Draw the circle with centre at **O** and radius
R = OA
This is known as the circumradius
Solution:
Given,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{3}$$

Let AD, BE, CF are altitudes of triangle from vertices **A**, **B**, **C** respectively.
Consider;

$$SlopeofBC = \frac{-5-6}{-3+4}m1 \qquad = -11 \tag{4}$$

Therefore, Slope of AD = $\frac{1}{11}$
Consider;

$$SlopeofAC = \frac{-5+1}{-3-1}m2 \qquad = 1 \tag{5}$$

Therefore, Slope of BE = -1
We know that,
Equation of line passing through a point **P** and having slope m is

$$\mathbf{r} = \mathbf{P} + t\mathbf{v} \tag{6}$$

where; **P** is given point
r is required vector
t is a scalar
v is $\begin{pmatrix} 1 \\ m \end{pmatrix}$
For equation of AD,

$$AD = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ \frac{1}{11} \end{pmatrix} \tag{7}$$

$$AD = \begin{pmatrix} 1+t \\ \left(-1+\frac{t}{11}\right) \end{pmatrix} \tag{8}$$

For equation of BE,

$$BE = \begin{pmatrix} -4 \\ 6 \end{pmatrix} + s\begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{9}$$

$$BE = \begin{pmatrix} -4+s \\ 6-s \end{pmatrix} \tag{10}$$

To get point of intersection of AD and BE, Equate above two equations;

$$\begin{pmatrix} (1+t) \\ \left(-1+\frac{t}{11}\right) \end{pmatrix} = \begin{pmatrix} -4+s \\ 6-s \end{pmatrix} \tag{11}$$

Now, compare like terms;
Consider,

$$1+t = 4+s \tag{12}$$

$$t = -5+s \tag{13}$$

Now, consider

$$-1+\frac{t}{11} = 6-s \tag{14}$$

$$-11-5+s = 66-11s \tag{15}$$

$$s = \frac{41}{6} \tag{16}$$

Therefore;

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4+s \\ 6-s \end{pmatrix} \tag{17}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \left(-4+\frac{41}{6}\right) \\ \left(6-\frac{41}{6}\right) \end{pmatrix} \tag{18}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{17}{6} \\ \frac{5}{6} \end{pmatrix} \tag{19}$$

Here, point of intersection $\begin{pmatrix} x \\ y \end{pmatrix}$ is Orthocentre(**O**)
Therefore,

$$\mathbf{O} = \begin{pmatrix} \frac{17}{6} \\ \frac{5}{6} \end{pmatrix} \tag{20}$$

Radius of circle with centre **O** is OA

$$OA = \sqrt{\left(\frac{17}{6}-1\right)^2 + \left(\frac{5}{6}+1\right)^2} \tag{21}$$

$$OA = \sqrt{(\frac{11}{6})^2 + (\frac{11}{6})^2} \tag{22}$$

$$OA = \frac{11\sqrt{22}}{6} \tag{23}$$

Therefore;
Radius of circle $= \frac{11\sqrt{22}}{6}$