Confirmatory factor analysis

The measurement model

Definitions

- Confirmatory factor analysis
  - SEM without causal paths among latent variables
- Exploratory factor analysis (EFA)
  - Principal components analysis (PCA)
  - Principal factor analysis (PFA)
- Path analysis
  - SEM without latent variables
- Structural equation modeling (SEM)
  - I reserve this terms for models with latent variables and causal paths

CFA

- The null hypothesis for model fit
  - The observed variance/covariance matrix is equal to the variance/covariance matrix implied by the model
  - $H_0 : R_{vv} = R'_{vv}$
  - $H_0 : \Sigma = \Sigma(\theta)$
    - Σ is the variance/covariance matrix
    - Θ is vector, containing the free parameters

CFA

- The null hypothesis for model fit
  - Evaluated with a chi-square test
  - If chi-square is significant then the model does not fit
    - Observed variance/covariance matrix is independent from the implied variance/covariance matrix
    - In short, significance is “bad”

CFA

- The null hypothesis for model fit
  - Can also compare the fit of two models, e.g., 1-factor (model 1) vs. 2-factor (model 2)
    - Chi-square (model 1) – Chi-square (model 2)
    - df (model 1) – df (model 2)

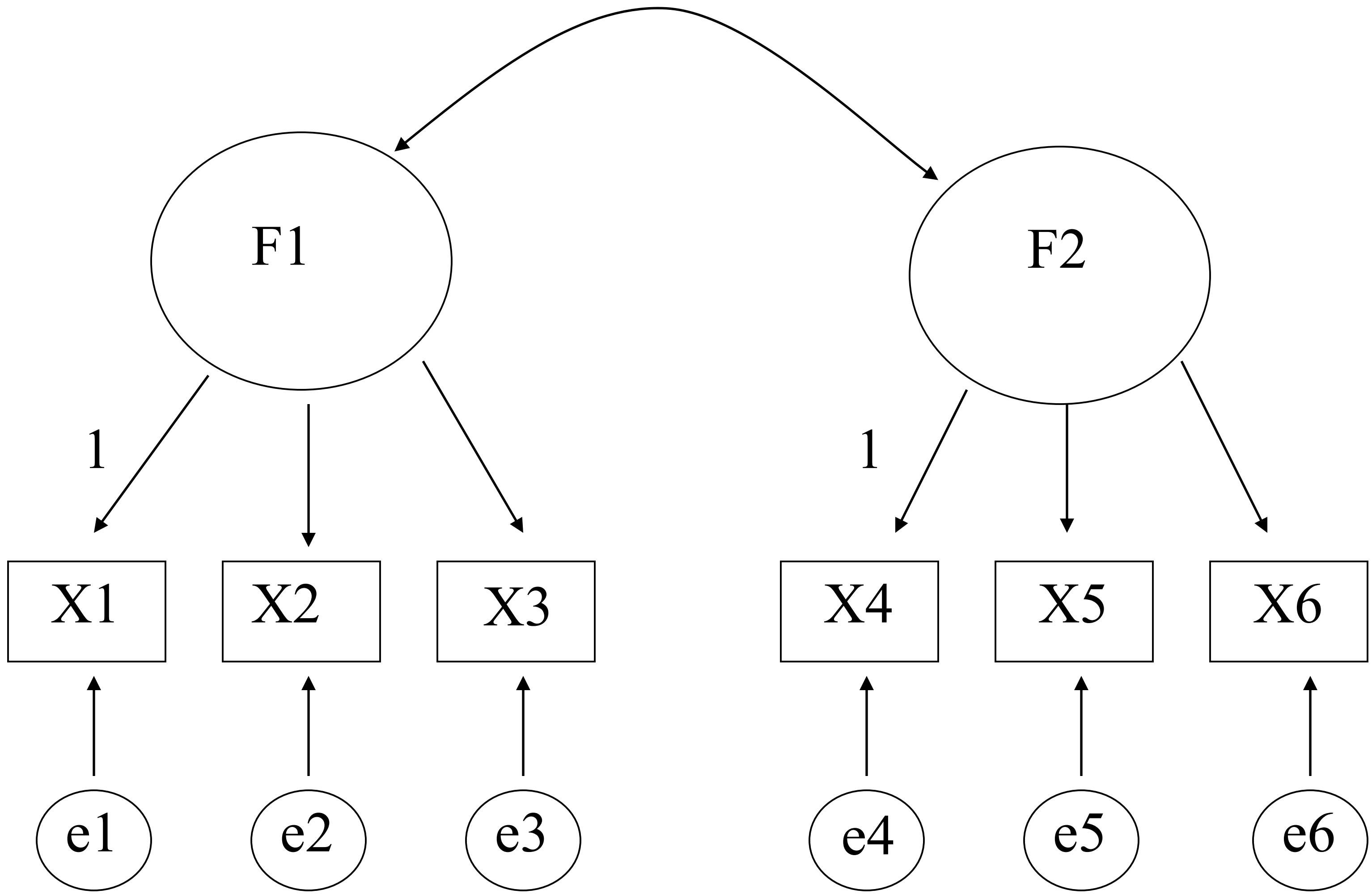
    - Can also compare models by comparing various fit indices (more on this later)

