

Question 11.16.3.35

The probability of an occurrence of event A is .7 and that of the occurrence of event B is .3 and the probability of occurrence of both is .4. Is this statement true or false?

Solution:

Given,

$$\Pr(A) = 0.7 \quad (1)$$

$$\Pr(B) = 0.3 \quad (2)$$

$$\Pr(AB) = 0.4 \quad (3)$$

Consider,

$$\Pr(A/B) = \frac{\Pr(AB)}{\Pr(B)} \quad (4)$$

Now, we know that

$$0 \leq \frac{\Pr(AB)}{\Pr(B)} \leq 1 \quad (5)$$

Since, probabilities are always between 0 and 1.

Multiply both sides by $\Pr(B)$:

$$0 \leq \Pr(AB) \leq \Pr(B) \quad (6)$$

Similarly, we can prove that

$$\Pr(AB) \leq \Pr(A) \quad (7)$$

by using the definition of conditional probability.

Therefore, we have:

$$\Pr(AB) \leq \Pr(A) \text{ and } \Pr(AB) \leq \Pr(B) \quad (8)$$

$$\therefore \Pr(AB) \leq \Pr(A) \times \Pr(B) \quad (9)$$

$$\implies \Pr(AB) \leq 0.7 \times 0.3 \quad (10)$$

$$\implies \Pr(AB) \leq 0.21 \quad (11)$$

But given that $\Pr(AB) = 0.4 > 0.21$

\therefore The given statement is false.