Lecture 19: Difference-in-Difference Models

ECON 480 - Econometrics - Fall 2018

Ryan Safner

December 5, 2018

Difference-in-Difference Models

Generalizing DND Models

Example: “The” Card-Kreuger Minimum Wage Study

Difference-in-Difference Models

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- Often, we want to examine the consequences of a change, such as a law or policy
    - e.g. States that implemented law X saw a change in Y
    - **Treatment**: States that implement law X
    - **Control**: States that did not implement law X
    - If we have **panel data** with observations for all states before *and* after the change:
- Simple logic: compare difference in outcomes of treatment group (before and after treatment) with those of non-treated group (before and after same treatment period)


DIFF-IN-DIFFS
SO HOT RIGHT NOW
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Difference-in-Difference Models III

- The difference-in-difference model (aka "diff-in-diff"" or "DND") identifies treatment effect by differencing the difference pre- and post-treatment between treatment and control groups

$$\widehat{Y_{it}} = \beta_0 + \beta_1 \text{Treated}_i + \beta_2 \text{After}_t + \beta_3 (\text{Treated}_i * \text{After}_t) + \epsilon_{it}$$

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HOOD COLLEGE

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	Control	Treatment	Group Diff. (ΔY_i)
Before	β_0	$\beta_0 + \beta_1$	β_1
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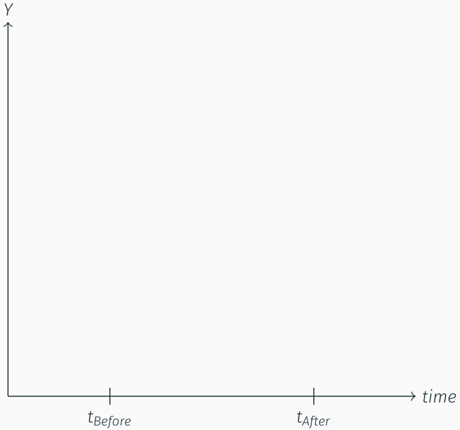
Time Diff. (ΔY_t)	β_2	$\beta_2 + \beta_3$	β_3
			Diff-in-diff ($\Delta_i \Delta_t Y$)

$$\widehat{Y_{it}} = \hat{\beta}_0 + \hat{\beta}_1 \text{Treated}_i + \hat{\beta}_2 \text{After}_t + \hat{\beta}_3 (\text{Treated}_i * \text{After}_t) + \hat{\epsilon}_{it}$$



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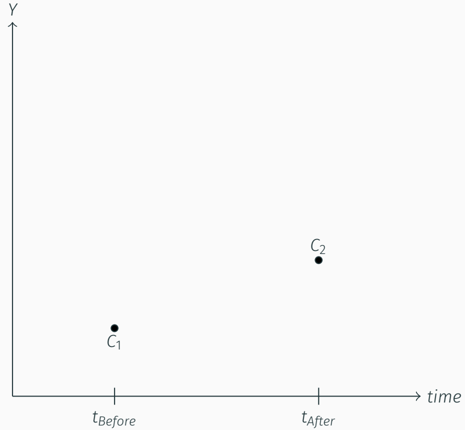

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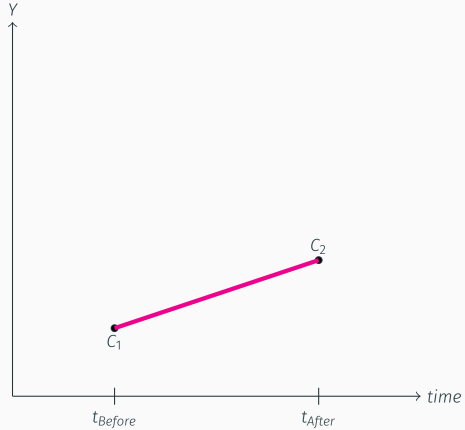
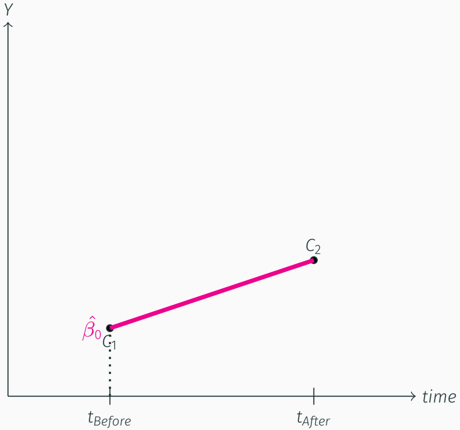


- Control ($Treated = 0$) group

Visualizing Diff-in-Diff

$$\widehat{Y_{it}} = \hat{\beta}_0 + \hat{\beta}_1 \text{Treated}_i + \hat{\beta}_2 \text{After}_t + \hat{\beta}_3 (\text{Treated}_i * \text{After}_t) + \hat{\epsilon}_{it}$$



- Control ($Treated = 0$) group
- $\hat{\beta}_0$: value of Y for control before treatment

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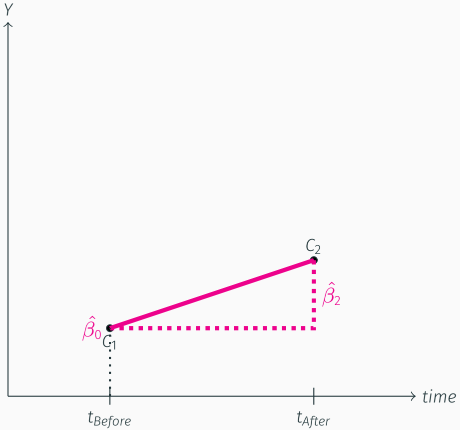

