

1) Inserting  $n$  elements using

a) aggregate method

The table doubles in size when it runs out of space. So if the original size is 1, after insertion it doubles the size to 2 after 2 more insertions it doubles to size 4 etc.

In general after  $k$  doublings the size is  $2^k$

Pseudo code:-

initialize table with capacity = 1

for  $i = 1$  to  $n$

if table is full:

newtable = create newtable with size  $2 \times$  current size.

copy elements then from old table to new table

table = newtable

insert element  $i$  into table

let  $k = \log(n+1) - 1$

Total cost =  $O(n)k$

=  $O(n \log n)$

cost per insertion =  $O(\log n)$

Runtime per insertion is  $O(\log n)$

Total time is  $O(n) * \log(n+1)$

in this method, cost includes both actual & potential cost.

we will use the potential function

$$\phi(T) = 2 * T : num$$

Initially the table has size  $(T : size = 1)$

$$\Rightarrow \text{Potential in } \phi(T) = 2 * T : num = 0$$

For inserting 1 element

$$\text{Actual cost} = 1$$

$$\text{Potential cost} = 2$$

$$\text{Amortized cost} = 1 + 2 = 3$$

The total amortized cost for inserting  $n$  elements is  $n * 3$ , & the amortized cost per insertion is

$$n * 3 / n = 3 \Rightarrow O(3)$$

The amortized runtime for inserting 'n' element  
=  $O(1)$