Sampling & Reconstruction

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Some Material Adopted from Peter Shirley and Steve Marshner

Recap

- What are Callbacks?
  - The program calls a function when something happens
  - Examples: Keyboad, Mouse motion
- What is an event loop
  - Handles inputs and displays, calls callback functions
  - Should run very fast (>30Hz)
- GLAD vs OpenGL vs SDL
  - SDL– Create windows
  - OpenGL – Access Hardware
  - GLAD – Access modern extensions

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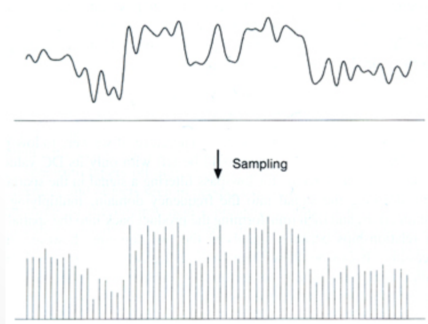
Agenda

- What is an image?
  - How can we best represent images
- What is Sampling?
- What is Reconstruction?
- What is Filtering?
- *What does any of this have to do with computer graphics...?*

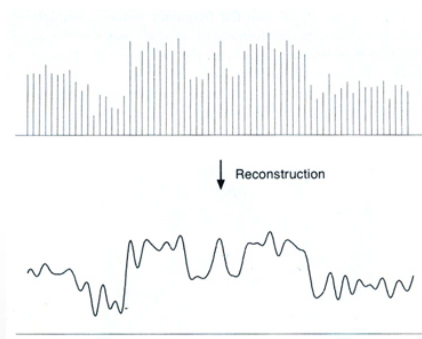
Sampled Representations

- Most of the world is analog/continuous
  - Computers think digitally/discretely
- How can we store continuous info in a computer?
  - Sampling!
  - Store the continuous value at many points



Reconstruction

- Turing samples back into a continuous functions
  - o For output (display technologies)
  - o For analysis or processing (math)
- Reconstructions = informed guessing



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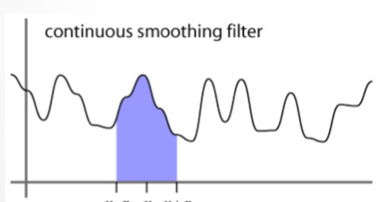
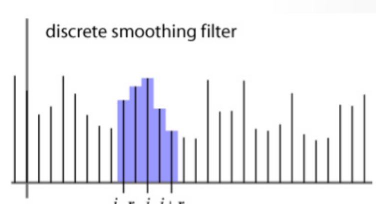
Filtering

- Processing done to a signal
  - o Can be continuous (analog circuit)
  - o Or done on sampled representation
- Ex: Smoothing





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Agenda

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History of Smapling

- Nyquist in 1928; Shannon in 1949
  - Famous results in information theory!
- Practical applications dominated by sound….

- 1940s – First practical uses (telecommunications)
- 1960s – Early digital audio systems
- 1970s – Commercial digital audio systems
- 1982 – Birth of the Compact Disc
  - First high profile consumer success of sampling

- Much of the terminology has an audio "flavor"
  - Audio is 1D signals, Graphics is 2D (images)

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Sampling in Digital Audio

- Recording:
  - Sound → Analog → Samples → Disk
- Playback:
  - Disk → Samples → Analog → Sound
  - <u>How do we know the gaps are filled correctly?</u>

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Undersampling

- Did we miss anything between the samples?
- Examples: A sine wave
  - Information in lost (not so surprising)
  - Indistinguishable form lower frequency wave!
  - Was indistinguishable from higher frequency too!
  - *Aliasing*: One signal disguised as another