- Control ($Treated = 0$) group
- $\hat{\beta}_0$: value of Y for control before treatment
- $\hat{\beta}_2$: time difference (for control group)

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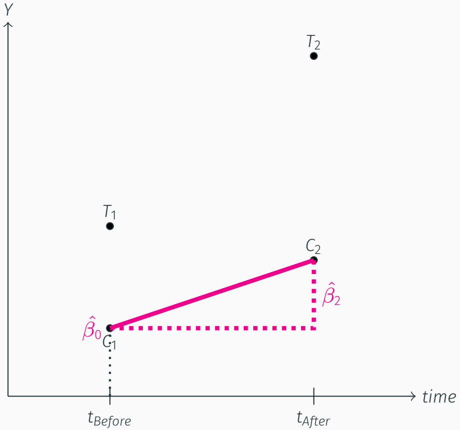

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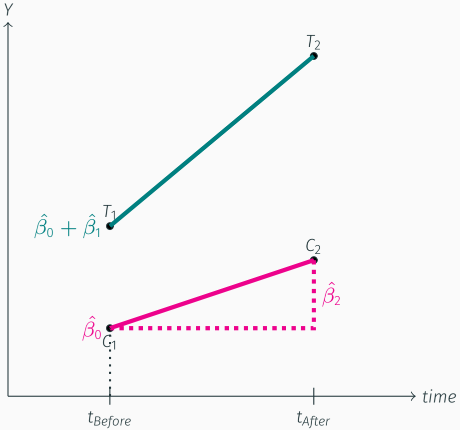

- Treatment (*Treated* $= 1$) group
- Control (*Treated* $= 0$) group
- $\hat{\beta}_0$: value of Y for control before treatment
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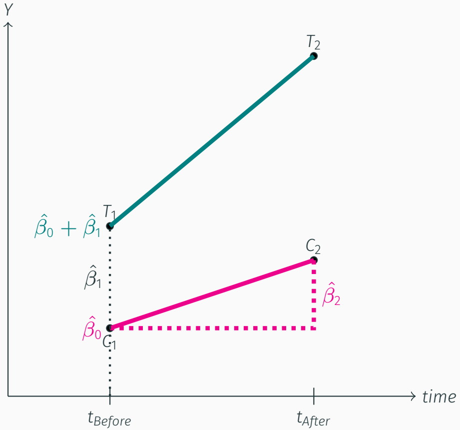

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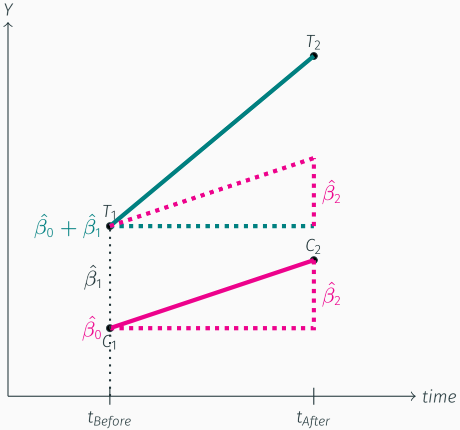

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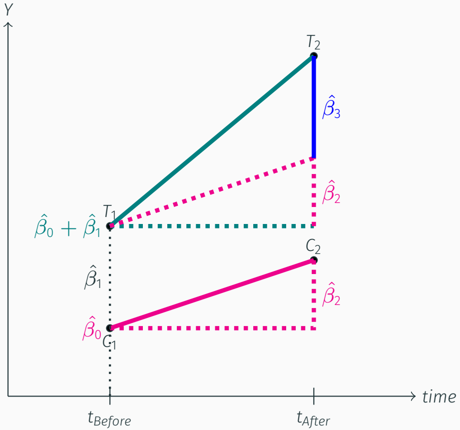

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- $\hat{\beta}_3$: difference-in-difference: effect of treatment

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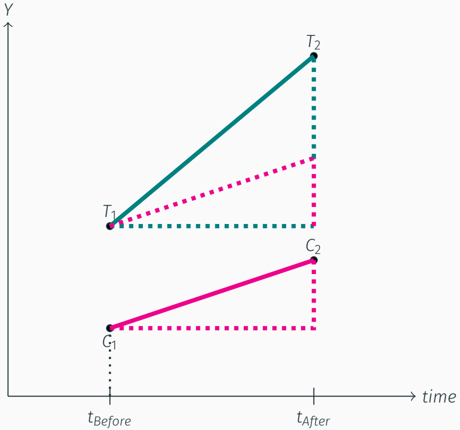

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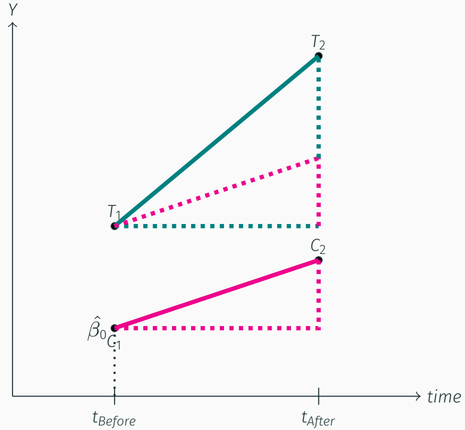

