Lecture 14: Categories and Interactions

ECON 480 - Econometrics - Fall 2018

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Dummy *Dependent* (Y) Variables

Interaction Effects

Interactions Between a Dummy and a Continuous Variable

Interactions Between Two Dummy Variables

Interactions Between Two Continuous Variables

Ordinal Variables

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    - e.g. Order of finalists in a competition (1st, 2nd, 3rd)
    - e.g. Highest education attained (1=elementary school, 2=high school, 3=bachelor's degree, 4=graduate degree)

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Can we run the following regression?

$$\widehat{Wages_i} = \hat{\beta}_0 + \hat{\beta}_1 Region_i + \epsilon_i$$

HOOD COLLEGE

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How do wages vary by region of the country?

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$$Region_i = \begin{cases} 1 & \text{if } i \text{ is in } \mathit{Northeast} \\ 2 & \text{If } i \text{ is in } \mathit{Midwest} \\ 3 & \text{if } i \text{ is in } \mathit{South} \\ 4 & \text{If } i \text{ is in } \mathit{West} \end{cases}$$

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How do wages vary by region of the country?

- Create dummy for each region:
    - $Northeast_i = 1$ if i is in Northeast, else 0
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- For every i: $Northeast_i + Midwest_i + South_i + West_i = 1$!

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- If we included *all* possible categories, they are perfectly multicollinear, an exact linear function of one another:

$$Northeast_i + Midwest_i + South_i + West_i = 1 \quad \forall i$$

- This is known as the dummy variable trap, a common source of perfect multicollinearity

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- β_1: difference between West and Northeast
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- β_3: difference between West and South

```r
dtreg<-lm(wage~noreast+northcen+south+west, data=wages)
summary(dtreg)
```

```
##
## Call:
## lm(formula = wage ~ noreast + northcen + south + west, data = wages)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -6.083 -2.387 -1.097  1.157 18.610
##
## Coefficients: (1 not defined because of singularities)
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   6.6134     0.3891  16.995  < 2e-16 ***
```

```
# run 4 regressions
no.noreast.reg<-lm(wage~northcen+south+west, data=wages)
no.northcen.reg<-lm(wage~noreast+south+west, data=wages)
no.south.reg<-lm(wage~noreast+northcen+west, data=wages)
no.west.reg<-lm(wage~noreast+northcen+south, data=wages)
```

```
# make output table
library("stargazer")
stargazer(no.noreast.reg, no.northcen.reg,
          no.south.reg, no.west.reg,
          type="latex", header=FALSE,
          float=FALSE, font.size="tiny")
```

Using Different Reference Categories II

	Dependent variable:			
	wage			
	(1)	(2)	(3)	(4)
northcen	−0.659		0.324	−0.903*
	(0.465)		(0.417)	(0.504)
noreast		0.659	0.983**	−0.244
		(0.465)	(0.432)	(0.515)
south	−0.983**	−0.324		−1.226***
	(0.432)	(0.417)		(0.473)
west	0.244	0.903*	1.226***	
	(0.515)	(0.504)	(0.473)	
Constant	6.370***	5.710***	5.387***	6.613***
	(0.338)	(0.320)	(0.268)	(0.389)
Observations	526	526	526	526
R^2	0.017	0.017	0.017	0.017
Adjusted R^2	0.012	0.012	0.012	0.012
Residual Std. Error (df = 522)	3.671	3.671	3.671	3.671
F Statistic (df = 3; 522)	3.099**	3.099**	3.099**	3.099**
Note:		*p<0.1; **p<0.05; ***p<0.01		

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- **It doesn't matter which category we omit**
- Same n, R^2, and SER; coefficients give same results

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  - requires special tools to properly interpret and extend this (**logit**, **probit**, etc)
- Feel free to write papers that have dummy Y variables (but you may have to ask me some more questions)!

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  - i.e. is there an interaction effect between sex and experience?

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- Do men *gain more* than women from an *additional* year of experience?
    - Note this is *not the same* as asking: "do men earn more than women with the same amount of experience?"

