"Theorem" (Absolute Value Theorem)

If $\sum |a_n|$ converges, then $\sum a_n$ converges

Corollary → If $\sum |a_n|$ diverges, then $\sum a_n$ "inconclusive"

Example: The alternating harmonic series

Process: $\sum_{n}^{\infty} |a_n| = \sum_{1}^{\infty} |(-1)^n \cdot a_n| = \sum_{1}^{\infty} a_n$

Note: make sure a_n is always positive

This can be anything that implies "strictly" alternating

From this point, use usual methods to determine convergence / divergence.

Tips: Something like $\sum_{n}^{\infty} (-4)^n a_n$ can be written as $\sum_{n}^{\infty} (-1)^n \cdot 4^n a_n$

This is useful so that when taking the abs. value, you are able to preserve the actual sequence

<u>Alternating Series Test</u>: "Glorified version of the n^{th} term test"

- Why? b/c it's always conclusive

Strictly

If $\sum_{1}^{\infty} (-1)^n a_n$ converges if

i) a_n is positive

ii) $a_n > a_{n+1}$ (decreasing)

iii) $\lim_{n \to \infty} a_n = 0$

or

① The series strictly alternates

② $\lim_{n \to \infty} a_n = 0$

$\sum_{1}^{\infty} (-1)^n a_n$ converges either way (same thing really)

Recognize certain series:

Geometric - $\sum_{n=1}^{\infty} a_1 (r)^{n-1}$ - keep in mind a geometric may not always look this neat. Look for patterns!

p-series $\sum_{1}^{\infty} \frac{1}{n^p}$

$p > 1$ converges; $p = 1$ is THE HARMONIC SERIES*

$p \leq 1$ diverges; less than 0 is actually just a polynomial

*When $p = 1$, it is preferable to use this justification

Other things to keep in mind:

$\sqrt{x^2} = |x| \Rightarrow$ useful for intervals of convergence

for $|ax + b| < 1$ radius is $\frac{1}{a}$, center is $-\frac{b}{a}$ and interval is $-\left(\frac{1+b}{a}\right) < x < \frac{1-b}{a}$

Always check endpoints of intervals when using the ♡ Ratio Test!!

Good luck y'all