SF2955 Homework 1

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1 A hidden Markov model for mobility tracking

A targets movement is described by the following model:

$$X_{n+1} = \Phi X_n + \Psi_n Z_n + \Psi_w W_{n+1} \tag{1}$$

Each X_n is a vector describing the target's state in position, velocity, and acceleration in the x_1 and x_2 directions. Z_n is the driving command modeled by a bivariate Markov chain, taking the values $(0,0), (3.5,0), (0,3.5), (0,-3.5), (-3.5,0)$. The states presents stationary, east, north, west, and south respectively. The chain evolves independently according to the probability matrix:

$$P = \frac{1}{20}\begin{pmatrix} 16 & 1 & 1 & 1 & 1 \\ 1 & 16 & 1 & 1 & 1 \\ 1 & 1 & 16 & 1 & 1 \\ 1 & 1 & 1 & 16 & 1 \\ 1 & 1 & 1 & 1 & 16 \end{pmatrix} \tag{2}$$

W_n is normally distributed noise variables, $N(0_{2x1}, \sigma^2 I)$, where $\sigma = 0.5$.

1.1 Problem 1

1.1.1 Is $\{X_n\}_{n\in N}$ a markov chain?

Markov chains are only dependant on what the current state is, and not what has happened previously. Rearranging equation 1 to:

$$\Phi X_n = \Phi X_{n+1} - \Psi_n Z_n - \Psi_w W_{n+1} \tag{3}$$

X_n is as seen in equation 3 dependent on X_{n+1}, which is not a previous state, W_{n+1}, which is random noise, and Z_n. The current value of Z_n is dependent on its previous state, and therefor X_n is not a markov chain.

1.1.2 Is $\{\tilde{X}_n\}_{n \in N}$ a markov chain?

In this scenario $\{\tilde{X}_n\} = (X_n^T, Z_n^T)^T$. Now Z_n is taken into account, and both the previous states of X_n and Z_n set the future value, therefor $\{\tilde{X}_n\}$ is a markov chain.

1.1.3 Simulated trajectory

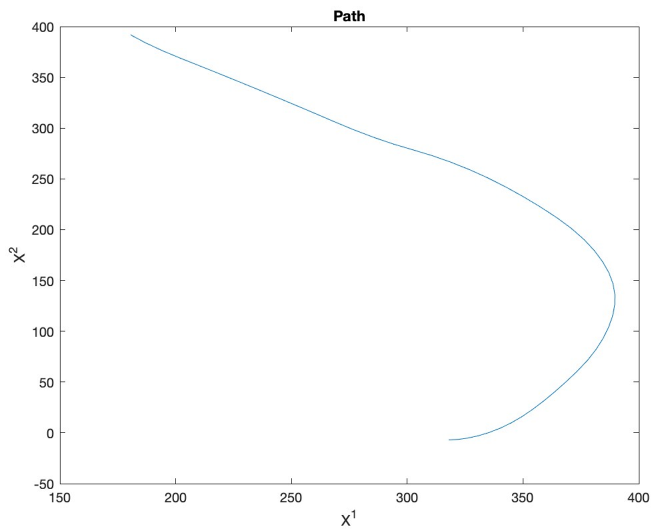

Figure 1: m = 50

In figure (1) a trajectory was simulated using Matlab and $m = 40$.

1.2 Problem 2

1.2.1 Finding the transition density of $p(y_n \mid \tilde{x}_n)$

As the target is moving it measures signal strengths (RSSI) from the base stations (BS) in a cellular network. There are 6 base stations whose positions $\{\pi_l\}$ are given in *stations.mat*. The received signal (dB) can be modeled as:

$$Y_n^l = v - 10\eta log_{10} \parallel (X_n^1, X_n^2)^T - \pi_l \parallel + V_n^l \tag{4}$$

Where $v = 90$ (dB) and $\eta = 3$. V_n^l are independent gaussian noise variables with mean 0 and standard deviation $\zeta = 1.5$ (dB).

$\{\tilde{X}_n, Y_n\}_{n \in N}$ forms a hidden markov model.

$$E[Y_n^l] = v - 10\eta log_{10} \parallel (X_n^1, X_n^2)^T - \pi_l \parallel \tag{5}$$

$$Var[Y_n^l] = Var[V_n^l] = \zeta^2 \tag{6}$$

And thus $p(y_n \mid \tilde{x}_n)$ forms a gaussian distribution as follows:

$$y_n \mid \tilde{x}_n \sim N(v - 10\eta log_{10} \parallel (X_n^1, X_n^2)^T - \pi_l \parallel_{1x6}, \zeta^2 I) \tag{7}$$

1.3 Problem 3

1.3.1 Estimating the target's position using SIS

Using the observation stream in *RSSI-measurements.mat* sequential importance sampling can be implemented to provide estimates of $\{\tau_n^1, \tau_n^2\}_{n=0}^m$, the target's positions.

