# *“Price discovery under limited information”*


by

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## Abstract:

In this thesis, we develop a model based on a closed homogeneous market to examine the existence of an equilibrium price in a market. Assumptions in our markets deviate from the assumptions of the Walrasian theorem market. The model employs a non-tatonnement process, where prices are adjusted iteratively. Different cost and price determination coefficients are checked into the simulation. The results reveal that, in the majority of cases, prices show a tendency to converge. However, further investigations are required in this area as there are instances where the average price values still significantly differ from each other.

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## Introduction

This thesis explores the concept of price convergence. A model distinct from the one proposed in the Walrasian theorem has been developed and simulated. The analysis of the results focuses on the main research question: Do prices converge to some clearing price?

Today's neoclassical economic model is based on the theory developed by Walras (1954). Schinkel (2006) questions the effectiveness of this theory. He states that Walrasian theory is built upon strong assumptions: the existence of some authority ('auctioneer') that behaves entirely rationally and knows all the information about the agents' prices, quantities and preferences; lack of trades until the equilibrium price is found. Schinkel states that these conditions are not realistic. He calls into question the fundamental tenets of the theory, namely the existence of an equilibrium price and the tendency of all prices to converge towards this price in a real market.

Gintis (2007) agrees with the view that the Walrasian theory is unrealistic. Specifically, he argues that Walras's tatonnement process and public prices could lead to market instability (Gintis (2007)). As evidence, Gintis (2007) gives Scarf's example in which all the conditions of the Walrasian theory are satisfied, but the prices do not converge to the equilibrium price. He then demonstrates a model based on a non-tatonnement process and private prices. The results of the iterations show that in such a model, prices do indeed converge to the clearing price with a stochastic error term.

In turn, Kirman and Vriend (2001) also have built a model based on the non-tatonnement process. They have added the variable loyalty to the classical model since this variable is often either simplified or ignored in research. In contrast, it can be relevant in real-life markets. It is also important to note that their model is only related to the Marseille fish market, i.e., this model can only show a partial equilibrium price and behaviour of only one market, not all economies. In their study, Kirman and Vriend (2001) have constructed a model in which customer loyalty depends on its effect on the price and the effect is determined by the sellers themselves. That is, if loyal customers get fish at better prices, then customers become loyal (that is, they always go to the same seller), but if the effect of loyalty is negative, then customers will buy from different sellers. Thus, Kirman and Vriend (2001) have complemented the neoclassical economic model and showed that prices in this model converge to equilibrium prices using a non-tatonnement process, and the obtained results of price and quantity behaviour are similar to those obtained from the real fish market in Marseille.

Additionally, Pinto, Ferreira, Finkenstädt, Oliveira, and Yannacopoulos (2012), Lahkar (2020), and D'Agata (2022) also argue that the Walrasian model, with its assumptions, is unrealistic. Therefore, Pinto et al. (2012) have developed their own random exchange economy model with two goods. The agents are randomly paired, and the Cobb-Douglas utility function characterises their preferences in the model. The agents have symmetry conditions for initial endowments and the distribution of initial preferences (Pinto et al., 2012). Eventually, Pinto et al. (2012) have proven that market prices converge to the Walrasian price using such a model.

In Lahkar's (2020) research, Cournot competition is employed to prove that the aggregate price in the market eventually equals the Walrasian equilibrium.

Lahkar (2020) uses a market with minimal information, where producers only know the market price and their own cost functions. As a result, Lahkar (2020) demonstrates that using best response dynamics under limited information in Cournot competition results in the market price converging to the Walrasian equilibrium.

D'Agata (2022) investigates the existence of the Walrasian equilibrium in a market without homogeneity and Walras' Law. D'Agata (2022) indeed proves the existence of the Walrasian equilibrium in a monetary economy with boundary conditions, which describe the relationship between prices and supply and demand excess functions.

Despite criticism of the Walrasian process, some studies demonstrate the effectiveness of the Walrasian tatonnement process. For example, Joyce (1984) has conducted an experiment in which he has shown that when all basic assumptions are met, prices converge to an equilibrium price. His experiment demonstrates that the Walrasian tatonnement mechanism is stable and highly efficient in practice. Based on Joyce's experiment, Bronfman, McCabe, Porter, Rassenti, and Smith (1996) conducted their own experiments. The results of both experiments held by Joyce (1984) and Bronfman et al. (1996) are similar. However, Bronfman et al. (1996) have noticed that the Walrasian tatonnement mechanism works much better in markets with a single commodity than in markets with multiple commodities.

