Elevator Problem

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Back story : i live at the 2nd floor of a building and i was wondering if i should always be the last one to enter the elevator because it's almost surely i will be the first one to exit the elevator

Problem : at the 0th floor of a building with M floors , $K > 1$ tenants each one wants to know where to stand in the elevator which has N spaces (1 being closest to the door , N being the furthest) without knowing which floor the rest of the tenants live , such that when they want to exit the elevator at the i'th floor , the number of tenants blocking the door will be minimal .

Algorithm 1 (K = N) :

- Input (M , K , i , { j })
- If M=N stand in the i'th place
- If M>N :
  - FOR $1 \leq j \leq N$ Calculate :
  - // unnormalized probability for "j" being the optimal position
  - // $\rightarrow$ (j-1) tenants live below or at i and (K-1-(j-1)=K-j) live
  - // above or at i
    - $P(Q = j) = \left[\left(\frac{i}{M}\right)^{j-1} \left(\frac{M-i+1}{M}\right)^{K-j} (K - 1 \; choose \; j - 1) \right]$
  - Choose $1 <= J <= N$ where $P(Q = J)$ is the largest .

Algorithm 0 (K=N):

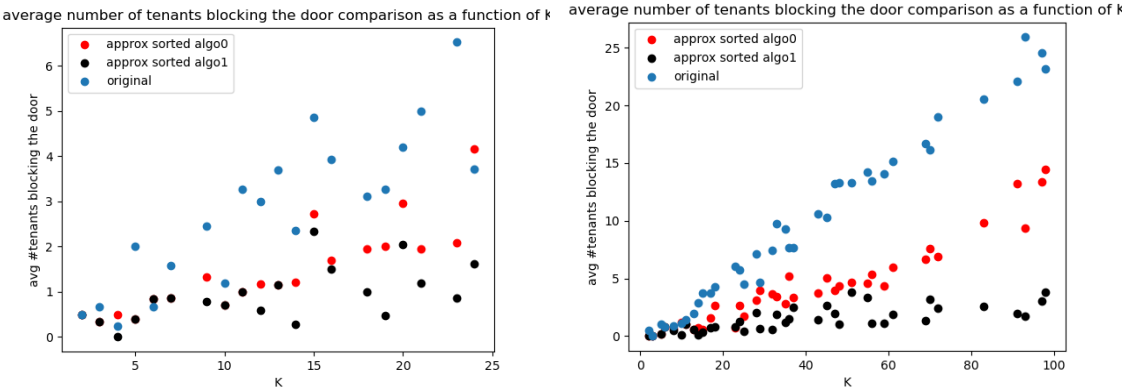
- If $i \leq N$:
  - If $i'th$ place is available stand there else :
    - Stand at the closest available place from either side
- Else:
  - Use $algorithm\_1$ to find where to stand

Algorithm approx sort (K=N) :

// let all K tenants run algorithm 1 now the problem is equivalent to sorting an array of
// size N where each tenant is represented by their floor number , the solution would be
// for T_k in $[T_1 ..... T_K]$ find "J"

- For T_k in $[T_1 ..... T_K]$:
  - $J_k = Algorithm\_1\,(M\,,\ K - (k-1)\,,\ \{j\}\backslash\{J_1 ..... J_{k-1}\})$
- Return $[J_1 ..... J_K]$

Results



For small Ks , algorithm 1 does slightly better than algorithm 0 , for really small Ks This line (If $i \leq N$:) is almost never true which means that algorithm 0 output is almost always the same as algorithm 1 which is almost always an improvement upon the original ordering

For larger Ks it's clear that the average number of tenants blocking the door for each tenant is considerably smaller for algorithm 1 .