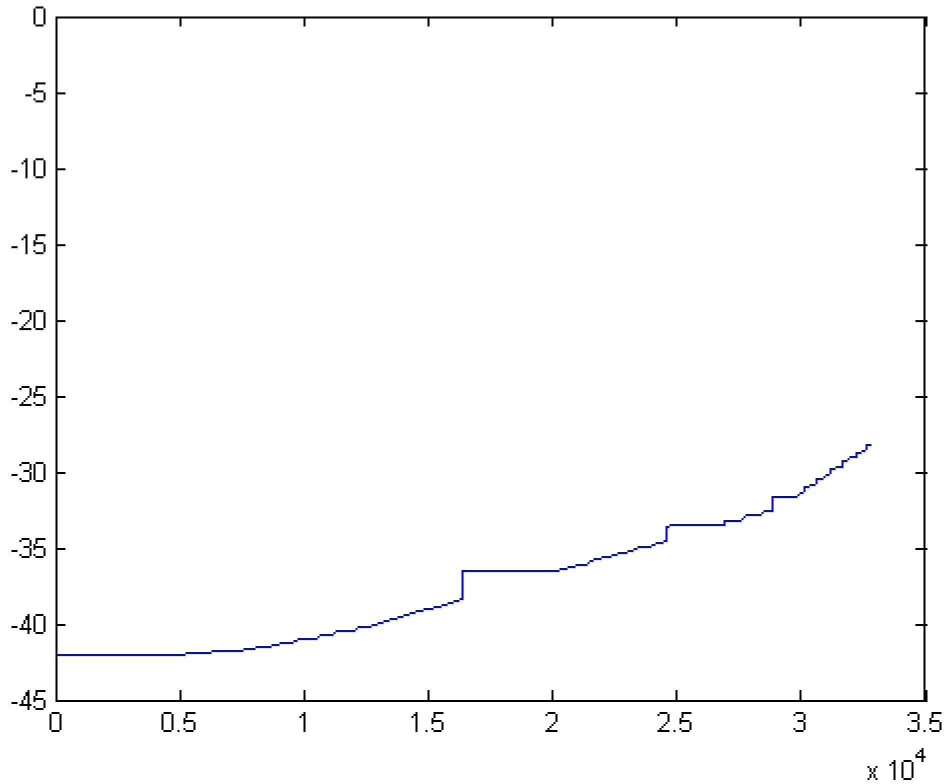
Exercise 3
Math foundation of
computer graphics and vision

Amrollah Seifoddini

Task1)

For this task I followed the algorithm of BnB described in lecture. The formulas are derived in task 2. The implementation is straightforward and can be run with main.m script. A function for linear programming computation and also two functions for convex and concave envelop calculations are implemented separately. For visualization I used a changed version of showMatchedFeatures2.m function.

Point matches



Task2)

For this task, I followed the guidelines of exercise sheet and lecture to formulate the problem as canonical linear programming with relaxed constraints. The steps to do that are depicted in this image.

Task 2

$$\max_{\theta, S_I} \ \text{card}(SI)$$
$$\text{s.t.} \quad |x_i + T_x - x_i'| \leq \delta \quad \forall i \in S_I \subseteq S$$
$$|y_i + T_y - y_i'| \leq \delta \quad \forall i \in S_I \subseteq S$$

$\Downarrow$

$$\max_{z, \theta} \sum_{i=1}^{N} z_i$$
$$\text{s.t.} \quad z_i\,|x_i + T_x - x_i'| \leq \delta z_i \quad \forall i = 1 \dots N$$
$$z_i\,|y_i + T_y - y_i'| \leq \delta z_i \quad \forall i = 1 \dots N$$
$$0 \leq z_i \leq 1 \quad \forall i = 1 \dots N$$

$\Downarrow$

$$\max_{z, \theta, w} \sum_{i=1}^{N} z_i$$
$$\text{s.t.} \quad |z_i x_i + w_{ix} - z_i x_i'| \leq z_i \delta \quad \forall i \dots N$$
$$|z_i y_i + w_{iy} - z_i y_i'| \leq z_i \delta \quad \forall i \dots N$$
$$0 \leq z_i \leq 1 \quad \forall i$$
$$\text{conv}(z_i, T_x) \leq w_{ix} \leq \text{conc}(z_i, T_x)$$
$$\text{conv}(z_i, T_y) \leq w_{iy} \leq \text{conc}(z_i, T_y)$$
$$\underline{\theta} \leq \theta \leq \bar{\theta}$$

$w_{ix} = z_i T_x$

$w_{iy} = z_i T_y$

$\Downarrow$

$$\max_{z, \theta, w} \sum_{i=1}^{N} z_i$$
$$\text{s.t.} \quad z_i(x_i - x_i' - \delta) + w_{ix} \leq 0$$
$$z_i(-x_i + x_i' - \delta) - w_{ix} \leq 0$$
$$\forall i \quad z_i(y_i - y_i' - \delta) + w_{iy} \leq 0$$
$$z_i(-y_i + y_i' - \delta) - w_{iy} \leq 0$$
$$0 \leq z_i \leq 1$$
$$\text{conv}(z_i, T_x) \leq w_{ix} \leq \text{conc}(z_i, T_x)$$
$$\text{conv}(z_i, T_y) \leq w_{iy} \leq \text{conc}(z_i, T_y)$$
$$\underline{\theta} \leq \theta \leq \bar{\theta}$$

$\Rightarrow$

$\boxed{\text{canonical form}}$

$$x = [\theta, z, w]^T$$
$$\min_x c^T x$$
$$c = [\text{zero}_{1\times 2}, -\mathbb{1}_{1\times N}, \text{Zero}_{1\times 2N}]^T$$
$$b = \text{zero}_{4\times 1}$$
$$Lb = [\underline{\theta}, \text{zero}_{1\times N}, \text{conv}(z, T_x), \text{conv}(z, T_y)]$$
$$Ub = [\bar{\theta}, \mathbb{1}_{1\times N}, \text{conc}(z, T_x), \text{conc}(z, T_y)]$$
$$A = \begin{bmatrix} 0, 0, (p_x - p_x' - \delta)_{1\times N}, \mathbb{1}_{1\times N}, \text{Zero}_{1\times N} \\ 0, 0, (-p_x + p_x' - \delta)_{1\times N}, -\mathbb{1}_{1\times N}, \text{Zero}_{1\times N} \\ 0, 0, (p_y - p_y' - \delta), \text{zero}_{1\times N}, \mathbb{1}_{1\times N} \\ 0, 0, (-p_y + p_y' - \delta), \text{Zero}_{1\times N}, -\mathbb{1}_{1\times N} \end{bmatrix}$$