Hash Tables

Computer Science S-111
Harvard Extension School

David G. Sullivan, Ph.D.

Data Dictionary Revisited

- We've considered several data structures that allow us to store and search for data items using their keys fields:

data structure	***searching for an item***	***inserting an item***
a list implemented using an array	*O*(log n) using binary search	*O*(n)
a list implemented using a linked list	*O*(n) using linear search	*O*(n)
binary search tree		
balanced search trees (2-3 tree, B-tree, others)		

- Today, we'll look at hash tables, which allow us to do better than *O*(log n).

Ideal Case: Searching = Indexing

- The optimal search and insertion performance is achieved when we can treat the key as an index into an array.
- Example: storing data about members of a sports team
  - key = jersey number (some value from 0-99).
  - class for an individual player's record:

```
public class Player {
    private int jerseyNum;
    private String firstName;
    …
}
```

  - store the player records in an array:

```
Player[] teamRecords = new Player[100];
```

- In such cases, we can perform both search and insertion in *O*(1) time. For example:

```
public Player search(int jerseyNum) {
    return teamRecords[jerseyNum];
}
```

Hashing: Turning Keys into Array Indices

- In most real-world problems, indexing is not as simple as it is in the sports-team example. Why?
  -
  -
  -
- To handle these problems, we perform *hashing*:
  - use a *hash function* to convert the keys into array indices
    `"Sullivan"` → `18`
  - use techniques to handle cases in which multiple keys are assigned the same hash value
- The resulting data structure is known as a *hash table*.

Hash Functions

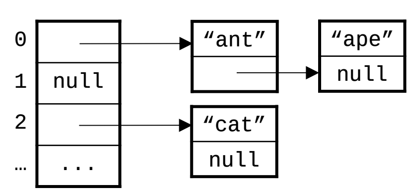
- A hash function defines a mapping from the set of possible keys to the set of integers.
- We then use the modulus operator to get a valid array index.

key value ⇨ **hash function** ⇨ integer $\overset{\%}{⇨}$ integer in [0, n – 1]
(n = array length)

- Here's a very simple hash function for keys of lower-case letters:
  h(key) = Unicode value of first char – Unicode value of ‘a’
  - examples:
    h(“ant”) = Unicode for ‘a’ – Unicode for ‘a’ = 0
    h(“cat”) = Unicode for ‘c’ – Unicode for ‘a’ = 2
- h(key) is known as the key's *hash code.*
- A *collision* occurs when items with different keys are assigned the same hash code.

Dealing with Collisions I: Separate Chaining

- If multiple items are assigned the same hash code, we “chain” them together.
- Each position in the hash table serves as a *bucket* that is able to store multiple data items.
- Two implementations:
  1. each bucket is itself an array
     - disadvantages:
       - large buckets can waste memory
       - a bucket may become full; *overflow* occurs when we try to add an item to a full bucket
  2. each bucket is a linked list
     - disadvantage:
       - the references in the nodes use additional memory



Dealing with Collisions II: Open Addressing

- When the position assigned by the hash function is occupied, find another open position.
- Example: “wasp” has a hash code of 22, but it ends up in position 23, because position 22 is occupied.
- We will consider three ways of finding an open position – a process known as *probing*.
- The hash table also performs probing to search for an item.
  - example: when searching for “wasp”, we look in position 22 and then look in position 23
  - we can only stop a search when we reach an empty position

0	“ant”
1	
2	“cat”
3	
4	“emu”
5	
6	
7	
…	...
22	“wolf”
23	**“wasp”**
24	“yak”
25	“zebra”

Linear Probing

- Probe sequence: h(key), h(key) + 1, h(key) + 2, …, wrapping around as necessary.
- Examples:
  - “ape” (h = 0) would be placed in position 1, because position 0 is already full.
  - “bear” (h = 1): try 1, 1 + 1, 1 + 2 – open!
  - where would “zebu” end up?
- Advantage: if there is an open position, linear probing will eventually find it.
- Disadvantage: “clusters” of occupied positions develop, which tends to increase the lengths of subsequent probes.
  - probe length = the number of positions considered during a probe