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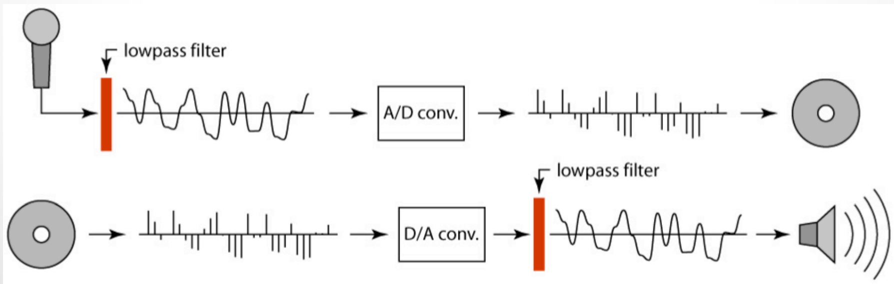
Preventing Aliasing

- Introduce lowpass filters:
  - Remove any high frequency signals before sampling
  - When reconstructing choose lowest valid frequency (disambiguate)



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What's a digital image?

- A (rectilinear) 2D array or pixels
- What's a pixel?
- A little square? **NO!**
- A pixel is a discrete sample of a continuous image!


Continuous Image


Digital Image

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Image Acquisition

- Pixels are **samples** of a continuous function
  - Photoreceptors in your eyes
  - CCD elements in a camera
  - Rays in a virtual camera

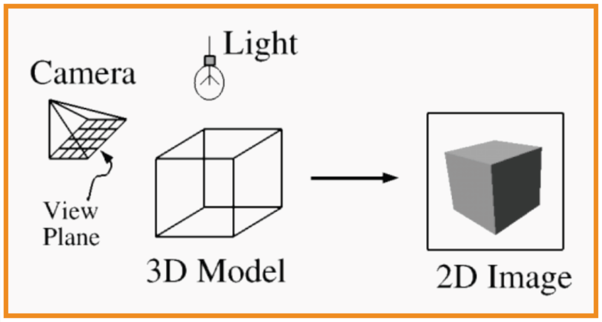

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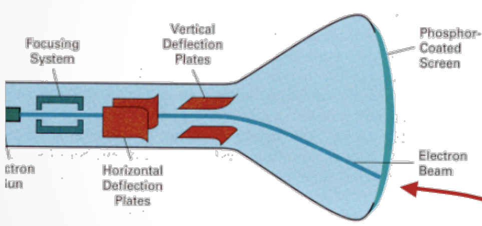
Image Reconstruction

- Generate continuous image from discrete samples
  - Ex: Cathode ray tube



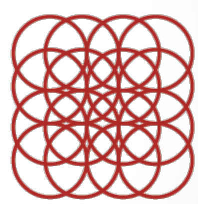


Image is reconstructed by displaying pixels with finite area (Gaussian)

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Image Resolution

- Resolution = Number of Samples
- Spatial Resolution
  - o Image has "Width" x "Height" pixels
- Intensity Resolution
  - o A pixel has "Depth" bits to represent color or intensity
- Temporal Resolution
  - o Monitors refresh at "Rate" Hz

Typical Resolutions

	Width x Height	Depth	Rate
NTSC	640 x 480	8	30
Workstation	1280 x 1024	24	75
Film	3000 x 2000	12	24
Laser Printer	6600 x 5100	1	-

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- **What is Filtering?**
  - o Linear Filtering
  - o Convolution
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Linear Filtering

- Represent transformations on signals:
  - o Bass/treble control on stereo
  - o Blurring/sharpening in image editing
  - o Smoothing in motion tracking
- Key properties
  - o Linearity: filter(f+g) = filter(f) + filter(g)
  - o Shift Invariance: filter behavior same after a shift
    - Delaying audio
    - Sliding around an image
- Can be modeled as a **convolution**

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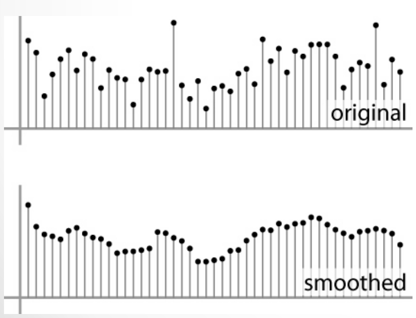
Convolution Intro.

