Pruitt, and Su 2018). It introduces a characteristic matrix through which traditional characteristics expected to drive returns can be incorporated as part of latent factor learning.

In this paper, we will extend the use of IPCA to construct factors that explain FX returns. We wish to study if compared to traditional factor construction, IPCA can capture extra information through statistical learning, and regime shifts in the FX market.

The research paper will be laid out as follows: In Chapter 2 we outline our methodology and provide a theoretical background of factor modeling. We will thoroughly discuss PCA and IPCA including motivation and model estimation. In Chapter 3 we describe the data we used in our analysis. In Chapter 4, we will carry out preliminary data exploration using

[1]Fama and French (see Fama and French 1993, pages 7-8) state that "Although size and book-to-market equity seem like ad-hoc variables for explaining average stock returns, we have reason to expect that they proxy for common risk factors in returns. ... But the choice of factors, especially the size and book-to-market factors, is motivated by empirical experience. Without a theory that specifies the exact form of the state variables or common factors in returns, the choice of any particular version of the factors is somewhat arbitrary. Thus detailed stories for the slopes and average premiums associated with particular versions of the factors are suggestive but never definitive."

PCA. Following this, we present the main results from IPCA in Chapter 5. We will conclude our results in Chapter 6 and suggest future research work in Chapter 7.

2 Methodology

In order to ground our analysis, we need a theoretical framework. We also need to understand key nuances in the FX market and provide an overview of the techniques and method of research we use. In this chapter, we will begin by laying out the background theory of factor modeling. We will then discuss factor modeling in FX followed by a description of PCA. This will lead to a broad discussion of IPCA including motivation, estimation and regularization. Finally, we will provide an overview of how we compare the different models.

2.1 Factor Modeling

Factors are succinctly described by Andrew Ang (see Ang 2014, pages 194-196). He suggests that assets are bundles of systematic factor risks and diversifiable "idiosyncratic" risks. Since the systematic factors *cannot* be diversified away, the extent to which an asset co-varies with the systematic factors is what ultimately drives the differences in *expected* returns.

Factor modeling is an attempt to describe and estimate these factors. The simplest version of this type of model is the famous single-factor Capital Asset Pricing Model (CAPM). CAPM states that the *expected* excess return of an asset is driven by how much it is exposed to the market factor (portfolio of all investible assets):

$$E(r_i) - r_f = \beta_{i,m}[E(r_m) - r_f] \tag{1}$$

where $E(r_i)$ is the expected return of asset i, r_f is the risk-free rate, $\beta_{i,m} = \frac{\text{cov}(r_i, r_m)}{\text{var}(r_m)}$ is asset i's measure of market exposure, i.e. undiversifiable risk, and $E(r_m)$ is the expected return of the market.

The framework developed above is amenable to being combined with the notion of the Stochastic Discount Factor (SDF). The SDF can be used to price any traded asset using the following equation:

$$1 = E[\tilde{m} \times (1 + r_i)] \tag{2}$$

where $\tilde{m}$ is the stochastic discount factor and r_i is the return of asset i.

As illustrated in Cochrane's paper linking asset pricing and SDFs (see Cochrane and Culp 2003, pages 59-65) the equation above can be re-written as:

$$E(r_i) - r_f = \beta_{i,\tilde{m}} \lambda_{\tilde{m}} \tag{3}$$

where $\tilde{m}$ is the SDF, $\beta_{i,\tilde{m}}$ is asset i's exposure to $\tilde{m}$ risk and $\lambda_{\tilde{m}} = -\frac{\mathrm{var}(\tilde{m})}{E(\tilde{m})}$ is the market price of taking $\tilde{m}$ risk. The negative sign appears since the expression is the inverse of the factor risk.

If the SDF is linear in the market return, then the CAPM can be derived immediately as a consequence. This raises the question of whether the market is the only factor that spans the SDF vector space.

Numerous multi-factor models have been proposed, the most renowned being the set of Fama-French factor models (see Fama and French 2016). These can all be subsumed under the framework developed above. The generalized version is given by:

$$\begin{aligned} r_i - r_f &= \beta_i^T f + \epsilon_i \\ E(r_i) - r_f &= \beta_i^T \lambda \end{aligned} \tag{4}$$

where β_i is the vector of factor loadings, f is the vector of corresponding factor returns and λ is the risk price of the factor (equal to its mean if the factor is tradable). Some approaches also admit an alpha intercept either explicitly or via having a vector of ones as one of the factors. Assuming that the factors linearly span the SDF vector space, it can be expressed as:

$$\tilde{m_t} = 1 - b^T(f_t - \lambda) \tag{5}$$

The above expression satisfies $E_t(m_{t+1}) = 1$ and is a widely used form for estimating the SDF (see Lustig, Roussanov, and Verdelhan 2011, page 14). An expression linking b and β can be obtained as:

$$b = \Sigma_f^{-1} \lambda \tag{6}$$

where Σ_f is the covariance matrix of the factor returns. For a detailed derivation, please refer to Appendix 2. Looked another way, the b can be interpreted as the factor holdings that maximize the mean-variance utility $b^T\lambda - \frac{\gamma}{2} b^T \Sigma_f b$ with the risk-aversion coefficient γ set to 1. Setting a risk target is equivalent to choosing the risk-aversion coefficient[2], and is another popular way to estimate the SDF ex-ante.

[2] For holdings h, covariance matrix V and risk target t, the mean-variance objective is $\max_h h^T\mu$ subject to $h^T V h = t^2$. Upon solving, it yields $h = \frac{1}{\gamma} V^{-1}\mu$ where γ is uniquely pinned down by $\frac{\sqrt{\mu^T V^{-1} \mu}}{t}$

2.2 Factor Modeling in FX Market

Factor modeling is extensively used today and has been substantially researched in the equity markets, ranging from the single-factor CAPM to the more complex Arbitrage Pricing Theory (APT). In the FX setting, the theoretical framework has similarities to that for equity markets which can be exploited to obtain a pre-defined factor framework. However, there are key differences which are summarized below:

- **Price quotes**: FX rates are quoted as relative values of exchange rates rather than absolute values. This is handled by introducing the notion of base currency (US Dollar - USD - in our case) and recording quotes per unit of the base currency.

- **Dividend yield**: While interest yields on the foreign currency side could be viewed as analogous to a stock's dividend yield, it is important to note that the base currency also pays an interest yield.

- **Excess returns**: These can be calculated as the excess returns earned by a USD investor that invests \$1 in a foreign currency for a single period and re-converts it back at the end. Denote the units of foreign currency per unit USD at time t as S_t, the foreign risk-free rate between t and t+1 as $r_{X,t}$, and the corresponding USD rate as $r_{\$,t}$. The log excess returns for the single period would be:

$$rx_{t+1} = \ln\left(\frac{e^{r_{X,t}} S_t}{S_{t+1}}\right) - r_{\$,t} = r_{X,t} - r_{\$,t} - \Delta \ln S_{t+1} \tag{7}$$

  The covered interest-rate parity formula relates the single period forward price of a currency with its current spot price as:

$$\frac{F_t}{S_t} = e^{r_{X,t} - r_{\$,t}} \tag{8}$$

Taking logs above and substituting equation (8) into (7), we get:

$$\begin{aligned} rx_{t+1} &= (\ln F_t - \ln S_t) - (\ln S_{t+1} - \ln S_t) \\ &= \ln F_t - \ln S_{t+1} = f_t - s_{t+1} \end{aligned} \tag{9}$$

The expression above is typically used in literature as the log realized excess return at time t+1 (see Lustig, Roussanov, and Verdelhan 2011, page 7). An advantage of defining excess returns in this way is that plugging in values for the risk-free rates becomes unnecessary. This is beneficial because assumptions regarding the choice of risk-free rate are avoided. A downside is that we need to assume that covered interest rate parity holds.

- **Numeraire**: It is also important to note that a sound model should be applicable to all choices of base currencies - numeraire invariant. However, we suspect that empirically this might not strictly hold, due to currency supply/demand differences, public policy and liquidity. For instance, an exchange of currencies between Egypt and Russia would most likely be executed by first converting the "base" to USD and then purchasing the "foreign" leg leading to differences between the direct Egypt-Russia quote and the implied Egypt-USD-Russia quote.

- **Systematic vs. Idiosyncratic risk**: The systematic risk in a currency setting is the undiversifiable risk of currency co-movement, whereas the idiosyncratic risk is captured by country-specific movements. The currency co-movement can be thought of as the analog to the CAPM market factor and captures the risk of the base currency movement. A classical example is currency co-movement in euro-zone currencies.

Empirical research was traditionally done with respect to macro and statistical factors, while more recent studies have also incorporated PCA models to capture co-movement. The table below shows the factors in all three aspects we plan to cover.

Table 1: FX Factors

Dynamic Factors	**Macro Factors**	**Latent Factors**
Carry	Interest Rate Differentials	PCA Slope Factor
Value	GDP Growth	
Momentum	Current Account Differentials	

2.3 PCA in FX Market

The general PCA specification is given by:

$$\begin{aligned} r_t &= \beta f_t + \epsilon_t \quad \forall\ t \\ \implies X &= \begin{bmatrix} r_1^T \\ \vdots \\ r_T^T \end{bmatrix} = \begin{bmatrix} f_1^T \beta^T \\ \vdots \\ f_T^T \beta^T \end{bmatrix} + \begin{bmatrix} \epsilon_1^T \\ \vdots \\ \epsilon_T^T \end{bmatrix} = F\beta^T + \epsilon \\ \text{subject to } & \beta^T \beta = I_K \end{aligned} \tag{10}$$

where $r_t \in \mathbb{R}^{N \times 1}$ is the vector of t returns for N assets, $f_t \in \mathbb{R}^{K \times 1}$ are the contemporaneous factor realizations and $\beta \in \mathbb{R}^{N \times K}$ are the static factor loadings.

The PCA structure is identical to standard regression with the difference being that both β and f_t are unknowns and need to be learned. This extra degree of freedom results in multiple solutions of the form βR^{-1} and FR where R is a rotation matrix. To pin down a unique solution, a common constraint added is the ortho-normality of loadings.

The factor realizations can be interpreted as coefficients of cross-sectional regressions between asset returns and loadings:

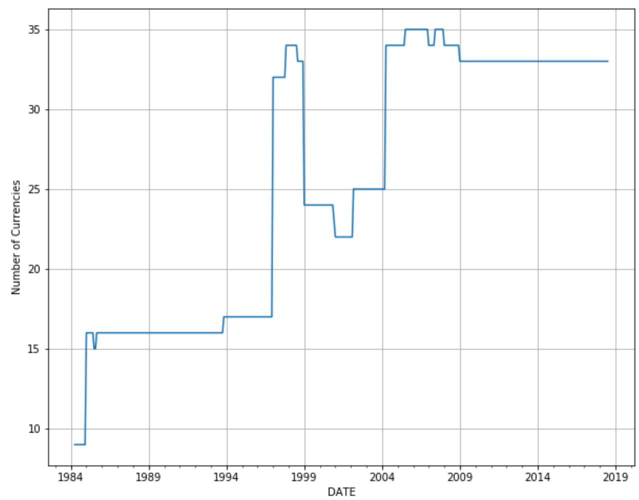
$$f_t = (\beta^T \beta)^{-1} \beta^T r_t \tag{11}$$

Since the β is constrained to be ortho-normal, equation (11) reduces to $\beta^T r_t$ which results in a convenient interpretation that the PCA loadings also serve as the asset weights of the factors. Since $\beta^T 1_N \neq 1_K$ in the general case, loadings result in portfolios that are not fully invested thus requiring a risk-free asset. Analyzing excess returns adjusts for this effect. The required borrowing/lending is not large since the L2 norm is constrained to 1.

There are numerous ways of solving the PCA specification. The most common methods use eigenvalue decomposition (EVD) on the covariance matrix of returns or equivalently singular-value decomposition (SVD) of the de-meaned returns matrix. When analyzing daily returns, it is often assumed that their mean is zero and there is no significant difference between the covariance and second-moment matrices. However, reducing observation frequency e.g. from daily to monthly could have a discernible impact. As studied by Lettau et al. (see Lettau and Pelger 2018, pages 6-7), the mean can sometimes contain information about the underlying factor structure and can improve the constructed latent factors if incorporated. They perform PCA on the matrix $\frac{1}{T}X^T X + \gamma \bar{X}^T \bar{X}$ (X is the data matrix, $\bar{X}$ is the matrix containing mean returns and γ is the weight given to the mean. The covariance matrix will have $\gamma = -1$.), and show that there is an optimal choice of γ which provides PCA factors that dominate covariance matrix PCA factors. Recreating the same analysis is beyond the scope of this paper. Hence, we restrict ourselves to constructing PCA factors using the covariance and second-moment matrices ($\gamma = -1$ and 0) respectively.

