Draw the circle with centre at O and radius

R = OA

This is known as the circumradius

Solution:

Given,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} -4\\6 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{3}$$

Let AD, BE, CF are altitudes of triangle from vertices A, B, C respectively. Equation of AD,

$$AD = \begin{pmatrix} 1+t\\ -1+\frac{t}{11} \end{pmatrix} \tag{4}$$

Equation of BE,

$$BE = \begin{pmatrix} -4 + s \\ 6 - s \end{pmatrix} \tag{5}$$

The point of intersection of AD and BE is O; Therefore,

$$\mathbf{O} = \begin{pmatrix} \frac{17}{6} \\ \frac{5}{6} \end{pmatrix} \tag{6}$$

Radius of circle with centre O

$$R = OA = \frac{11\sqrt{22}}{6} \tag{7}$$

Therefore;

Equation of circle is

$$\left(x - \frac{17}{6}\right)^2 + \left(y - \frac{5}{6}\right)^2 = \left(\frac{11\sqrt{22}}{6}\right)^2 \tag{8}$$