Simulation of Gaussian Processes

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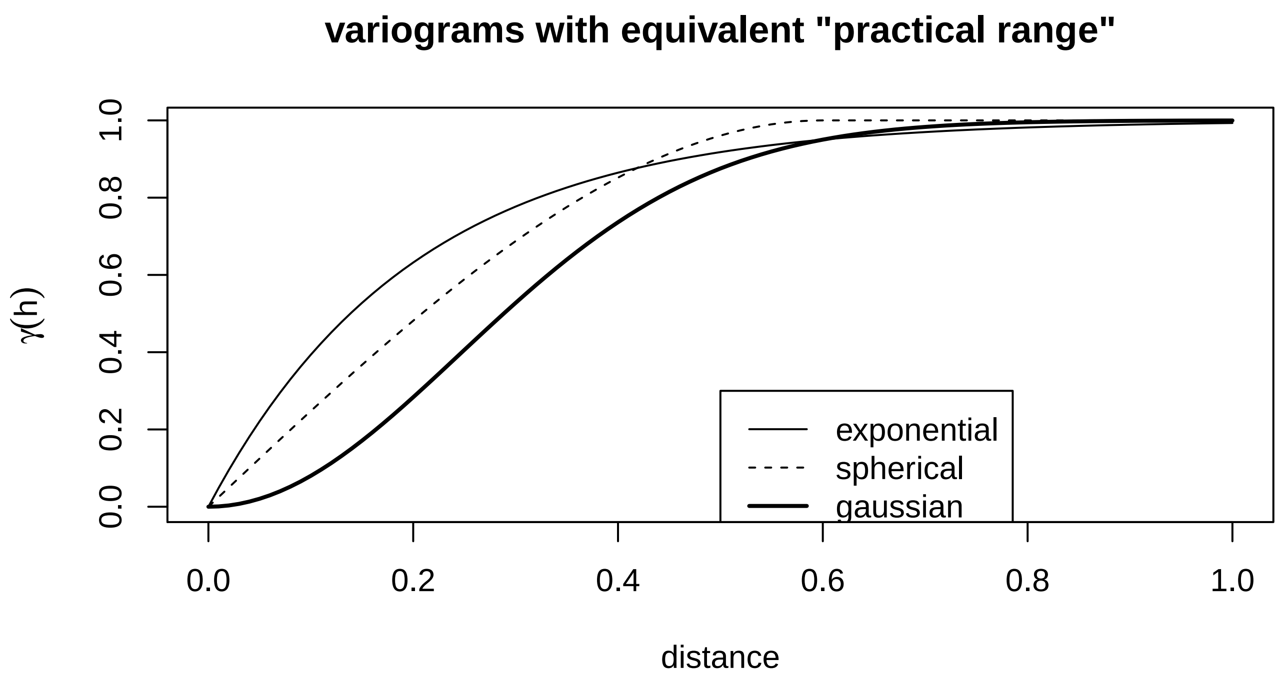
This document shows simulations of Gaussian processes (GP) for different correlation functions, different values of the parameters for a given correlation function. The idea is to visualize how the covariance structure influences the smoothness of the resultant GP.

First we start by using the `geoR` package to show different variograms.