CFA

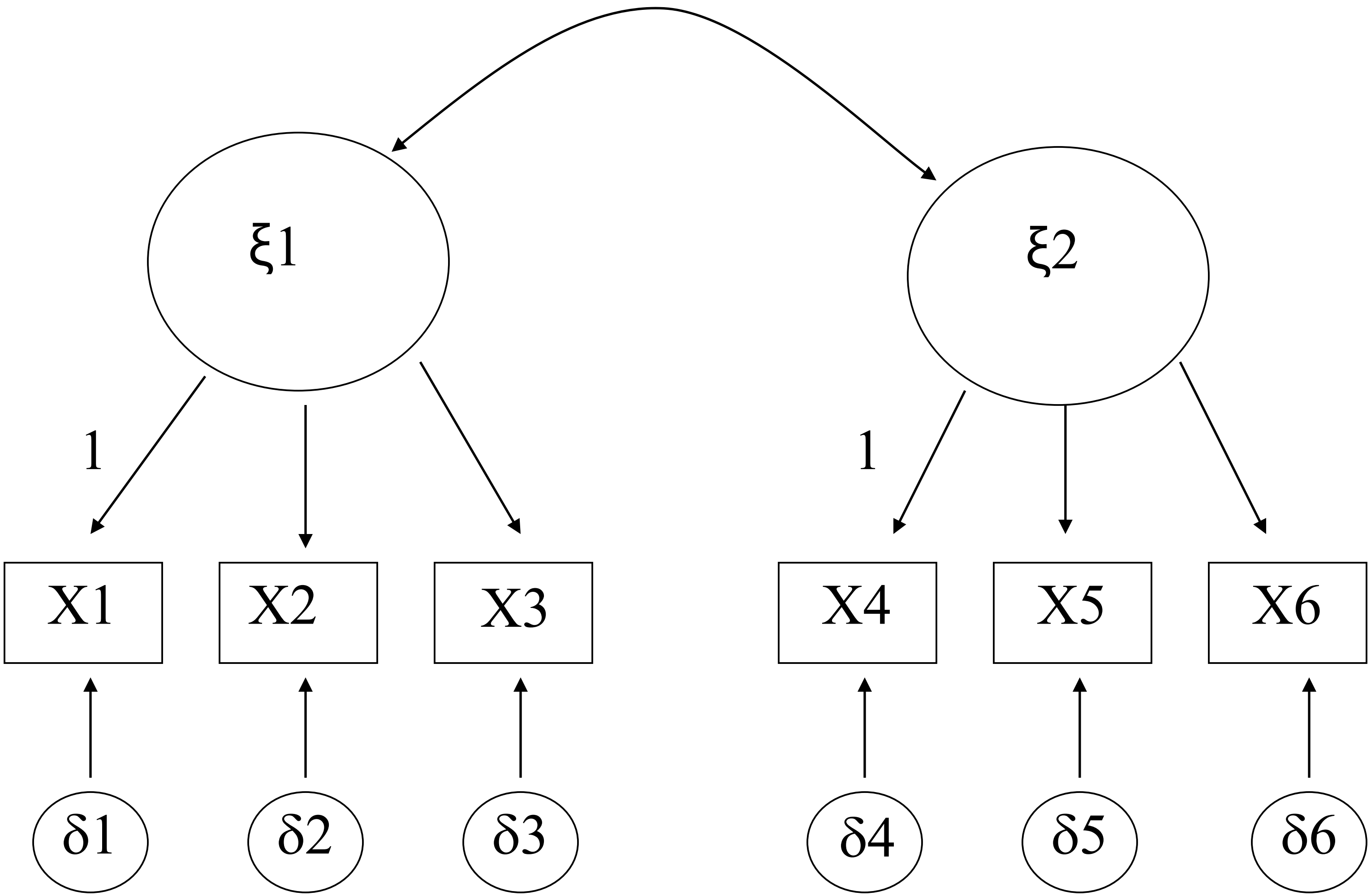
- The null hypothesis for specific parameters
  - The parameter = 0
  - $H_0 : \lambda_{11} = 0$
  - Where λ_{11} is the factor loading relating Factor 1 to Variable 1