- Y for Control Group Before: $\hat{\beta}_0$

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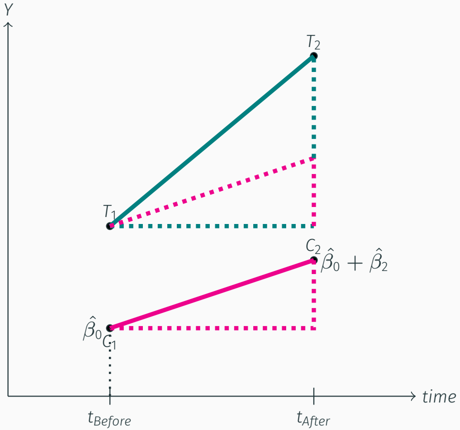

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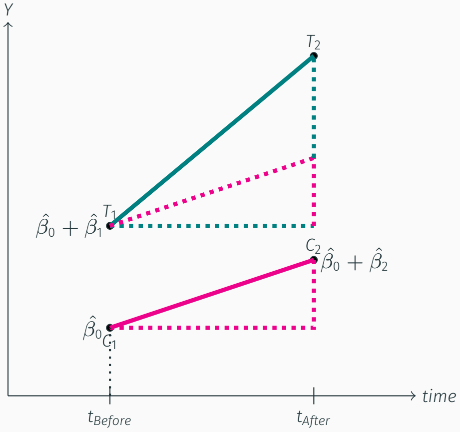
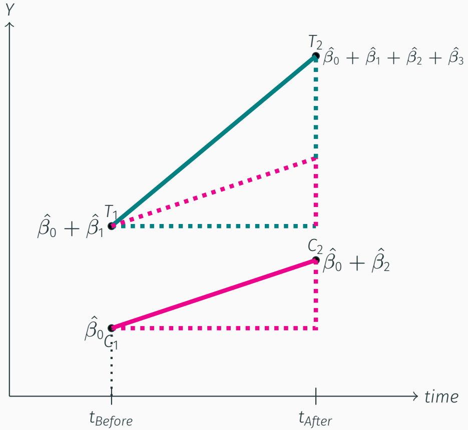


- Y for Control Group Before: $\hat{\beta}_0$
- Y for Control Group After: $\hat{\beta}_0 + \hat{\beta}_2$
- Y for Treatment Group Before: $\hat{\beta}_0 + \hat{\beta}_1$

Visualizing and Comparing Group Means

$$\widehat{Y_{it}} = \hat{\beta}_0 + \hat{\beta}_1 \text{Treated}_i + \hat{\beta}_2 \text{After}_t + \hat{\beta}_3 (\text{Treated}_i * \text{After}_t) + \hat{\epsilon}_{it}$$



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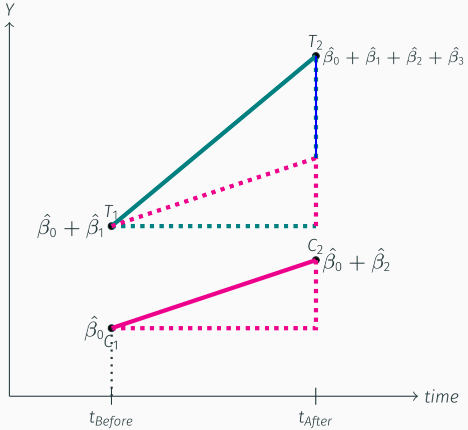

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- Y for Treatment Group Before: $\hat{\beta}_0 + \hat{\beta}_1$
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- Treatment Effect: $\hat{\beta}_3$

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Key Assumption About Counterfactual

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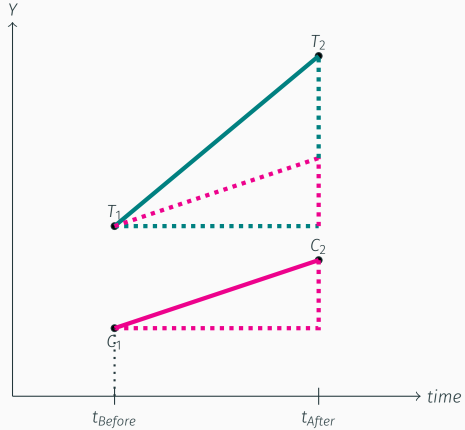

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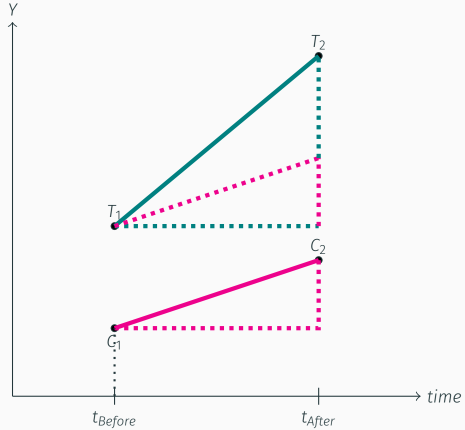

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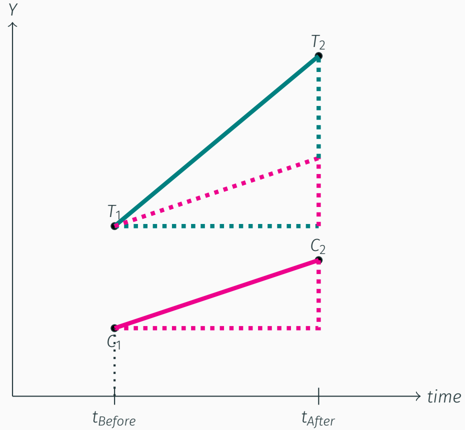