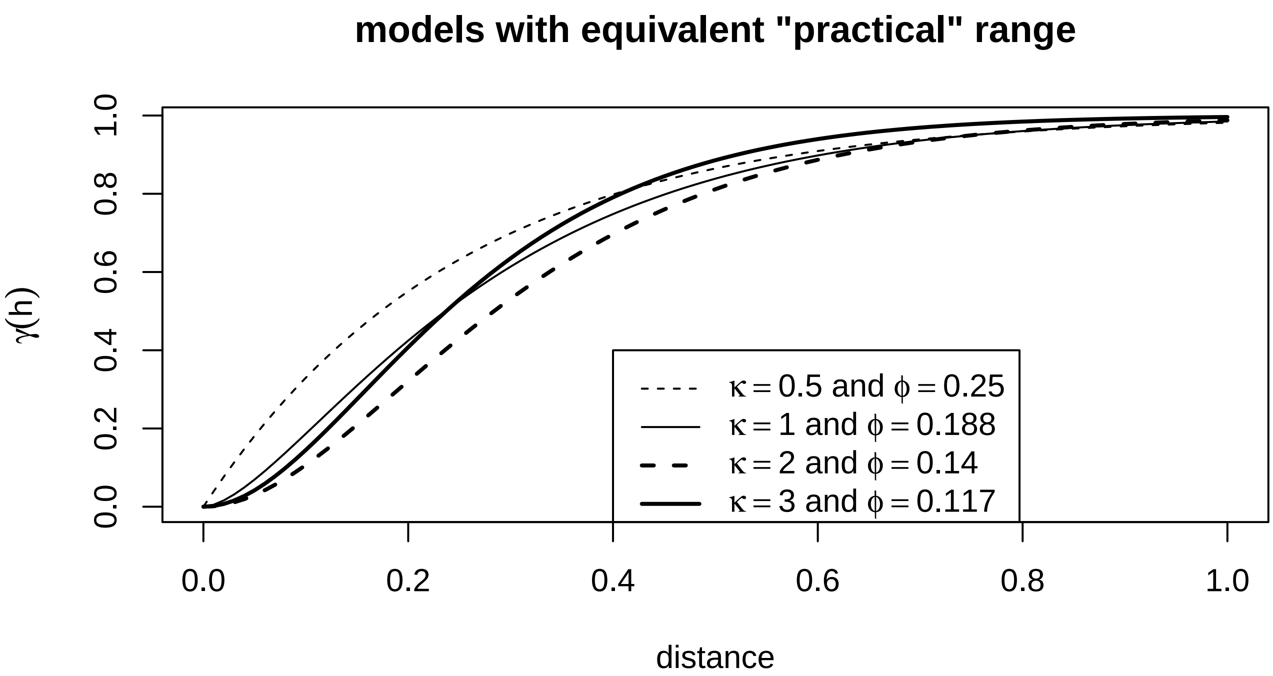
```
library(geoR)

#
#
# Variogram models with the same "practical" range:
#

v.f <- function(x, ...){1-cov.spatial(x, ...)}

curve(v.f(x, cov.pars=c(1, .2)), from = 0, to = 1,
      xlab = "distance", ylab = expression(gamma(h)),
      main = "variograms with equivalent \"practical range\"")
curve(v.f(x, cov.pars = c(1, .6), cov.model = "sph"), 0, 1,
      add = TRUE, lty = 2)
curve(v.f(x, cov.pars = c(1, .6/sqrt(3)), cov.model = "gau"),
      0, 1, add = TRUE, lwd = 2)
legend(0.5,.3, c("exponential", "spherical", "gaussian"),
       lty=c(1,2,1), lwd=c(1,1,2))
```



Now we plot the Matérn correlation function using equivalent "practical ranges" but varying the smoothness parameter ($\kappa = 1, 2, 3$).

```
#
# Matern models with equivalent "practical range"
# and varying smoothness parameter
#
par(mfrow=c(1,1))
curve(v.f(x, cov.pars = c(1, 0.25), kappa = 0.5),from = 0, to = 1,
      xlab = "distance", ylab = expression(gamma(h)), lty = 2,
      main = "models with equivalent \"practical\" range")
curve(v.f(x, cov.pars = c(1, 0.188), kappa = 1),from = 0, to = 1,
      add = TRUE)
curve(v.f(x, cov.pars = c(1, 0.14), kappa = 2),from = 0, to = 1,
      add = TRUE, lwd=2, lty=2)
curve(v.f(x, cov.pars = c(1, 0.117), kappa = 2),from = 0, to = 1,
      add = TRUE, lwd=2)
legend(0.4,.4, c(expression(paste(kappa == 0.5, " and ",
                                  phi == 0.250)),
                 expression(paste(kappa == 1, " and ", phi == 0.188)),
                 expression(paste(kappa == 2, " and ", phi == 0.140)),
                 expression(paste(kappa == 3, " and ", phi == 0.117))),
```

```r
      lty=c(2,1,2,1), lwd=c(1,1,2,2))
```



