MIE404 Homework 1

In this problem set, we will take a look at modelling different parts of an electric vehicle—practicing dynamical modelling and Laplace transforms. The homework solutions are **due Tue 9/27 at 11:59pm** on **Quercus**. You can submit scans of your handwritten answers, MATLAB livescripts, Latex files, Word files, etc. **Ensure all your answers are in one PDF file** on submission.

The total marks of this homework are 40, which will count towards 4% of your final grade. Clearly show your work, assumptions, and steps, and box your final $\boxed{answer}$ to each part.

1 Problem 1: Longitudinal motion

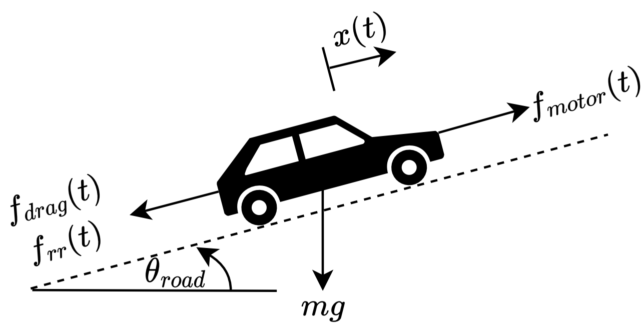

Figure 1: Vehicle diagram

The car in Fig. 1 is moving up a road with slope θ_{road} [degrees]. The car's motor applies a force f_{motor} to drive the car forward. The car is subject to air drag f_{drag} and rolling resistance f_{rr} forces. The air drag can be modelled as follows:

$$f_{drag} = \rho C_d A \frac{v^2}{2} \text{ [N]}$$

where $\rho = 1.2041$ kg/m^3 is the air density, $C_d A = 0.58$ m^2 is the drag coefficient times the front facing cross section of the car, and $v = \dot{x}$ [m/s] is the car's velocity.

The rolling resistance can be modelled as follows:

$$f_{rr} = C_{rr1} v + C_{rr2} F_N \text{ [N]}$$

where $C_{rr1} = 2.75$ N·s/m and $C_{rr2} = 0.018$ are rolling resistance coefficients. F_N is normal force exerted by the road on the vehicle's tires.

Assume that the car's mass is $m = 1400$ kg, $g = 9.98$ N/kg and that the car can be modelled as a point mass.

(a) (6 marks) Write the equation(s) of motion of the car.

(b) (5 marks) To obtain a transfer function from the equation(s) of motion in (a), we need to linearize the equation(s). We will linearize the air drag around a velocity 1 m/s. i.e., we will assume

$$f_{drag} = \frac{d}{dv}\left(\rho C_d A \frac{v^2}{2}\right)\Bigg|_{v=1\text{m/s}} v = \rho C_d A v \text{ [N]}$$

Find the transfer function $G_{car}(s)$ and the expression of F_r in Fig. 2. $F_{motor}(s) = \mathcal{L}\{f_{motor}(t)\}$, and $V(s) = \mathcal{L}\{v(t)\}$

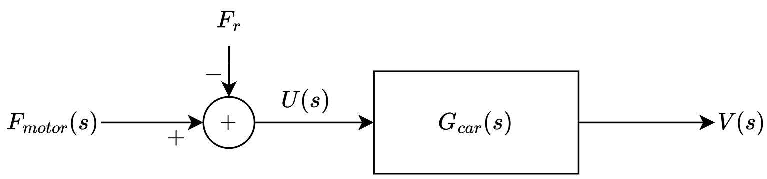

Figure 2: Vehicle block diagram

(c) (4 marks) Calculate the final value (i.e., $t \to \infty$) of the car's velocity $v(t)$ if $u(t) = \mathcal{L}^{-1}\{U(s)\}$ is a step input with magnitude 4 (i.e., the net force input to $G_{car}(s)$ is 4 N) and $\theta_{road} = 10^o$. Show your calculations.

2 Problem 2: Suspension

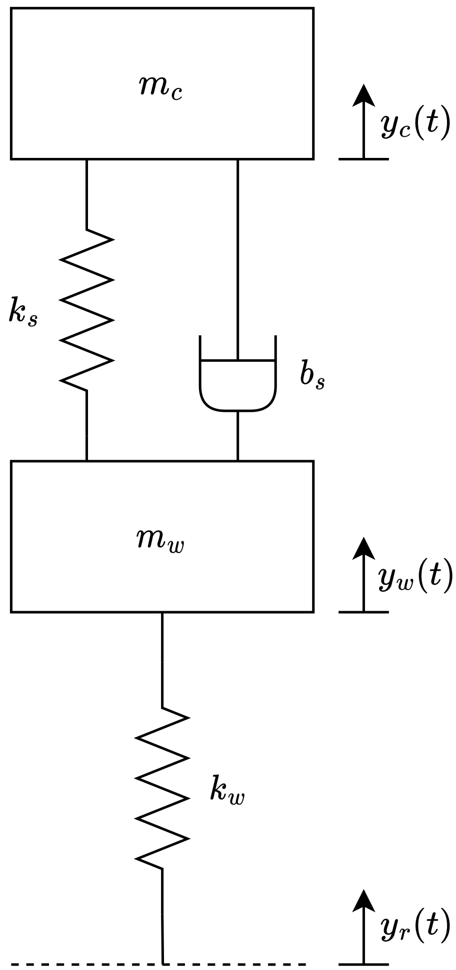

Figure 3: Suspension

Fig. 3 illustrates the suspension of each wheel. The car, modelled as a point mass m_c [kg], lies on the suspension, modelled by a spring and damper. The suspension connects to the wheels. The wheel tire is modelled as a spring on top of the road.