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Interactions Between a Dummy and a Continuous Variable

- We can model this interaction by introducing a variable that is an interaction term capturing the interaction between two variables:

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Model with Interaction Effects

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- β_3 estimates the interaction term (in this case between a dummy variable and a continuous variable)
- What do the different coefficients (β's) tell us?
    - Again, think logically by examining each group ($D_i = 0$ or $D_i = 1$)

Interaction Effects as Two Regressions

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- When $D_i = 1$ (Treatment group):

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Interaction Effects as Two Regressions

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$$\hat{Y}_i = (\hat{\beta}_0 + \hat{\beta}_2) + (\hat{\beta}_1 + \hat{\beta}_3) X_i$$

HOOD COLLEGE

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i * D_i$$

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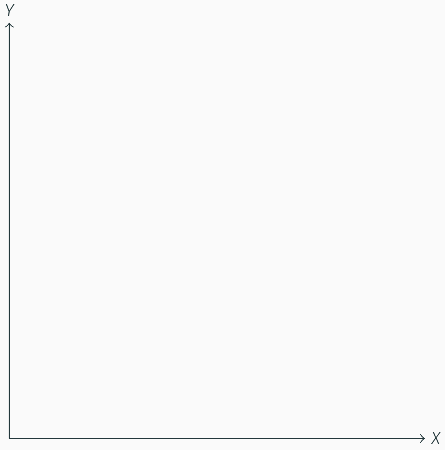
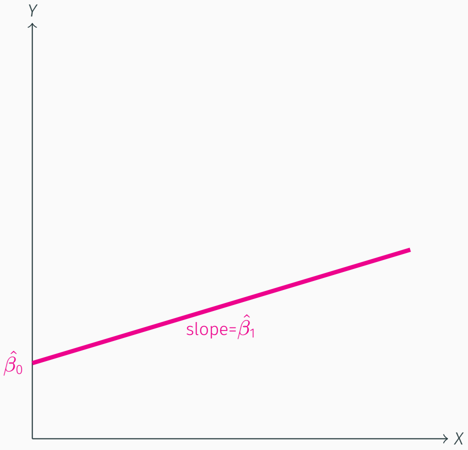
- So what we really have is *two* regression lines!

Interaction Effects as Two Regressions II

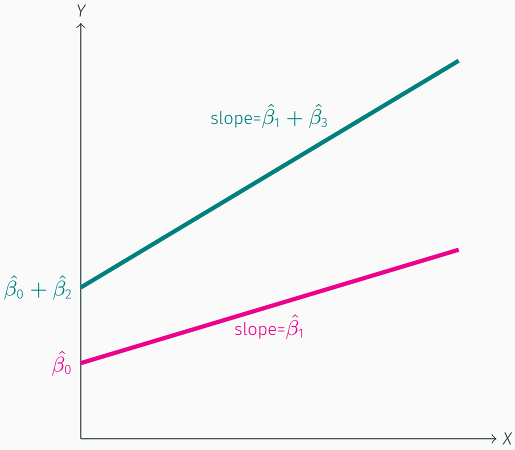


Interaction Effects as Two Regressions II



- $D_i = 0$ group:
  $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$

Interaction Effects as Two Regressions II



- $D_i = 1$ group:
  $\hat{Y}_i = (\hat{\beta}_0 + \hat{\beta}_2) + (\hat{\beta}_1 + \hat{\beta}_3)X_i$
- $D_i = 0$ group:
  $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$

- Three distinct possibilities for the two lines $D_i = 0$ and $D_i = 1$:

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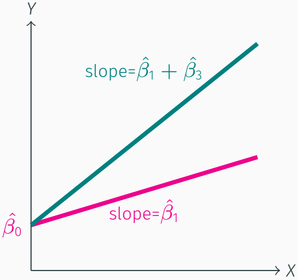
Same intercept, different slopes



- Three distinct possibilities for the two lines $D_i = 0$ and $D_i = 1$:

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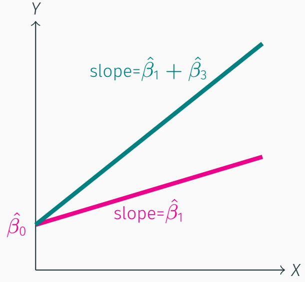
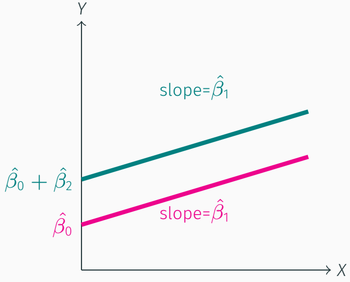
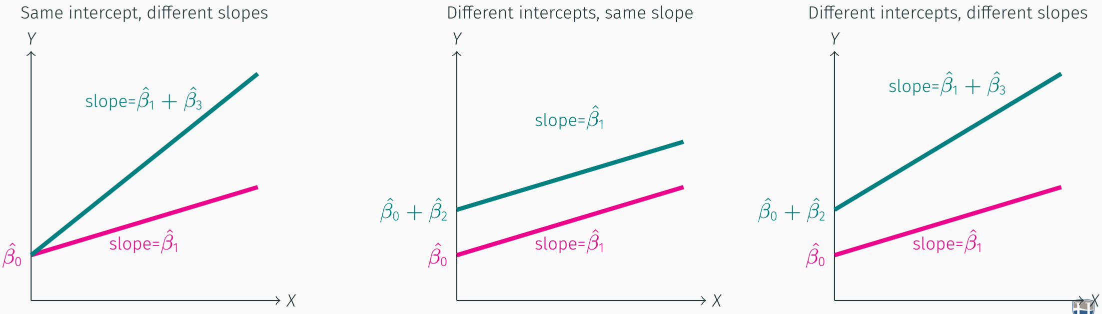


Different intercepts, same slope



Interaction Effects as Two Regressions III

- Three distinct possibilities for the two lines $D_i = 0$ and $D_i = 1$:



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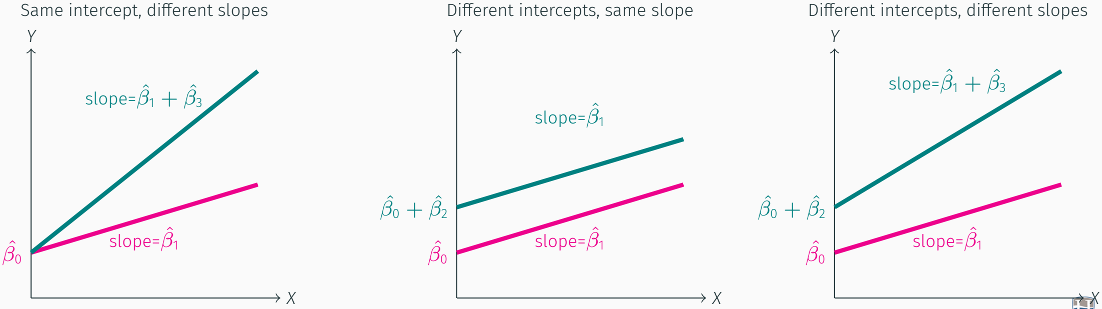

- Well...four, but: what if they had the same slope and same intercept?

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i * D_i$$

- To interpret the coefficients, compare cases after changing X by ΔX_i:

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Interpretting the Coefficients

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- To interpret the coefficients, compare cases after changing X by ΔX_i:

$$Y_i + \Delta Y_i = \beta_0 + \beta_1 (X_i + \Delta X_i) \beta_2 D_i + \beta_3 \big((X_i + \Delta X_i) D_i \big)$$

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i * D_i$$

- To interpret the coefficients, compare cases after changing X by ΔX_i:

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- Subtracting the two equations, the difference is:

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- Subtracting the two equations, the difference is:

$$\Delta Y_i = \beta_1 \Delta X_i + \beta_3 D_i \Delta X_i$$

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i * D_i$$

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$$\frac{\Delta Y_i}{\Delta X_i} = \beta_1 + \beta_3 D_i$$

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- **The effect of $X \rightarrow Y$ depends on the value of D_i!**
- β_3: *increment* to the effect of $X_i \rightarrow Y_i$ when $D_i = 1$ (vs. $D_i = 0$)

Interpretting the Coefficients II

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i * D_i$$

- β_0: Y_i for $X_i = 0$ and $D_i = 0$

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i * D_i$$

- β_0: Y_i for $X_i = 0$ and $D_i = 0$
- β_1: Marginal effect of $X_i \rightarrow Y_i$ for $D_i = 0$

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i * D_i$$

- β_0: Y_i for $X_i = 0$ and $D_i = 0$
- β_1: Marginal effect of $X_i \rightarrow Y_i$ for $D_i = 0$
- β_2: Marginal effect on Y_i of difference between $D_i = 0$ and $D_i = 1$

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i * D_i$$

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- β_1: Marginal effect of $X_i \rightarrow Y_i$ for $D_i = 0$
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- β_3: The **difference** of the marginal effect of $X_i \rightarrow Y_i$ between $D_i = 0$ and $D_i = 1$