  - Evaluated with a z-test
  - In short, significance is "good"

CFA



CFA



CFA

$x = \Lambda_x \xi + \delta$

$$
\begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \\ x6 \end{pmatrix}
=
\begin{pmatrix} 1 & 0 \\ \lambda_{21} & 0 \\ \lambda_{31} & 0 \\ 0 & 1 \\ 0 & \lambda_{52} \\ 0 & \lambda_{62} \end{pmatrix}
\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}
+
\begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{pmatrix}
$$

CFA

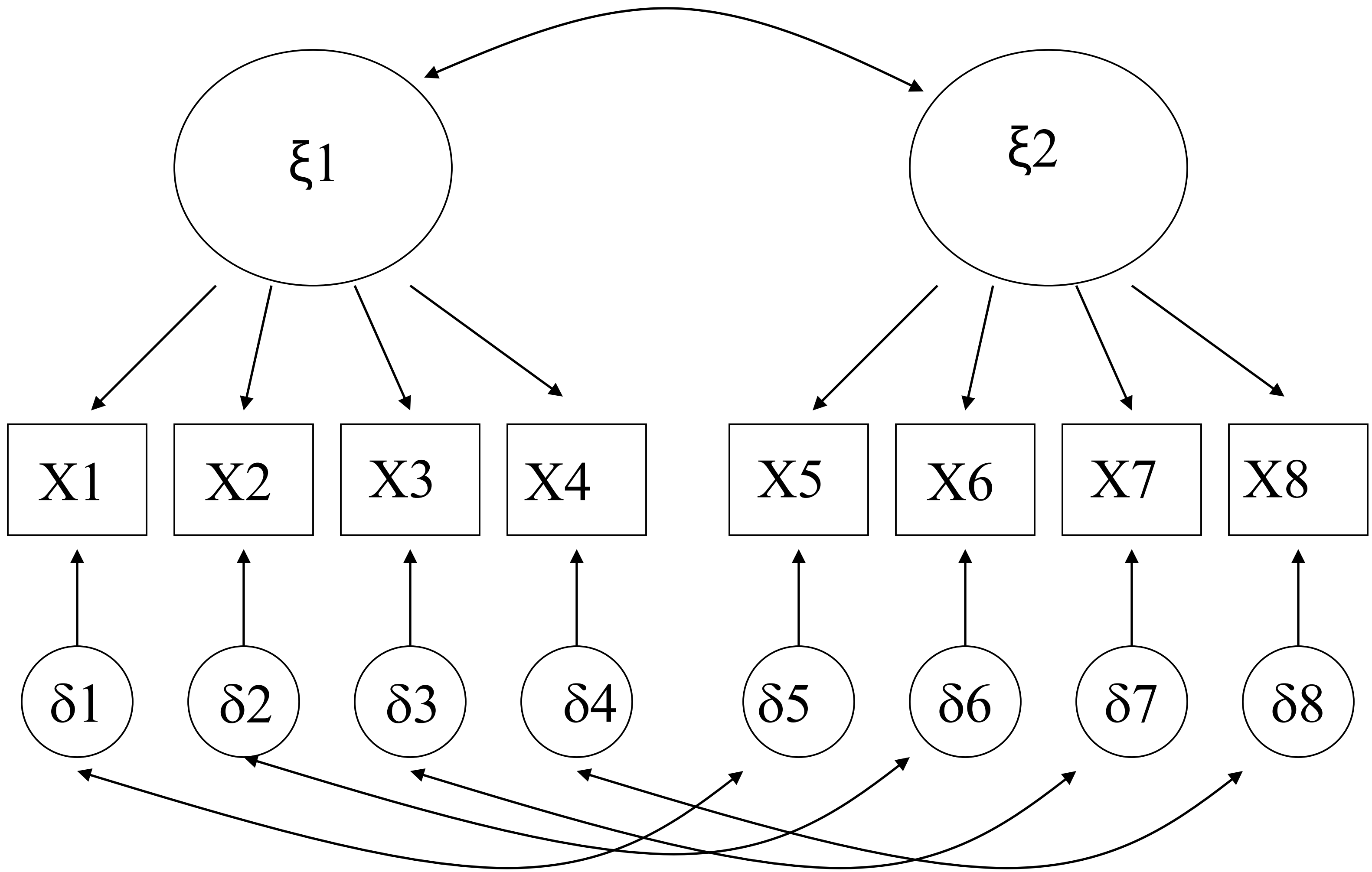
$\Sigma(\Theta) = E(xx') = [(\Lambda_x \xi + \delta)\ (\xi' \Lambda_x' + \delta')]$
$= \Lambda_x \phi \Lambda_x' + \Theta_\delta$