PCA can be applied on both portfolios constructed using the FFR approach and the individual currencies themselves. The former approach was employed by Lustig et al. for constructing their PCA implied carry factor. It is the second principal component since the first one is always a level factor which does not co-vary differentially with the portfolios (needed to explain carry). The latter approach has been discarded in recent literature due to the issue of changing cross-sectional breadths. Figure 1 shows how cross-sectional breadth changes over time.

Figure 1: Cross-Sectional Counts of Currencies



Events such as the adoption of the Euro at different points in time by different countries, poor quality in forwards data and removal of data points where the Covered Interest Rate Parity was violated (as mentioned in Chapter 2.2, our definition of excess returns requires this assumption) contribute to changes in cross-sectional breadth.

Another possible issue with fitting PCA on a rolling time window basis is that the covariance matrix estimated as an intermediate step would suffer from serious estimation errors i.e. the matrix will not be full rank if the number of time samples $<$ number of currencies. This issue is alleviated somewhat since strictly speaking, K-component PCA only requires the optimal K-rank approximation of the covariance matrix for which the data requirement falls from $O(N^2)$ to $O(K \ln N)$ (see Vershynin 2010, page 33).

2.4 IPCA

The general IPCA model specification as stated by Kelly, Pruitt, and Su 2018, page 10 is presented below:

$$\begin{gathered} r_{t+1} = \beta_t f_{t+1} + \epsilon_{t+1} \\ \beta_t = Z_t \Gamma_\beta + \nu_{\beta,t} \\ \text{subject to } \Gamma_\beta^T \Gamma_\beta = I_K;\ E(f_t^T f_t) = \text{diagonal matrix} \end{gathered} \tag{12}$$

where $r_{t+1} \in \mathbb{R}^{N \times 1}$ is the vector of t+1 returns for N assets, $f_{t+1} \in \mathbb{R}^{K \times 1}$ are the contemporaneous latent K-factors similar to those observed in PCA, $Z_t \in \mathbb{R}^{N \times L}$ is the matrix at time t of L characteristics for each of the N assets, $\beta_t \in \mathbb{R}^{N \times K}$ is the matrix of loadings for the latent factors and $\Gamma_\beta \in \mathbb{R}^{L \times K}$ is the invariant mapping between characteristics and the loadings. Another important criteria is that $K \leq L$. It is not possible to learn more latent factors than the number of characteristics provided.

2.4.1 Why IPCA?

We identify a few commonly faced modeling challenges that are addressed to some extent by IPCA.

- **Fundamental vs. statistical approach**: As referred to in the Introduction, the FFR approach, by construction, uses fundamental factors to explain the variation across the returns. This in a way introduces a specification bias. The PCA approach relies excessively on the returns data itself. The loadings are static and do not have any influence from macro-economic factors. IPCA combines key elements in these approaches by making the factors an explicit part of the estimation process while also relating them to economic fundamentals through the characteristics matrix.

- **Time variation**: Coefficients of FX factor models are unstable across time. This is consistent with our intuition that exchange rates are highly regime-dependent. For example, models with high explanatory power before 2007 may become ineffective due to the unprecedented global monetary policies since then (see Rossi 2013). IPCA handles this by making loadings an explicit function of the time-varying characteristics.

- **Dimensionality**: Estimating PCA on limited samples of time series is problematic as mentioned before. IPCA solves this by incorporating more data apart from just returns. These additional data are characteristics that are expected to drive currency returns. Suppose we have N assets and returns data with T time samples and L characteristics per asset. We wish to estimate K latent factors. To calculate principal components from the covariance matrix without *any* estimation errors, we will need to estimate $\frac{N(N+1)}{2}$ parameters from NT data points. IPCA is able to incorporate L characteristics per asset (NLT more data points) and requires the estimation of $K(T+L)$ parameters. For example, values of $K = 3, L = 5, N = 30, T = 36$, we see that the number of data points per parameter increases from a paltry 2.3 for PCA to 52.7 for IPCA.

2.4.2 Estimating IPCA

In the IPCA formulation (equation (12)), if we assume Z_t is time-invariant, β_t would be constant and the equations would be precisely standard PCA. However, it varies across time and the system of equations cannot leverage SVD directly. As a result, IPCA does not have a closed form solution. It requires the use of the Alternating Minimization with the following objective function:

$$\min_{\Gamma_\beta, f} \sum_{t=1}^{T-1} (r_{t+1} - Z_t \Gamma_\beta f_{t+1})^T (r_{t+1} - Z_t \Gamma_\beta f_{t+1}) \tag{13}$$

Analogous to PCA, the factor realizations are defined as the regression coefficients of cross-sectional returns on the loadings:

$$\begin{aligned}\hat{f}_{t+1} &= (\beta_t^T \beta_t)^{-1} \beta_t^T r_{t+1} \\ &= (\hat{\Gamma}_\beta^T Z_t^T Z_t \hat{\Gamma}_\beta)^{-1} \hat{\Gamma}_\beta^T Z_t^T r_{t+1} \;\; \forall \; t\end{aligned} \tag{14}$$

The expression above satisfies the first order condition for f_{t+1} given by:

$$\sum_{t=1}^{T-1} \Gamma_\beta^T Z_t^T (r_{t+1} - Z_t \Gamma_\beta f_{t+1}) = 0 \tag{15}$$

Substituting the above into the objective function and minimizing the resulting expression with respect to Γ_β results in the following expression:

$$\text{vec}(\hat{\Gamma}_\beta^T) = \left(\sum_{t=1}^{T-1} (Z_t^T Z_t) \otimes (\hat{f}_{t+1} \hat{f}_{t+1}^T) \right)^{-1} \left(\sum_{t=1}^{T-1} (Z_t \otimes \hat{f}_{t+1}^T)^T r_{t+1} \right) \tag{16}$$

where $\otimes$ is the Kronecker product and vec(X) is the flattened matrix X (see Kelly, Pruitt, and Su 2018, page 13).

The IPCA parameters f and Γ_β are initialized by converting the equation to the most similar PCA equation. This is achieved by assuming that $Z_t^T Z_t = I_L$. This results in:

$$\begin{aligned}x_{t+1} = Z_t^T r_{t+1} &= Z_t^T Z_t \Gamma_\beta f_{t+1} + Z_t^T \epsilon_{t+1} \\ &= \Gamma_\beta f_{t+1} + \epsilon_{t+1}^* \\ \text{subject to } \Gamma_\beta^T \Gamma_\beta &= I_K\end{aligned} \tag{17}$$

The above is interpreted as PCA on “characteristics managed" portfolios, i.e. portfolios with the characteristics (Z_t) as the weights. This yields the initial values for Γ_β and f which are then updated using equations (14) and (16).

2.4.3 Regularizing IPCA

Similar to equation (11), the IPCA weights at time t are given by:

$$(\beta_t^T \beta_t)^{-1} \beta_t^T = (\Gamma_\beta^T Z_t^T Z_t \Gamma_\beta)^{-1} \Gamma_\beta^T Z_t^T \tag{18}$$

Γ_β is constrained to be orthonormal and is not going to drive weights that have unreasonable leverage. However, Z_t has observable characteristics that may have various orders of magnitude. Hence, the columns of Z_t (cross-sectional characteristics) need to be normalized. This has the additional benefit of making the corresponding Γ_β entries comparable across characteristics. We used different techniques to achieve this, such as:

1. **Orthonormalization**: This results in Z_t becoming an orthonormal matrix and equation (18) reduces to $\Gamma_\beta^T Z_t^T$. This is the product of rotation matrices and hence will not explode during optimization. An issue with this method is the loss of interpretability since all preceding columns of Z_t are used to normalize each of its columns. The new "characteristics" are not exactly comparable to the original version.

2. **Unit L2 norm**: The columns of Z_t are unit normalized with respect to the L2 norm resulting in a matrix that is reasonably well behaved and interpretable.

3. **Z-Score**: Similar to above except that the columns are de-meaned prior to being normalized to unit norm.

4. **X-Rank**: The cross-sectional characteristics are converted into a specific number of quantile ranks. For example, the vector $[12.4, 45.6, 12.8, 56.7]$ would be converted to $[1, 2, 1, 2]$ if the chosen number of quantile ranks is 2. This method is particularly useful because it only looks at relative instead of absolute magnitudes and is robust to outliers. It is similar to the FFR approach of creating quantile portfolios after sorting.

After experimenting with many combinations of normalization techniques, we settled on x-ranking and unit L2 normalization of Z_t. The convergence of the algorithm is very quick with these specifications (after 3 steps the absolute error is $O(10^{-10})$).

2.5 General Comparison Framework

We choose to use FFR and PCA as our base models to compare to IPCA. While PCA has been discussed in detail in this section, we have covered FFR in Appendix 1 as it is a standard method and is more straightforward in the FX setting. The summary of these methods is presented below with a comparison of their characteristics.

Approach	β_t	f_t	Constraints	Estimation Method
FFR	None	None	None	Quantile portfolios based on factors
PCA	Constant over the time sample	Learned from data	$\beta^T \beta = I_K$	SVD for β and f
IPCA	Time-varying	Learned from data	$\Gamma_\beta^T \Gamma_\beta = I_K$ $E(f_t^T f_t) =$ diagonal matrix	Alternating Minimization for β and f

3 Data

3.1 Overview

- **Universe of currencies**: We downloaded and analyzed data for 47 currencies from Datastream. As a side note, developing world currencies are often not fully convertible. A fully convertible currency is one which can be converted to other currencies without any restrictions. We include the full universe of fully and partially convertible currencies in our analysis.

- **Begin date**: Different countries in Europe began adopting the Euro at different points in time. This limits the data available for these currencies.

- **Contemporaneous data**: Unlike equity markets, the FX market trades 24×7. As a consequence, there is no equivalent notion of "closing" price. The trading volume of a country's currency does tend to drop during its non-business hours, but the movements in prices can still be substantial. To reduce the impact of incorporating data at different times during the day, we set the frequency of our data to monthly.

- **Interest rate parity exclusions**: As detailed in Chapter 2.2, our definition of excess returns assumes that covered interest rate parity holds. Consequently, we have removed the following data points for some currencies in specific time spans, a procedure also used by Lustig et al. (see Lustig, Roussanov, and Verdelhan 2011, page 9).

Country	**Start Date**	**End Date**
South Africa	July 1985	August 1985
Malaysia	August 1998	June 2005
Indonesia	December 2000	May 2007
Turkey	October 2000	November 2001
UAE	June 2006	November 2006

3.2 Summary Statistics

Summary statistics of monthly returns on 47 currencies against USD are reported in (Appendix 3). Mean monthly returns across the currencies range from -0.59% to 1.11% and volatility, measured by standard deviation, ranges from 0.11% to 8.41%. Three currencies have very low volatility (below 1%): Hong Kong Dollar, Saudi Arabia Riyal and Kuwait Dinar. This is expected as these currencies are either pegged to USD or a basket of currencies i.e. they are not free floating. On the other end of the spectrum, developing countries tend to have higher volatilities. This is expected as developing countries, in general, are considered riskier. Indonesia, South Africa, Poland, Brazil, Egypt, Russia, and Iceland have monthly volatilities above 4% with Indonesia being somewhat of an outlier at 8.41%. Close to 75% of the currencies have volatilities between 2% and 4%.

In general, monthly returns of currencies have negative skewness. Only Switzerland, Japan, Ireland, and South Africa have returns with positive skewness. Skewness ranges from -6.64 to 0.47 across the currencies. Excess kurtosis ranges from -0.41 to 77.35.