Interpretting the Coefficients II

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- β_0: Y_i for $X_i = 0$ and $D_i = 0$
- β_1: Marginal effect of $X_i \rightarrow Y_i$ for $D_i = 0$
- β_2: Marginal effect on Y_i of difference between $D_i = 0$ and $D_i = 1$
- β_3: The **difference** of the marginal effect of $X_i \rightarrow Y_i$ between $D_i = 0$ and $D_i = 1$
- This is a bit awkward, easier to think about the two regression lines:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i * D_i$$

- For $D_i = 0$ Group: $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i * D_i$$

- For $D_i = 0$ Group: $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$
  - Intercept (Y_i for $X_i = 0$): $\hat{\beta}_0$

Interpretting the Coefficients III

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i * D_i$$

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    - Intercept (Y_i for $X_i = 0$): $\hat{\beta}_0$
    - Slope (Marginal effect of X_i on Y_i for $D_i = 0$ group): $\hat{\beta}_1$

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i * D_i$$

- For $D_i = 0$ Group: $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$
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    - Slope (Marginal effect of X_i on Y_i for $D_i = 0$ group): $\hat{\beta}_1$
- For $D_i = 1$ Group: $\hat{Y}_i = (\hat{\beta}_0 + \hat{\beta}_2) + (\hat{\beta}_1 + \hat{\beta}_3) X_i$

Interpretting the Coefficients III

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  - Intercept (Y_i for $X_i = 0$): $\hat{\beta}_0$
  - Slope (Marginal effect of X_i on Y_i for $D_i = 0$ group): $\hat{\beta}_1$
- For $D_i = 1$ Group: $\hat{Y}_i = (\hat{\beta}_0 + \hat{\beta}_2) + (\hat{\beta}_1 + \hat{\beta}_3) X_i$
  - Intercept (Y_i for $X_i = 0$): $\hat{\beta}_0 + \hat{\beta}_2$

Interpretting the Coefficients III

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- How can we determine if the two lines have the same slope and/or intercept (and distinguish between the 3 cases)?

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  - Same intercept? t-test H_0: $\beta_2 = 0$

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  - Intercept (Y_i for $X_i = 0$): $\hat{\beta}_0$
  - Slope (Marginal effect of X_i on Y_i for $D_i = 0$ group): $\hat{\beta}_1$
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  - Intercept (Y_i for $X_i = 0$): $\hat{\beta}_0 + \hat{\beta}_2$
  - Slope (Marginal effect of X_i on Y_i for $D_i = 1$ group): $\hat{\beta}_1 + \hat{\beta}_3$
- How can we determine if the two lines have the same slope and/or intercept (and distinguish between the 3 cases)?
  - Same intercept? t-test H_0: $\beta_2 = 0$
  - Same slope? t-test H_0: $\beta_3 = 0$

Example

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1 exper_i + \hat{\beta}_2 female_i + \hat{\beta}_3 exper_i * female_i$$

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- For Males ($female = 0$):

$$\widehat{wage}_i = \hat{\beta}_0 + \hat{\beta}_1 exper$$

Example

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1 exper_i + \hat{\beta}_2 female_i + \hat{\beta}_3 exper_i * female_i$$

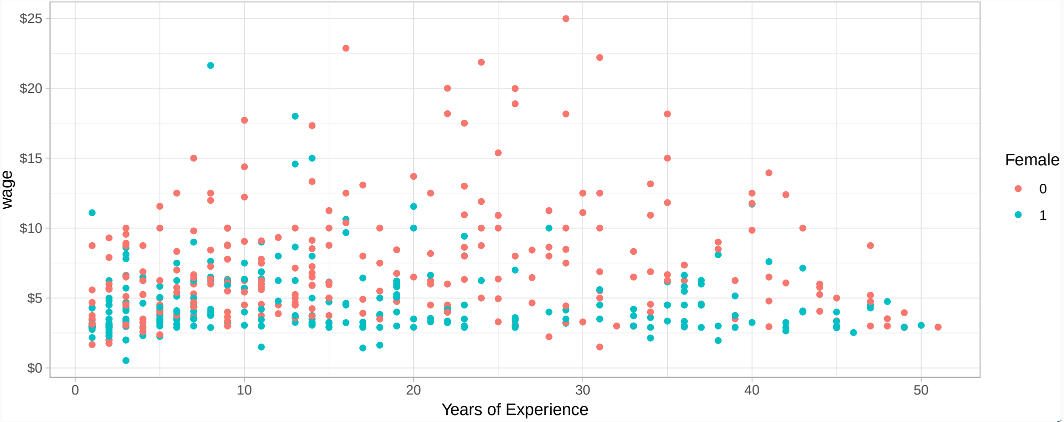
- For Males ($female = 0$):

