Torque Minimization for Redundant Manipulators

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Robotics II Course Project
Project #6 – A.A. 2018/19

January 22, 2020

Outline

Abstract

Introduction

Dynamic Resolution
- Minimum Torque Norm
- Peak Torque Reduction
- Stable Torque Minimization Using Short Preview

Simulations

Conclusion

We study torque minimization as a criterion for resolving kinematic redundancy in manipulators. The minimization is studied, locally, at different time instants (current and/or future) in order to address the torque instability problem.

Preliminaries:

- Solving inverse kinematics for redundant robots yields infinite solutions
- Secondary objective: minimum joint torques
- Take into account manipulator dynamics

Different methods for torque optimization

- Hollerbach – Instantaneous torque minimization
  - Inclusion of torque bounds
- Li – Use current torque to minimize torque at next instant
- Al Khudir – Minimize torque over two successive instants
  - Inclusion of damping in the null space

Dynamic Model:

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q})$$

$$\boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q})$$

Velocity relationship:

$$\dot{\boldsymbol{x}} = \boldsymbol{J}\dot{\boldsymbol{q}}$$

Acceleration relationship:

$$\ddot{\boldsymbol{x}} = \boldsymbol{J}\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}\dot{\boldsymbol{q}} = \boldsymbol{J}\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}})$$

Hollerbach minimizes the instantaneous motor torques by resolving the redundancy at the acceleration level.

$$\begin{aligned}
\min_{\ddot{q}_k} \quad & H_1 = \frac{1}{2}\|\boldsymbol{\tau}_k\|^2 \\
\text{s.t.} \quad & \boldsymbol{\tau}_k = \boldsymbol{M}_k \ddot{\boldsymbol{q}}_k + \boldsymbol{n}_k, \\
& \ddot{\boldsymbol{x}}_k = \boldsymbol{J}_k \ddot{\boldsymbol{q}}_k + \boldsymbol{h}_k
\end{aligned}$$

MTN – with bounds (MTNB)

Extension:
Consider the upper and lower joint torque limits are $\boldsymbol{\tau}^+$ and $\boldsymbol{\tau}^-$, respectively. The goal is to place the joint torques closest to the midpoints of the limits $\frac{1}{2}(\boldsymbol{\tau}^+ + \boldsymbol{\tau}^-)$.

$$
\begin{aligned}
\min_{\ddot{q}_k} \quad & H_2 = \frac{1}{2}\left(\boldsymbol{\tau}_k - \frac{\boldsymbol{\tau}^+ + \boldsymbol{\tau}^-}{2}\right)^T \boldsymbol{W} \left(\boldsymbol{\tau}_k - \frac{\boldsymbol{\tau}^+ + \boldsymbol{\tau}^-}{2}\right) \\
\text{s.t.} \quad & \boldsymbol{\tau}_k = \boldsymbol{M}_k \ddot{\boldsymbol{q}}_k + \boldsymbol{n}_k, \\
& \ddot{\boldsymbol{x}}_k = \boldsymbol{J}_k \ddot{\boldsymbol{q}}_k + \boldsymbol{h}_k
\end{aligned}
$$

Consider the interval $t_k \leq t \leq t_{k+1}$, where t_k, t_{k+1} are two immediate time instants. The goal is to use the future state $k+1$ as an objective and use current torque $\boldsymbol{\tau}_k$ to accelerate/decelerate the joint velocity from $\dot{\boldsymbol{q}}_k$ to $\dot{\boldsymbol{q}}_{k+1}^{opt}$ (thus, minimizing $\boldsymbol{\tau}_{k+1}$), while keeping $\boldsymbol{\tau}_k$ within limits.

$$
\begin{aligned}
\min_{\dot{q}} \quad & H_3^{'} = \frac{1}{2}\left(\boldsymbol{\tau}_{k+1}(\boldsymbol{q}_{k+1}, \dot{\boldsymbol{q}}_{k+1}) - \frac{\boldsymbol{\tau}^{+} + \boldsymbol{\tau}^{-}}{2}\right)^T \\
& \quad \times \boldsymbol{W}\left(\boldsymbol{\tau}_{k+1}(\boldsymbol{q}_{k+1}, \dot{\boldsymbol{q}}_{k+1}) - \frac{\boldsymbol{\tau}^{+} + \boldsymbol{\tau}^{-}}{2}\right) \\
\text{s.t.} \quad & \dot{\boldsymbol{x}}_{k+1} = \boldsymbol{J}(\boldsymbol{q}_{k+1})\dot{\boldsymbol{q}}_{k+1}, \\
& \boldsymbol{\tau}_k^{-} \leq \boldsymbol{\tau}_k^{i} \leq \boldsymbol{\tau}_k^{+}
\end{aligned}
$$