- Basic Idea: A weighted, sliding average
  - o Different weights are different functions
- Example: Smoothing
  - o Equally weighted moving average



$$b_{\text{smooth}}[i] = \frac{1}{2r+1} \sum_{j=i-r}^{i+r} b[j]$$

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Discrete Convolution

- Simple Moving Average:

$$b_{\text{smooth}}[i] = \frac{1}{2r+1} \sum_{j=i-r}^{i+r} b[j]$$

- Convolution is a **weighted moving average**:

$$(a \star b)[i] = \sum_j a[j] b[i-j]$$

  - Each sample *b* has a weight (or zero outside the signal)
  - The weights of *a* define the filter

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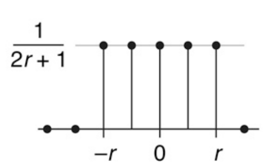
Filters

- Sequence of weights a[j] is a fliter
- *Region of support*: Non-zero portion of filter
  - Usually centered at zero with support radius r
- *Normalized* to sum to 1
  - So it's a weighted average not a weighted sum
- Typically *symmetric* about 0



a box filter

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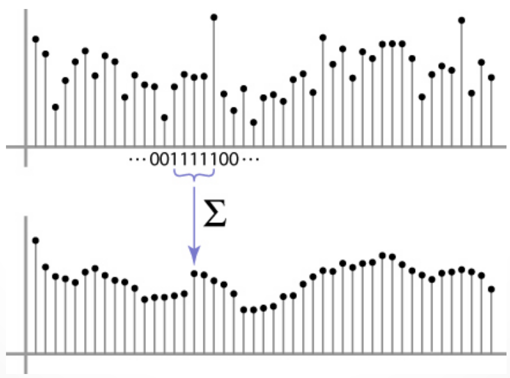
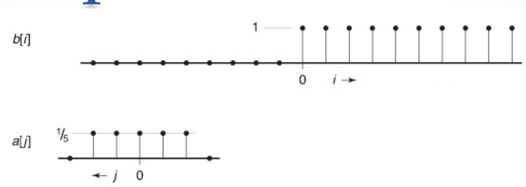
Convolution & Filtering

- Sliding average is convolution with a box filter
  - a_{box} = [..., 0, 1, 1, 1, 1, 1, 0, ...]/(2r+1)

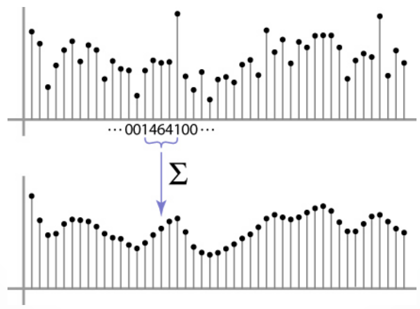


Example: *Box* and *Step*



Convolution & Filtering

- Convolution possible with any sequence of weights
- Ex: Bell curve (guassian-like) [..., 1, 4, 6, 4, 1, ...]/16



Pseudocode

function convolve(sequence a, sequence b, int r, int i)
  $s = 0$
  for $j = -r$ to r
    $s = s + a[j]b[i - j]$
  return s

Discrete convolution

- Notation: $b = c \star a$
- Convolution is a multiplication-like operation

commutative	$a \star b = b \star a$
associative	$a \star (b \star c) = (a \star b) \star c$
distributes over addition	$a \star (b + c) = a \star b + a \star c$
scalars factor out	$\alpha a \star b = a \star \alpha b = \alpha (a \star b)$
identity: unit impulse	$a \star e = a$ e = [..., 0, 0, 1, 0, 0, ...]

- Conceptually no distinction between filter and signal
  - a*b = b*a

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Readings

- Please read Chapter 9 of the textbook!
- It covers all of this in good detail

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