Thus, the debate about the efficacy and plausibility of the Walrasian theory and the models built on its basis continues. This study proposes a model based on the production economy using a non-tatonnement process, where buyers know all the prices on the market and firms know only their own cost coefficients and the number of goods sold. The model is simulated, and the results are analysed. The main research question: Do prices converge to some clearing price in the market?

**Model**

There are a set of producers $J = \{1, \dots, J\}$ indexed by $j$ and a set of buyers $I = \{1, \dots, I\}$ indexed by $i$ — different combinations of $J$ and $I$ are checked to see their influence on the results. The market is homogeneous, i.e., firms produce the same product. Each period consists of $T$ episode. An episode or iteration is just one step in a period, whereas a period means a complete cycle. In terms of real-life analogy, an episode or iteration can be considered as a small time interval within which changes can occur in the model, typically daily, weekly, or monthly. On the other hand, a period represents a larger time interval that is used to analyse results obtained over that duration, such as a quarter, a year, or several years.

The seller aims to maximise their profit function. To achieve this, every company uses the following profit formula:

$$\pi_{j,t} = p_{j,t} \sum_{i} x_{i,j,t} - \mathrm{c}_j \hat{q}_{j,t}^2 - \theta_j q_{j,t}^{left}$$

Where $\sum_i x_{i,j,t}$ is the total quantity of products purchased from seller $j$ by all consumers at time $t$, $p_{j,t}$ is the price set by the seller $j$ at time $t$, $q_{j,t}^{left}$ is the number of goods left from the previous period; $\hat{q}_{j,t}$ is the number of goods produced by firm $j$ at time $t$, $c_j$ is cost coefficient of the producer $j$ and $\theta_j$ is a cost coefficient for keeping products left from the last episode. The coefficients

$\theta_j$ and $c_j$ are set uniformly at random for each producer at the beginning of the simulation.

The price $p_{j,t}$ is calculated using the quantities from the previous episode and constants:

$$p_{j,t} = \begin{cases} p_{j,t-1} + \lambda\left(\sum_i x_{i,j,t-1} - q_{j,t-1}^{left} - \hat{q}_{j,t-1}\right), \text{if } \sum_i x_{i,j,t-1} < q_{j,t-1}^{left} + \hat{q}_{j,t-1} \\ p_{j,t-1} + \varphi \qquad , \text{if } \sum_i x_{i,j,t-1} = q_{j,t-1}^{left} + \hat{q}_{j,t-1} \end{cases}$$

$\sum_i x_{i,j,t-1}$ is the total quantity of products purchased from seller $j$ by all consumers at time $t-1$. Price $p_{j,0}$ is chosen randomly at the beginning of the simulation. Different constants $\lambda \in \mathbb{R}^+$ and $\varphi \in \mathbb{R}^+$ combinations are examined to investigate their impact on the final outcomes. Basically, the concept of a company selling all its products indicates a high demand for those products in the market. As a result, the seller can take advantage of this situation by raising the prices of their products without negatively impacting their profits. Conversely, if the demand for the products is not met, it means that there is an oversupply in the market. In such a scenario, the seller must adjust the price and quantity of their produced goods to ensure that all their inventory is sold, and they can achieve their expected profits.

The maximisation occurs with respect to the variable $\hat{q}_{j,t}$, since all the remaining variables are available. The cost coefficient $c_j$ is known since it is derived randomly at the beginning of the simulation. And the price $p_{j,t}$ is also available. The solution for $\hat{q}_{j,t}$ is:

$$\hat{q}_{j,t} = \frac{p_{j,t}}{2c_j}$$

The number of sellers $J$ is constant. Each producer has its own capital $K_{j,t}$, which is the sum of the firm's profits over the entire period of its existence: $K_{j,t} = \sum_{\tau=1}^{t} \pi_{j,\tau}$. If the firm's capital becomes negative, the producer goes bankrupt and disappears from the market, and a new producer takes his place with new coefficients for the cost functions.

