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Geometric Series: $\sum_{n=1}^{\infty} a_1(r)^{n-1} = a_1 + a_1(r) + a_1(r)^2 + \dots a_1(r)^{n-1} + \dots = \frac{a_1}{1-r}$

GIVEN $|r| < 1$

Diverges otherwise

N^{th} term test: $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges

Intuition: If limit exists, then eventually you keep adding a finite number. If it's ∞, then the number you are adding is always getting bigger

DCT: Direct Comparison Test & LCT: Limit Comparison Test

Known Diverger

i) a_n & b_n are positive, cont, decreasing* on $[a, \infty)$

ii) $\sum_{a}^{\infty} b_n$ diverges (JUSTIFY)

iii) $a_n \geq b_n$ on $[a, \infty)$

$\therefore \sum_{a}^{\infty} a_n$ diverges by DCT Q.E.D.

$\longleftarrow$ DCT $\longrightarrow$

*Not strictly necessary

iii) $\lim_{n\to\infty} \frac{a_n}{b_n} \in \mathbb{R}$

$\longleftarrow$ LCT $\longrightarrow$

$\therefore \sum_{a}^{\infty} a_n$ diverges by LCT Q.E.D.

Known Converger:

i) a_n & b_n are positive, cont, decreasing* on $[a, \infty)$

ii) $\sum_{a}^{\infty} b_n$ converges (JUSTIFY)

iii) $a_n \leq b_n$ on $[a, \infty)$

$\therefore \sum_{a}^{\infty} a_n$ converges by DCT Q.E.D.

iii) $\lim_{n\to\infty} \frac{a_n}{b_n} \in \mathbb{R}$

$\therefore \sum_{a}^{\infty} a_n$ converges by LCT Q.E.D.

Integral Test: (IT) Always conclusive!!

$\sum_{a}^{\infty} a_n$ let $f(n) = a_n$ $f(x) =$ "stuff"

$f(x)$ is positive, continuous, and decreasing on $[a, \infty)$

EVALUATE the improper integral: $\int_{a}^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_{a}^{b} f(x)\,dx$

If the integral converges, so does the series.

If the integral diverges, so does the series.

Note: It does not matter what the integral converges to, that is not indicative of what the series converges to.

Ratio Test - Remember the heart!

i) a_n is positive

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$, $\sum a_n$ Converges

or

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$ $\sum a_n$ diverges

or

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ Inconclusive

Intuition: When you take the limit as it goes to infinity, you're essentially finding the common "ratio", almost like a geometric. So it makes sense that above 1 diverges, 1 is not conclusive, and under 1 converges.