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1 exper$$

- For Females ($female = 1$):

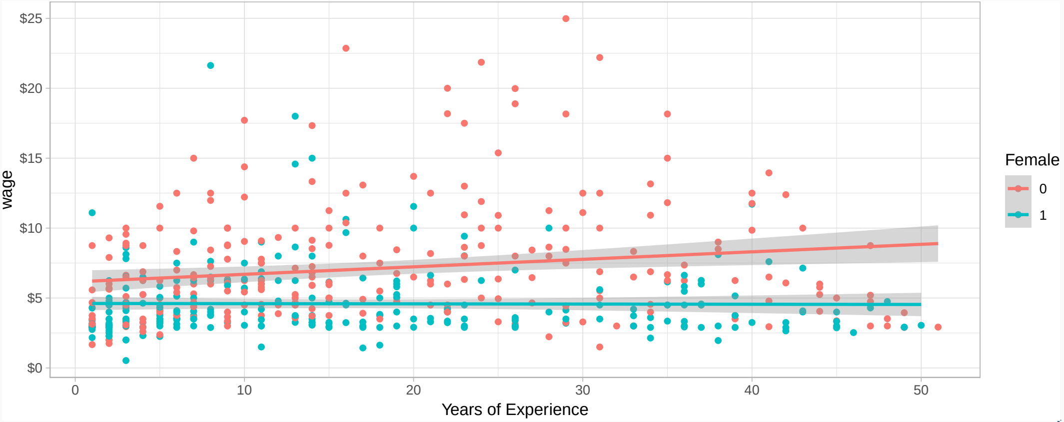
$$\widehat{wage_i} = \underbrace{(\hat{\beta}_0 + \hat{\beta}_2)}_{\text{intercept}} + \underbrace{(\hat{\beta}_1 + \hat{\beta}_3)}_{\text{slope}} exper$$

Example II



$25
$20
$15
$10
$5
$0
wage
0
10
20
30
40
50
Years of Experience
Female
0
1

Example III



$25
$20
$15
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$0
wage
0
10
20
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40
50
Years of Experience
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0
1

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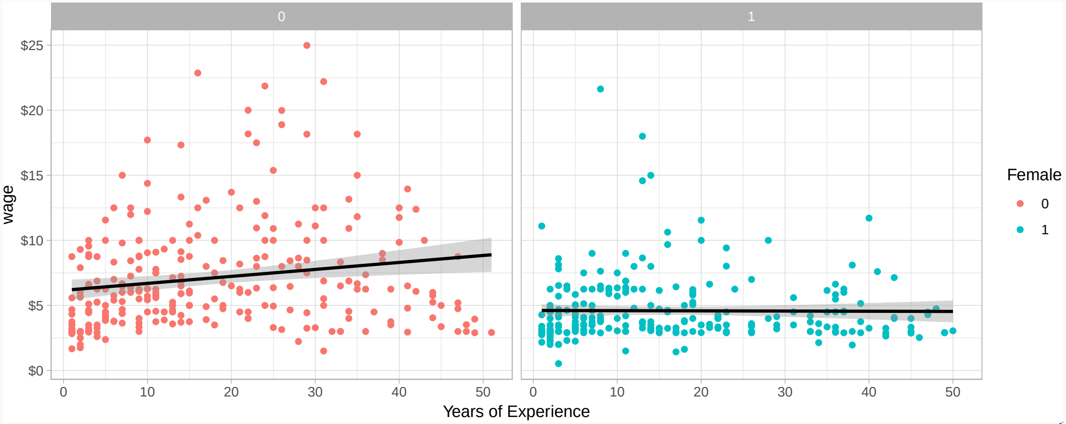