Al Khudir uses two successive discrete-time samples to minimize the joint torque norm.

$$
\begin{aligned}
\min_{\ddot{q}_k, \ddot{q}_{k+1}} \quad & H_4 = \frac{1}{2}\left(\omega_k \|\boldsymbol{\tau}_k\|^2 + \omega_{k+1}\|\boldsymbol{\tau}_{k+1}\|^2\right) \\
\text{s.t.} \quad & \boldsymbol{\tau}_k = \boldsymbol{M}_k \ddot{\boldsymbol{q}}_k + \boldsymbol{n}_k, \\
& \ddot{\boldsymbol{x}}_k = \boldsymbol{J}_k \ddot{\boldsymbol{q}}_k + \boldsymbol{h}_k, \\
& \boldsymbol{\tau}_{k+1} = \boldsymbol{M}_{k+1} \ddot{\boldsymbol{q}}_{k+1} + \boldsymbol{n}_{k+1}, \\
& \ddot{\boldsymbol{x}}_{k+1} = \boldsymbol{J}_{k+1} \ddot{\boldsymbol{q}}_{k+1} + \boldsymbol{h}_{k+1}
\end{aligned}
$$

But the problem loses the original Linear-Quadratic formulation...

By applying dynamic approximations to the future state (namely, $\ddot{\boldsymbol{x}}_{k+1}$ and $\boldsymbol{\tau}_{k+1}$), we can obtain a linear dependence of the constraints and a quadratic dependence of the objective on $\ddot{\boldsymbol{q}}_k$. Thus...

$$
\begin{aligned}
\min_{\ddot{q}_k, \ddot{q}_{k+1}} \quad & H_4 = \frac{1}{2}\left(\omega_k \|\boldsymbol{\tau}_k\|^2 + \omega_{k+1}\|\boldsymbol{\tau}_{k+1}\|^2\right) \\
\text{s.t.} \quad & \boldsymbol{\tau}_k = \boldsymbol{M}_k \ddot{\boldsymbol{q}}_k + \boldsymbol{n}_k, \\
& \ddot{\boldsymbol{x}}_k = \boldsymbol{J}_k \ddot{\boldsymbol{q}}_k + \boldsymbol{h}_k, \\
& \boldsymbol{\tau}_{k+1} = \boldsymbol{M}_{k^+} \ddot{\boldsymbol{q}}_{k+1} + \boldsymbol{S}_{k^+}(\dot{\boldsymbol{q}}_k + \ddot{\boldsymbol{q}}_k T) + \boldsymbol{g}_{k^+}, \\
& \ddot{\boldsymbol{x}}_{k+1} = \boldsymbol{J}_{k^+} \ddot{\boldsymbol{q}}_{k+1} + (\boldsymbol{J}_{k^+} - \boldsymbol{J}_k)\ddot{\boldsymbol{q}}_k + \boldsymbol{h}_{k^+}
\end{aligned}
$$

Dynamic Damping in the Null Space

Minimize the difference in norm with respect to a suitable desired target torque. Define

$$\boldsymbol{\tau}_{D_k} = -\boldsymbol{D}_k \boldsymbol{M}_k \dot{\boldsymbol{q}}_k$$

When $\boldsymbol{\tau}_k = \boldsymbol{\tau}_{D_k}$ and the damping matrix is in the form $\boldsymbol{D}_k = d\boldsymbol{I} > 0$, the joint acceleration $\ddot{\boldsymbol{q}}_k$ becomes

$$\ddot{\boldsymbol{q}}_k = -\boldsymbol{M}_k^{-1}(\boldsymbol{S}_k \dot{\boldsymbol{q}}_k + \boldsymbol{g}_k) - d\dot{\boldsymbol{q}}_k$$

Effect of $\boldsymbol{\tau}_{D_k}$ is opposite to the current joint velocity, thus acting as a damper on the joint motion.