The number of buyers $I$ is also constant. Consumers choose quantities to maximise their utility function:

$$\mathrm{u}_i(x_{i,j,t}, m_i) = b_i \, log\left(\sum_j x_{i,j,t}\right) - m_i$$

subject to

$$\sum_j p_{j,t} \, x_{i,j,t} + m_i \leq \omega_i$$

Where $\sum_j x_{i,j,t}$ is the sum of all products bought by buyer $i$ from all the seller at time $t$, $\omega_i$ is the salary of the buyer $i$, $m_i$ is consumption of money, $b_i$ is random uniformly distributed coefficients, which are different for each buyer. $b_i$ is set at the beginning of the simulation for each buyer. And $x_{i,j,t}$ is the number of goods the buyer $j$ wants to buy from producer $j$ at time $t$. The solution for $x_{i,j,t}$ is then:

$$x_{i,j,t} = \frac{b_i}{p_{j,t}} - \sum_{k=1}^{j-1} x_{i,k,t}$$

where $\sum_{k=1}^{j-1} x_{i,k,t}$ is the sum of the ordered quantities of purchases from each of the previous firms. The order depends on the price. The firm with the lowest price is the first in this queue for each purchaser; if this firm runs out of goods, the buyer goes to the next firm with the lowest price and so on, until the consumer is satisfied or until the market runs out of goods. The exact process is repeated for each consumer. The buyers are randomly shuffled each episode, and their order is constant for all sellers. The consumer's income is constant for the whole simulation and is defined randomly initially. It is assumed that if the buyer does not spend all the money in this market, the consumer can spend it in other markets. In other words, consumers do not have savings, and the amount of money they can spend is constant each period.

The equilibrium price is the price at which the demand equals the supply in the market. In our model, this can be described by the following formula:

$$\sum_i \frac{b_i}{p} - \sum_j \frac{p}{2c_j} = 0$$

where $p$ represents the equilibrium price. Thus, the obtained value of $p$ from this formula is used as the clearing price in each simulation to analyze the price dynamics of the sellers.

Thus, this model aims to simulate agents' behaviour and price convergence dependence on various factors in real markets. However, it is necessary to note that this model only investigates partial market equilibrium, and further research is necessary to obtain a comprehensive understanding of the market dynamics.

**Results**

Our research is aimed to answer the question: "Do prices eventually converge to a clearing price?" To achieve this, we have developed a market model explained in the previous section. A Python program is used to simulate the model and obtain the results described in this section.

Figure 1 shows the price change in the market with $\lambda = 0.02$ and $\varphi = 0.01$. The number of iterations in the simulation is 1000. The red dashed line shows the equilibrium price in the market. As can be seen, prices do converge to the equilibrium price. It takes them about 400 iterations to converge due to the small values of the $\varphi$ and $\lambda$ coefficients, as prices change slightly at each iteration. The cost coefficients do not differ significantly, but the coefficient of seller 1 is smaller than that of seller 0. After stabilisation, the variance of both prices is extremely small. However, variance still does not equal zero due to the discrete nature of the price determination function. However, over time the average values converge to the clearing price. The clearing price is significantly higher than the cost factors due to the market's imbalance of a large number of buyers and a small number of sellers.

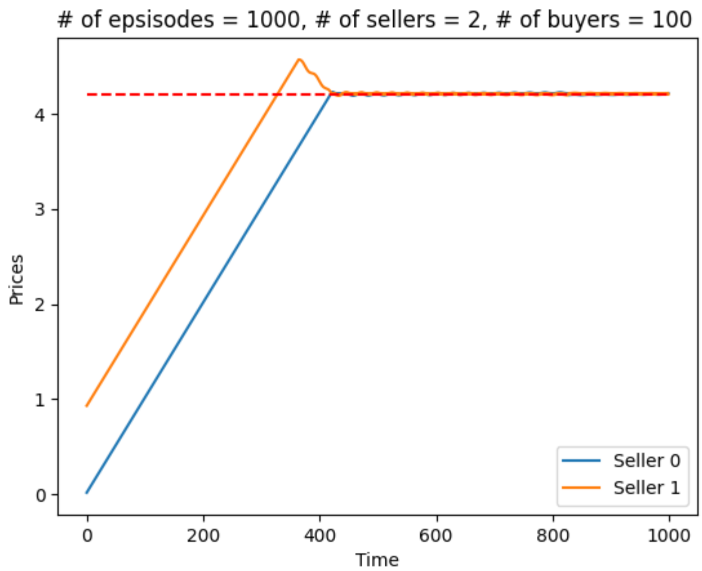


*Figure 1* $(\lambda = 0.02; \varphi = 0.01)$

Figure 2 shows a simulation chart with identical starting prices and cost coefficients for the two sellers, as in the first figure. However, the $\lambda$ and $\varphi$ have been increased. As can be seen on the graph, the sellers' prices do indeed converge to the clearing price much more quickly, but the variance of their fluctuations has become much more remarkable. After achieving stabilisation, both price variances become approximately 100 times higher than those in the previous simulation. This indeed confirms the dependence of price variation on $\lambda$ and $\varphi$ coefficients. It is also important to note that the price variance of seller 1 is almost twice that of seller 0. The mean values of prices after the stabilisation of both sellers have remained nearly the same as in the previous simulation and are still close to the equilibrium price. Also, the equilibrium price itself has not changed either, as the values of the $\lambda$ and $\varphi$ coefficients do not affect it in any way.