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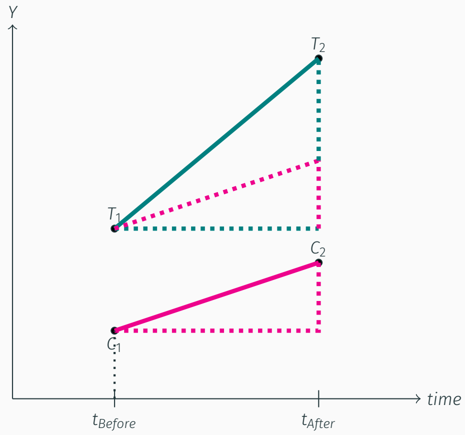

- Key assumption in DND models is that the time trend is parallel
- Treatment & control groups must be similar over time *except for treatment*
- Counterfactual if the treatment group were *not* treated, they would change the same as control group over time $\hat{\beta}_2$

Key Assumption about Counterfactual II

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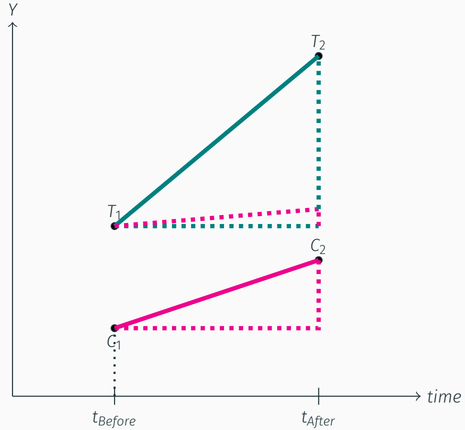

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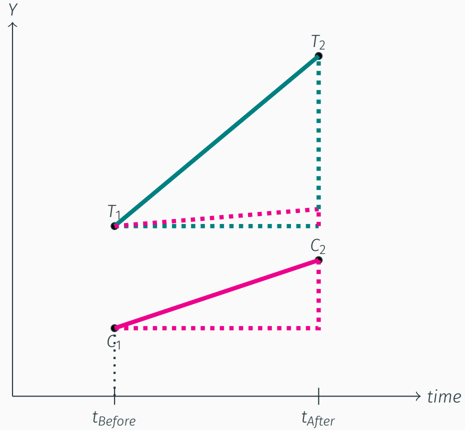

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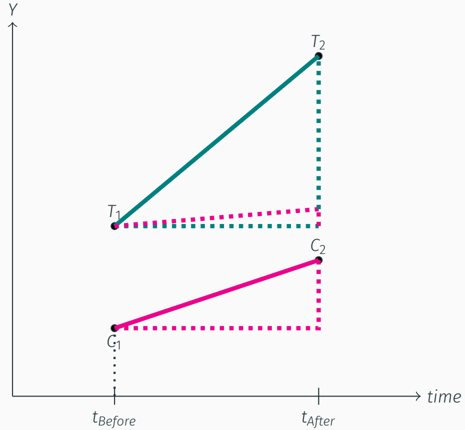

- If time trend is different between treatment and control groups
- Treatment effect may be over/under-estimated!

Example

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  - $\text{After}_{t} = \begin{cases} 1 \text{ if } t \text{ is after 1992} \\ 0 \text{ if } t \text{ is after 1992} \end{cases}$
  - Note: With a dummy *dependent* variable (Y), coefficients estimate the probability $Y = 1$, i.e. the *probability* a person is enrolled in college

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Diff-in-Diff Example II

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- Treatment: Georgia

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- Georgia and nearby states: if not for HOPE, changes should be the same over time
- Treatment period: 1992
- Treatment: Georgia

$$\Delta_i \Delta_t Enrolled = (\text{GA}_{after} - \text{GA}_{before}) - (\text{neighbors}_{after} - \text{neighbors}_{before})$$

Diff-in-Diff Example II

Example

In 1993 Georgia initiated a HOPE scholarship program to let state residents with at least a B average in high school attend public college in Georgia for free. Did it increase college enrollment?

- We can use a diff-in-diff model to measure the effect of HOPE scholarship on enrollments
- Georgia and nearby states: if not for HOPE, changes should be the same over time
- Treatment period: 1992
- Treatment: Georgia

$$\Delta_i \Delta_t Enrolled = (\text{GA}_{after} - \text{GA}_{before}) - (\text{neighbors}_{after} - \text{neighbors}_{before})$$

$$\widehat{Enrolled}_{it} = \beta_0 + \beta_1 Georgia_{it} + \beta_2 After_{it} + \beta_3 Georgia_{it} * After_{it}$$

Diff-in-Diff Example III

```
DND<-lm(InCollege~Georgia+After+AfterGeorgia, data=HOPE)
summary(DND)
```

```
##
## Call:
## lm(formula = InCollege ~ Georgia + After + AfterGeorgia, data = HOPE)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.4058 -0.4058 -0.4013  0.5942  0.6995
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.40578    0.01092  37.146  < 2e-16 ***
## Georgia      -0.10524    0.03778  -2.785  0.00537 **
## After        -0.00446    0.01585  -0.281  0.77848
## AfterGeorgia  0.08933    0.04889   1.827  0.06776 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4893 on 4287 degrees of freedom
## Multiple R-squared:  0.001872,   Adjusted R-squared:  0.001174
## F-statistic: 2.681 on 3 and 4287 DF,  p-value: 0.04528
```

$$\widehat{Enrolled_{it}} = 0.406 - 0.105 Georgia_i - 0.004 After_t + 0.089 Georgia_i * After_t$$

