

Draw the circle with centre at **O** and radius

$R = OA$

This is known as the circumradius

Solution:

Given,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (1)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (2)$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (3)$$

Let AD, BE, CF are altitudes of triangle from vertices **A**, **B**, **C** respectively.

Consider;

$$\text{Slope of } BC = \frac{-5 - 6}{-3 - 4} m_1 = -11 \quad (4)$$

Therefore, Slope of AD = $\frac{1}{11}$

Consider;

$$\text{Slope of } AC = \frac{-5 + 1}{-3 - 1} m_2 = 1 \quad (5)$$

Therefore, Slope of BE = -1

We know that,

Equation of line passing through a point **P** and having slope m is

$$\mathbf{r} = \mathbf{P} + t\mathbf{v} \quad (6)$$

where; **P** is given point

r is required vector

t is a scalar

v is $\begin{pmatrix} 1 \\ m \end{pmatrix}$

For equation of AD,

$$AD = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + t \begin{pmatrix} 1 \\ \frac{1}{11} \end{pmatrix} \quad (7)$$

$$AD = \begin{pmatrix} 1 + t \\ -1 + \frac{t}{11} \end{pmatrix} \quad (8)$$

For equation of BE,

$$BE = \begin{pmatrix} -4 \\ 6 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (9)$$

$$BE = \begin{pmatrix} -4 + s \\ 6 - s \end{pmatrix} \quad (10)$$

To get point of intersection of AD and BE, Equate above two equations;

$$\begin{pmatrix} 1+t \\ -1+\frac{t}{11} \end{pmatrix} = \begin{pmatrix} -4+s \\ 6-s \end{pmatrix} \quad (11)$$

Now, compare like terms;

Consider,

$$1+t = 4+s \quad (12)$$

$$t = -5+s \quad (13)$$

Now, consider

$$-1+\frac{t}{11} = 6-s \quad (14)$$

$$-11-5+s = 66-11s \quad (15)$$

$$s = \frac{41}{6} \quad (16)$$

Therefore;

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4+s \\ 6-s \end{pmatrix} \quad (17)$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4+\frac{41}{6} \\ 6-\frac{41}{6} \end{pmatrix} \quad (18)$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{17}{6} \\ \frac{5}{6} \end{pmatrix} \quad (19)$$

Here, point of intersection $\begin{pmatrix} x \\ y \end{pmatrix}$ is Orthocentre(**O**)

Therefore,

$$\mathbf{O} = \begin{pmatrix} \frac{17}{6} \\ \frac{5}{6} \end{pmatrix} \quad (20)$$

Radius of circle with centre **O** is OA

$$OA = \sqrt{\left(\frac{17}{6}-1\right)^2 + \left(\frac{5}{6}+1\right)^2} \quad (21)$$

$$OA = \sqrt{\left(\frac{11}{6}\right)^2 + \left(\frac{11}{6}\right)^2} \quad (22)$$

$$OA = \frac{11\sqrt{22}}{6} \quad (23)$$

Therefore;

Radius of circle = $\frac{11\sqrt{22}}{6}$