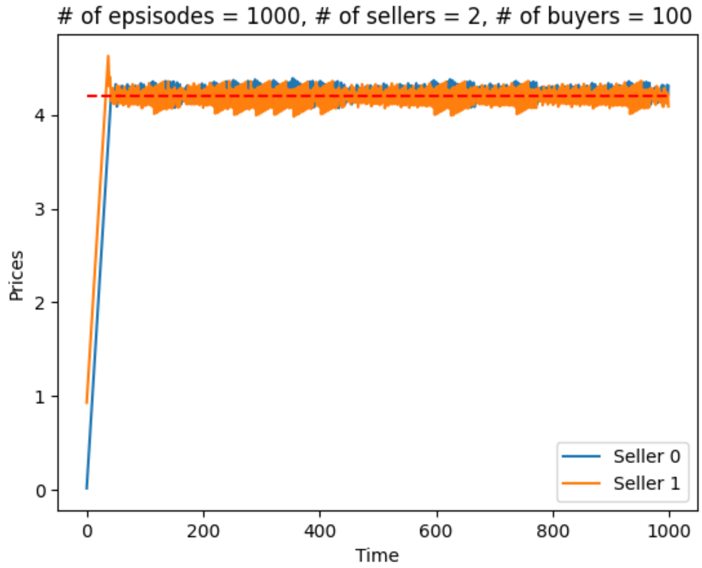


*Figure 2* $(\lambda = 0.5; \varphi = 0.1)$

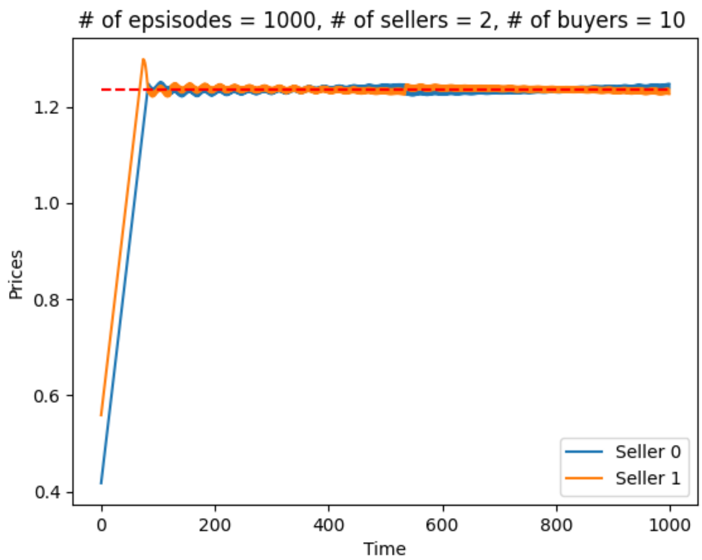


*Figure 3* $(\lambda = 0.02; \varphi = 0.01)$

The graph in Figure 3 has the same parameters as Figure 1, except it illustrates a decrease in the number of buyers from 100 to 10. Since all the coefficients have remained the same and only the number of buyers in the market has changed, the variances of the prices have remained almost the same, approximately equal to zero. The prices are still converging to the clearing prices. However, the average value of the prices, as well as the equilibrium price itself, have changed. As expected, they have fallen due to increased competition, i.e. fewer buyers for the same number of sellers. Also, compared to the first simulation, prices have reached the clearing price much faster. However, this can be explained by the fact that the equilibrium price itself decreased by a factor of 3.4 since the prices of the first simulation are at approximately the same level at the 80th iteration.