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- β_0: A **non-Georgian before** 1992 was 40.6% likely to be a college student

Diff-in-Diff Example: Interpreting Coefficients

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- β_1: **Georgians before** 1992 were 10.5% less likely to be college students than neighboring states

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- β_3: **After** 1992, **Georgians** are 8.9% more likely to enroll in colleges than neighboring states
- Treatment effect: HOPE increased enrollment likelihood by 8.9%

Diff-in-Diff Example: Comparing Group Means

$$\widehat{Enrolled}_{it} = 0.406 - 0.105 Georgia_i - 0.004 After_t + 0.089 Georgia_i * After_t$$

- A group mean for a dummy Y is $E[Y = 1]$, i.e. the probability a student is enrolled:

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  - **Non-Georgian enrollment probability post-1992**: $\beta_0 + \beta_2 = 0.406 - 0.004 = 0.402$
  - **Georgian enrollment probability post-1992**: $\beta_0 + \beta_1 + \beta_2 + \beta_3 = 0.406 - 0.105 - 0.004 + 0.089 = 0.386$

Diff-in-Diff Example: Comparing Group Means in R

```
# group mean for non-Georgian before 1992
HOPE %>%
  filter(Georgia==0 & After==0) %>%
  summarize(prob=mean(InCollege))
```

```
##        prob
## 1 0.4057827
```

```
# group mean for Georgian before 1992
HOPE %>%
  filter(Georgia==1 & After==0) %>%
  summarize(prob=mean(InCollege))
```

```
##        prob
## 1 0.3005464
```

```
# group mean for non-Georgian AFTER 1992
HOPE %>%
  filter(Georgia==0 & After==1) %>%
  summarize(prob=mean(InCollege))
```

```
##       prob
## 1 0.401323
```

```
# group mean for Georgian AFTER 1992
HOPE %>%
  filter(Georgia==1 & After==1) %>%
  summarize(prob=mean(InCollege))
```

```
##        prob
## 1 0.3854167
```

$$\widehat{Enrolled}_{it} = 0.406 - 0.105Georgia_{it} - 0.004After_{it} + 0.089Georgia_{it} * After_{it}$$

	Neighbors	Georgia	Group Diff. (ΔY_i)
Before	0.406	0.301	-0.105
After	0.402	0.386	0.016
Time Diff. (ΔY_t)	-0.004	0.085	0.089
			Diff-in-diff ($\Delta\Delta Y$)

Diff-in-Diff Example Summary

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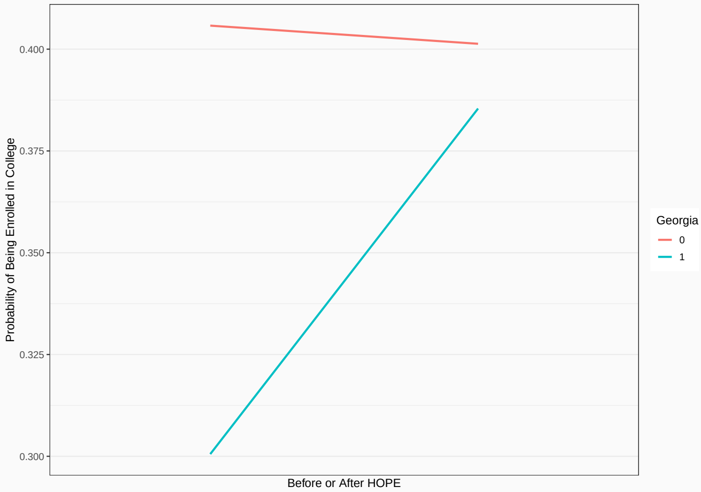
$$\begin{aligned}\Delta_i \Delta_t Enrolled &= (\text{GA}_{after} - \text{GA}_{before}) - (\text{neighbors}_{after} - \text{neighbors}_{before}) \\ &= (0.386 - 0.301) - (0.402 - 0.406) \\ &= (0.085) - (-0.004) \\ &= 0.089\end{aligned}$$

Diff-in-Diff Time Graph



HOOD COLLEGE

Generalizing DND Models

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$$\widehat{Y_{it}} = \alpha_i + \theta_t + \beta_3(\text{Treated}_i * \text{After}_{it}) + \nu_{it}$$

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  - θ_t: **time fixed effects** (pre/post treatment)
- Allows *many* periods, and treatment(s) can occur at different times to different units (so long as some do not get treated)
- Can also add control variables that vary within units and over time

$$\widehat{Y_{it}} = \alpha_i + \theta_t + \beta_3(\text{Treated}_i * \text{After}_{it}) + \beta_4 X_{it} + \nu_{it}$$

Example

$$\widehat{Enrolled}_{it} = \alpha_i + \theta_t + \beta_3 Georgia_{it} * After_{it}$$

The Georgia HOPE Example, Generalized II

- `StateCode` is a variable for the State $\implies$ create State fixed effect

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- `StateCode` is a variable for the State $\implies$ create State fixed effect
- `Year` is a variable for the year $\implies$ create year fixed effect
- Using LSDV method (note we must ensure both `StateCode` and `Year` are `factor` variables!):