0	“ant”
1	“ape”
2	“cat”
3	“bear”
4	“emu”
5	
6	
7	
…	...
22	“wolf”
23	“wasp”
24	“yak”
25	“zebra”

Quadratic Probing

- Probe sequence: h(key), h(key) + 1, h(key) + 4, h(key) + 9, …, wrapping around as necessary.
  - the offsets are perfect squares: h + 1^2, h + 2^2, h + 3^2, …
- Examples:
  - “ape” (h = 0): try 0, 0 + 1 – open!
  - “bear” (h = 1): try 1, 1 + 1, 1 + 4 – open!
  - “zebu”?
- Advantage: reduces clustering
- Disadvantage: it may fail to find an existing open position. For example:

table size = 10
x = occupied

trying to insert a key with h(key) = 0

offsets of the probe sequence in italics

0	x	
1	x	*1 81*
2		
3		
4	x	*4 64*

5	x	*25*
6	x	*16 36*
7		
8		
9	x	*9 49*

0	“ant”
1	“ape”
2	“cat”
3	
4	“emu”
5	“bear”
6	
7	
…	...
22	“wolf”
23	“wasp”
24	“yak”
25	“zebra”

Double Hashing

- Use two hash functions:
  - h1 computes the hash code
  - h2 computes the increment for probing
  - probe sequence: h1, h1 + h2, h1 + 2*h2, …
- Examples:
  - h1 = our previous h
  - h2 = number of characters in the string
  - “ape” (h1 = 0, h2 = 3): try 0, 0 + 3 – open!
  - “bear” (h1 = 1, h2 = 4): try 1 – open!
  - “zebu”?
- Combines the good features of linear and quadratic probing:
  - reduces clustering
  - will find an open position if there is one, provided the table size is a prime number

0	“ant”
1	“bear”
2	“cat”
3	“ape”
4	“emu”
5	
6	
7	
…	...
22	“wolf”
23	“wasp”
24	“yak”
25	“zebra”

Removing Items Under Open Addressing

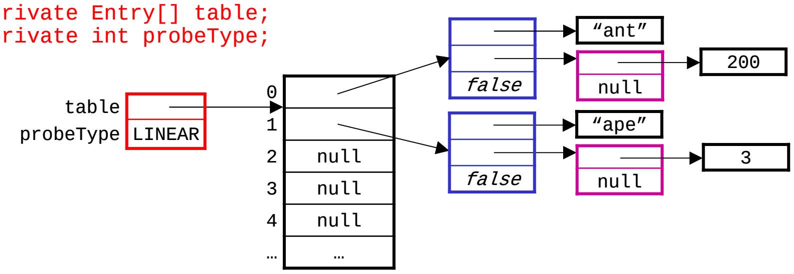
- Consider the following scenario:
  - using linear probing
  - insert “ape” (h = 0): try 0, 0 + 1 – open!
  - insert “bear” (h = 1): try 1, 1 + 1, 1 + 2 – open!
  - remove “ape”
  - search for “ape”: try 0, 0 + 1 – no item
  - search for “bear”: try 1 – no item, but “bear” is further down in the table

0	“ant”
1	
2	“cat”
3	“bear”
4	“emu”
5	
…	...
22	“wolf”
23	“wasp”
24	“yak”
25	“zebra”

- When we remove an item from a position, we need to leave a special value in that position to indicate that an item was removed.
- Three types of positions: occupied, empty, “removed”.
- We stop probing when we encounter an empty position, but not when we encounter a removed position.
- We can insert items in either empty or removed positions.

Implementation

```
public class HashTable {
    private class Entry {
        private String key;
        private LLList valueList;
        private boolean hasBeenRemoved;
        …
    }
    …
    private Entry[] table;
    private int probeType;
}
```



- We use a private inner class for the entries in the hash table.
- To handle duplicates, we maintain a list of values for each key.
- When we remove a key and its values, we set the `Entry`’s `hasBeenRemoved` field to `true`; this indicates that the position is a removed position.