Now we simulate different partial realizations from different Gaussian processes. We start by defining a regular grid. And we use an exponential correlation function with different values of the range parameter.

```r
#
# Different values of phi
#
  i<-1
  set.seed(234+i)
  ap1 <- grf(961, grid="reg", cov.pars=c(1, 0))
  set.seed(234+i)
  ap2 <- grf(961, grid="reg", cov.pars=c(1, .1))
  set.seed(234+i)
  ap3 <- grf(961, grid="reg", cov.pars=c(1, .25))
  set.seed(234+i)
  ap4 <- grf(961, grid="reg", cov.pars=c(1, .75))
```

```r
   par(mfrow=c(2,2), mar=c(1.5,.5,1.5,0), mgp=c(1, .5, 0))
   iis <- range(c(ap1$data, ap2$data, ap3$data, ap4$data))
  image(ap1, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
```

```
  mtext(expression(phi==0), cex=1.5)
  image(ap2, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
  mtext(expression(phi==0.10), cex=1.5)
  image(ap3, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
  mtext(expression(phi==0.25), cex=1.5)
  image(ap4, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
  mtext(expression(phi==0.75), cex=1.5)
```

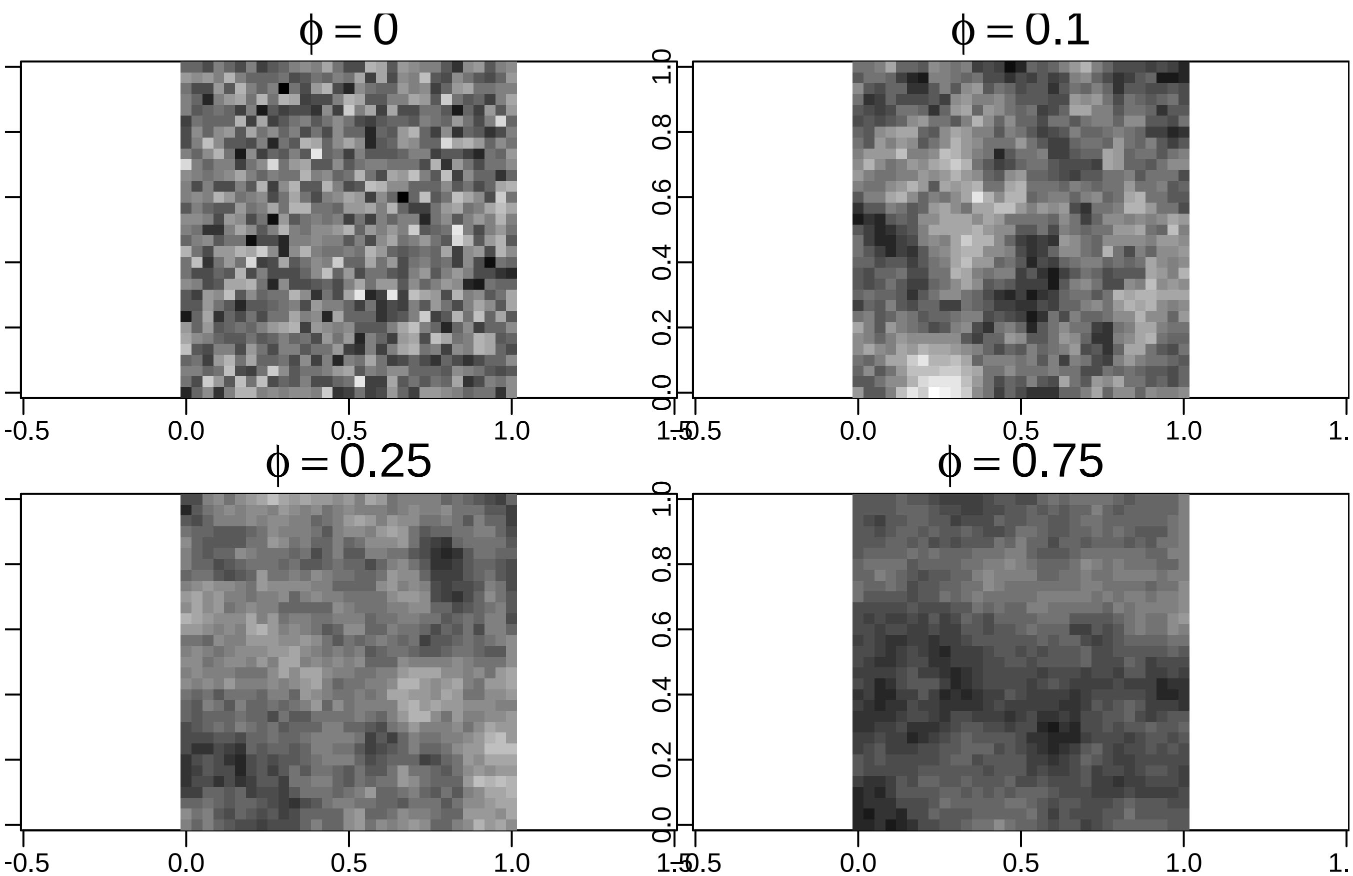


Figure 1: Four realizations from a GP with exponential correlation function and different range parameters.

We now simulate realizations from GPs by keeping the range parameter fixed, and varying the variance of the GP.

```
##
## Different values of sigma^2
##

  set.seed(234+i)
  ap1 <- grf(961, grid="reg", cov.pars=c(1, 0.3))
  set.seed(234+i)
```

```
ap2 <- grf(961, grid="reg", cov.pars=c(2, .3))
set.seed(234+i)
ap3 <- grf(961, grid="reg", cov.pars=c(3, .3))
set.seed(234+i)
ap4 <- grf(961, grid="reg", cov.pars=c(5, .3))
```

```
iis <- range(c(ap1$data, ap2$data, ap3$data, ap4$data))
par(mfrow=c(2,2), mar=c(1.5,.5,1.5,0), mgp=c(1, .5, 0))
image(ap1, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
mtext(expression(sigma^2==1), cex=1.5)
image(ap2, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
mtext(expression(sigma^2==2), cex=1.5)
image(ap3, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
mtext(expression(sigma^2==3), cex=1.5)
image(ap4, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
mtext(expression(sigma^2==5), cex=1.5)
```