0
1
wage
$25
$20
$15
$10
$5
$0
0
10
20
30
40
50
Years of Experience
Female
0
1

- Syntax for interaction term is easy, simply add `var1*var2` to the regression

```
interactionreg<-lm(wage~female+exper+female*exper, data=wages)
summary(interactionreg)
```

```
##
## Call:
## lm(formula = wage ~ female + exper + female * exper, data = wages)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.3200 -1.8191 -0.9708  1.4132 17.2672
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   6.15828    0.34167  18.024  < 2e-16 ***
## female       -1.54655    0.48186  -3.210 0.001411 **
## exper         0.05360    0.01544   3.472 0.000559 ***
## female:exper -0.05507    0.02217  -2.483 0.013325 *
## ---
```

Example Regression in R II

```r
library("stargazer")
stargazer(interactionreg,
          type="latex", header=FALSE,
          float=FALSE, font.size="tiny")
```

	Dependent variable:
	wage
female	−1.547***
	(0.482)
exper	0.054***
	(0.015)
female:exper	−0.055**
	(0.022)
Constant	6.158***
	(0.342)
Observations	526
R^2	0.136
Adjusted R^2	0.131
Residual Std. Error	3.443 (df = 522)
F Statistic	27.307*** (df = 3; 522)
Note:	*p<0.1; **p<0.05; ***p<0.01

$$\underset{}{\widehat{\text{wage}_i}} = \underset{(0.34)}{6.16} + \underset{(0.02)}{0.05}\,\text{Experience}_i - \underset{(0.49)}{1.55}\,\text{Female}_i - \underset{(0.02)}{0.06}\,\text{Experience}_i \times \text{Female}_i$$

- $\hat{\beta}_0$:

$$\underset{}{\widehat{\text{wage}_i}} = \underset{(0.34)}{6.16} + \underset{(0.02)}{0.05}\,\text{Experience}_i - \underset{(0.49)}{1.55}\,\text{Female}_i - \underset{(0.02)}{0.06}\,\text{Experience}_i \times \text{Female}_i$$

- $\hat{\beta}_0$: Males with experience of 0 years earn \$6.16
- $\hat{\beta}_1$:

Example Regression in R: Interpretting Coefficients

$$\widehat{\text{wage}_i} = \underset{(0.34)}{6.16} + \underset{(0.02)}{0.05}\,\text{Experience}_i - \underset{(0.49)}{1.55}\,\text{Female}_i - \underset{(0.02)}{0.06}\,\text{Experience}_i \times \text{Female}_i$$

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- $\hat{\beta}_0$: Males with experience of 0 years earn \$6.16
- $\hat{\beta}_1$: For every additional year of experience, *males* earn \$0.05
- $\hat{\beta}_2$:

$$\widehat{\text{wage}_i} = \underset{(0.34)}{6.16} + \underset{(0.02)}{0.05}\,\text{Experience}_i - \underset{(0.49)}{1.55}\,\text{Female}_i - \underset{(0.02)}{0.06}\,\text{Experience}_i \times \text{Female}_i$$

- $\hat{\beta}_0$: Males with experience of 0 years earn \$6.16
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Example Regression in R: Interpretting Coefficients

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- Regression for females ($female = 1$):

$$\begin{aligned}\widehat{\text{wage}_i} &= 6.16 + 0.05 Experience_i - 1.55(1) - 0.06 Experience_i * (1) \\ &= (6.16 - 1.55) + (0.05 - 0.06) Experience \\ &= 4.61 - 0.01 Experience\end{aligned}$$

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- Females with no experience earn \$4.61
- For every year of experience, females' wages decrease by \$0.01

$$\underset{(0.34)}{\widehat{\text{wage}_i}} = \underset{(0.34)}{6.16} + \underset{(0.02)}{0.05}\,\text{Experience}_i - \underset{(0.49)}{1.55}\,\text{Female}_i - \underset{(0.02)}{0.06}\,\text{Experience}_i * \text{Female}_i$$

- How can we test significant differences between two regressions' slopes and intercepts?

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    - Is β_2 **significant**?
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- **Different slopes**?
    - Differences between male vs. female change in wages per 1 year of experience?
    - Is β_3 **significant**?
    - Yes, $t = \frac{0.06}{0.02} \approx -2.483$, $p(T > t) = 0.01$ (from R output, above)

Interactions Between Two Dummy Variables

Interactions Between Two Dummies

$$Y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + \beta_3 D_{1i} * D_{2i}$$

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Interactions Between Two Dummies

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- β_3: **increment** to effect on Y of going from $D_{1i} = 0$ to $D_{1i} = 1$ when $D_{2i} = 1$ (vs. when $D_{2i} = 0$)