Probing Using Double Hashing

```
private int probe(String key) {
    int i = h1(key);     // first hash function
    int h2 = h2(key);    // second hash function

    // keep probing until we get an empty position or match
    // (write this together)




    return i;
}
```

- We'll assume that removed positions have a key of null.
  - thus, for non-empty positions, it's always okay to compare the probe key with the key in the `Entry`

Avoiding an Infinite Loop

- The while loop in our probe method could lead to an infinite loop.

```
while (table[i] != null && !key.equals(table[i].key)) {
    i = (i + h2) % table.length;
}
```

- When would this happen?

- We can stop probing after checking n positions (n = table size), because the probe sequence will just repeat after that point.
  - for quadratic probing:

    $(h1 + n^2) \% n = h1 \% n$

    $(h1 + (n+1)^2) \% n = (h1 + n^2 + 2n + 1) \% n = (h1 + 1) \% n$
  - for double hashing:

    $(h1 + n*h2) \% n = h1 \% n$

    $(h1 + (n+1)*h2) \% n = (h1 + n*h2 + h2) \% n = (h1 + h2) \% n$

Avoiding an Infinite Loop (cont.)

```
private int probe(String key) {
    int i = h1(key);    // first hash function
    int h2 = h2(key);   // second hash function
    int positionsChecked = 1;

    // keep probing until we get an
    // empty position or a match
    while (table[i] != null && !key.equals(table[i].key)) {
        if (positionsChecked == table.length)
            return -1;
        i = (i + h2) % table.length;
        positionsChecked++;
    }

    return i;
}
```

Handling the Other Types of Probing

```
private int probe(String key) {
    int i = h1(key);    // first hash function
    int h2 = h2(key);   // second hash function
    int positionsChecked = 1;

    // keep probing until we get an
    // empty position or a match
    while (table[i] != null && !key.equals(table[i].key)) {
        if (positionsChecked == table.length)
            return -1;
        i = (i + probeIncrement(positionsChecked, h2))
               % table.length;
        positionsChecked++;
    }

    return i;
}
```

Handling the Other Types of Probing (cont.)

- The `probeIncrement()` method bases the increment on the type of probing:

```
private int probeIncrement(int n, int h2) {
    if (n <= 0)
       return 0;

    switch (probeType) {
    case LINEAR:
        return 1;
    case QUADRATIC:
        return (2*n - 1);
    case DOUBLE_HASHING:
        return h2;
    }
}
```

Handling the Other Types of Probing (cont.)

- For quadratic probing, `probeIncrement(n, h2)` returns
  `2*n – 1`
  Why does this work?
- Recall that for quadratic probing:
  - probe sequence = h1, h1 + 1^2, h1 + 2^2, …
  - nth index in the sequence = h1 + n^2
- The increment used to compute the nth index
  = nth index – (n – 1)st index
  = (h1 + n^2) – (h1 + $(n – 1)^2$)
  = $n^2 – (n – 1)^2$
  = $n^2 – (n^2 – 2n + 1)$
  = $2n – 1$

Search and Removal

- Both of these methods begin by probing for the key.

```
public LLList search(String key) {
    int i = probe(key);
    if (i == -1 || table[i] == null)
        return null;
    else
        return table[i].valueList;
}

public void remove(String key) {
    int i = probe(key);
    if (i == -1 || table[i] == null)
        return;

    table[i].key = null;
    table[i].valueList = null;
    table[i].hasBeenRemoved = true;
}
```

Insertion

- We begin by probing for the key.
- Several cases:
  1. the key is already in the table (we're inserting a duplicate)
     → add the value to the valueList in the key's Entry
  2. the key is not in the table: three subcases:
     a. encountered 1 or more removed positions while probing
        → put the (key, value) pair in the *first* removed position that we encountered while searching for the key.
        why does this make sense?

     b. no removed position; reached an empty position
        → put the (key, value) pair in the empty position

     c. no removed position or empty position encountered
        → overflow; throw an exception

Insertion (cont.)

