Question 11.16.3.35

The probability of an occurrence of event A is .7 and that of the occurrence of event B is .3 and the probability of occurrence of both is .4.Is this statement true or false?

Solution:

Consider,

$$\Pr(A/B) = \frac{\Pr(AB)}{\Pr(B)} \tag{1}$$

Now, we know that

$$0 \le \frac{\Pr(AB)}{\Pr(B)} \le 1 \tag{2}$$

Since, probabilities are always between 0 and 1.
Multiply both sides by P(B):

$$0 \le \Pr(AB) \le \Pr(B) \tag{3}$$

Similarly, we can prove that

$$\Pr(AB) \le \Pr(A) \tag{4}$$

by using the definition of conditional probability.
Therefore, we have:

$$\Pr(AB) \le \Pr(A) \text{ and } \Pr(AB) \le \Pr(B) \tag{5}$$

$$\therefore \Pr(AB) \le \Pr(A) \times \Pr(B) \tag{6}$$

$$\implies \Pr(AB) \le 0.7 \times 0.3 \tag{7}$$

$$\implies \Pr(AB) \le 0.21 \tag{8}$$

But given that $\Pr(AB) = 0.4 > 0.21$
$\therefore$ The given statement is false.