$m_w = 50$ kg is the wheel's mass. Assume that the weight of the car is distributed equally across the wheels so that $m_c = 300$ kg.

$k_s = k_t = 50 \times 10^3$ N/m are the suspension and tire spring constants, and $b_s = 2 \times 10^3$ N·s/m is the suspension damping.

y_r, y_w, and y_c are the vertical displacements of the road, wheel and car, respectively. Note that $y_r(t)$ is a function of time to model road irregularities, such as speed bumps, uneven paving, gravel, etc.

(a) (11 marks) The car's wheel drives over a stone that is 2 cm high. What is the maximum vertical displacement that the passengers in the car will feel? Assume that stone's action on the wheel can be modelled as an impulse, and that the passengers feel the displacement y_c. You can use MATLAB or any other mathematical software to compute the final answer. Show your calculations and steps.

3 Problem 3: Electric motor

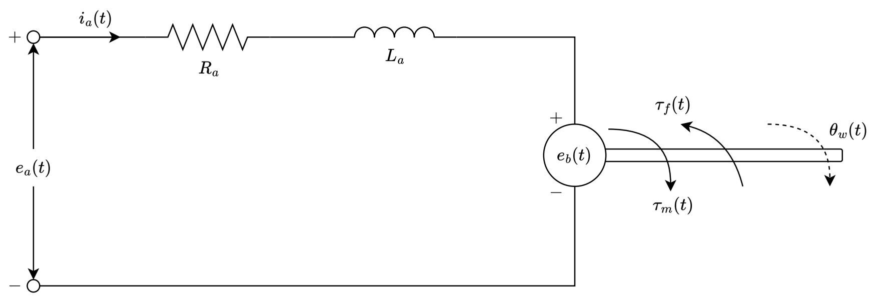

Figure 4: DC motor circuit

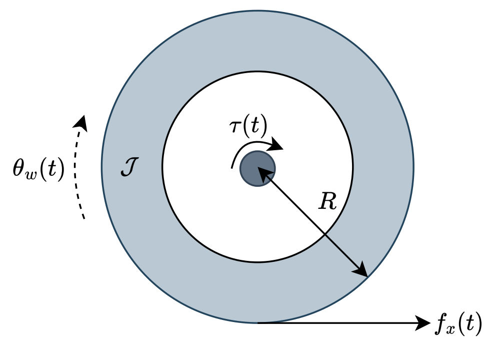

Figure 5: Wheel

Fig. 4 illustrates the internal circuit of an electric DC motor. The electric vehicle has one motor connected to each wheel. The motor is subjected to an armature voltage e_a [V]. The armature resistance R_a [Ohm] and inductance L_a [S] model non-idealities in the motor, including losses. The motor is connected to the wheel shaft. The wheel and its shaft are shown in Fig. 5 and have inertia $\mathcal{J}$. The back EMF e_b [V] generated by the motor is a function of the shaft's angular speed of rotation

$$e_b = k_e \omega_w$$

where k_e is the motor's speed constant, and $\omega_w = \dot{\theta}_w$ is the shaft/wheel's angular velocity.

The mechanical torque τ_m applied to the shaft by the motor is a function of the armature current i_a

$$\tau_m = k_t i_a$$

where k_t is the motor's torque constant.

Assume that the friction on the shaft τ_f can be modelled as a damping force

$$\tau_f = b\omega_w$$

(a) (14 marks) Fill in the transfer functions in each of the blocks (1 to 6) in Fig. 6. The symbols represent the following:

- $\mathcal{L}\{e_a(t)\} = E_a(s)$,
- $\mathcal{L}\{e_b(t)\} = E_b(s)$,
- $\mathcal{L}\{i_a(t)\} = I_a(s)$,
- $\mathcal{L}\{\tau_m(t)\} = T_m(s)$,
- angular velocity $\mathcal{L}\{\omega_w(t)\} = \Omega(s)$,
- longitudinal velocity $\mathcal{L}\{v(t)\} = V(s)$,
- longitudinal displacement $\mathcal{L}\{x(t)\} = X(s)$

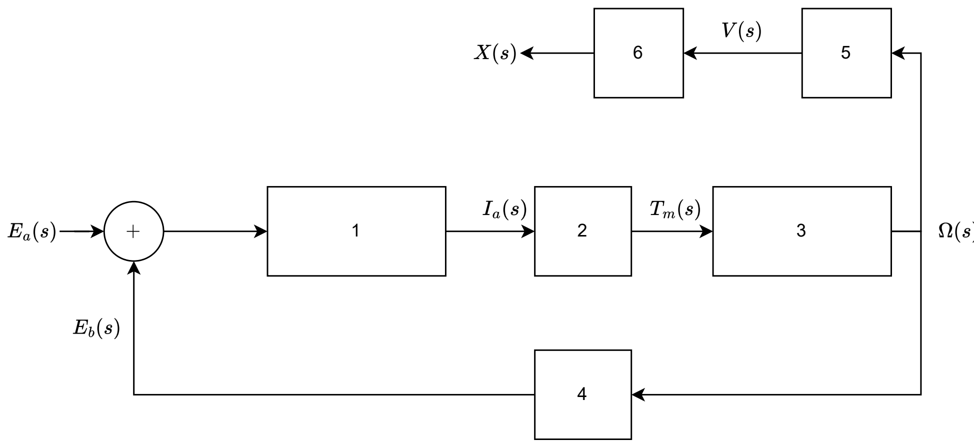

Figure 6: Motor block diagram