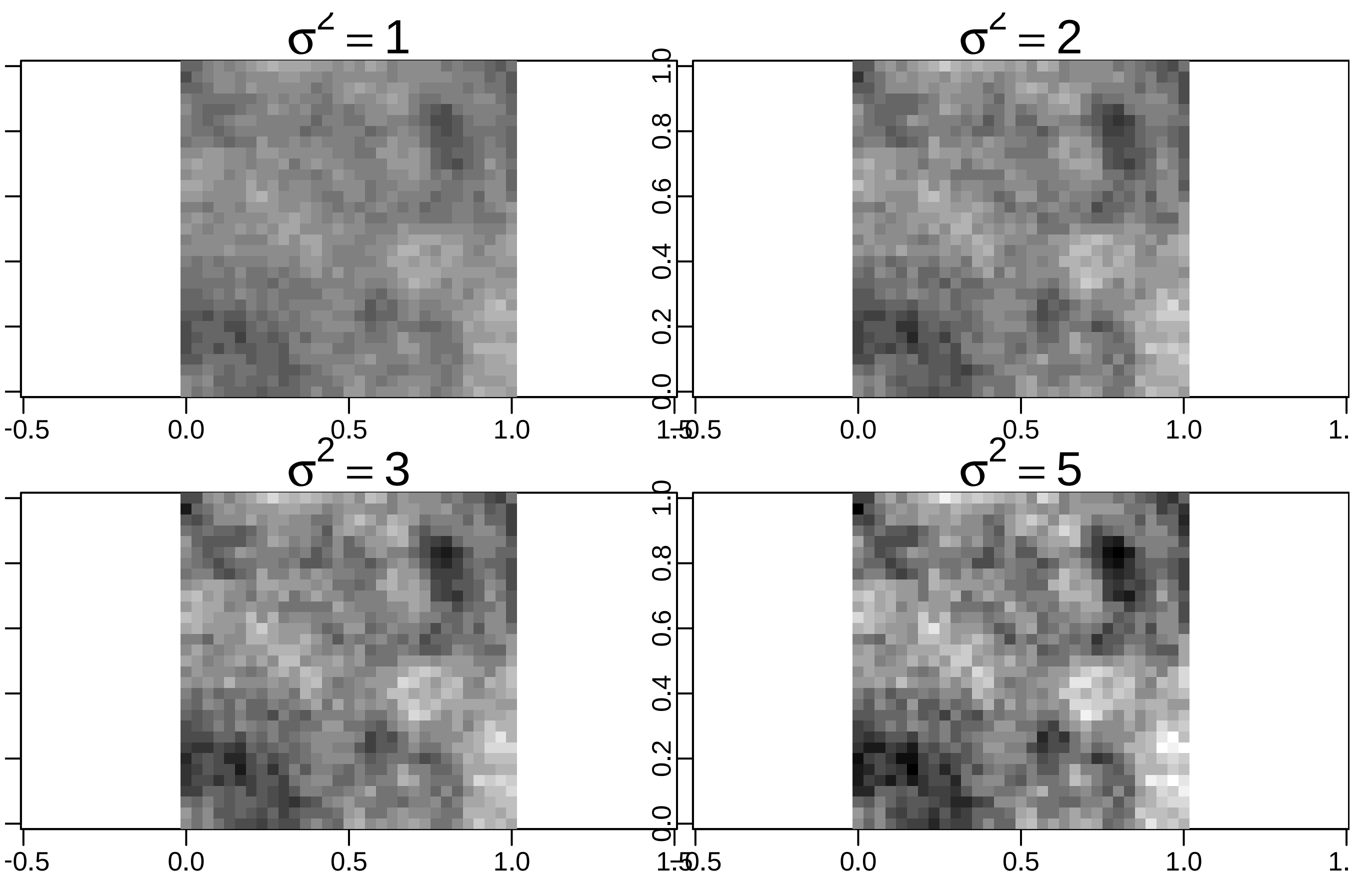


Figure 2: Four realizations from a GP with exponential correlation function and different variances.

Now we simulate from GPs with the same exponential correlation function, variance, and range parameters, and change the nugget effect.

```
##
## Different nugget effects
##
  set.seed(234+i)
  ap1 <- grf(961, grid="reg", cov.pars=c(1, 0.3), nug=0)
  set.seed(234+i)
  ap2 <- grf(961, grid="reg", cov.pars=c(.75, .3), nug=0.25)
  set.seed(234+i)
  ap3 <- grf(961, grid="reg", cov.pars=c(.5, .3), nug=0.5)
  set.seed(234+i)
  ap4 <- grf(961, grid="reg", cov.pars=c(.1, .3), nug=.9)
```

```
par(mfrow=c(2,2), mar=c(1.5,.5,1.5,0), mgp=c(1, .5, 0))
iis <- range(c(ap1$data, ap2$data, ap3$data, ap4$data))
image(ap1, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
mtext(expression(paste(sigma^2==1, " and ", tau^2 == 0), cex=1.5))
image(ap2, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
mtext(expression(paste(sigma^2==0.75, " and ", tau^2 == 0.25), cex=1.5))
image(ap3, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
mtext(expression(paste(sigma^2==0.5, " and ", tau^2 == 0.5), cex=1.5))
image(ap4, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
mtext(expression(paste(sigma^2==0.1, " and ", tau^2 == 0.9), cex=1.5))
```