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- β_3: **increment** to effect on Y of going from $D_{1i} = 0$ to $D_{1i} = 1$ when $D_{2i} = 1$ (vs. when $D_{2i} = 0$)
- Again, best to think logically about the possibilities (when each dummy $= 0$ or $= 1$)

$$Y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + \beta_3 D_{1i} * D_{2i}$$

- To interpret the coefficients, compare cases:

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- To interpret the coefficients, compare cases:

$$E(Y_i | D_{1i} = 0, D_{2i} = d_2) = \beta_0 + \beta_2 d_2$$

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- **The effect of $D_{1i} \rightarrow Y_i$ depends on d_{2i}**

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- **The effect of** $D_{1i} \rightarrow Y_i$ **depends on** d_{2i}
- β_3: **increment** to the effect of D_1 when $D_2 = 1$

Example

Return to the gender pay gap: does it matter if person is married or single?

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1 female_i + \hat{\beta}_2 married_i + \hat{\beta}_3 female_i * married_i$$

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Return to the gender pay gap: does it matter if person is married or single?

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  3. **Unmarried females** ($\text{female}_i = 1$, $\text{married}_i = 0$)
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Interactions Between Two Dummies: Example

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  4. **Married females** ($\text{female}_i = 1$, $\text{married}_i = 1$)
     $\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3$

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Interactions Between Two Dummies: Conditionally Looking at the Data

```r
# get average wage for unmarried men
mean(wages$wage[wages$married==0 & wages$female==0])
```

```
## [1] 5.168023
```

```r
# get average wage for married men
mean(wages$wage[wages$married==1 & wages$female==0])
```

```
## [1] 7.983032
```

```r
# get average wage for unmarried women
mean(wages$wage[wages$married==0 & wages$female==1])
```

```
## [1] 4.611583
```

```r
# get average wage for married wommen
mean(wages$wage[wages$married==1 & wages$female==1])
```

```
## [1] 4.565909
```

$$\widehat{wage}_i = \hat{\beta}_0 + \hat{\beta}_1 female_i + \hat{\beta}_2 married_i + \hat{\beta}_3 female_i * married_i$$

	Unmarried	Married
Male	\$5.17	\$7.98
Female	\$4.61	\$4.57

Average Wage for each Grouping

Interactions Between Two Dummies: Regression in R

```r
reg.2dummies.interact<-lm(wage~female+married+female*married, data=wages)
summary(reg.2dummies.interact)
```

```
##
## Call:
## lm(formula = wage ~ female + married + female * married, data = wages)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.7530 -1.7327 -0.9973  1.2566 17.0184
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)      5.1680     0.3614  14.299  < 2e-16 ***
## female          -0.5564     0.4736  -1.175    0.241
## married          2.8150     0.4363   6.451 2.53e-10 ***
## female:married  -2.8607     0.6076  -4.708 3.20e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```r
library("stargazer")
stargazer(reg.2dummies.interact,
          type="latex", header=FALSE,
          float=FALSE, font.size="tiny")
```

	Dependent variable:
	wage
female	−0.556
	(0.474)
married	2.815***
	(0.436)
female:married	−2.861***
	(0.608)
Constant	5.168***
	(0.361)
Observations	526
R^2	0.181
Adjusted R^2	0.176
Residual Std. Error	3.352 (df = 522)
F Statistic	38.451*** (df = 3; 522)
Note:	*p<0.1; **p<0.05; ***p<0.01

$$\widehat{\text{wage}_i} = \underset{(0.36)}{5.17} - \underset{(0.47)}{0.56}\,\text{Female}_i + \underset{(0.44)}{2.82}\,\text{Married}_i - \underset{(0.61)}{2.86}\,\text{Female}_i * \text{Married}_i$$

Average Wage for each Grouping

	Unmarried	Married
Male	\$5.17	\$7.98
Female	\$4.61	\$4.57

- Wage for **unmarried males**: $\hat{\beta}_0 = \$5.17$

Interactions Between Two Dummies: Interpretting the Coefficients

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- Wage for **unmarried males**: $\hat{\beta}_0 = \$5.17$
- Wage for **married males**: $\hat{\beta}_0 + \hat{\beta}_2 = 5.17 + 2.82 = \$7.98$

$$\widehat{\text{wage}_i} = \underset{(0.36)}{5.17} - \underset{(0.47)}{0.56}\,\text{Female}_i + \underset{(0.44)}{2.82}\,\text{Married}_i - \underset{(0.61)}{2.86}\,\text{Female}_i * \text{Married}_i$$