Θ_δ = covariance matrix of δ
ϕ = covariance matrix of ξ

Λ_x consists of 4 parameters (4 weights)
ϕ consists of 3 parameters (2 variances, 1 covariance)
Θ_δ consists of 12 parameters (6 variances, 0 covariances)

df = 21 - 13 = 8

CFA



CFA

$x = \Lambda_x \xi + \delta$

$$\begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \\ x6 \\ x7 \\ x8 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \lambda_{21} & 0 \\ \lambda_{31} & 0 \\ \lambda_{41} & 0 \\ 0 & 1 \\ 0 & \lambda_{62} \\ 0 & \lambda_{72} \\ 0 & \lambda_{82} \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \end{pmatrix}$$

CFA

$\Sigma(\Theta) = E(xx') = [(\Lambda_x\xi + \delta)(\xi'\Lambda_x' + \delta')]$

$= \Lambda_x\phi\Lambda_x' + \Theta_\delta$

Θ_δ = covariance matrix of δ

ϕ = covariance matrix of ξ

Λ_x consists of 6 parameters (6 weights)

ϕ consists of 3 parameters (2 variances, 1 covariance)

Θ_δ consists of 12 parameters (8 variances, 4 covariances)

df = 36 - 21 = 15

CFA Model Fit

- Model fit
  - Types of fit indices
    - Absolute fit indices
    - Incremental fit indices
    - Parsimony-adjusted indices
  - Chi-square/df ratio
    - See Kline p. 272
  - For more on the math of model fit:
    - http://www.davidakenny.net/cm/fit.htm

CFA Model Fit

- Model fit
  - Kline recommends:
    - Chi-square, df, p
    - SRMR (absolute)
    - CFI (incremental)
    - RMSEA (other)

Fit indices

- Fit indices consider the fit of the model relative to the
  - Saturated model
    - Perfectly reproduces the sample covariance matrix because all relations are specified (df = 0)
  - Independence model
    - Predicts no relations among variables

Absolute fit indices

- Evaluate the fit of the model relative to the saturated model
  - GFI
  - AGFI
    - GFI and AGFI = 1 for the saturated model
  - SRMR (standardized root mean square residual)
    - SRMR = 0 for the saturated model
    - The larger the value the worse the fit

Incremental (comparative) fit indices

- Evaluate the fit of the model relative to a simpler baseline model, typically the independence model
  - NFI
  - TLI
  - CFI

Other measures

- RMSEA
  - No penalty for model complexity
- AIC
  - Akaike Information Criterion
  - Intended for model comparisons
  - Considers fit and complexity

What is “good” model fit?

- Chi-square

  $p \geq .05$

  $(\chi^2 / df) \leq 3.00$
- Absolute fit indices

  GFI, AGFI $\geq .95$

  SRMR $\leq .08$
- Incremental fit indices

  CFI, TLI $\geq .95$
- Other measures

  RMSEA $\leq .06$ to $.08$

What is “good” model fit?

- Hu, L., & Bentler, P.M. Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Struct. Equ. Model. A Multidiscip. J.* **1999**, *6*, 1–55.

- Schreiber, J.B., Nora, A., Stage, F.K., Barlow, E.A., & King, J. Reporting structural equation modeling and confirmatory factor analysis results: A review. *J. Educ. Res.* **2006**, *99*, 323–338

Model modification

- If a path is non-significant then drop and re-run (aka Wald test)
- If a path is suggested (by modification indices) then add and re-run (aka Lagrange test)

Model comparison

- Model A: more constrained (more parsimonious)
- Model B: less constrained (more complex)

  – $\Delta\chi^2 = \chi^2{}_a - \chi^2{}_b$

  – $df = df_a – df_b$

  – If $\Delta\chi^2$ is significant then choose Model B
  – If $\Delta\chi^2$ is NOT significant then choose Model A