MIE404 Homework 2

In this problem set, we will take a look at system modelling and first and second order systems. The homework solutions are **due Tue 10/11 at 11:59pm** on **Quercus**. You can submit scans of your handwritten answers, MATLAB livescripts, Latex files, Word files, etc. **Ensure all your answers are in one PDF file** on submission. You are encouraged to use MATLAB/Simulink to understand the problems, plot figures and verify your answers.

The total marks of this homework are 40, which will count towards 4% of your final grade. Clearly show your work, assumptions, and steps, and box your final $\boxed{answer}$ to each part.

1 Problem 1: Cruise Control

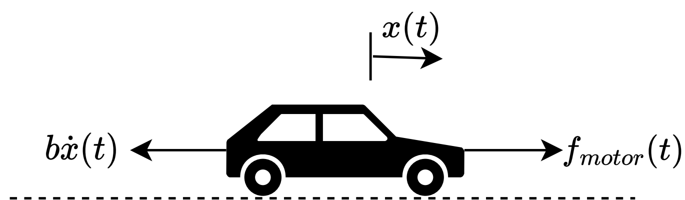

Figure 1: Car

We want to design a cruise controller for the car in Fig. 1. The driver sets the desired speed of the car and the cruise controller must adjust the engine force $f(t)$ to maintain the speed of the car at the desired speed.

Assume that the car (control plant) can be modelled as a point mass $m = 1000$ kg. The damping coefficient is $b = 20$ N·s/m.

(a) (1 marks) Write the equation(s) of motion of the car.

(b) (1 marks) Write the transfer function, $G(s)$, relating the engine force to the car's velocity.

(c) (2 marks) Later in the course, we will study PID (proportional-integral-derivative) controllers. The general form of the transfer function of a PID controller is

$$C(s) = k_p + \frac{k_i}{s} + k_d s$$

We will design an integral controller as the car's cruise controller, i.e., $k_p = k_d = 0$. Find the transfer function, $H(s)$, of the closed loop system in Fig. 2.

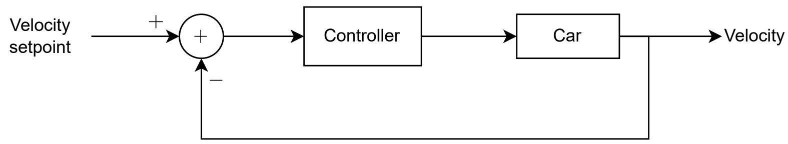

Figure 2: Control block diagram

(d) (3 marks) Design the integral controller such that $H(s)$ is critically damped.

(e) (3 marks) You're driving on the highway at 85 km/hr and increase your cruise control's desired speed up by 5 km/hr. Following the push on the cruise control's button in your car's dashboard, the input signal to $H(s)$ steps up the desired speed. Plot the response of $H(s)$ to the desired speed step change. Does it match what you expect of a critically damped system? What is a shortcoming in the current system?

We'll learn how to deal with such shortcomings later in the course.

(f) (3 marks) You are driving at your desired speed of 50 km/hr when the road slopes down at an angle of 10 degrees (see Fig. 3). Model the force applied on the car due to the slope as a disturbance to the car that influences the input to $G(s)$ (see Fig. 4). Plot the controller's output and the car's velocity as the car starts descending the slope. Is the system able handle the disturbance and keep the car driving at 50 km/hr?

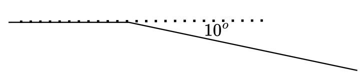

Figure 3: Road slope

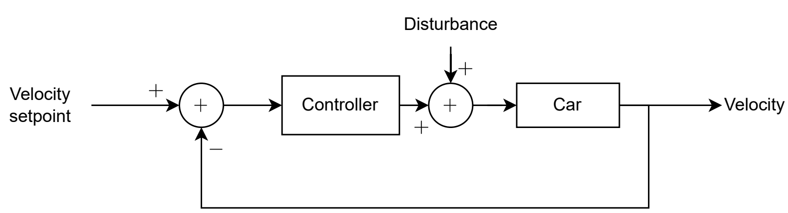

Figure 4: Control block diagram with disturbance

2 Problem 2: Grandfather clock

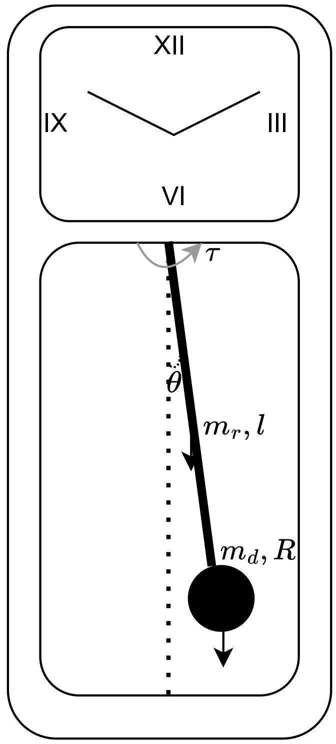

Figure 5: Grandfather clock

Grandfather clocks are tall-standing old clocks that used a compound pendulum to drive their clock gears. Fig. 5 is an illustration of such a clock. The compound pendulum, consisting of a rod and ball, oscillates with a time period of 2 seconds (frequency of π rad/s). In this question, we'll re-design a grandfather clock We have obtained a ball of mass $m_d = 2.5$ kg and diameter 6 cm. We have also obtained a long rod from which we can cut the pendulum rod we'll be using in the clock. Each meter of rod has a mass of 100 grams.