Average Wage for each Grouping

	Unmarried	Married
Male	\$5.17	\$7.98
Female	\$4.61	\$4.57

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- Wage for **unmarried females**: $\hat{\beta}_0 + \hat{\beta}_1 = 5.17 - 0.56 = \$4.61$

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- Wage for **married females**: $\hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3 = 5.17 - 0.56 + 2.82 - 2.86 = \$4.57$

Interactions Between Two Continuous Variables

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 (X_{1i} X_{2i})$$

- To interpret the coefficients, compare cases after changing ΔX_1:

$$Y + \Delta Y = \beta_0 + \beta_1 (X_1 + \Delta X_1) \beta_2 X_2 + \beta_3 ((X_1 + \Delta X_1) X_2)$$

-The difference is:

$$\Delta Y = \beta_1 \Delta X_1 + \beta_3 X_2 \Delta X_1$$
$$\frac{\Delta Y}{\Delta X_1} = \beta_1 + \beta_3 X_2$$

- The effect of X_1 depends on X_2

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- The effect of X_1 depends on X_2
- β_3: increment to the effect of X_1 from a 1 unit change in X_2

Example
Wages on education and experience: Do education & experience interact?

$$\widehat{wage_i} = \hat{\beta}_0 + \hat{\beta}_1 educ_i + \hat{\beta}_2 exper_i + \hat{\beta}_3 educ_i \times exper_i + \epsilon_i$$

Example
Wages on education and experience: Do education & experience interact?

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- Estimated effect of education on wages depends on the amount of experience (and vice versa)!

$$\frac{\Delta wage}{\Delta educ} = \hat{\beta}_1 + \beta_3 exper$$

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Wages on education and experience: Do education & experience interact?

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- Estimated effect of education on wages depends on the amount of experience (and vice versa)!

$$\frac{\Delta wage}{\Delta educ} = \hat{\beta}_1 + \beta_3 exper$$

- This is a type of nonlinearity (we will examine nonlinearities next lesson)

```
reg.2x.interact<-lm(wage~educ+exper+educ*exper, data=wages)
summary(reg.2x.interact)
```

```
##
## Call:
## lm(formula = wage ~ educ + exper + educ * exper, data = wages)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.6747 -1.9683 -0.6991  1.2803 15.8067
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.859916   1.181080  -2.421   0.0158 *
```

Interactions Between Two Continuous Variables: Regression in R II

```r
library("stargazer")
stargazer(reg.2x.interact,
          type="latex", header=FALSE,
          float=FALSE, font.size="tiny")
```

	Dependent variable:
	wage
educ	0.602^{***}
	(0.090)
exper	0.046
	(0.043)
educ:exper	0.002
	(0.003)
Constant	-2.860^{**}
	(1.181)
Observations	526
R^2	0.226
Adjusted R^2	0.221
Residual Std. Error	3.259 (df = 522)
F Statistic	50.713^{***} (df = 3; 522)
Note:	$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

$$\underset{(1.181)}{\widehat{\text{wage}_i} = -2.86} + \underset{(0.090)}{0.60}\ \text{educ}_i + \underset{(0.043)}{0.05}\ \text{exper}_i + \underset{(0.003)}{0.002}\ \text{educ}_i \times \text{exper}_i$$

Interactions Between Two Continuous Variables: Interpretting Coefficients

$$\widehat{\text{wage}_i} = \underset{(1.181)}{-2.86} + \underset{(0.090)}{0.60}\ \text{educ}_i + \underset{(0.043)}{0.05}\ \text{exper}_i + \underset{(0.003)}{0.002}\ \text{educ}_i \times \text{exper}_i$$

Changes in Education

Exper	$\frac{\Delta wage}{\Delta educ}$
5	$0.60 + 0.002(5) = \$0.61$
10	$0.60 + 0.002(10) = \$0.62$
15	$0.60 + 0.002(15) = \$0.63$

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$$\widehat{\text{wage}_i} = \underset{(1.181)}{-2.86} + \underset{(0.090)}{0.60}\ \text{educ}_i + \underset{(0.043)}{0.05}\ \text{exper}_i + \underset{(0.003)}{0.002}\ \text{educ}_i \times \text{exper}_i$$

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- Marginal effect of education $\rightarrow$ wages **increases** with more experience (but very insignificantly)