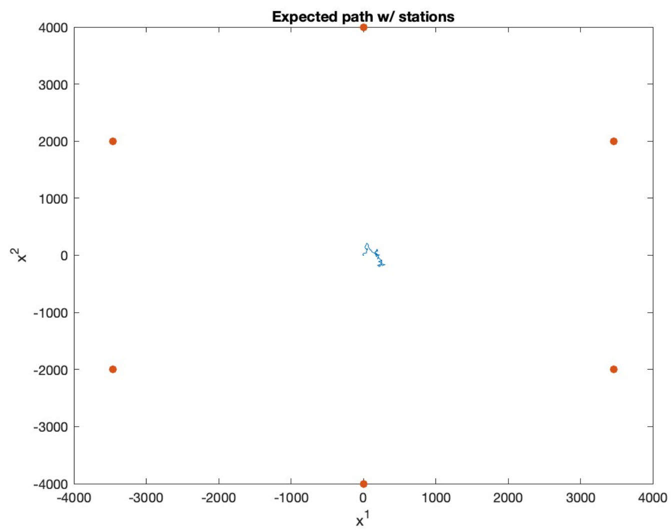

Figure 2: Trajectory and stations, N = 10000.

The stations look very symmetric and the trajectory does not show a lot of movement. Now plotting histgrams of the importance weights:

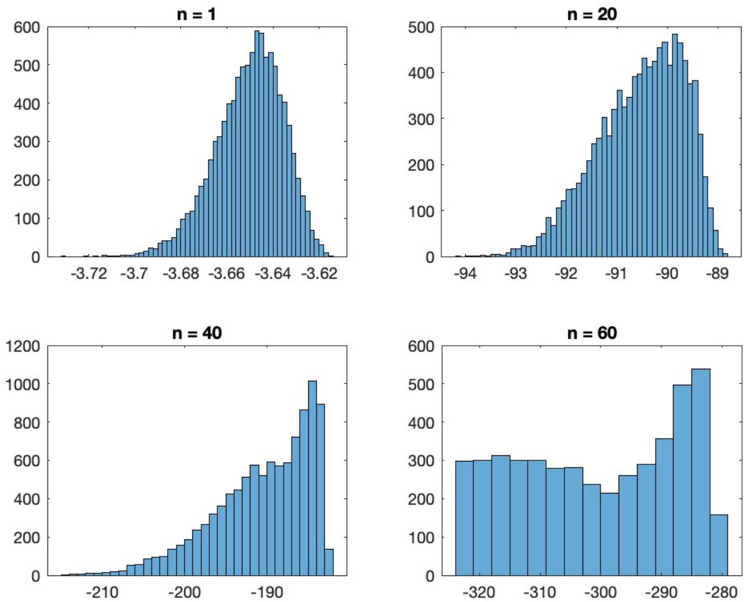

Figure 3: Histograms of different weights.

In order to compute the efficient sample sizes first CV was calculated with:

$$CV = (\frac{1}{N}\sum(N\frac{\omega_n^i}{\sum\omega_n^j} - 1)^2)^{1/2} \tag{8}$$

Now computing the efficient sample size with the equation:

$$ESS = \frac{N}{1 + CV^2} \tag{9}$$

And finally get the plot:

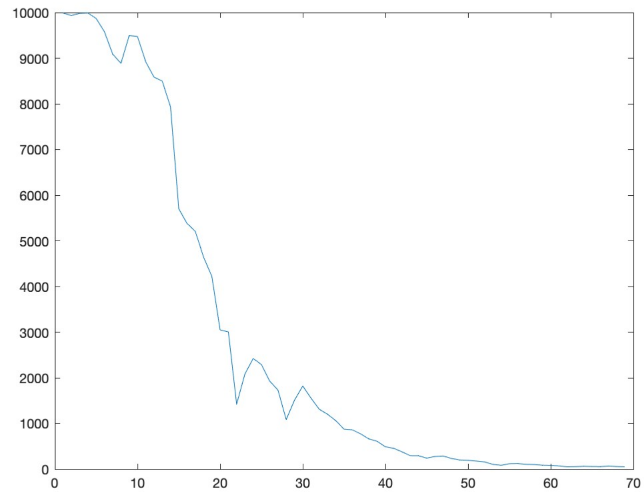

Figure 4: ESS for N = 10000.

Where the lowest efficient sample sizes are by ≥ 60.

1.4 Problem 4

1.4.1 Trajectory using SISR

Now the trajectory using the same data as before is going to be computed using sequential importance sampling with resampling. Again, $N = 10000$.

