Question

A box has 100 pens of which 10 are defective . What is the probability that out of a sample of 5 pens drawn one by one with replacement at most one is defective?

(A)$\left(\frac{9}{10}\right)^5$
(B)$\frac{1}{2}\left(\frac{9}{5}\right)^4$
(C)$\frac{1}{2}\left(\frac{9}{10}\right)^5$
(D)$\frac{1}{2}\left(\frac{9}{5}\right)^4+\left(\frac{9}{10}\right)^5$

Solution:

Parameter	Values	Description
n	5	Number of defective pens
p	0.1	probability of drawing a defective pen
μ	0.5	np
σ	0.671	$\sqrt{np(1-p)}$
X		Defective pens

Using Binomial

Given,
Probability of drawing a defective pen = $\frac{1}{10}$
Probability of drawing a non-defective pen = $\frac{9}{10}$
Let,
Probability of drawing atmost one pen out of 5 defective with replacement = $\Pr(X \leq 1)$

$$\Pr(X \leq 1) = p_X(0) + p_X(1) \tag{1}$$
$$\implies \Pr(X \leq 1) = \binom{5}{0}\left(\frac{9}{10}\right)^5 + \binom{5}{1}\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right) \tag{2}$$
$$= \left(\frac{9}{10}\right)^5 + 5\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right) \tag{3}$$
$$= \left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4 \tag{4}$$

Gaussian

$$Y \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{5}$$

CDF of Y is

$$F_Y(y) = \Pr(Y \leq y) \tag{6}$$

We know that

$$Q(x) = \Pr(X > x), x > 0, X \sim N(0,1) \tag{7}$$
$$Q(-x) = \Pr(X > -x), x < 0, X \sim N(0,1) \tag{8}$$
$$= 1 - Q(x) \tag{9}$$

Hence,
CDF :

$$F_Y(y) = \begin{cases} 1 - Q\left(\frac{y-\mu}{\sigma}\right), & \text{if } y > \mu \\ 1 - Q\left(\frac{y-\mu}{\sigma}\right) = Q\left(\frac{\mu-y}{\sigma}\right), & \text{if } y < \mu \end{cases} \tag{10}$$

$$F_Y(1) = \Pr(Y \leq 1) \tag{11}$$
$$= 1 - Q\left(\frac{1 - 0.5}{\sqrt{0.671}}\right) \tag{12}$$
$$= 1 - Q\left(\frac{0.5}{0.819}\right) \tag{13}$$
$$= 1 - Q(0.6104) \tag{14}$$
$$= 0.729198876 \tag{15}$$

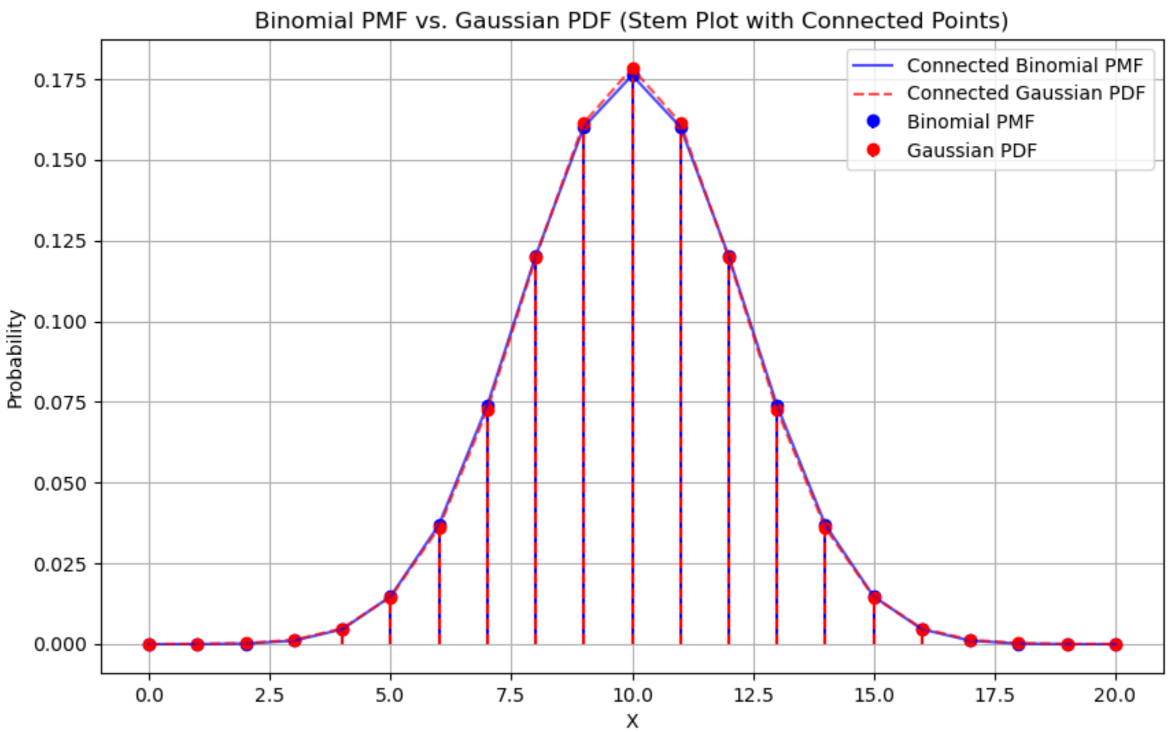

Fig. 0. pmf of binomial and pdf of Gaussian of X and Y marked balls