3.3 Factor Construction

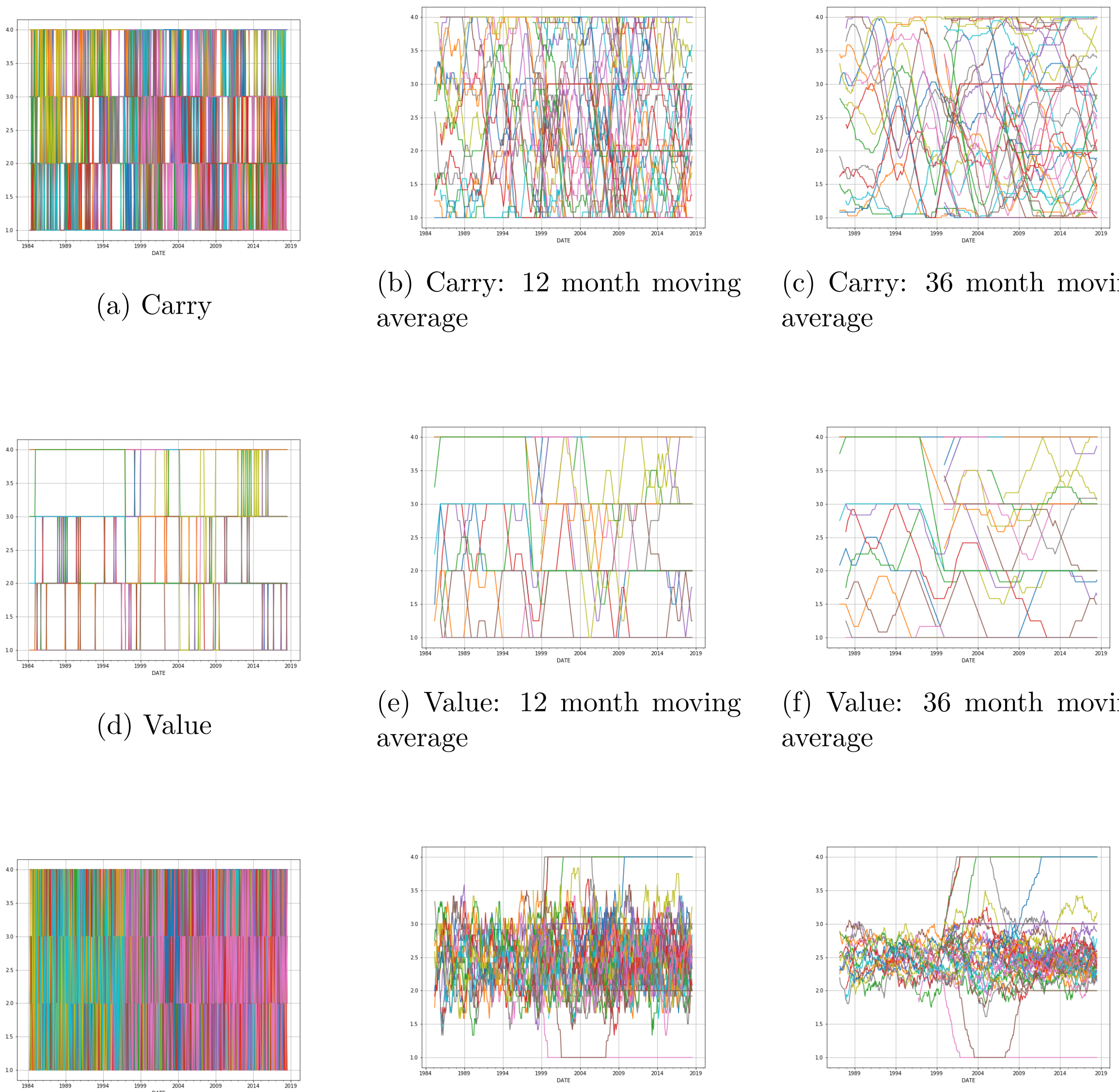
There has been significant work done on the construction of latent factors in literature, most notably by Lustig et al. (Lustig, Roussanov, and Verdelhan 2011, see). They create six equally weighted portfolios of currencies sorted on the basis of their forward discounts defined as $f_t - s_t$ where f_t is the log of the forward price and s_t is the log of the spot price. The portfolios are rebalanced monthly. The latent factors are constructed by fitting PCA to these portfolios on a 36-month rolling window. The first factor is a level factor and is the premium earned by currencies to account for the risk of the US Dollar appreciating. The second factor is a slope factor and is the one that co-varies differently with the carry portfolios. It is the PCA analog of the FFR carry factor. The Lustig method of fitting PCA on sorted portfolios can be extended to the other metrics as well, but the results do not seem to be very encouraging. Perhaps that is why we have not seen many published papers applying this technique to other metrics.

Instead of choosing six portfolios, we choose to use four portfolios since the cross-section at the start of the data is only 8 currencies. The FFR carry factor is constructed by going long the fourth rank carry portfolio and short the first rank carry portfolio.

For constructing value and momentum, we use the metrics defined by Aloosh and Bekaert (see Aloosh and Bekaert 2019, page 17). The value metric exploits a known tendency of currencies to revert to their long-term Purchasing Power Parity (PPP) values (see Mark 1995). It is defined as $\frac{<S_t^{X/USD}>}{PPP_{t-12}^{X/USD}}$, where $< S_t^{X/USD} >$ refers to the 3 month moving average of the daily X/USD spot rate. $PPP_{t-12}^{X/USD}$ is the average annual PPP for X/USD over the past year. Momentum is relatively straightforward and the metric is defined as the past month returns.

IPCA factor construction does not require any portfolio sorts. The characteristic metrics need to be provided to the matrix Z_t, and the algorithm is able to return loadings and factors upon convergence. The ranks of the currencies for each characteristic through time have been provided in the figure on the next page. As expected, momentum has the highest variation while value has the least.

Figure 2: Characteristic Ranks



(a) Carry

(b) Carry: 12 month moving average

(c) Carry: 36 month moving average

(d) Value

(e) Value: 12 month moving average

(f) Value: 36 month moving average

(g) Momentum

(h) Momentum: 12 month moving average

(i) Momentum: 36 month moving average

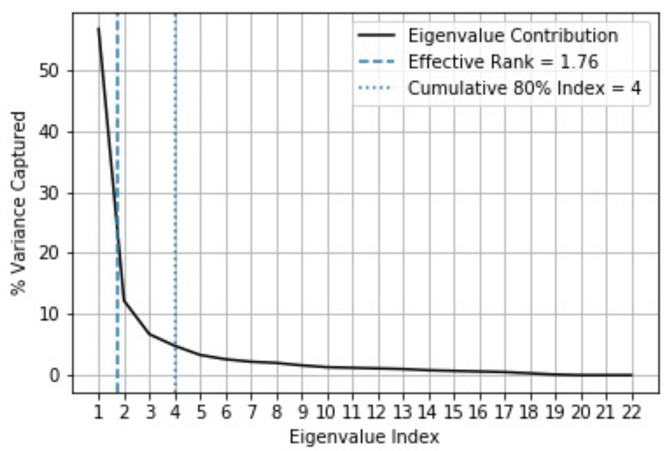
4 Preliminary Data Exploration

In this chapter, we will use PCA to perform an initial exploration of the data. Specifically, we will estimate the number of latent factors through EVD, study clustering and motivate IPCA by analyzing time variation of factor loadings.

4.1 Estimating Number of Latent Factors

To start off, it is important to ascertain the number of significant latent factors present in our data. This is made challenging due to a fluctuating cross-section of currencies. We ultimately settled on currency data starting in January 1999 for this analysis because it contains the Euro and has 22 currencies with monthly returns available for all the dates. The eigenvalues of the resulting covariance matrix have the familiar put option payoff-like figure as shown in Figure 3.

Figure 3: PCA: Ratio of Eigenvalues to Sum of Eigenvalues



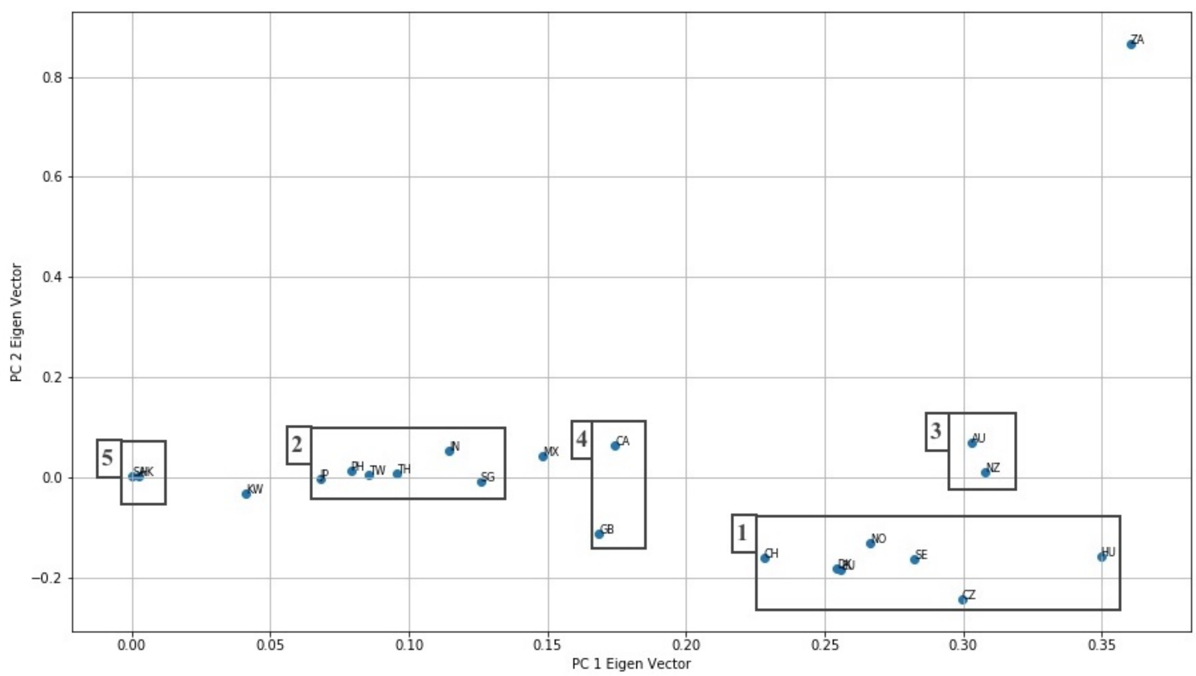
The effective rank is calculated as the ratio of the sum of all eigenvalues to the largest eigenvalue. The cumulative variance captured up to and including the fourth eigenvalue is in excess of 80%. This suggests that there is a factor structure in the data that can potentially be exploited to construct priced factors.

We use the iterative algorithm proposed by Onatski (see Onatski 2010) to determine the number of factors from the empirical distribution of the eigenvalues. The algorithm requires an initial estimate of the number of significant latent factors. We run it with initial values ranging from 6 to 15 and the optimal number always converged to 3. 76% cumulative variance is captured by the third eigenvalue and it seems to be a reasonable number of latent factors to extract.

4.2 Clustering in Currencies

As shown in the previous section, the first two components of PCA capture close to 68% of the total variance of the data and are largely representative of it. To further understand the trends and clusters in currency data, we plot the loadings of these two components in the figure below.

Figure 4: PCA Principal Component Similarity



Overall, we observe five major clusters described below:

1. **Europe**: Euro, Danish Krone, Norwegian Krone, Swedish Krona, Swiss Franc, Czech Koruna, Hungarian Forint.

2. **Asia**: Japanese Yen, Philippine Peso, New Taiwan Dollar, Thai Baht, Indian Rupee, Singapore Dollar.

3. **Oceania**: Australian Dollar and New Zealand Dollar.

4. **Anglosphere**: Canadian Dollar and British Pound.

5. **Pegged Currencies**: Hong Kong Dollar and Saudi Riyal.

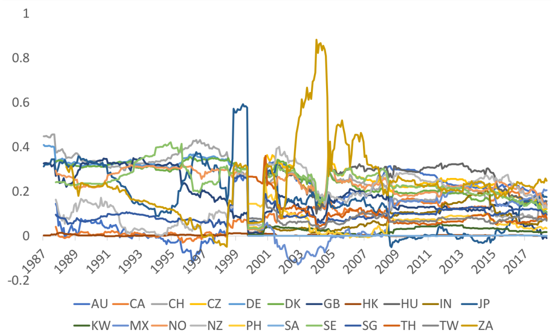
Most of these clusters are a result of geographical proximity. This is likely because of trade volumes being typically inversely related to distance. Nations at similar distances from the US are likely to have similar trade volumes with it, which introduces a common source of variation in their exchange rates. In the fifth cluster, the Hong Kong Dollar and Saudi Riyal picked up close to zero weights as these currencies are pegged and exhibit low volatility. In addition, we observe the South African Rand as an outlier. This is most likely due to its relatively higher volatility leading PCA to give it higher weights.