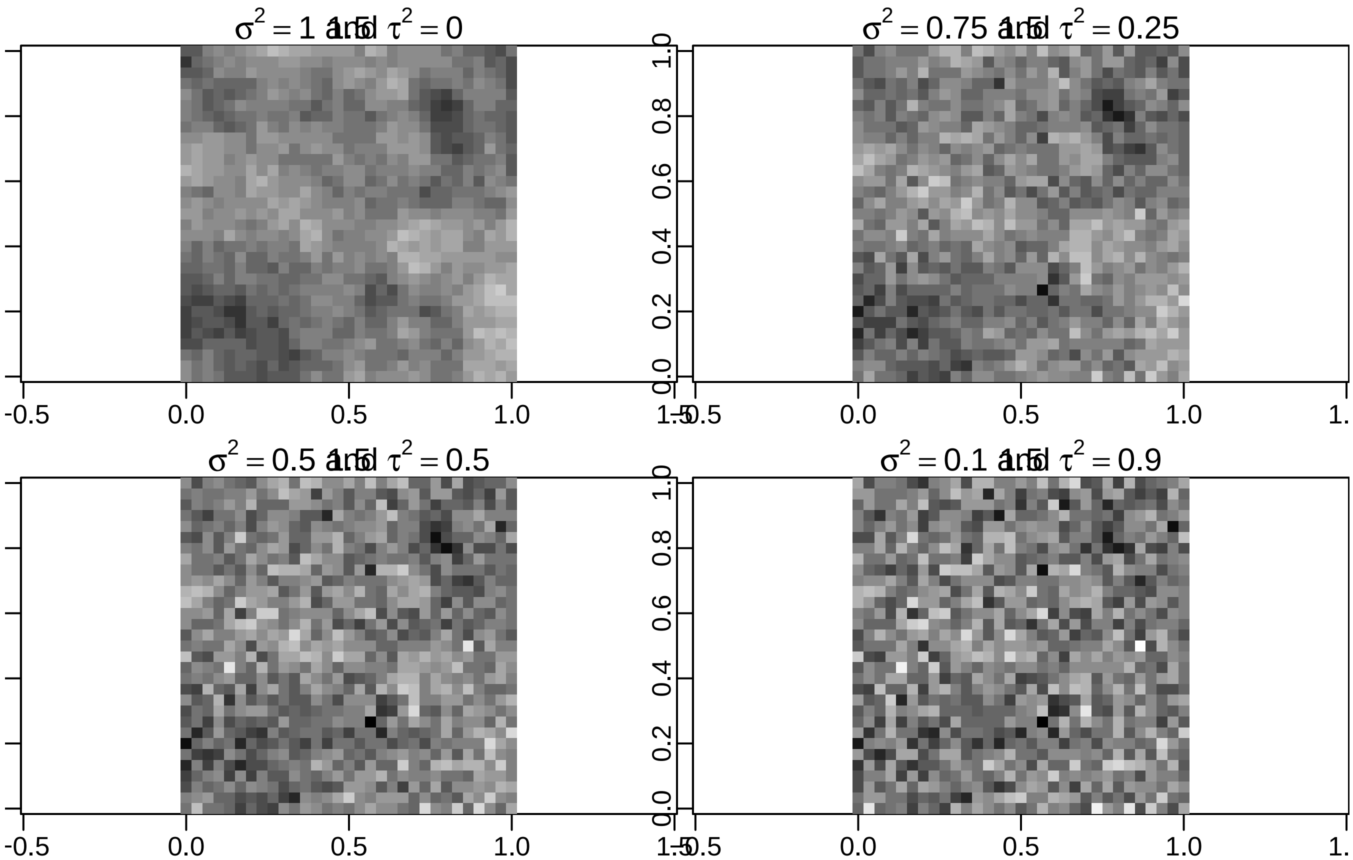


Figure 3: Four realizations from a GP with exponential correlation function and different nugget effects.

Now we simulate partial realizations from GPs with different correlation functions.

```
##
## Different correlation functions
##


  set.seed(234+i)
  ap1 <- grf(961, grid="reg", cov.pars=c(1, .25))
  set.seed(234+i)
  ap2 <- grf(961, grid="reg", cov.pars=c(1, .75), cov.model="sph")
  set.seed(234+i)
```

```
par(mfrow=c(1,2), mar=c(1.5,.5,1.5,0), mgp=c(1, .5, 0))
iis <- range(c(ap1$data, ap2$data, ap3$data, ap4$data))
image(ap1, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
mtext("exponential", cex=1.5)
image(ap2, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
mtext("spherical", cex=1.5)
```