HOOD COLLEGE

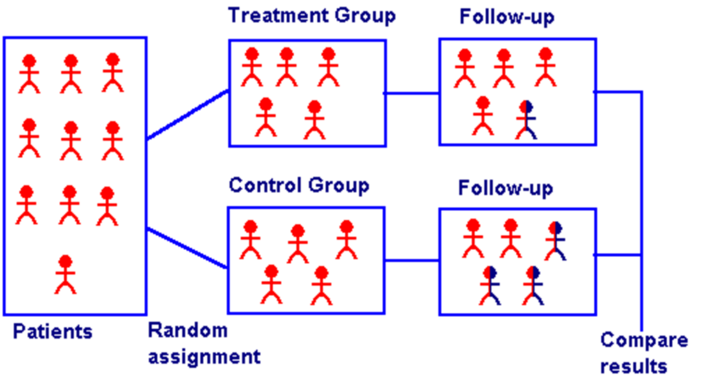
The Georgia HOPE Example, Generalized II

- `StateCode` is a variable for the State $\implies$ create State fixed effect
- `Year` is a variable for the year $\implies$ create year fixed effect
- Using LSDV method (note we must ensure both `StateCode` and `Year` are `factor` variables!):

```r
DND_fe<-lm(InCollege~AfterGeorgia+factor(StateCode)+factor(Year), data=HOPE)
summary(DND_fe)
```

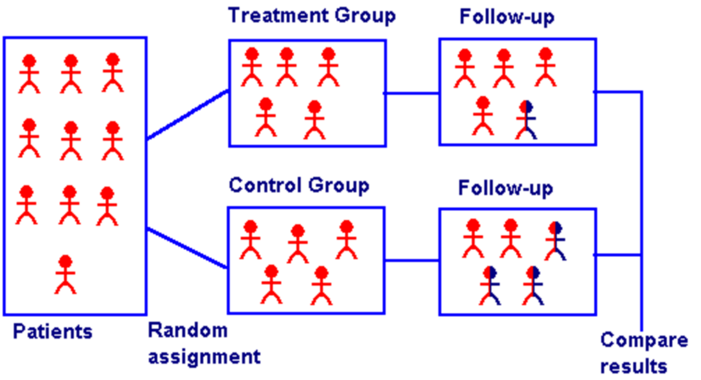
```
##
## Call:
## lm(formula = InCollege ~ AfterGeorgia + factor(StateCode) + factor(Year),
##     data = HOPE)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.4934 -0.4148 -0.3344  0.5690  0.7359
##
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)          0.418057   0.022611  18.489  < 2e-16 ***
## AfterGeorgia         0.091420   0.048761   1.875 0.060879 .
## factor(StateCode)57 -0.014181   0.027397  -0.518 0.604754
## factor(StateCode)58 -0.141501   0.039361  -3.595 0.000328 ***
## factor(StateCode)59 -0.062379   0.019543  -3.192 0.001424 **
## factor(StateCode)62 -0.132650   0.028061  -4.727 2.35e-06 ***
## factor(StateCode)63 -0.005104   0.026278  -0.194 0.846007
## factor(Year)90       0.046609   0.028336   1.645 0.100075
## factor(Year)91       0.032276   0.028569   1.130 0.258642
## factor(Year)92       0.023536   0.029846   0.789 0.430403
## factor(Year)93       0.030161   0.030154   1.000 0.317254
## factor(Year)94       0.014505   0.030574   0.474 0.635220
```

- Diff-in-diff models are the quintessential example of exploiting natural experiments

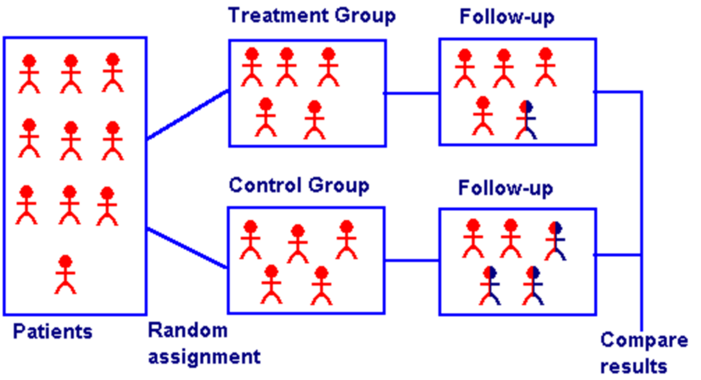


Intuition Behind DND Models

- Diff-in-diff models are the quintessential example of exploiting natural experiments
  - A major change at a point in time (change in law, a natural disaster, political crisis) separates groups where one is affected and another is not—identifies the effect of the change (treatment)



- Diff-in-diff models are the quintessential example of exploiting natural experiments
  - A major change at a point in time (change in law, a natural disaster, political crisis) separates groups where one is affected and another is not—identifies the effect of the change (treatment)
- One of the cleanest and clearest causal **identification strategies**



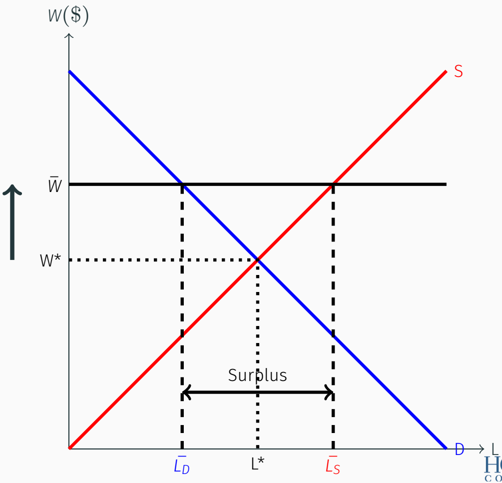
Example: "The" Card-Kreuger Minimum Wage Study

Example

The controversial minimum wage study, Card & Kreuger (1994) is a quintessential (and clever) diff-in-diff.