4.3 Time Variation

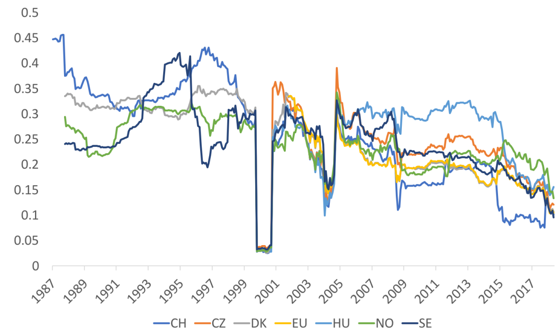
After understanding the static structure of the data, we analyze FX returns across time and study potential regime shifts. Figure 5 shows the results from a 36-month rolling window PCA.

Plot (a) shows loadings of the first PCA factor across time for all the currencies. In general, loadings are positive and vary significantly across time. The positive loadings confirm the first PCA factor as a level factor. The time variation of loadings shows how at different times, different currencies have explanatory power. For example, in the early 2000s, the South African Rand loading increased to 0.9. This may have been due to the South African Reserve Bank (SARB) officially adopting an inflation targeting framework for monetary policy in February

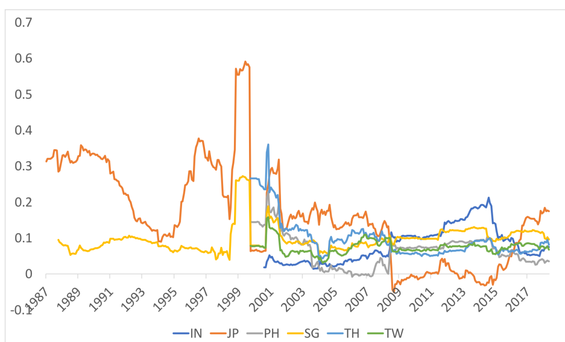
Figure 5: Time Variation: 36-Month Rolling-PCA



(a) Rolling PCA: All Currencies



(b) Rolling PCA: EU Cluster



(c) Rolling PCA: Asia Cluster

(d) Rolling PCA: AUD, NZD

2000. Prior to that, the SARB used a discretionary monetary policy framework that included monetary-aggregate and exchange-rate targeting. This was a regime shift.

Plot (b) shows loadings of the EU cluster of currencies while plot (c) shows loadings of the Asia cluster. Loadings of the EU cluster have been trending lower with a downward shock in the late 1990s. On the other hand, loadings of the Asia cluster had an upward shock during the same period. We can attribute these shocks to the Asia crisis that happened in the late 1990s. A number of Asian countries devalued their currencies and this caused volatility in FX markets. Asian currencies therefore played a bigger role in explaining currency movements during that time.

Plot (d) shows a slight upward trend in loadings of the Australian Dollar and the New Zealand Dollar. We speculate that this was likely due to an increased popularity of the carry trade in these currencies. Increased trading in these currencies mean they progressively became more important in explaining overall FX returns.

Regime shifts and the time-varying nature of factor loadings are further motivations for using IPCA to try to understand drivers of FX returns.

5 Results

In this section, we aim to analyze our IPCA approaches using two performance metrics:

- Total and Predictive R^2.
- Factor Spanned SDFs.

We will see that although IPCA gives a better R^2, implying the IPCA factors are well-priced in currency markets, it did not strictly outperform FFR on an SDF basis. After understanding the overall performance of IPCA factors, we will then study them in more detail. We will analyze IPCA factor portfolios across time and characteristic ranks and comment on the trends we observe.

5.1 Total and Predictive R^2

A sound factor model should generate factors that are priced. To test this for IPCA, we make use of two R-Squared measures calculated by Kelly et al. (see Kelly, Pruitt, and Su 2018, page 5):

1. **Total $\mathbf{R}^2$**: This panel R^2 captures the extent to which realized returns are described by the estimated loadings and factors. It is given by:

$$1 - \frac{\Sigma_t ||r_{t+1} - \hat{\beta}_t \hat{f}_{t+1}||_2^2}{\Sigma_t ||r_{t+1}||_2^2} \tag{19}$$

where $\hat{\beta}_t \in \mathbb{R}^{N \times K}$ are the estimated loadings and $\hat{f}_{t+1} \in \mathbb{R}^{K \times 1}$ are the estimated contemporaneous factor returns.

2. **Predictive $\mathbf{R}^2$**: The success of any asset pricing model is ascertained by how well it describes the differences in expected returns of assets. If the forecast returns are assumed to be factor exposures ($\hat{\beta}_t$) times the risk price of the factor $\hat{\lambda}_t$ ($= \frac{1}{t}\Sigma_{k=1}^{t} f_k$ since the factors are returns), we can compute another panel R^2 that would be a measure of how well the cross-section is fit by the factors. It is given by:

$$1 - \frac{\Sigma_t ||r_{t+1} - \hat{\beta}_t \hat{\lambda}_t||_2^2}{\Sigma_t ||r_{t+1}||_2^2} \tag{20}$$

The results for the FFR, PCA and IPCA approaches have been provided below. As shown, IPCA yields factors that explain returns and fit the cross-section better than both FFR and PCA. The outperformance of PCA and IPCA in terms of the total R^2, can be explained by the fact that the objective function for both explicitly maximizes it. This seems to have an impact on the predictive R^2 which is also higher for both. The results for PCA on the covariance matrix and the second-moment matrix are largely the same and markedly better than the correlation matrix. This is expected if covariance with factors gets priced rather than correlation.

Figure 6: R-Squared Comparison

	FFR		PCA		IPCA	
	Predictive Rsq	*Total Rsq*	*Predictive Rsq*	*Total Rsq*	*Predictive Rsq*	*Total Rsq*
Carry	-0.38%	17.55%	-	-	1.04%	86.00%
Momentum	0.01%	9.97%	-	-	1.13%	44.23%
Value	-0.23%	17.50%	-	-	1.18%	40.74%
Curr Acc	0.07%	24.80%	-	-	0.55%	26.37%
GDP Growth	-0.21%	4.20%	-	-	0.71%	24.86%
Carry, Momentum	-0.19%	23.56%	-	-	1.29%	48.03%
Carry, Value	1.09%	38.03%	-	-	1.33%	45.66%
Curr Acc, GDP Growth	-0.35%	29.49%	-	-	1.13%	42.98%
Momentum, Value	-0.58%	25.30%	-	-	0.87%	46.33%
Carrry, Momentum, Value	0.54%	41.95%	-	-	1.29%	49.06%
All Factors Combined	0.44%	56.20%	-	-	1.32%	49.12%
2nd Moment Matrix	-	-	0.82%	76.49%	-	-
Correlation Matrix	-	-	0.58%	63.66%	-	-
Covariance Matrix	-	-	0.81%	75.91%	-	-

5.2 SDF Estimation

As mentioned in Chapter 2.1, given a set of factors, the SDF can be calculated as the portfolio resulting from an ex-ante mean-variance optimization with a risk target of 1%. The weights of the corresponding factors are $\frac{t}{\sqrt{\lambda^T \Sigma_f^{-1} \lambda}} \Sigma_f^{-1} \lambda$ where t is the risk target, λ is the price of the factor risk (or its mean since we are dealing with tradable portfolios) and Σ_f is the covariance matrix of the factors. We need to forecast the λ and Σ_f to construct the SDF ex-ante. We calculate the mean returns and the covariance matrix using a rolling window of the past 36 months between $t-35$ and t and assume that these values remain constant over the next period. The ex-post risk (standard deviation) is also close to 1%.

Given a risk target, the mean-variance optimization can be interpreted as the maximal Sharpe portfolio[3]. This implies that the set of priced factor combination with the maximum Sharpe

[3] For holdings h, covariance matrix V and risk target t, the mean-variance objective is $\max_h h^T \mu$ subject to $h^T V h = t^2$. Since the t is constant, we can divide the objective by it without impacting the optima. The

ratio will span the SDF and is commonly used in the literature to contrast competing factor creation models (see Lettau and Pelger 2018, page 3).

5.2.1 Effect of Adding a Constant Characteristic to IPCA

We observed that adding a constant vector of 1s to the characteristics matrix Z_t generates results that are significantly better than without. We interpret this as the inclusion of the risk that the US Dollar moves against the universe currencies. From an econometric standpoint, it can be thought of as allowing for a level correction that leads to more robust results similar to the inclusion of an intercept.

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new objective would be $\max_h \frac{h^T \mu}{\sqrt{h^T V h}}$ subject to $h^T V h = t^2$. This is precisely maximizing Sharpe ratio with a risk target.

Figure 7: IPCA SDF With and Without Constant

	Approach	SDF (ex-ante)					SDF (ex-post)				
		Leverage	*Mean*	*Sharpe*	*Std Dev*	*Turnover*	*Leverage*	*Mean*	*Sharpe*	*Std Dev*	*Turnover*
	Constant	1.38	0.12%	0.11	1.05%	0.17	1.38	0.17%	0.17	1.00%	0.16
Without Constant Factor	Carry	1.36	0.14%	0.12	1.09%	0.28	1.35	0.20%	0.20	1.03%	0.25
	Momentum	1.34	0.13%	0.12	1.05%	0.56	1.35	0.20%	0.20	1.00%	0.55
	Value	1.28	0.06%	0.06	1.05%	0.21	1.28	0.14%	0.14	1.00%	0.20
	Curr Acc	1.32	0.07%	0.06	1.08%	0.20	1.33	0.17%	0.16	1.02%	0.19
	GDP Growth	1.44	0.13%	0.11	1.09%	0.27	1.44	0.20%	0.19	1.02%	0.27
	Carry, Momentum	1.77	0.08%	0.07	1.12%	0.95	1.78	0.29%	0.29	1.00%	0.95
	Carry, Value	2.19	0.17%	0.14	1.20%	0.37	2.19	0.35%	0.34	1.03%	0.38
	Carry, Momentum, Value	2.28	0.17%	0.14	1.15%	0.78	2.27	0.39%	0.41	0.95%	0.77
With Constant Factor	Carry	2.07	0.26%	0.22	1.14%	0.38	2.09	0.38%	0.37	1.02%	0.37
	Momentum	1.72	0.16%	0.16	1.01%	1.05	1.72	0.31%	0.34	0.93%	1.04
	Value	2.32	0.15%	0.13	1.13%	0.32	2.31	0.28%	0.27	1.03%	0.33
	Curr Acc	2.61	0.12%	0.10	1.18%	0.39	2.61	0.29%	0.27	1.08%	0.40
	GDP Growth	2.11	0.11%	0.1	1.10%	0.51	2.11	0.27%	0.27	0.99%	0.51
	Carry, Momentum	2.27	0.32%	0.27	1.18%	0.79	2.28	0.49%	0.47	1.03%	0.77
	Carry, Value	2.70	0.29%	0.24	1.18%	0.43	2.70	0.46%	0.45	1.02%	0.43
	Carry, Momentum, Value	2.24	0.31%	0.26	1.20%	0.84	2.24	0.48%	0.46	1.05%	0.84

5.2.2 Comparison Across Models

Figures 8, 9 and 10 show the performance summaries of the SDF in various FFR, PCA and IPCA combinations. We have calculated the ex-ante and ex-post Sharpe ratios to get a sense of the loss in performance due to errors in forecasts of expected returns (the covariance matrix forecast errors are relatively smaller). The Approach column in the figures contains the names of the characteristics that were used for constructing the factors which in turn were utilized for the SDF estimation.