- To handle the special cases, we give this method its own implementation of probing:

```
void insert(String key, int value) {
    int i = h1(key);
    int h2 = h2(key);
    int positionsChecked = 1;
    int firstRemoved = -1;

    while (table[i] != null && !key.equals(table[i].key)) {
        if (table[i].hasBeenRemoved && firstRemoved == -1)
            firstRemoved = i;
        if (positionsChecked == table.length)
            break;
        i = (i + probeIncrement(positionsChecked, h2))
              % table.length;
        positionsChecked++;
    }

    // deal with the different cases (see next slide)
}
```

- firstRemoved remembers the first removed position encountered

Insertion (cont.)

```
void insert(String key, int value) {
    ...
    int firstRemoved = -1;

    while (table[i] != null && !key.equals(table[i].key) {
        if (table[i].hasBeenRemoved && firstRemoved == -1)
            firstRemoved = i;
        if (++positionsChecked == table.length)
            break;
        i = (i + h2) % table.length;
    }

    // deal with the different cases
    if (table[i] != null && key.equals(table[i].key))  // 1
        table[i].valueList.addItem(value, 0);
    else if (firstRemoved != -1)                        // 2a
        table[firstRemoved] = new Entry(key, value);
    else if (table[i] == null)                          // 2b
        table[i] = new Entry(key, value);
    else throw an exception…                            // 2c
}
```

Tracing Through Some Examples

- Start with the hashtable at right with:
  - double hashing
  - our earlier hash functions h1 and h2
- Perform the following operations:
  - insert "bear"
  - insert "bison"
  - insert "cow"
  - delete "emu"
  - search "eel"
  - insert "bee"

0	“ant”
1	
2	“cat”
3	
4	"emu"
5	"fox"
6	
7	
8	
9	
10	

Dealing with Overflow

- Overflow = can't find a position for an item
- When does it occur?
  - linear probing:
  - quadratic probing:
    -
    -
  - double hashing:
    - if the table size is a prime number: same as linear
    - if the table size is not a prime number: same as quadratic
- To avoid overflow (and reduce search times), grow the hash table when the percentage of occupied positions gets too big.
  - problem: if we're not careful, we can end up needing to rehash **all** of the existing items
  - approaches exist that limit the number of rehashed items

Implementing the Hash Function

- Characteristics of a good hash function:
  1) efficient to compute
  2) uses the entire key
     - changing any char/digit/etc. should change the hash code
  3) distributes the keys more or less uniformly across the table
  4) must be a function!
     - a key must always get the same hash code
- In Java, every object has a `hashCode()` method.
  - the version inherited from `Object` returns a value based on an object's memory location
  - classes can override this version with their own

Hash Functions for Strings: version 1

- h_a = the sum of the characters' Unicode values
- Example: h_a("eat") = 101 + 97 + 116 = 314
- All permutations of a given set of characters get the same code.
  - example: h_a("tea") = h_a("eat")
  - could be useful in a Scrabble game
    - allow you to look up all words that can be formed from a given set of characters
- The range of possible hash codes is very limited.
  - example: hashing keys composed of 1-5 lower-case char's (padded with spaces)
  - 26*27*27*27*27 = over 13 million possible keys
  - smallest code = h_a("a   ") = 97 + 4*32 = 225
    largest code = h_a("zzzzz") = 5*122 = 610
    $\}$ 610 – 225 = 385 codes

Hash Functions for Strings: version 2

- Compute a *weighted* sum of the Unicode values:

  $h_b = a_0 b^{n-1} + a_1 b^{n-2} + \ldots + a_{n-2} b + a_{n-1}$

  where a_i = Unicode value of the ith character
  b = a constant
  n = the number of characters

- Multiplying by powers of b allows the *positions* of the characters to affect the hash code.
  - different permutations get different codes

- We may get arithmetic overflow, and thus the code may be negative. We adjust it when this happens.