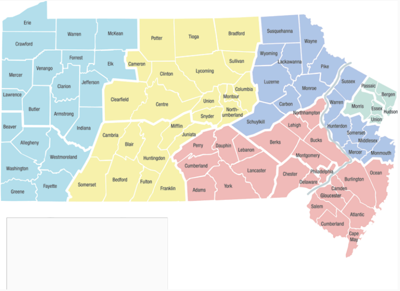
Card, David, Krueger, Alan B, (1994), "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania," *American Economic Review* 84 (4): 772–793

- Economic theory: increases in minimum wage ($\bar{W}$) move us up a downward-sloping demand curve for labor
- A surplus of labor: disemployment



Card & Kreuger (1994): Background

- Card & Kreuger (1994) compare employment in fast food restaurants on New Jersey and Pennsylvania sides of border between February and November 1992.



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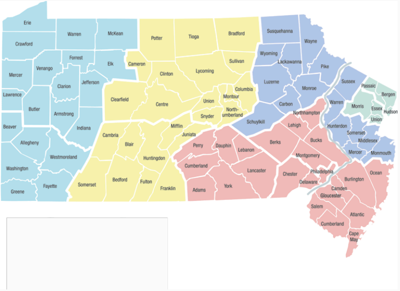
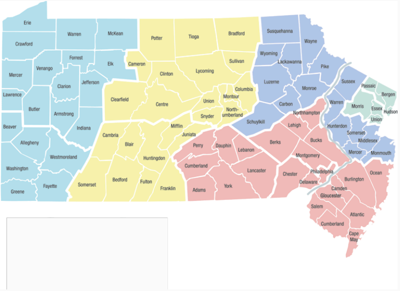
- Pennsylvania & New Jersey both had a minimum wage of $4.25 before February 1992



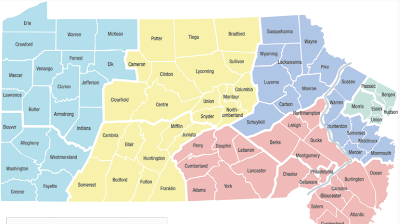
Card & Kreuger (1994): Background

- Card & Kreuger (1994) compare employment in fast food restaurants on New Jersey and Pennsylvania sides of border between February and November 1992.
- Pennsylvania & New Jersey both had a minimum wage of $4.25 before February 1992
- In February 1992, New Jersey raised minimum wage from $4.25 to $5.05



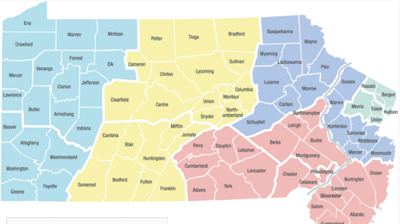
Card & Kreuger (1994): Background II

- If we look only at New Jersey before & after change:

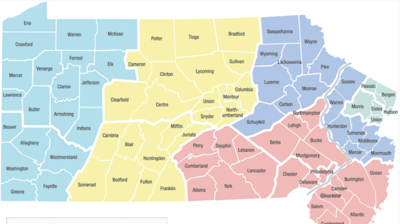


Card & Kreuger (1994): Background II

- If we look only at New Jersey before & after change:
    - **Omitted variable bias**: macroeconomic variables (there's a recession!), weather, etc.



- If we look only at New Jersey before & after change:
  - **Omitted variable bias**: macroeconomic variables (there's a recession!), weather, etc.
    - Including PA as a control will control for these time-varying effects if they are national trends



- If we look only at New Jersey before & after change:
  - **Omitted variable bias**: macroeconomic variables (there's a recession!), weather, etc.
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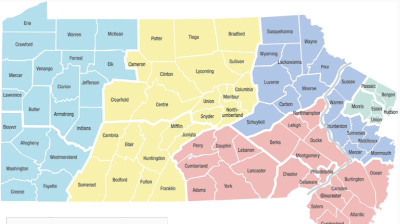
- Surveyed 400 fast food restaurants on each side of the border, before & after min wage increase



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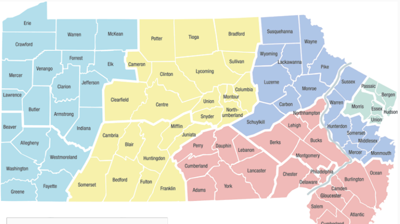
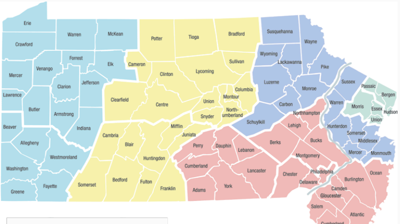
- If we look only at New Jersey before & after change:
  - **Omitted variable bias**: macroeconomic variables (there's a recession!), weather, etc.
    - Including PA as a control will control for these time-varying effects if they are national trends
- Surveyed 400 fast food restaurants on each side of the border, before & after min wage increase
  - Key assumption: Pennsylvania and New Jersey follow parallel trends,



- If we look only at New Jersey before & after change:
    - **Omitted variable bias**: macroeconomic variables (there's a recession!), weather, etc.
        - Including PA as a control will control for these time-varying effects if they are national trends
- Surveyed 400 fast food restaurants on each side of the border, before & after min wage increase
    - Key assumption: Pennsylvania and New Jersey follow parallel trends,
        - **Counterfactual**: if not for the minimum wage increase, NJ employment would have changed similar to PA employment