The IPCA factors implied SDFs have ex-ante Sharpe ratios that are slightly worse than the FFR factors implied SDFs for comparable leverage and turnover. PCA has the worst metrics. The Sharpe ratios likely have errors since the expected returns/covariance matrix forecast models we used were rudimentary. Good forecasts of the expected returns would require models and data that are beyond the scope of this paper. To get a sense of how good the performance could be, we also calculated SDFs ex-post. A downside of introducing the look-ahead bias is that the Sharpe ratios tend to increase with additional factors since the expected returns forecasts are now perfect. Comparing both these Sharpe ratios, we find that carry and momentum have the best ex-ante Sharpe ratios for both FFR and IPCA with FFR marginally better. However, the ex-post for IPCA is much better.

On balance, this evidence is inconclusive over which approach is definitively better. The best we can say is that IPCA retains the better cross-sectional explanatory power of PCA while providing ex-ante performance comparable to FFR.

Figure 8: FFR SDF Performance Metrics

Approach	*Leverage*	*Mean*	*Sharpe*	*Std Dev*	*Turnover*	*Leverage*	*Mean*	*Sharpe*	*Std Dev*	*Turnover*
	SDF (ex-ante)					**SDF (ex-post)**				
Carry	1.98	0.24%	0.22	1.10%	0.26	1.98	0.28%	0.27	1.04%	0.26
Momentum	1.88	0.18%	0.17	1.05%	1.19	1.88	0.23%	0.23	0.98%	1.19
Value	2.64	0.05%	0.05	1.17%	0.17	2.63	0.12%	0.12	1.05%	0.18
Curr Acc	2.22	0.00%	0.00	1.02%	0.23	2.22	0.15%	0.15	0.97%	0.24
GDP Growth	1.99	0.08%	0.07	1.03%	0.37	1.99	0.15%	0.15	0.97%	0.37
Carry, Momentum	2.13	0.33%	0.28	1.16%	0.73	2.13	0.41%	0.39	1.06%	0.73
Carry, Value	2.64	0.24%	0.18	1.31%	0.28	2.65	0.39%	0.35	1.11%	0.28
Carry, Curr Acc	2.40	0.25%	0.19	1.27%	0.29	2.40	0.37%	0.33	1.12%	0.29
Carry, GDP Growth	2.21	0.22%	0.20	1.11%	0.37	2.21	0.35%	0.37	0.96%	0.37
Momentum, Value	2.45	0.20%	0.18	1.11%	0.77	2.45	0.33%	0.32	1.01%	0.77
Curr Acc, GDP Growth	2.29	0.05%	0.05	1.08%	0.35	2.29	0.22%	0.23	0.95%	0.36
Carry, Momentum, Value	2.61	0.33%	0.27	1.25%	0.56	2.62	0.49%	0.46	1.07%	0.57
All Factors Combined	2.99	0.32%	0.24	1.30%	0.58	2.99	0.57%	0.55	1.04%	0.58

Figure 9: PCA SDF Performance Metrics

SVD Matrix	*Leverage*	*Mean*	*Sharpe*	*Std Dev*	*Turnover*	*Leverage*	*Mean*	*Sharpe*	*Std Dev*	*Turnover*
	SDF (ex-ante)					**SDF (ex-post)**				
Second Moment	2.04	-0.06%	-0.05	1.30%	0.47	2.04	0.31%	0.32	0.99%	0.46
Correlation	2.32	0.03%	0.02	1.42%	0.59	2.30	0.40%	0.37	1.09%	0.58
Covariance	2.01	-0.05%	-0.03	1.41%	0.51	2.00	0.35%	0.32	1.08%	0.50

Figure 10: IPCA SDF Performance Metrics

Approach	*Leverage*	*Mean*	*Sharpe*	*Std Dev*	*Turnover*	*Leverage*	*Mean*	*Sharpe*	*Std Dev*	*Turnover*
	SDF (ex-ante)					**SDF (ex-post)**				
Carry	2.07	0.26%	0.22	1.14%	0.38	2.09	0.38%	0.37	1.02%	0.37
Momentum	1.72	0.16%	0.16	1.01%	1.05	1.72	0.31%	0.34	0.93%	1.04
Value	2.32	0.15%	0.13	1.13%	0.32	2.31	0.28%	0.27	1.03%	0.33
Curr Acc	2.61	0.12%	0.10	1.18%	0.39	2.61	0.29%	0.27	1.08%	0.40
GDP Growth	2.11	0.11%	0.1	1.10%	0.51	2.11	0.27%	0.27	0.99%	0.51
Carry, Momentum	2.27	0.32%	0.27	1.18%	0.79	2.28	0.49%	0.47	1.03%	0.77
Carry, Value	2.70	0.29%	0.24	1.18%	0.43	2.70	0.46%	0.45	1.02%	0.43
Carry, Curr Acc	2.65	0.29%	0.23	1.25%	0.44	2.64	0.48%	0.46	1.05%	0.44
Carry, GDP Growth	2.34	0.28%	0.21	1.30%	0.5	2.35	0.48%	0.43	1.11%	0.51
Momentum, Value	2.42	0.15%	0.13	1.08%	0.85	2.43	0.0034	0.35	0.0098	0.85
Curr Acc, GDP Growth	2.62	0.10%	0.09	1.21%	0.51	2.6	0.0037	0.36	0.0102	0.49
Carry, Momentum, Value	2.24	0.31%	0.26	1.20%	0.84	2.24	0.48%	0.46	1.05%	0.84
All Factor Combined	2.09	0.24%	0.20	1.22%	0.85	2.10	0.44%	0.43	1.03%	0.85

5.3 Currency Weights Interpretation

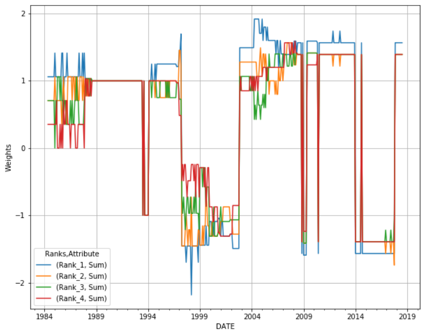
A common downside of latent factors is that there is no obvious interpretability in terms of exposure to characteristics. In this section, we study in more detail how the IPCA portfolios are weighted across time and how these weights signal changing exposure to characteristics and cast influence on its performance relative to FFR. The model is evaluated using a 36-month rolling window and the reported factor portfolios are constructed using next period realizations of the previous period's weights. We bucket currencies into four portfolios based on characteristic ranks and analyze how much weight IPCA has attributed to them.

In order to make the comparison, it is important to understand that in an FFR setting, we long the rank 4 portfolio, short the rank 1 portfolio, and assign no weights to portfolios rank 2 and 3. This is equivalent to setting the sum of portfolio weights to 1 for rank 4, -1 for rank 1 and 0 for ranks 2 and 3. This should be used as the baseline for the IPCA factors and differences in weights can be utilized to explain IPCA's relative performance.

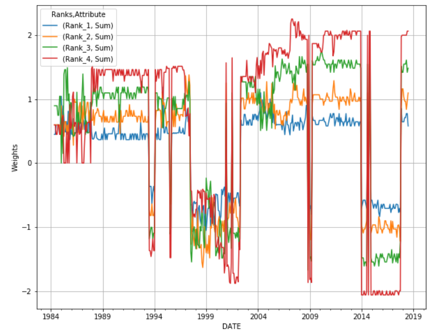
We start with the simplest case of IPCA construction using single characteristics: constant, carry, value, and momentum. In Figure 11, we report the sum of weights bucketed by carry ranks through time for these factor portfolios.

We observe sharp changes of net long and short regimes which are noticeably similar regardless of the characteristic used. The short bias coincides with the Asian currency crisis during which the US Dollar appreciated due to its status as a safe haven. If we look at IPCA carry, then the positive bias regimes see higher weights on the higher ranks. However, if we look at the negative bias phases, the directions seem to flip with the most negative weights on the higher ranks. This indicates that the factor is long carry in the former phase while short carry in the latter phase signifying that it has learned to short carry during the disaster of

Figure 11: IPCA sum of Coefficients bucketed by Carry Ranks

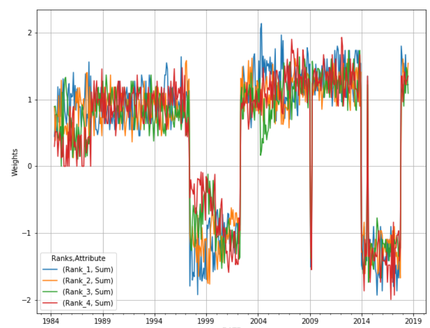


(a) IPCA, Characteristics: Constant

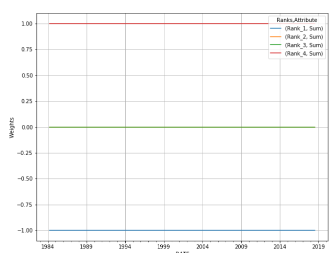


(b) IPCA, Characteristics: Carry

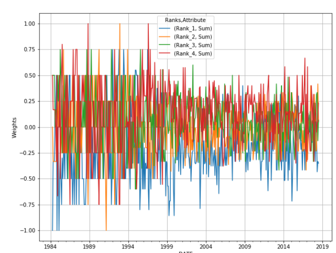
(c) IPCA, Characteristics: Value



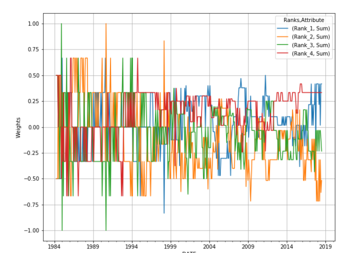
(d) IPCA, Characteristics: Momentum



(e) FFR Carry Factor



(f) FFR Momentum Factor



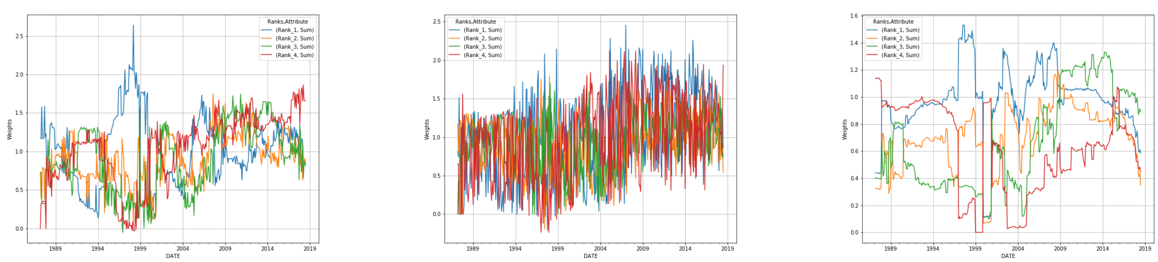
(g) FFR Value Factor

the Asian currency crisis. This reduction in weights is not predictive and suggests that IPCA is particularly vulnerable to factor shocks.

The value-based factor has the same broad regimes but its composition in terms of carry ranks is much more ambiguous as expected. The momentum based factor has more noise compared to others which is in line with the variance of the momentum ranks.

We can use the same approach to interpret the PCA factors as well. The first factor is short value for most of the historical period as can be seen in Figure 12 (c). It also seems to be short carry in some regimes. It is likely picking up the risk of the US Dollar appreciating and seems to be long the lowest characteristic rank portfolios.