- Java uses this hash function with b = 31 in the `hashCode()` method of the `String` class.

Hash Table Efficiency

- In the best case, search and insertion are *O*(1).

- In the worst case, search and insertion are linear.
  - open addressing: *O*(m), where m = the size of the hash table
  - separate chaining: *O*(n), where n = the number of keys

- With good choices of hash function and table size, complexity is generally better than *O*(log n) and approaches *O*(1).

- *load factor* = # keys in table / size of the table.
  To prevent performance degradation:
  - open addressing: try to keep the load factor < 1/2
  - separate chaining: try to keep the load factor < 1

- Time-space tradeoff: bigger tables have better performance, but they use up more memory.

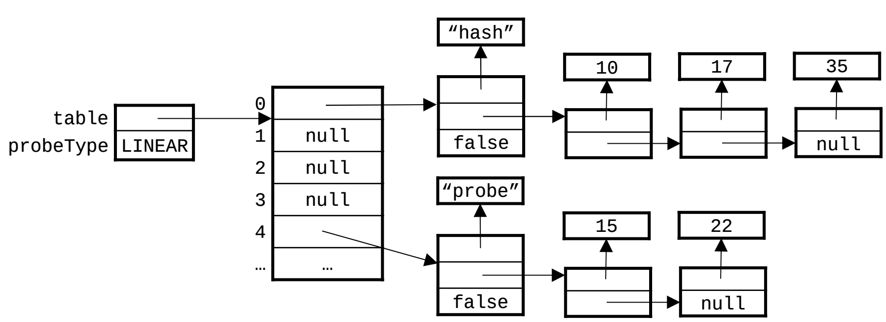
Hash Table Limitations

- It can be hard to come up with a good hash function for a particular data set.
- The items are not ordered by key. As a result, we can't easily:
  - print the contents in sorted order
  - perform a range search
  - perform a rank search – get the kth largest item

  We *can* do all of these things with a search tree.

Application of Hashing: Indexing a Document

- Read a text document from a file and create an index of the line numbers on which each word appears.
- Use a hash table to store the index:
  - key = word
  - values = line numbers in which the word appears



- See `WordIndex.java`

Optional: Computing h_b More Efficiently

- Use Horner's method of evaluating a polynomial:
  - $a_0b^{n-1} + a_1b^{n-2} + \ldots + a_{n-2}b^{n-1} + a_{n-1}$
    $= (\ldots((a_0b + a_1)b + a_2)b + \ldots + a_{n-2})b + a_{n-1}$
  - example: $101*31^2 + 97*31 + 116 = ((101*31 + 97)*31 + 116$
  - here it is in Java for the string `s`:

```
int hash = 0;
for (int i = 0; i < s.length(); i++)
    hash = hash * b + s.charAt(i);
```

- Use the left-shift operator (<<) to multiply by 31:
  - `n << i` shifts the binary representation of `n` left by `i` places
  - `n << i = n * 2`^{i}
  - `n * 31 = n * (32 – 1) = (n * 32) – n = (n << 5) – n`
  - example:

```
     n = 100 = 0000000001100100₂
100 << 5 = 0000110010000000₂ = 3200
100 * 31 = 3200 – 100 = 3100
```

Optional: Hash Functions for Numeric Keys

- If the keys are `ints` (or a smaller numeric type – e.g., `byte`), we can use the keys themselves as the hash codes.
- If the keys are `longs` or `doubles` (64 bits), we could cast the keys to `ints`, but that means that half of the bits in the keys won't contribute to the hash codes.
- Instead, use folding – as we did in h_a for strings.
  - break the 64 bits into two 32-bit pieces
  - combine the pieces by adding them or by applying the exclusive-or operator (^) – see the textbooks for more info.
- Example:

```
long key;
long leftMostBits = key >> 32;  // shift bits right by 32
int hash = (int)(key ^ leftMostBits);
```