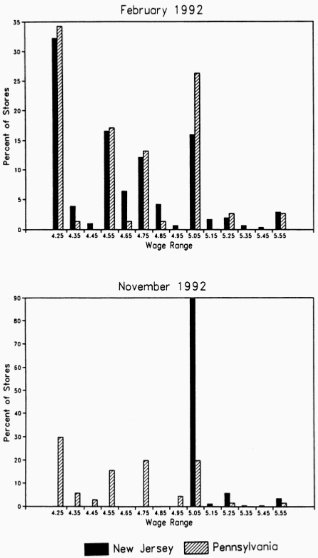


Figure 1. Distribution of Starting Wage Rates

Table 1—Sample Design and Response Rates

	All	Stores in: NJ	PA
Wave 1, February 15 – March 4, 1992:			
Number of stores in sample frame:[a]	473	364	109
Number of refusals:	63	33	30
Number interviewed:	410	331	79
Response rate (percentage):	86.7	90.9	72.5
Wave 2, November 5 – December 31, 1992:			
Number of stores in sample frame:	410	331	79
Number closed:	6	5	1
Number under rennovation:	2	2	0
Number temporarily closed:[b]	2	2	0
Number of refusals:	1	1	0
Number interviewed:[c]	399	321	78

Table 2—Means of Key Variables

Variable	Stores in: NJ	Stores in: PA
1. *Distribution of Store Types (percentages):*		
a. Burger King	41.1	44.3
b. KFC	20.5	15.2
c. Roy Rogers	24.8	21.5
d. Wendy's	13.6	19.0
e. Company-owned	34.1	35.4

Card & Kreuger (1994): Identification Strategy and Model

$$\widehat{Employment_{it}} = \beta_0 + \beta_1 NJ_i + \beta_2 After_t + \beta_3 (NJ_i * After_t)$$

- PA Before: β_0

	PA	NJ	State Diff. (ΔY_i)
Before	β_0	$\beta_0 + \beta_1$	β_1
After	$\beta_0 + \beta_2$	$\beta_0 + \beta_1 + \beta_2 + \beta_3$	$\beta_1 + \beta_3$
Time Diff. (ΔY_t)	β_2	$\beta_2 + \beta_3$	β_3
			Diff-in-diff ($\Delta\Delta Y$)

$$\widehat{Employment_{it}} = \beta_0 + \beta_1 NJ_i + \beta_2 After_t + \beta_3 (NJ_i * After_t)$$

- PA Before: β_0
- PA After: $\beta_0 + \beta_2$

	PA	NJ	State Diff. (ΔY_i)
Before	β_0	$\beta_0 + \beta_1$	β_1
After	$\beta_0 + \beta_2$	$\beta_0 + \beta_1 + \beta_2 + \beta_3$	$\beta_1 + \beta_3$
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$$\widehat{Employment_{it}} = \beta_0 + \beta_1 NJ_i + \beta_2 After_t + \beta_3 (NJ_i * After_t)$$

- PA Before: β_0
- PA After: $\beta_0 + \beta_2$
- NJ Before: $\beta_0 + \beta_1$

	PA	NJ	State Diff. (ΔY_i)
Before	β_0	$\beta_0 + \beta_1$	β_1
After	$\beta_0 + \beta_2$	$\beta_0 + \beta_1 + \beta_2 + \beta_3$	$\beta_1 + \beta_3$
Time Diff. (ΔY_t)	β_2	$\beta_2 + \beta_3$	β_3
			Diff-in-diff ($\Delta\Delta Y$)

$$\widehat{Employment}_{it} = \beta_0 + \beta_1 NJ_i + \beta_2 After_t + \beta_3 (NJ_i * After_t)$$

- PA Before: β_0
- PA After: $\beta_0 + \beta_2$
- NJ Before: $\beta_0 + \beta_1$
- NJ After: $\beta_0 + \beta_1 + \beta_2 + \beta_3$

	PA	NJ	State Diff. (ΔY_i)
Before	β_0	$\beta_0 + \beta_1$	β_1
After	$\beta_0 + \beta_2$	$\beta_0 + \beta_1 + \beta_2 + \beta_3$	$\beta_1 + \beta_3$
Time Diff. (ΔY_t)	β_2	$\beta_2 + \beta_3$	β_3
			Diff-in-diff ($\Delta\Delta Y$)

$$\widehat{Employment}_{it} = \beta_0 + \beta_1 NJ_i + \beta_2 After_t + \beta_3 (NJ_i * After_t)$$

- PA Before: β_0
- PA After: $\beta_0 + \beta_2$
- NJ Before: $\beta_0 + \beta_1$
- NJ After: $\beta_0 + \beta_1 + \beta_2 + \beta_3$
- **Diff-in-diff**: $(NJ_{after} - NJ_{before}) - (PA_{after} - PA_{before})$

	PA	NJ	State Diff. (ΔY_i)
Before	β_0	$\beta_0 + \beta_1$	β_1
After	$\beta_0 + \beta_2$	$\beta_0 + \beta_1 + \beta_2 + \beta_3$	$\beta_1 + \beta_3$
Time Diff. (ΔY_t)	β_2	$\beta_2 + \beta_3$	β_3
			Diff-in-diff ($\Delta\Delta Y$)

Variable	PA (i)	NJ (ii)	Difference, NJ – PA (iii)
1. FTE employment before, all available observations	23.33 (1.35)	20.44 (0.51)	−2.89 (1.44)
2. FTE employment after, all available observations	21.17 (0.94)	21.03 (0.52)	−0.14 (1.07)
3. Change in mean FTE employment	−2.16 (1.25)	0.59 (0.54)	2.76 (1.36)