MTN – with damping (MTND)

$$\begin{aligned} H_1^{'} &= \frac{1}{2}\left\|\boldsymbol{\tau}_k - \boldsymbol{\tau}_{D_k}\right\|^2 \\ &= \frac{1}{2}\left\|\boldsymbol{M}_k\ddot{\boldsymbol{q}}_k + (\boldsymbol{S}_k + \boldsymbol{D}_k\boldsymbol{M}_k)\dot{\boldsymbol{q}}_k + \boldsymbol{g}_k\right\|^2 \end{aligned}$$

MBP – with damping (MBPD)

$$H_4^{'} = \frac{1}{2}\left(\omega_k\left\|\boldsymbol{\tau}_k - \boldsymbol{\tau}_{D_k}\right\|^2 + \omega_{k+1}\left\|\boldsymbol{\tau}_{k+1} - \boldsymbol{\tau}_{D_{k+}}\right\|^2\right)$$

To obtain new solutions replace $\boldsymbol{S}_k$ with $(\boldsymbol{S}_k + \boldsymbol{D}_k\boldsymbol{M}_k)$

3R planar manipulator on horizontal plane

- 3 uniform links, each of length $l = 1$ [m], mass $m_l = 10$ [kg], and moment of inertia $I_l = m_l l^2/12$
- upper and lower torque limits for joints 1-3 are $\pm 54, \pm 24\, and\, \pm 6$ [N.m]

Other parameters:

- Integration step $T_s = 0.001$ [s]
- Cartesian task error gains $\boldsymbol{K}_P = 10\boldsymbol{I}$ and $\boldsymbol{K}_D = \boldsymbol{I}$
- Damping matrix $\boldsymbol{D}_k = 10\boldsymbol{I}$

The simulated end-effector motion is a rest-to-rest straight-line Cartesian trajectory, with bang-bang acceleration profile.

1. Short path: path length $\|L_1\| = 0.2828$ [m] and acceleration bound $\|A_1\| = 2.8284$ $[m/s^2]$
2. Long path: path length $\|L_2\| = 1.1738$ [m] and acceleration bound $\|A_2\| = 1.1412$ $[m/s^2]$

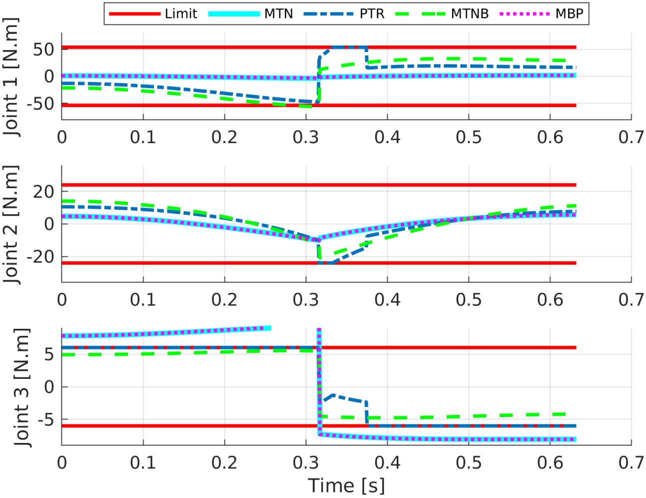

Figure: Torque profiles

Simulations – Short path

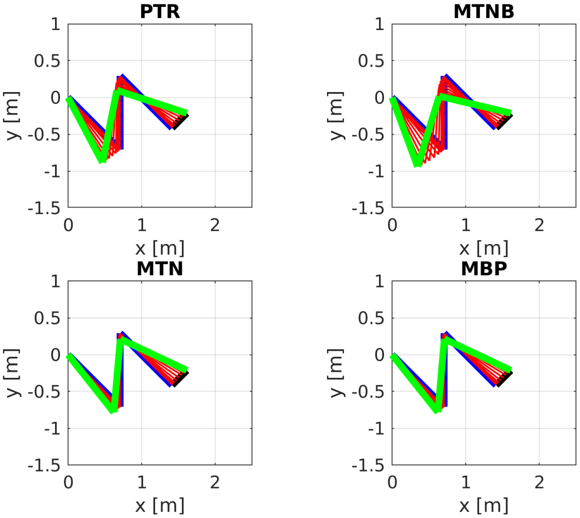

Figure: Continual arm motions (initial in blue, final in green)