Figure 12: PCA 1 Sum of Coefficients bucketed by Carry, Value and Momentum



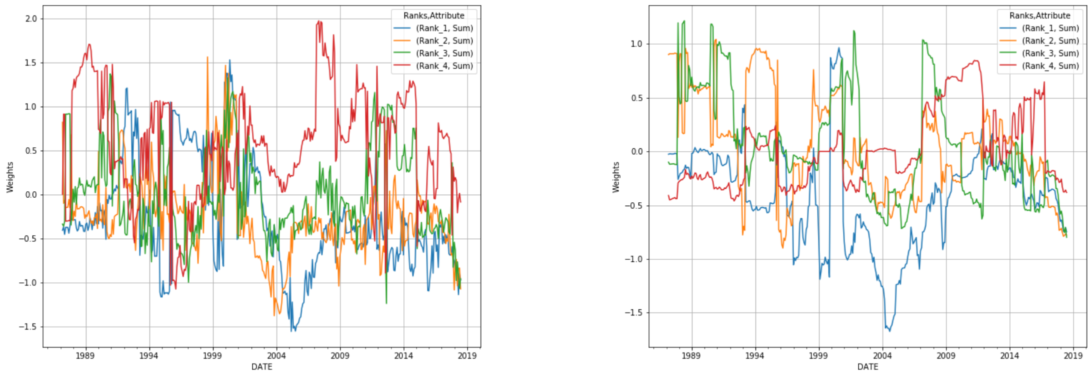
(a) PCA Component 1, bucketed by Carry

(b) PCA Component 1, bucketed by Momentum

(c) PCA Component 1, bucketed by Value

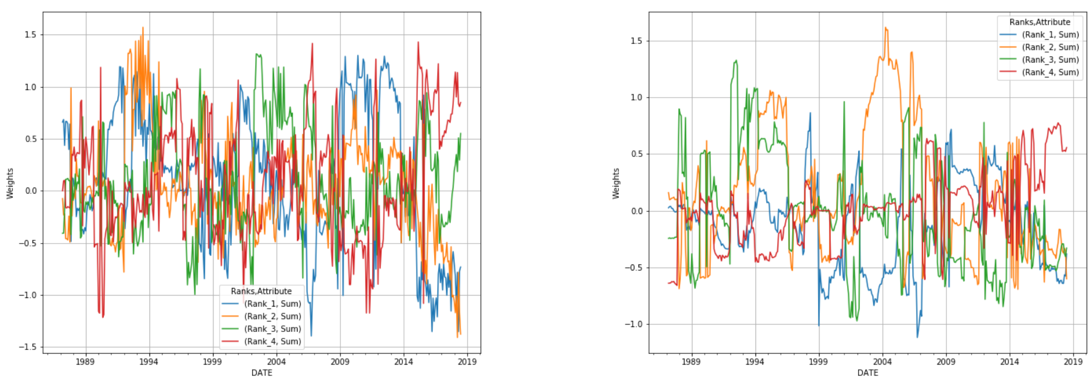
The second factor portfolio has a higher sum of weights for the carry rank 4 bucket. This implies that it has a decisive long carry exposure. Exposure to momentum is as noisy as the metric, and nothing definitive can be said about it. Exposure to value tends to oscillate between clearly marked positive and negative regimes. The third factor portfolio does not seem to have a decisive exposure to any of the FFR components.

Figure 13: PCA 2 Sum of Coefficients bucketed by Carry, Value and Momentum



(a) PCA Component 2, bucketed by Carry (b) PCA Component 2, bucketed by Value

Figure 14: PCA 3 Sum of Coefficients bucketed by Carry, Value and Momentum

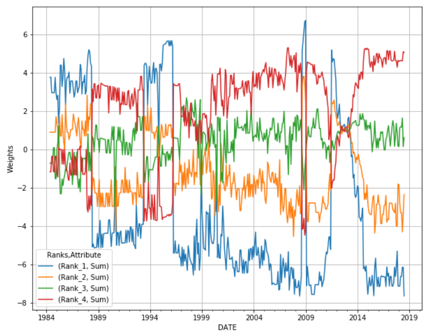


(a) PCA Component 3, bucketed by Carry (b) PCA Component 3, bucketed by Value

As was seen in Figure 3, the first principal component explains more than 50% of the variation. It has a prolonged short value exposure. The second principal component has the next highest contribution to variance of around 10% and seems to have a long carry exposure. The third and fourth components do not seem to have explicit exposures to any FFR factor as there is no observable pattern.

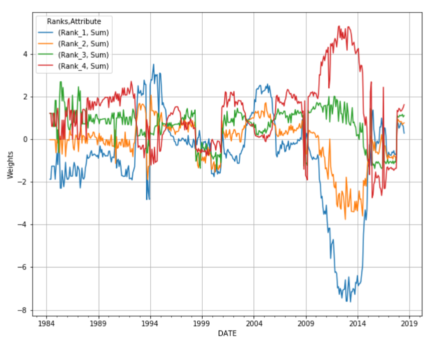
In Figure 15, we examine factors constructed using two sets of characteristics: 1) constant and carry; 2) constant, carry, and value. As dictated by the IPCA constraint, the maximum number of factors for these are 2 and 3 respectively. An immediate observation is that the regimes of net long and short biases disappear with the addition of the constant. The first factor in each of the case has a dominant exposure to carry. We can also notice that the addition of value does not majorly impact the IPCA factors. They are similar for both configurations. The third component for constant, carry, value configuration has a long value exposure. This is observed in Figure 24 in (Appendix 6).The ranking of weights by value is presented in the appendix.

Figure 15: IPCA Sum of Coefficients bucketed by Carry Ranking for No. of Characteristics = 2 and 3

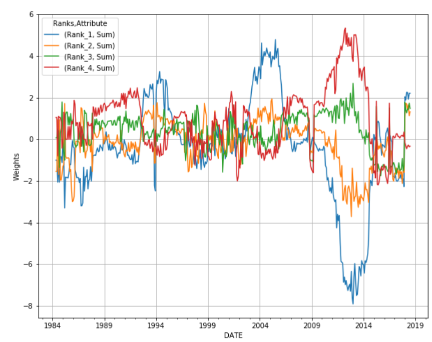


(a) IPCA Component 1, Characteristics: constant, carry

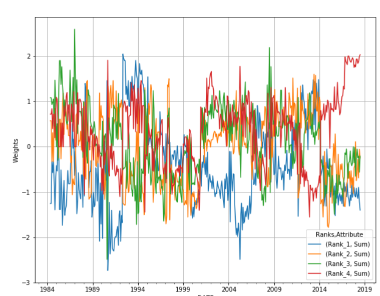
(b) IPCA Component 1, Characteristics: constant, carry, value



(c) IPCA Component 2, Characteristics: constant, carry



(d) IPCA Component 2,Characteristics: constant, carry, value

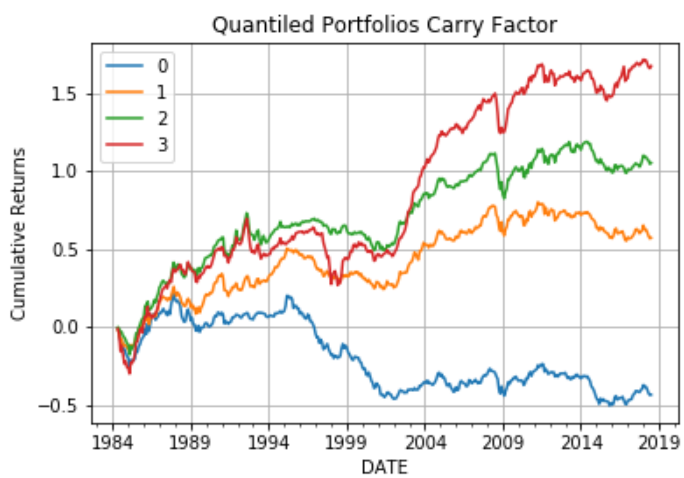


(e) IPCA Component 3, Characteristics: constant, carry, value

The carry portfolios have different performance in terms of cumulative returns as showcased in Figure 16. It shows why the FFR model chooses to long rank 4 and short rank 1.

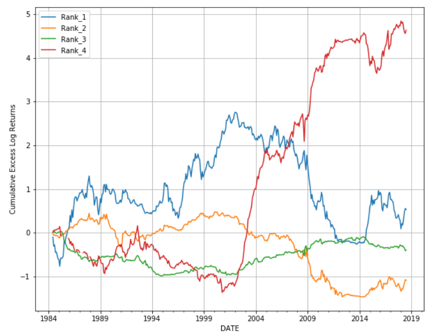
While bucketing the sum of coefficients by carry, value and momentum ranks helps understand exposure to the characteristics, we also wish to understand the impact on returns. For this purpose, we look at the cumulative returns after bucketing by characteristic ranks. Figure 17 has configurations for IPCA similar to those in Figure 15. It shows the cumulative sum of log excess returns bucketed by carry ranks. Figure 17 therefore has the IPCA components decomposed into the four carry portfolios.

Figure 16: Quantiled Portfolios : Carry



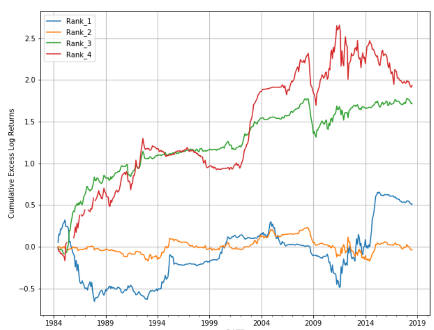
The constant and carry configuration in Figure 17 (a) gets most of its returns through the Rank 4 carry portfolio (long carry portfolio) post 2000. The corresponding returns of each carry rank portfolio are provided in Figure 16. Comparing the relative magnitudes of returns, it becomes apparent that the first IPCA factor seems to have leveraged longs in rank 4. Its losses are due to the rank 2 portfolio where its consistently negative weighting results in losses. The second IPCA factor seems to be very judicious in its allocation of weights to carry ranks with none of the portfolios losing money on average. Its quality of returns seems to be superior to the first factor. This analysis could be used for choosing factors from disparate characteristic configurations with desirable properties.

Figure 17: IPCA Cumulative Returns in Carry Ranked buckets for No. of Characteristics = 2 and 3



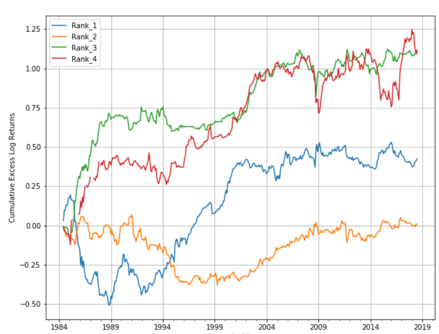
(a) IPCA Component 1, Characteristics: Constant, Carry

(b) IPCA Component 1,Characteristics: Constant, Carry, Value



(c) IPCA Component 2, Characteristics: Constant, Carry

(d) IPCA Component 2,Characteristics: Constant, Carry, Value



(e) IPCA Component 3, Characteristics: Constant, Carry, Value

6 Conclusion

We demonstrated that by incorporating characteristics, IPCA yields factors that fit the cross-section better than both PCA and FFR. This result is in line with the results presented by Kelly et al. (see Kelly, Pruitt, and Su 2018, page 6) for equities. However, this better explanation of the cross-section does not necessarily translate into higher Sharpe ratio portfolios as compared to the FFR approach in the currency market.

7 Future Research

The IPCA methodology by design can take as input characteristics that can affect the FX market. While we worked on a handful of characteristics due to time and data constraints, a gamut of characteristics could be used. These include Macro factors such as volatility, inflation, trade deficits; Technical Factors such as 52-week highs, Moving averages etc; Alternative data sets such as news sentiment. These characteristics can help in improving the total and predictive R-squared. It could also be possible to use the analysis tools developed by us in Chapter 5.3 to ascertain factors with good quality of returns (not necessarily captured by Sharpe ratios) that could be combined to create a better benchmark.

Acknowledgements

This project would not have been possible without the guidance of Professors Martin Lettau and Eric Reiner. We thank you for your time, help, and mentoring and hope that we were exactly the right amount of bother.

8 Appendix

Appendix 1

The Fama-French Approach:

A typical factor construction methodology following the Fama-French Approach would involve the following steps:

- Take a known/discovered anomaly such as smaller companies on average earns a risk premium due to their lower ability to raise capital from the debt market.

- Construct a portfolio that takes advantage of this "size" anomaly-

  1. Come up with a quantifiable figure such as the market cap that captures the characteristics of size and is comparable in the cross-section of stocks.
  2. Sort companies on the basis of said figure in increasing order into a fixed number of groups (say 10).
  3. Create portfolios of these groups using simplistic approaches such as equal or market cap weighting. The factor portfolio is taking on a long position of the first portfolio and a short position of the last.

Appendix 2

Link SDF Coefficients:

$$
\begin{aligned}
m_{t+1} &= 1 - b^T(f_{t+1} - \lambda) \\
m_{t+1}(f_{t+1} - \lambda)^T &= (f_{t+1} - \lambda)^T - b^T(f_{t+1} - \lambda)(f_{t+1} - \lambda)^T \\
E_t[m_{t+1}(f_{t+1} - \lambda)^T] &= E_t[(f_{t+1} - \lambda)^T] - b^T E_t[(f_{t+1} - \lambda)(f_{t+1} - \lambda)^T] \\
0 - \lambda^T &= 0 - b^T \Sigma_f \\
\lambda &= \Sigma_f b
\end{aligned}
$$

The above holds only if the factors are returns of a tradable asset since that implies $E_t(m_{t+1} f_{t+1}) = 0$.

