MIE404 Homework 3

In this problem set, we will take a look at controller design and root locus. Besides the root locus material covered in class, solving this homework will require the use of additional root locus plotting rules which are covered here: `https://lpsa.swarthmore.edu/Root_Locus/RootLocusReviewRules.html`

While not required, you can read about the rules derivation here: `https://lpsa.swarthmore.edu/Root_Locus/DeriveRootLocusRules.html` to understand the concepts further.

The homework solutions are **due Fri 11/4 at 11:59pm** on **Quercus**. You can submit scans of your handwritten answers, MATLAB livescripts, Latex files, Word files, etc. **Ensure all your answers are in one PDF file** on submission. In this homework, you will have to derive (mathematically) all of the numerical solutions; but you are welcome to use MATLAB/Simulink to understand the problems, plot figures and verify your answers.

The total marks of this homework are 40, which will count towards 4% of your final grade. Clearly show your work, assumptions, and steps, and box your final $\boxed{\textit{answer}}$ to each part.

1 Problem 1: Fighter jet

Fighter jets are designed to have lightly damped poles (and sometimes even designed to be slightly unstable) to enable complex maneuvers. Below is a block diagram for the roll dynamics of a jet aircraft.

(a) Assuming a proportional controller ($G_c(s) = K$), for what range of K would the closed-loop system be stable? (6 marks)

(b) Draw the root locus for the system. Clearly show all important calculations (including any asymptotes, departure angles from poles, arrival angles at zeros, break-in/break-away points, and imaginary-axis crossings). (10 Marks)

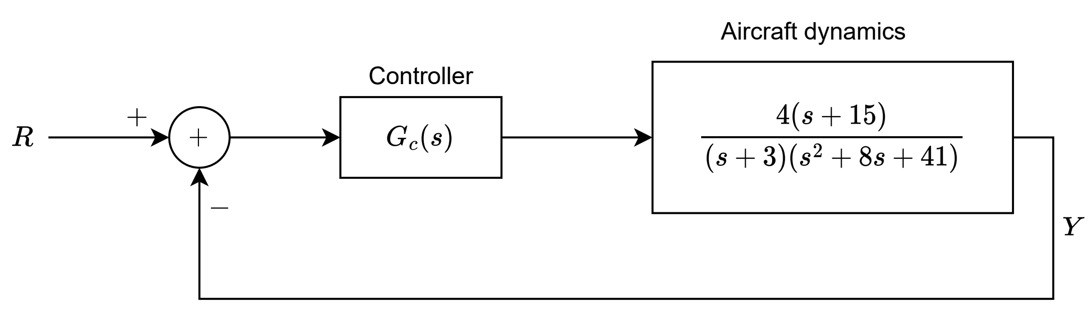

Figure 1: Fighter jet block diagram

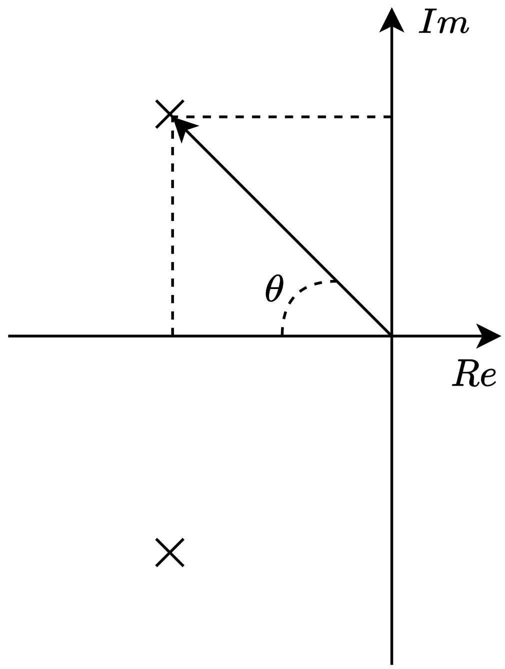

Figure 2: Complex plane

(c) In this part, we will identify the region of the complex plane where the system poles would need to be to satisfy some control specifications. (4 Marks)

(i) Recall that the characteristic equation of a general second-order system is

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

Derive the roots of the characteristic equation in terms of the variables ζ and ω_n (for an under-damped system).

(ii) In Fig. 2, we plot the two complex conjugate roots of the characteristic equation. What is the value of the real and imaginary parts of the roots?

(iii) Write the equation relating the damping ratio ζ to the angle θ marked in Fig. 2.

(iv) Show the region of the complex plane where the system poles would need to be to have an overshoot of less than 5% and a settling time (within 2% of steady state) of less than 0.5 seconds for a step input.

Notice that while the above concepts are relevant to second order systems, we can and do generalize them to higher order systems. We impose that all poles of the higher order system be in the region.

(v) With the proportional controller, would it be possible to achieve these control specifications?

(d) Design a controller that places the dominant, imaginary poles at $-15 \pm j15$ (it is in the region outlined in part (c) above). Use one of the standard control approaches outlined in the class (proportional, integral, derivative). Any choice is fine as long as it satisfies the criteria. Box the transfer function $G_c(s)$ of your controller. (10 Marks)

2 Problem 2: Root Locus

The open-loop transfer function in a unity feedback system is

$$G_{OL}(s) = \frac{s^2 + 2s + 2}{s(s^4 + 9s^3 + 33s^2 + 51s + 26)}$$

Draw the root locus for the system. Clearly show all important calculations (including any asymptotes, departure angles from poles, arrival angles at zeros, break-in/break-away points, and imaginary-axis crossings). (10 Marks)