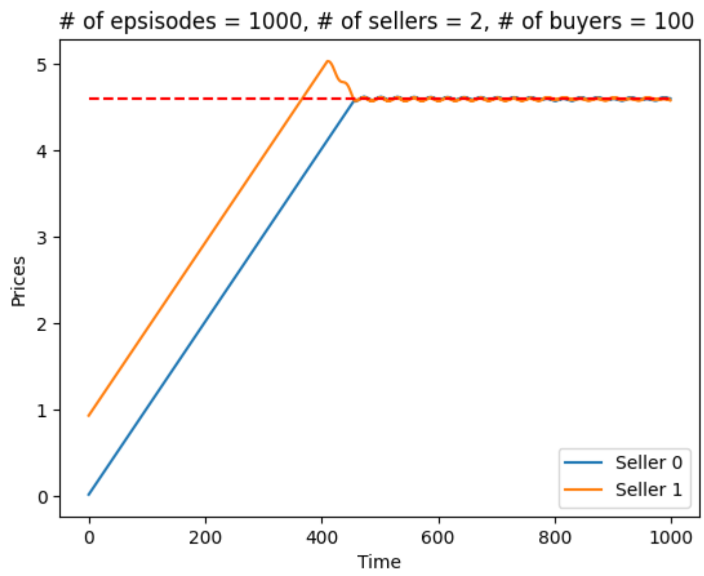


*Figure 4* ($\lambda = 0.02; \varphi = 0.01$)

Following, we use the same coefficients and parameters as in the first simulation, but now both cost coefficients are equal to each other. Specifically, the cost coefficient of seller 1 is equal to the cost coefficient of seller 0 from the first simulation. Since in the first simulation the cost coefficients for both sellers do not differ dramatically, the trends in price changes remain similar even with equal coefficients, as can be observed in Figure 4. However, the prices take longer to converge to the clearing price. The clearing price has increased. This could be due to the fact that neither firm can achieve higher profits at a lower price, indicating less competition in the market. After price stabilization, the profits become approximately equal between the firms. The average profits of both firms in the fourth simulation depicted in Figure 4 are approximately equal to the mean of the average profits of both firms in the first simulation. The price variances after stabilization are approximately equal to each other but larger than in the first simulation. The average price values after stabilization are also approximately equal. However, in general, seller 0 has a higher price than seller 1. This may be related to the initial price values. A zoomed-in version of the graph from Figure 4 can be seen in Figure 5, where the difference in the average price values between the two sellers becomes more noticeable.

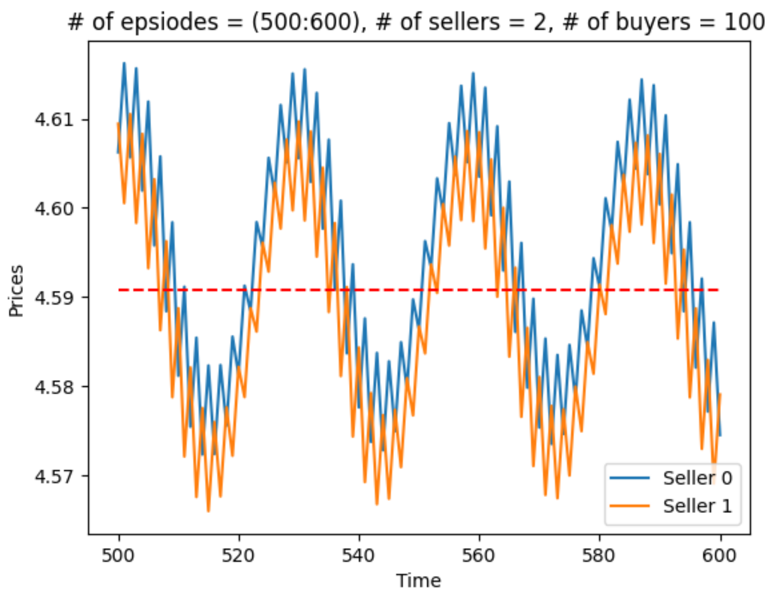


*Figure 5* ($\lambda = 0.02; \varphi = 0.01$)

Figure 6 presents the results of the fifth simulation. In this simulation, the initial parameters remain the same as in the first simulation, including the cost coefficients for the first two sellers, φ and λ coefficients, and the number of buyers. The only change is the number of sellers, which has increased. As expected, the equilibrium price has decreased compared to the first simulation.

However, the price is lower than in the third simulation (Figure 3). This suggests that the equilibrium price depends not only on the ratio of sellers to buyers but also on the actual number of sellers in the market. In the third simulation, the sellers-to-buyers ratio was $\frac{2}{10} = 0.2$, while in the fifth simulation, depicted in Figure 6, this ratio is $\frac{15}{100} = 0.15$. If the clearing price solely depended on the sellers-to-buyers ratio, it should have been higher in the fifth simulation compared to the third simulation. However, this is not the case. On the other hand, the price variances have significantly increased compared to the first and third simulations. This could be caused by increased competition in the market due to the substantial increase in the number of sellers.