Appendix 3

Descriptive Statistics of Currency Universe:

Table Legend:

1. Start, End: Start and End dates of the currency data
2. μ, σ, γ, κ: Mean, Standard Deviation, Skewness, Excess Kurtosis respectively

Currency	Start	End	μ	σ	Median	γ	κ	Count
British Pound	1983-11	2018-07	0.10%	2.94%	0.00%	-0.20	2.29	417
Swiss Franc	1983-11	2018-07	0.05%	3.30%	0.04%	0.06	0.66	417
Japanese Yen	1983-11	2018-07	-0.02%	3.20%	-0.07%	0.33	1.57	417
Canadian Dollar	1985-01	2018-07	0.07%	2.14%	0.13%	-0.54	4.26	403
Australian Dollar	1985-01	2018-07	0.22%	3.39%	0.39%	-0.69	2.44	403
New Zealand Dollar	1985-01	2018-07	0.43%	3.56%	0.52%	-0.29	1.75	403
Swedish Krona	1985-01	2018-07	0.12%	3.23%	0.21%	-0.40	1.20	403
Norwegian Krone	1985-01	2018-07	0.19%	3.13%	0.35%	-0.38	1.01	403
Danish Krone	1985-01	2018-07	0.21%	3.06%	0.34%	-0.15	0.65	403
Euro	1999-01	2018-07	-0.03%	2.88%	0.06%	-0.17	0.96	235
Deutsche Mark	1983-11	1998-12	0.17%	3.38%	0.22%	-0.20	0.13	182
Italian Lira	1984-04	1998-12	0.32%	3.26%	0.61%	-0.60	1.62	177
French Franc	1983-11	1998-12	0.33%	3.25%	0.51%	-0.24	0.32	182
Dutch Guilder	1983-11	1998-12	0.19%	3.38%	0.29%	-0.19	0.22	182
Belgian Franc	1997-01	1998-12	-0.50%	2.74%	0.56%	-0.26	-0.19	24
Finnish Markka	1997-01	1998-12	-0.59%	2.80%	0.33%	-0.27	-0.30	24
Irish Punt	1993-11	1998-12	-0.13%	2.23%	-0.48%	0.47	0.25	62
Hong Kong Dollar	1983-11	2018-07	-0.02%	0.19%	-0.01%	-0.51	7.30	417

Currency	Start	End	μ	σ	Median	γ	κ	Count
South African Rand	1983-11	2018-07	0.46%	4.92%	0.20%	0.36	2.46	415
Singapore Dollar	1985-01	2018-07	0.03%	1.60%	0.04%	-0.26	2.78	403
Austrian Schilling	1997-01	1998-12	-0.50%	2.76%	0.53%	-0.26	-0.17	24
Czech Koruna	1997-01	2018-07	0.14%	3.55%	0.27%	-0.27	0.48	259
Greek Drachma	1997-01	2000-12	-0.40%	3.13%	-0.67%	-0.03	0.96	48
Hungarian Forint	1997-11	2018-07	0.27%	3.92%	0.61%	-1.04	3.79	249
Indian Rupee	1997-11	2018-07	0.14%	2.06%	0.29%	-0.33	2.93	249
Indonesian Rupiah	1997-01	2018-07	-0.25%	8.41%	0.06%	-2.54	23.58	181
Kuwaiti Dinar	1997-01	2018-07	0.05%	0.66%	0.03%	-1.15	14.94	259
Malaysian Ringgit	1997-01	2018-07	-0.16%	2.88%	0.13%	-0.42	4.39	176
Mexican Peso	1997-01	2018-07	0.25%	3.02%	0.41%	-0.88	2.67	259
Philippine Peso	1997-01	2018-07	0.05%	2.37%	0.20%	-1.04	5.91	259
Polish Zloty	2002-03	2018-07	0.27%	4.07%	0.39%	-0.80	1.90	197
Portuguese Escudo	1997-01	1998-12	-0.44%	2.62%	0.61%	-0.19	-0.27	24
Saudi Riyal	1997-01	2018-07	0.01%	0.11%	0.01%	-5.31	77.35	259
South Korean Won	2002-03	2018-07	0.16%	3.25%	0.44%	-0.40	4.48	197
Spanish Peseta	1997-01	1998-12	-0.41%	2.71%	0.58%	-0.25	-0.10	24
New Taiwan Dollar	1997-01	2018-07	-0.12%	1.57%	-0.08%	-0.06	3.47	259
Thai Baht	1997-01	2018-07	0.08%	3.10%	0.24%	-0.56	17.93	259
Brazilian Real	2004-04	2018-07	0.60%	4.43%	0.93%	-0.53	1.51	172
Egyptian Pound	2004-04	2018-07	0.90%	4.67%	0.74%	-6.64	77.3	172
Russian Ruble	2004-04	2018-07	0.08%	4.23%	0.35%	-0.73	3.96	172
Slovak Koruna	2002-03	2008-12	1.11%	3.36%	1.11%	-0.29	1.21	82
Croatian Kuna	2004-04	2018-07	0.08%	2.98%	0.23%	-0.52	1.17	172
Cypriot Pound	2004-04	2007-12	0.40%	2.02%	0.34%	-0.15	-0.32	45

Currency	Start	End	μ	σ	Median	γ	κ	Count
Israeli New Shekel	2004-04	2018-07	0.15%	2.37%	0.19%	-0.26	0.66	172
Icelandic Króna	2004-04	2018-07	0.25%	4.21%	0.36%	-1.27	7.59	172
Slovenian Tolar	2004-04	2006-12	0.22%	2.17%	-0.03%	-0.02	-0.41	33
Bulgarian Lev	2004-04	2018-07	-0.02%	2.89%	0.20%	-0.36	1.49	172

Appendix 4

Performance Metrics: IPCA, PCA, FFR:

In the figure below We contrast various variations for the Fama French approach based on the performance metrics. Combinations of factors: carry, momentum, value, current account, GDP growth are compared.

Figure 18: FFR Performance Metrics

Approach	**Equal Weight Portfolio**				
	Leverage	*Mean*	*Sharpe*	*Std Dev*	*Turnover*
Carry	3.00	0.01	0.22	0.02	0.25
Momentum	2.28	0.01	0.23	0.02	1.21
Value	2.63	0.00	0.15	0.02	0.06
Curr Acc	2.62	0.00	0.02	0.02	0.07
GDP Growth	2.70	0.00	-0.02	0.02	0.29
Carry, Momentum	2.19	0.01	0.33	0.02	0.83
Carry, Value	3.08	0.00	0.25	0.02	0.18
Carry, Curr Acc	3.00	0.00	0.23	0.01	0.23
Carry, GDP Growth	2.71	0.00	0.14	0.02	0.32
Momentum, Value	3.00	0.00	0.27	0.01	0.78
Curr Acc, GDP, Growth	3.00	0.00	0.00	0.02	0.20
Carry, Momentum, Value	3.15	0.00	0.35	0.01	0.63
All Factor Combined	3.00	0.00	0.25	0.01	0.55

In Figure 19, we contrast variations of the IPCA approach based on the performance metrics. Various combinations of factors: carry, momentum, value, GDP growth, current account are compared. We compare by adding factors one by one. The naive way to combine the factors is equally weighing the factors (Figure 20). We also generate a stochastic discount factor (SDF) to combine these factors in an optimal way. The Sharpe ratio for the equal-weighted version goes up as we add more factors. The leverage and turnovers also go up as we add more factors.

If we compare these metrics between the FFR, IPCA and PCA versions, we find that the PCA version performs poorly even in all cases. The IPCA equal weighted combination works almost at par with the FFR combinations. The leverage however is higher for IPCA compared to the FFR combinations.

Figure 19: IPCA Performance Metrics

Approach	Factor 1					Factor 2					Factor 3				
	Leverage	*Mean*	*Sharpe*	*Std Dev*	*Turnover*	*Leverage*	*Mean*	*Sharpe*	*Std Dev*	*Turnover*	*Leverage*	*Mean*	*Sharpe*	*Std Dev*	*Turnover*
Constant	8.81	1.18%	0.12	9.56%	0.15										
Carry	7.89	1.50%	0.16	9.18%	0.24										
Momentum	7.89	1.28%	0.15	8.66%	0.52										
Value	6.41	1.02%	0.12	8.16%	0.17										
GDP Growth	6.94	1.17%	0.15	7.79%	0.24										
Curr Acc	7.24	0.99%	0.12	8.01%	0.13										
Constant Carry	15.57	1.68%	0.15	11.56%	2.84	7.76	1.57%	0.25	6.31%	1.14					
Constant Momentum	17.37	2.10%	0.13	15.61%	3.65	5.99	1.08%	0.19	5.62%	1.03					
Constant Value	8.90	1.06%	0.13	8.46%	1.23	8.28	0.65%	0.19	3.45%	1.73					
Constant GDP Growth	11.45	1.46%	0.14	10.72%	1.94	7.27	0.74%	0.17	4.28%	1.26					
Constant Curr Acc	12.88	1.11%	0.09	12.13%	1.63	7.90	0.36%	0.11	3.33%	1.29					
Carry Value	7.90	1.32%	0.15	8.60%	1.39	7.21	0.62%	0.16	3.91%	1.70					
Carry Momentum	10.57	0.71%	0.07	10.23%	2.63	6.93	1.09%	0.19	5.82%	1.05					
Carry GDP Growth	8.09	0.93%	0.11	8.83%	1.60	7.44	0.64%	0.16	3.97%	1.35					
Carry Curr Acc	7.83	1.63%	0.19	8.55%	1.22	7.75	0.59%	0.14	4.26%	1.16					
Momentum Value	7.70	1.17%	0.15	7.89%	1.58	7.19	0.56%	0.12	4.77%	1.73					
Curr Acc GDP Growth	6.55	1.07%	0.14	7.51%	1.35	7.58	0.57%	0.14	4.13%	1.76					
Constant Carry Value	15.71	1.94%	0.17	11.68%	2.23	8.58	1.08%	0.17	6.40%	1.12	7.42	0.74%	0.24	3.04%	1.61
Constant Carry Momentum	18.61	2.86%	0.19	14.81%	3.70	12.68	1.26%	0.14	8.82%	1.31	5.97	1.08%	0.24	4.47%	1.02
Constant Momentum Value	17.86	2.28%	0.15	15.01%	2.46	6.93	0.72%	0.13	5.68%	1.06	7.42	0.56%	0.17	3.29%	1.74
Carry Momentum Value	11.03	1.31%	0.13	10.01%	2.13	7.66	0.93%	0.16	6.02%	1.11	6.92	0.61%	0.18	3.44%	1.69
Constant Curr Acc GDP Growth	14.62	1.71%	0.14	12.54%	2.39	8.47	0.85%	0.16	5.27%	1.28	6.88	0.33%	0.12	2.63%	1.28
Constant Carry Momentum Value	17.84	2.87%	0.20	14.48%	3.72	12.25	1.62%	0.19	8.61%	1.31	5.85	0.95%	0.21	4.55%	0.60
All Factor Combined	18.40	2.66%	0.17	15.30%	3.33	12.32	1.58%	0.17	9.35%	1.29	6.43	0.91%	0.17	5.34%	1.16