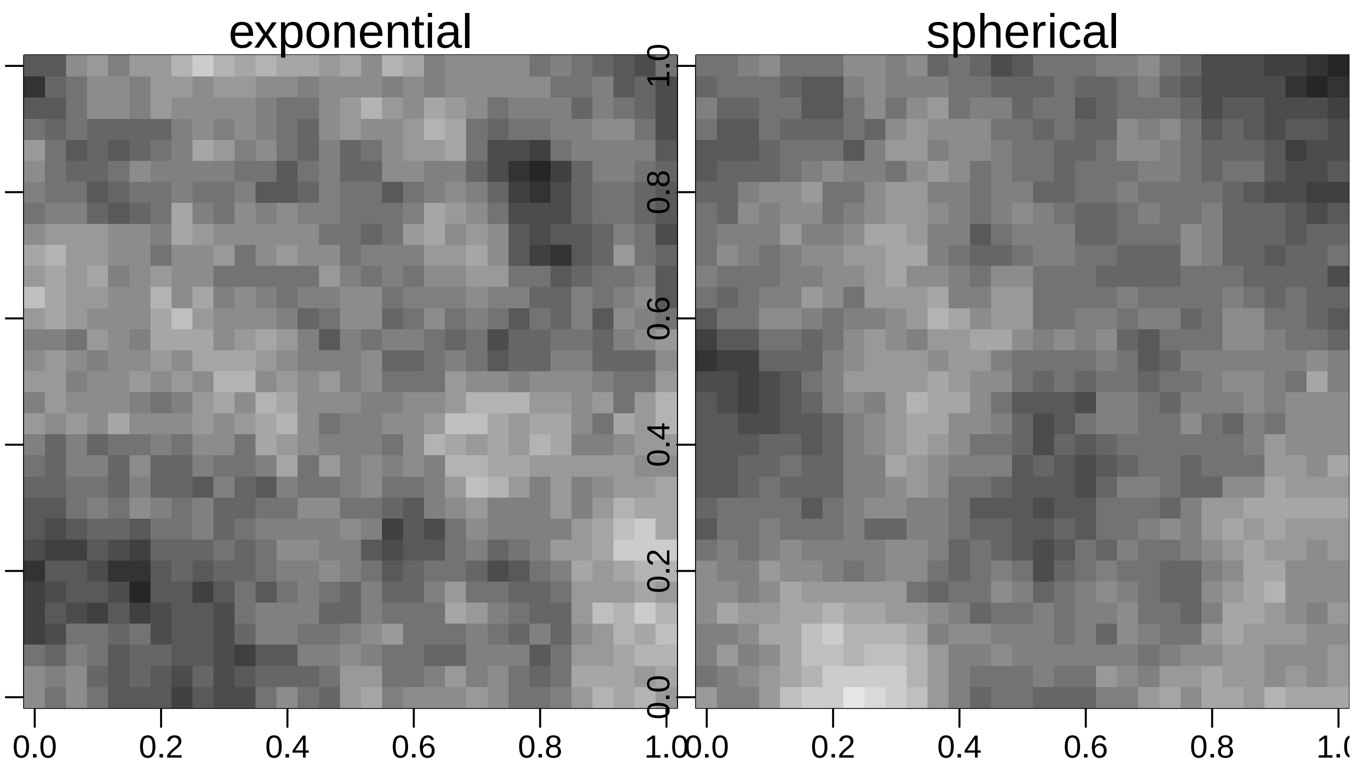


Figure 4: Two realizations from a GP with different correlation functions.

And now we simulate GPs considering different levels of geometrical anisotropy.

```
##
## Different anisotropy
##

  set.seed(234+i)
  ap1 <- grf(961, grid="reg", cov.pars=c(1, .25), aniso.pars=c(pi/4, 2))
  set.seed(234+i)
  ap2 <- grf(961, grid="reg", cov.pars=c(1, .25), aniso.pars=c(pi/4, 4))
  set.seed(234+i)
  ap3 <- grf(961, grid="reg", cov.pars=c(1, .25), aniso.pars=c(2*pi/3, 2))
  set.seed(234+i)
  ap4 <- grf(961, grid="reg", cov.pars=c(1, .25), aniso.pars=c(2*pi/3, 4))
```

```
  par(mfrow=c(2,2), mar=c(1.5,.5,1.5,0), mgp=c(1, .5, 0))
  iis <- range(c(ap1$data, ap2$data, ap3$data, ap4$data))
  image(ap1, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
  mtext(expression(paste(phi[a]==pi/4, " \ ,\ ", phi[r] == 2), cex=1.5))
  image(ap2, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
```

```
mtext(expression(paste(phi[a]==pi/4, " \ ,\ ", phi[r] == 4), cex=1.5))
image(ap3, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
mtext(expression(paste(phi[a]== 2*pi/3, " \ ,\ ", phi[r] == 2), cex=1.5))
image(ap4, xlab="", ylab="", col=gray(seq(1,0,l=21)), zlim=iis)
mtext(expression(paste(phi[a]==2*pi/3, " \ ,\ ", phi[r] == 4), cex=1.5))
```

