Question

A box has 100 pens of which 10 are defective . What is the probability that out of a sample of 5 pens drawn one by one with replacement at most is defective?

(A)$\left(\frac{9}{10}\right)^5$

(B)$\frac{1}{2}\left(\frac{9}{5}\right)^4$

(C)$\frac{1}{2}\left(\frac{9}{10}\right)^5$

(D)$\frac{1}{2}\left(\frac{9}{5}\right)^4 + \left(\frac{9}{10}\right)^5$

Solution:

The gaussian distribution function is defined as:

Parameter	Values	Description
n	10	Number of defective pens
p	0.1	probability of drawing a defective pen
μ	1	np
σ	0.948	$\sqrt{np(1-p)}$
X		Defective pens

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad (x \in Y) \tag{1}$$

Probability that out of a sample of 5 pens drawn one by one with replacement at most is defective is

$$Y = 5 \tag{2}$$

$$p_Y(2) = \frac{1}{\sqrt{2\pi(0.9)}} e^{-\frac{(5-1)^2}{2(0.9)}} \tag{3}$$

$$= \frac{1}{\sqrt{2\pi(0.9)}} e^{-8.88} \tag{4}$$

$$= 5.85 \times 10^{-5} \tag{5}$$

Q function

Solving using Q function is defined

$$Q(x) = \int_x^\infty f(x)\,dx \tag{6}$$

then CDF of Y is:

$$\Pr(Y < x) = \int_{-\infty}^x f(x)\,dx \tag{7}$$

$$= 1 - \int_x^\infty f(x)\,dx \tag{8}$$

$$= 1 - Q(x) \tag{9}$$

and for finding $\Pr\left(Z = \frac{X-\mu}{\sigma}\right)$ Using approximation,

$$\Pr\left(Z = \frac{Y-\mu}{\sigma}\right) \approx \Pr\left(\frac{Y+0.5-\mu}{\sigma} < Z < \frac{Y-0.5-\mu}{\sigma}\right) \tag{10}$$

$$\approx \Pr\left(Z < \frac{Y+0.5-\mu}{\sigma}\right) - \Pr\left(Z < \frac{Y-0.5-\mu}{\sigma}\right) \tag{11}$$

$$\approx Q\left(\frac{Y-0.5-\mu}{\sigma}\right) - Q\left(\frac{Y+0.5-\mu}{\sigma}\right) \tag{12}$$

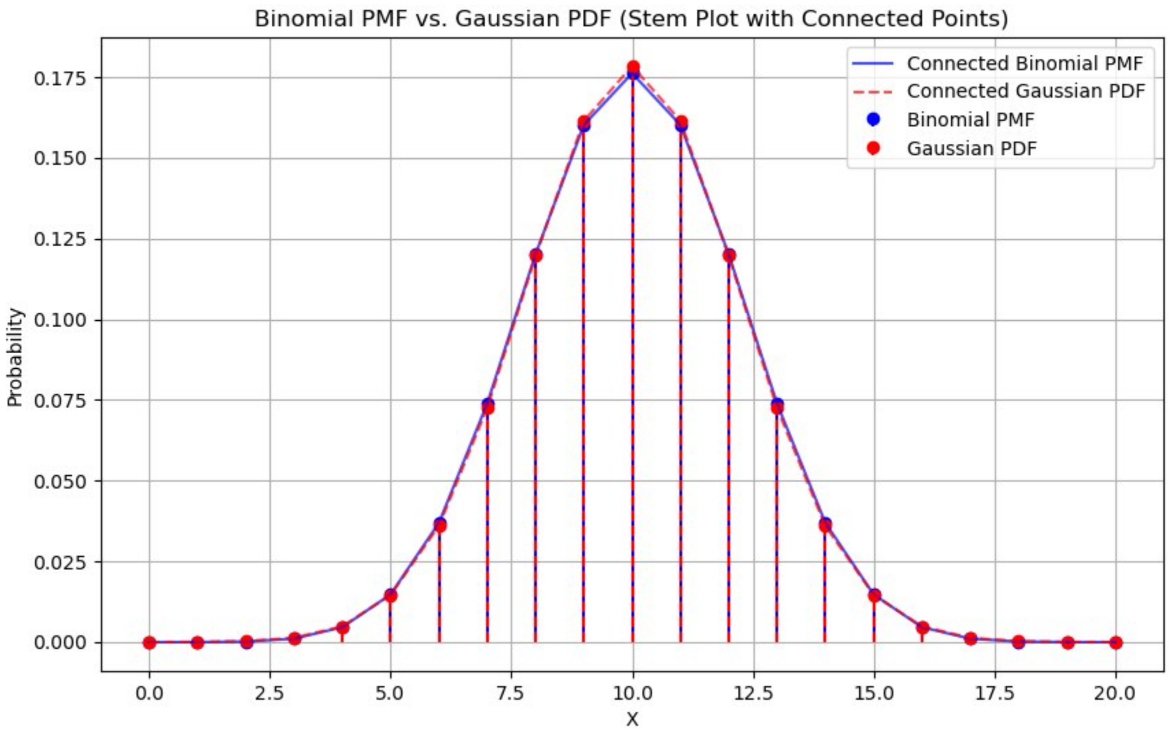

Fig. 0. pmf of binomial and pdf of Gaussian of X and Y marked balls

$$Y = 5 \tag{13}$$

$$\Pr(Z = 4.219) \approx Q(3.69) - Q(4.74) \tag{14}$$

$$\approx 0.000112127 - 1.06859 \times 10^{-}6 \tag{15}$$

$$\approx 1.11 \times 10^{-4} \tag{16}$$