Figure 20: IPCA Performance Metrics Equal Weighted

Approach	Equal Weight Portfolio				
	Leverage	*Mean*	*Sharpe*	*Std Dev*	*Turnover*
Constant	8.81	1.18%	0.12	9.56%	0.15
Carry	7.89	1.50%	0.16	9.18%	0.24
Momentum	7.89	1.28%	0.15	8.66%	0.52
Value	6.41	1.02%	0.12	8.16%	0.17
GDP Growth	6.94	1.17%	0.15	7.79%	0.24
Curr Acc	7.24	0.99%	0.12	8.01%	0.13
Constant Carry	11.67	1.63%	0.25	6.60%	0.32
Constant Momentum	11.68	1.59%	0.20	8.08%	1.20
Constant Value	8.59	0.85%	0.18	4.62%	0.19
Constant GDP Growth	9.36	1.10%	0.19	5.86%	0.36
Constant Curr Acc	10.39	0.73%	0.12	6.24%	0.34
Carry Value	7.56	0.97%	0.21	4.72%	0.32
Carry Momentum	8.75	0.90%	0.15	5.86%	1.12
Carry GDP Growth	7.77	0.79%	0.17	4.70%	0.44
Carry Curr Acc	7.79	1.11%	0.23	4.77%	0.34
Momentum Value	7.45	0.86%	0.20	4.33%	0.68
Curr Acc GDP Growth	7.07	0.82%	0.20	4.18%	0.32
Constant Carry Value	10.57	1.25%	0.28	4.42%	0.36
Constant Carry Momentum	12.42	1.73%	0.30	5.76%	0.87
Constant Momentum Value	10.74	1.18%	0.23	5.23%	1.13
Carry Momentum Value	8.54	0.95%	0.24	4.01%	1.03
Constant Curr Acc GDP Growth	9.99	0.96%	0.21	4.57%	0.39
Constant Carry Momentum Value	11.98	1.82%	0.32	5.73%	0.92
All Factor Combined	12.38	1.72%	0.27	6.37%	0.92

Figure 21: PCA Performance Metrics

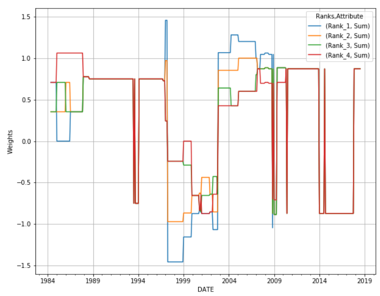
Matrix	Factor 1					Factor 2					Factor 3					Equal Weight Portfolio				
	Leverage	*Mean*	*Sharpe*	*Std Dev*	*Turnover*	*Leverage*	*Mean*	*Sharpe*	*Std Dev*	*Turnover*	*Leverage*	*Mean*	*Sharpe*	*Std Dev*	*Turnover*	*Leverage*	*Mean*	*Sharpe*	*Std Dev*	*Turnover*
2nd Moment	6.80	0.87%	0.08	10.65%	1.68	4.45	0.34%	0.07	4.73%	1.39	4.01	-0.01%	0.00	3.31%	1.40	5.09	0.40%	0.10	3.94%	0.24
Correlation	7.72	0.62%	0.06	10.30%	1.79	4.38	0.32%	0.10	3.40%	1.33	4.24	0.07%	0.03	2.87%	1.40	5.45	0.34%	0.09	3.72%	0.34
Covariance	6.82	0.85%	0.08	10.64%	1.66	4.36	0.34%	0.07	4.65%	1.38	4.03	0.08%	0.02	3.16%	1.40	5.07	0.42%	0.11	3.89%	0.23

Appendix 6

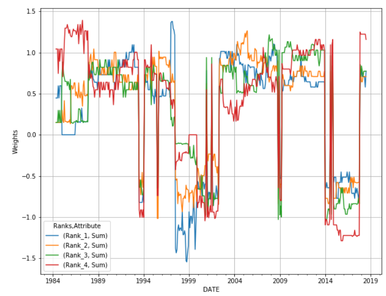
Coefficients based Ranking:

In Figure 22 we have coefficients ranked by value for number of characteristics as 1. We observe similar trends in the weights as seen for carry ranking.

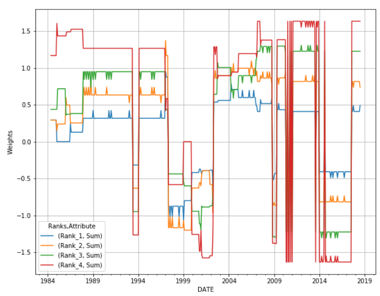
Figure 22: IPCA Coefficients bucketed by Value Ranking for No. of Characteristics = 1



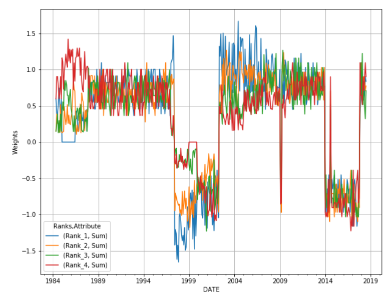
(a) Characteristics: Constant



(b) Characteristics: Carry



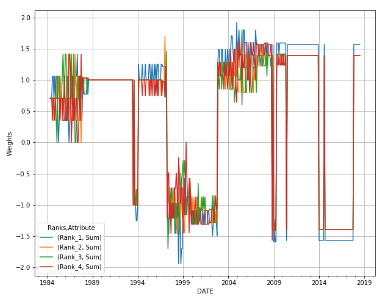
(c) Characteristics: Value



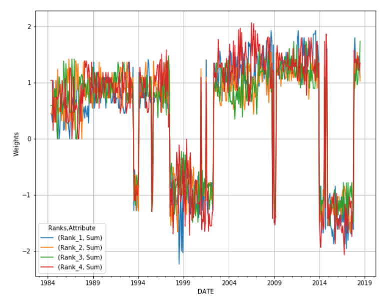
(d) Characteristics: Momentum

In Figure 23 we have coefficients ranked by momentum for number of characteristics as 1. We observe similar trends in the weights as seen for carry and value ranking only with more noise.

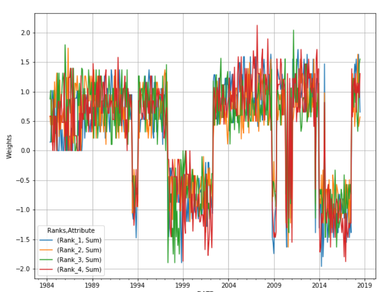
Figure 23: IPCA Coefficients bucketed by Momentum Ranking for No. of Characteristics = 1



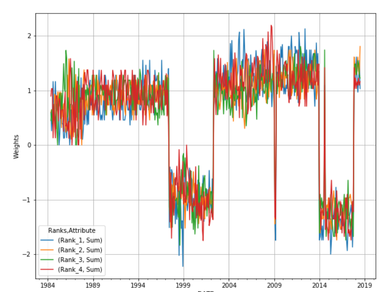
(a) Characteristics: Constant



(b) Characteristics: Carry



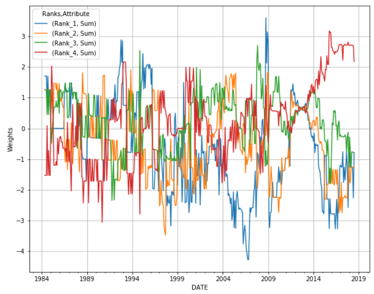
(c) Characteristics: Value



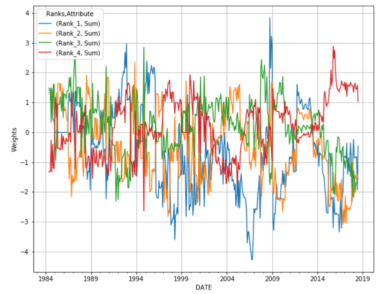
(d) Characteristics: Momentum

In Figure 24 we have coefficients ranked by value for number of characteristics as 2 and 3. The weights have more noise compared to value and carry.

Figure 24: IPCA Sum of Coefficients bucketed by Value Ranking for No. of Characteristics = 2 and 3

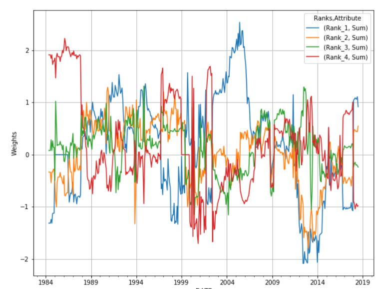


(a) IPCA Component 1, Characteristics: Constant, Carry

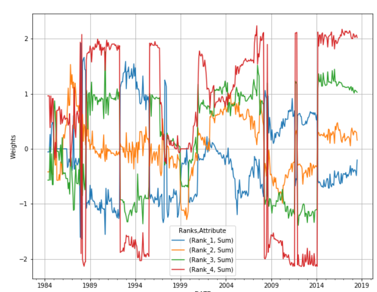


(b) IPCA Component 1,Characteristics: Constant, Carry, Value

(c) IPCA Component 2, Characteristics: Constant, Carry



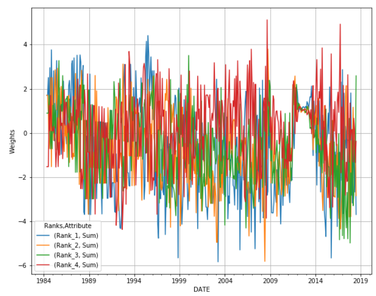
(d) IPCA Component 2,Characteristics: Constant, Carry, Value



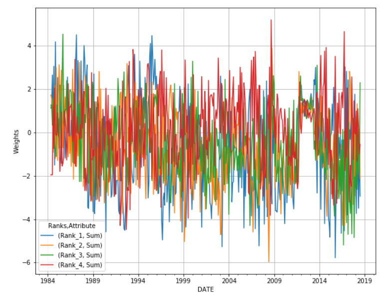
(e) IPCA Component 3, Characteristics: Constant, Carry, Value

In Figure 25 we have coefficients ranked by momentum for number of characteristics as 2 and 3. This is the counterpart of the carry sum of coefficients presented in the paper.

Figure 25: IPCA Sum of Coefficients bucketed by Momentum Ranking for No. of Characteristics = 2 and 3



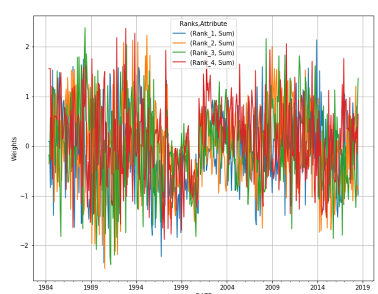
(a) IPCA Component 1, Characteristics: Constant, Carry



(b) IPCA Component 1,Characteristics: Constant, Carry, Value

(c) IPCA Component 2, Characteristics: Constant, Carry

(d) IPCA Component 2,Characteristics: Constant, Carry, Value



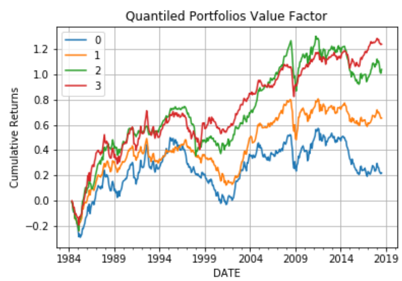
(e) IPCA Component 3, Characteristics: Constant, Carry, Value

Appendix 7

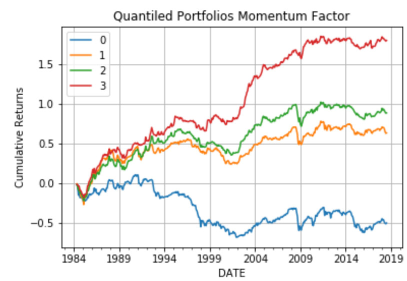
Quantiled Portfolios:

In Figure 26 we present the quantile portfolios for the characteristics we have used. We notice that the last two characteristics, portfolios constructed using the current account and GDP growth, do not exhibit a clear quantile returns as value and momentum portfolios. This may be interpreted by the fact that value and momentum are priced factors, whereas macroeconomic factors such as GDP growth and current account are not.

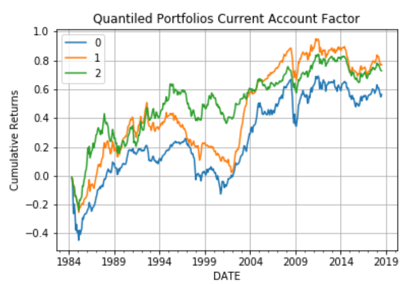
Figure 26: Quantiled Portfolios for Value, Momentum, Current Account and GDP Growth



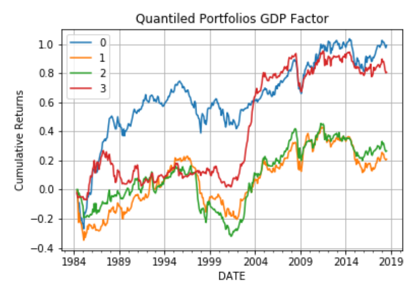
(a) Quantiled Portfolios for Value Factor



(b) Quantiled Portfolios for Momentum Factor



(c) Quantiled Portfolios for Current Account



(d) Quantiled Portfolios for GDP Growth Factor

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