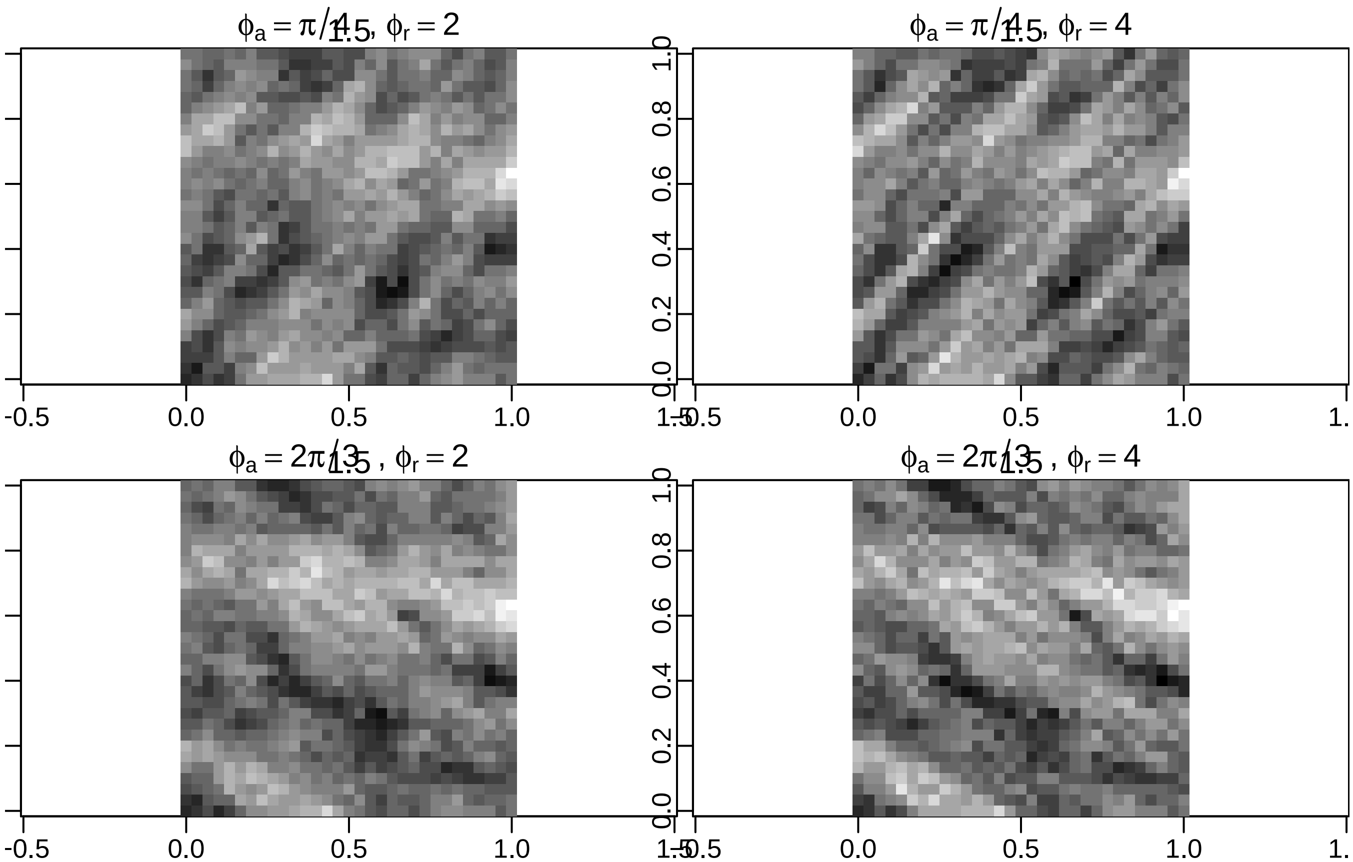


Figure 5: Four realizations from a GP with different geometrical anisotropic structure.