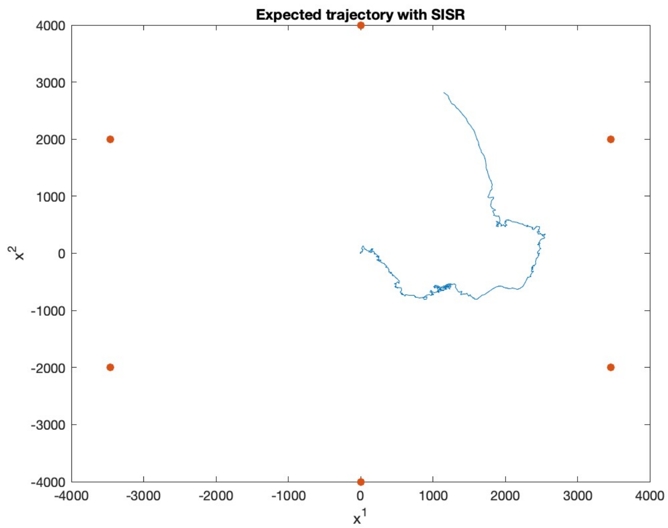

Figure 5: Expected trajectory with SISR.

In figure (5) the expected trajectory is longer than before, since SISR prioritizes larger weights and does not move towards 0. Now plotting the highest probability direction:

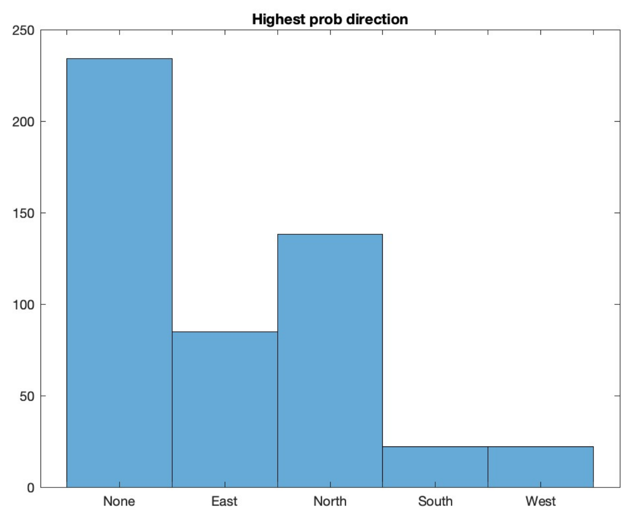

Figure 6: Highest probability direction each step

North was the direction with the highest probability, which coincides with the calculated trajectory using SISR.

1.5 Problem 5

Realistically the parameters are unknown and needs to be estimated. In problem 5 all parameters except ζ are now calibrated previously.

1.5.1 Maximum likelihood estimation of ζ.

The standard deviation is somewhere on the interval $\zeta = (0, 3)$, and will be estimated using SISR and monte carlo. A log likelihood function will be computed for each standard deviation.

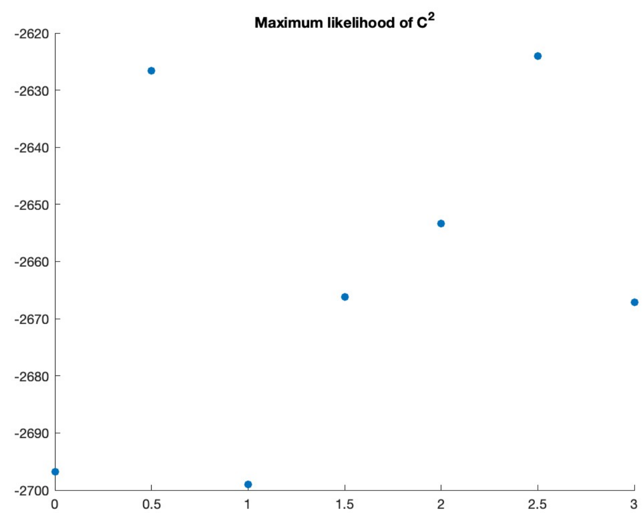

Figure 7: Caption

Looking at figure (7), $\zeta^2 = 2.5$ has the highest likelihood. Now computing the trajectory with the new ζ^2:

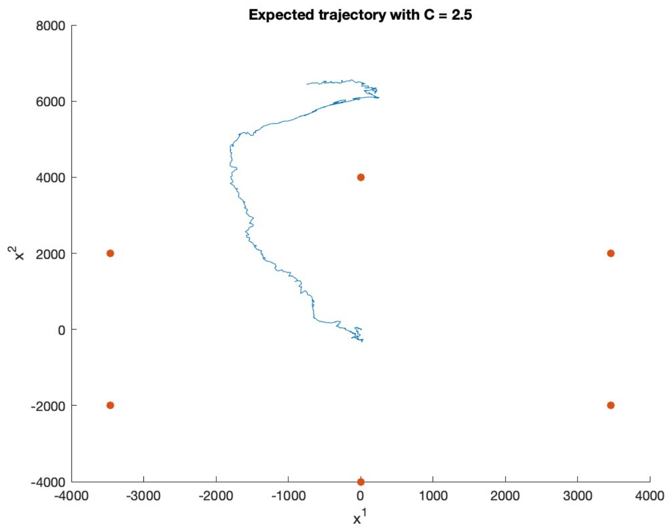

Figure 8: Caption