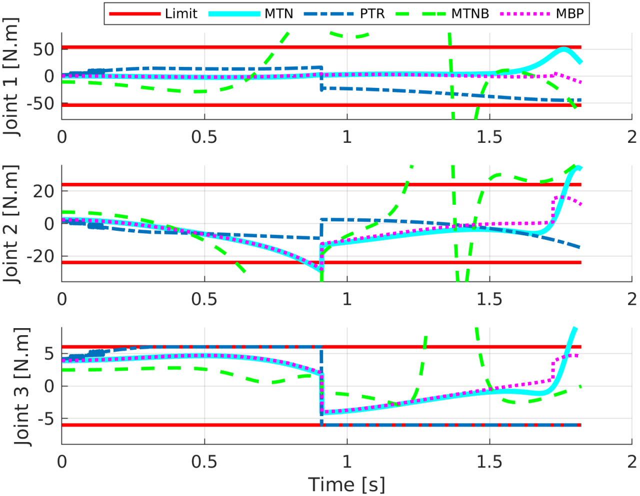

Figure: Torque profiles

Simulations – Long path

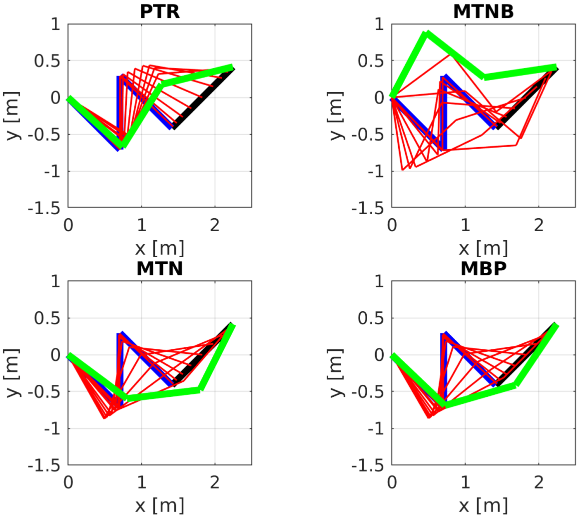

Figure: Continual arm motions (initial in blue, final in green)