Assume that (1) the mass of the rod is uniformly distributed and can be modelled at the center of the rod, (2) the rod connects directly to the surface of the ball at one end and a pivot point at the other end, (3) the ball's mass can be modelled at its center of mass, and (4) $g = 9.98$ N/kg.

(a) (4 marks) What is the moment of inertia, $\mathcal{J}$, of the compound pendulum about its pivot point.

(b) (2 marks) Write the equation(s) of motion of the compound pendulum. The pendulum's angle is 0 rad when the pendulum is vertical.

(c) (3 marks) Write the transfer function, $G(s)$, relating the torque applied to the compound pendulum at the pivot point to the pendulum's angle. This will require you to linearize about an equilibrium point. Take 0 rad as the equilibrium and calculate the derivative with respect to θ at $\theta = 0$ of the forces acting on the compound pendulum due to its weight, i.e.,

$$\frac{d}{d\theta} f|_{\theta=0} \cdot \theta$$

.

(d) (4 marks) How long should the pendulum's rod be for the pendulum's natural frequency to be π rad/s?

(e) (3 marks) Friction at the pivot point leads to the pendulum losing its energy over time.

Model the friction at the pivot as a damping force with coefficient $b = 0.01$ N·s, i.e., $f_{damper} = b\dot{\theta}$ [N], and re-write the transfer function, $G(s)$, relating the torque applied to the compound pendulum at the pivot point to the pendulum's angle. How long should the pendulum's rod be for the pendulum's damped frequency to be π rad/s?

(f) (4 marks) A torque needs to be re-applied to the pendulum at the pivot point regularly to keep it moving. When the pendulum's maximum angle of deviation drops below 0.05 rad, a torque must be applied on the pendulum compound to 'reset' the clock by driving the angle up to 0.1 rad. The pendulum is then left again to lose its energy over time. How should we schedule the resets, i.e., how long is the time between resets?

3 Problem 3: Block diagrams

(a) (3 marks) Find the transfer function $\frac{Y}{R}$

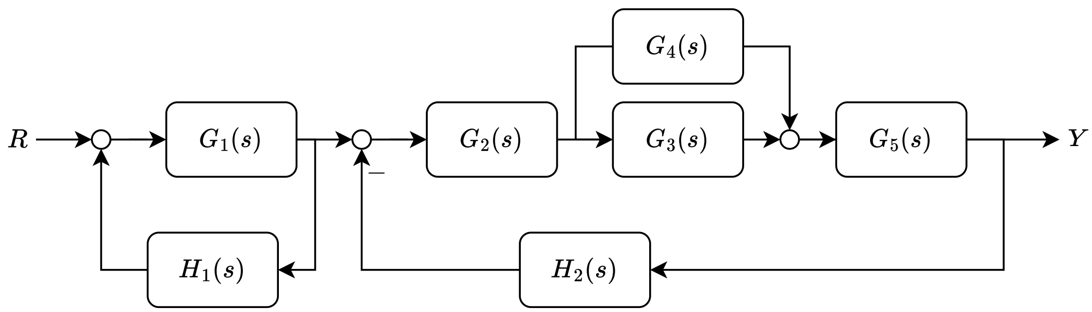

Figure 6: Block diagram 1

(b) (4 marks) Find the transfer function $\frac{Y_1}{R}$

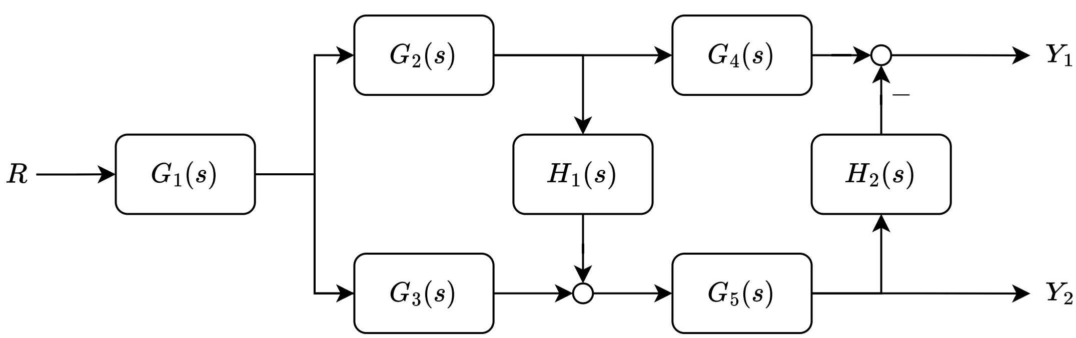

Figure 7: Block diagram 2