Question: A student says that if you throw a die, it will show up 1 or not 1. Therefore, the probability of getting 1 and the probability of getting 'not 1' each is equal to $\frac{1}{2}$. Is this correct? Give reasons.

Solution:

Given,

A die is thrown

Total number of outcomes = 6

Hence, Probability of getting any number be $p_X(X = i) = \frac{1}{6}$

Then, by using pmf

$$p_X(2 \le X \le 6) = \sum_{i=2}^6 p_X(X=i) \tag{1}$$

$$p_X(X=1) = 1 - p_X(2 \le X \le 6) \tag{2}$$

$$=1-\sum_{i=2}^{6}p_{X}(X=i) \tag{3}$$

$$= 1 - p_X(X = 2) + p_X(X = 3) + p_X(X = 4) + p_X(X = 5) + p_X(X = 6)$$
 (4)

$$=1-\frac{5}{6}$$
 (5)

$$=\frac{1}{6}\tag{6}$$

$$\implies \Pr(2 \le X \le 6) \ne \Pr(X = 1) 7 \tag{7}$$

Since, Pr(X = 1) and $Pr(2 \le X \le 6)$ are not equal.

... The given statement is not true