Simulations – Damping

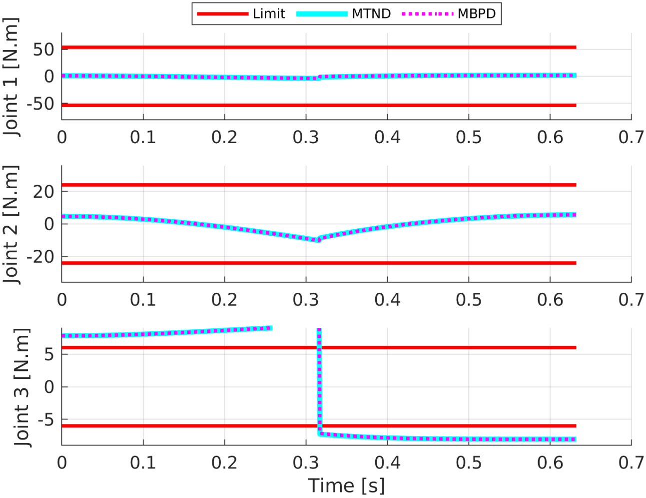

Figure: Torque profiles for short path

Simulations – Damping

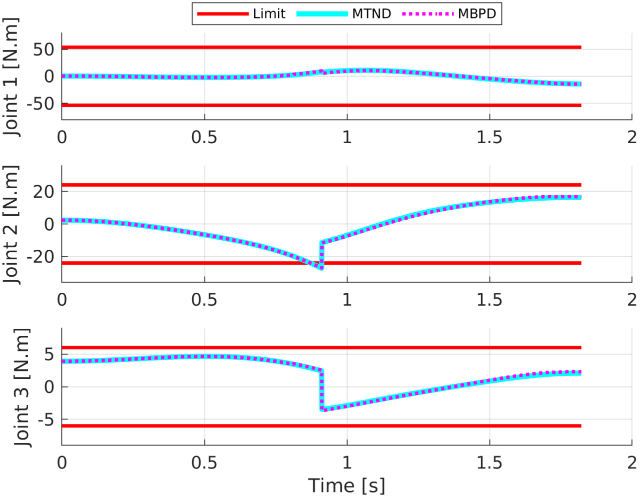

Figure: Torque profiles for long path

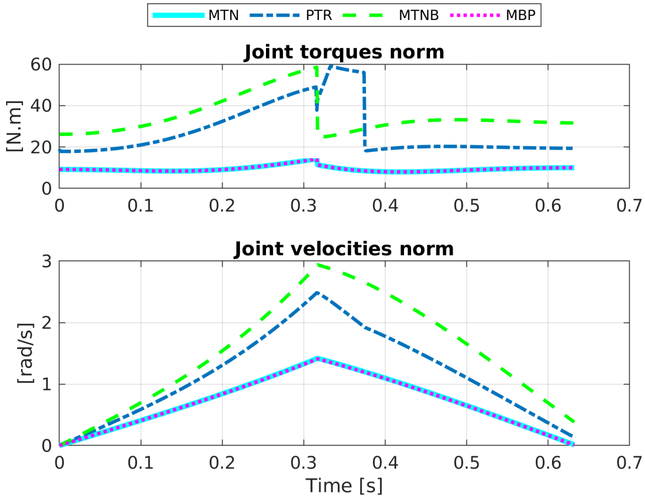

Figure: Joint torque and velocity norms for short path

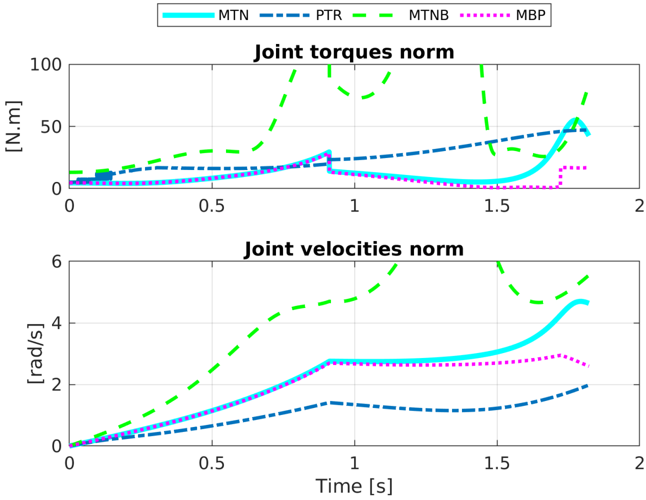

Figure: Joint torque and velocity norms for long path

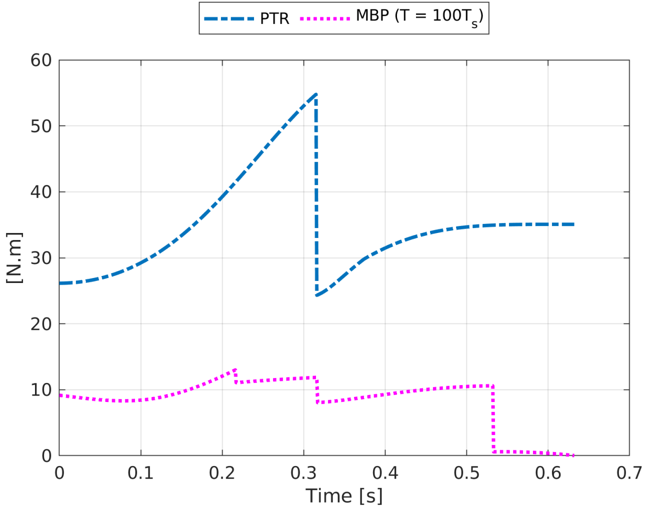

Figure: Future torque estimated by each method (short path)

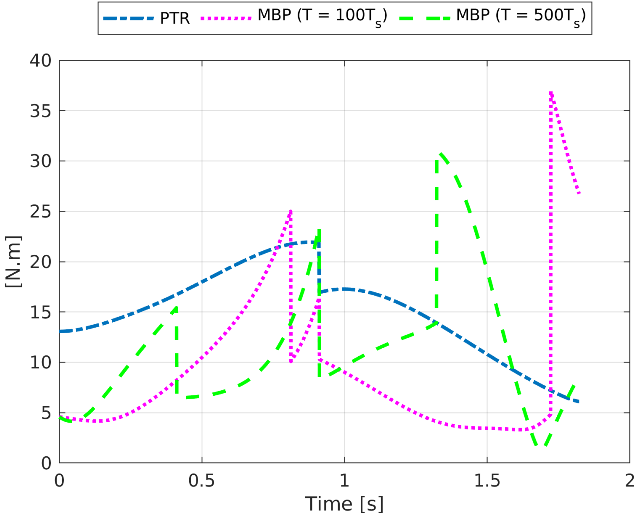

Figure: Future torque estimated by each method (long path)

- PTR can't be used for real-time control
- PTR always respects torque bounds, while MTNB has unstable behaviour for longer paths
- PTR has smoother future torque prediction, while MBP has lower norm
- No method guarantees stability and respecting the bounds at the same time.