Elevator Problem

Amro Asali

Back story : i live at the 2nd floor of a building and i was wondering if i should always be the last one to enter the elevator because it's almost surely i will be the first one to exit the elevator

Problem : at the 0th floor of a building with M floors , $K > 1$ tenants each one wants to know where to stand in the elevator which has N spaces (1 being closest to the door , N being the furthest) without knowing which floor the rest of the tenants live , such that when they want to exit the elevator at the i'th floor minimal number of tenants will block the door .

Algorithm 1 (K = N) :

- Input (M , K , i , { j })
- If M=N stand in the i'th place
- If M>N :
  - Let $1 \leq j \leq N$ Calculate :
  - // unnormalized probability for "j" being the optimal position
  - // $\rightarrow$ (j-1) tenants live below or at i and (K-1-(j-1)=K-j) live
  - // above or at i
  - $P(Q = j) = \left[\left(\frac{i}{M}\right)^{j-1} \left(\frac{M-i+1}{M}\right)^{K-j} (K - 1 \; choose \; j - 1) \right]$
  - Choose $1 <= J <= N$ where $P(Q = J)$ is the largest .

Algorithm 0 (K=N):

- If $i \leq N$:
  - If $i'th$ place is available stand there else :
    - Stand at the closest available place from either side
- Else:
  - Use $algorithm\_1$ to find where to stand

Algorithm approx sort (K=N) :

// let all K tenants run algorithm 1 now the problem is equivalent to sorting an array of
// size N where each tenant is represented by their floor number , the solution would be
// for T_k in $[T_1 ..... T_K]$ find "J"

- For T_k in $[T_1 ..... T_K]$:
  - $J_k = Algorithm\_1\,(M\,,\ K - (k-1)\,,\ \{j\}\backslash\{J_1 ..... J_{k-1}\})$
- Return $[J_1 ..... J_K]$

Results