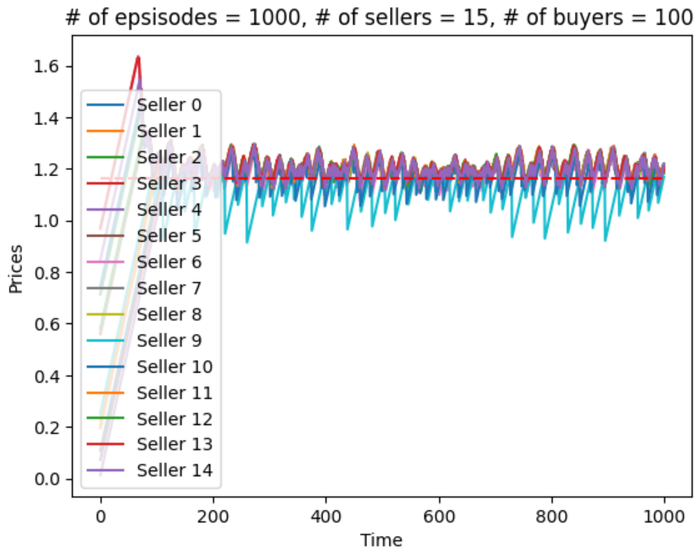


*Figure 6* $(\lambda = 0.02; \varphi = 0.01)$

The seller with the lowest cost coefficient, seller 9, has the highest price variance. Conversely, seller 13, whose coefficient is not the highest but is above the mean of the cost coefficients, has the lowest price variance. Seller 13 has the highest starting price in the market. However, further research is needed to check if there is a connection between the price variance and the initial price of a seller.

The overall trends are consistent with the previous simulations: all prices converge to the clearing price and then fluctuate within a range determined by their variances. In the beginning, sellers with the highest initial prices peak in market prices, as other sellers produce fewer goods than consumers desire, and there is excess demand in the market. However, as prices of all sellers have approached the clearing price, prices above the clearing price start to decrease. The average values of prices after stabilization tend to approach the equilibrium price, but due to big variances, they still differ. This can also be observed in Figure 7, which displays an enlarged segment of the graph from Figure 6.

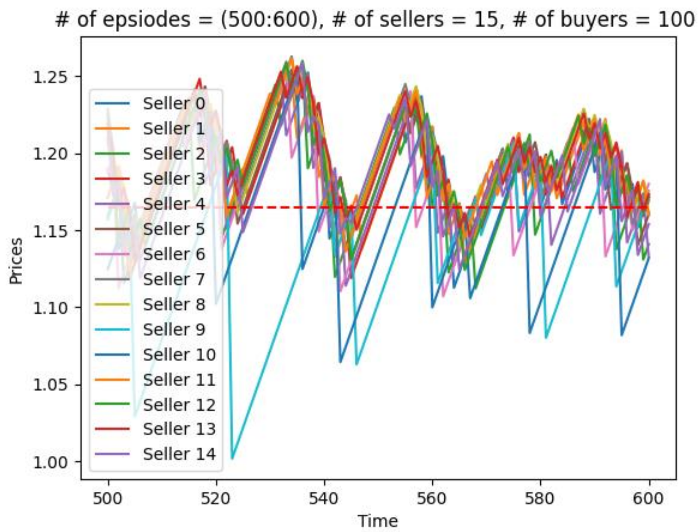


*Figure 7* ($\lambda = 0.02; \varphi = 0.01$)

Figure 8 displays the simulation results with the same parameters as Figures 6 and 7, except that all cost coefficients are equal to the cost coefficient of seller 0. The equilibrium price has increased 1.4 times compared to simulation 5, depicted in Figures 6 and 7. A similar increase in the equilibrium price occurs in simulation 4, depicted in Figure 4, where there are only two sellers in the market with identical cost coefficients. This indicates a possible decrease in price competition, as all sellers have the same cost coefficients. Price variances are also significantly lower than in the previous simulation but higher than in the simulation with fewer sellers in the market, suggesting that price variance partially depends on the number of sellers in the market. Due to relatively high price variances compared to the market with two sellers, the average profit values of the firms also fluctuate significantly, but to a lesser extent than in the previous simulation with sellers having different cost coefficients. However, the overall average profit value among firms is approximately equal to the mean value of the average profits from the previous simulation, as in simulation 4. The general price trends remain consistent, with prices still converging to the clearing price.

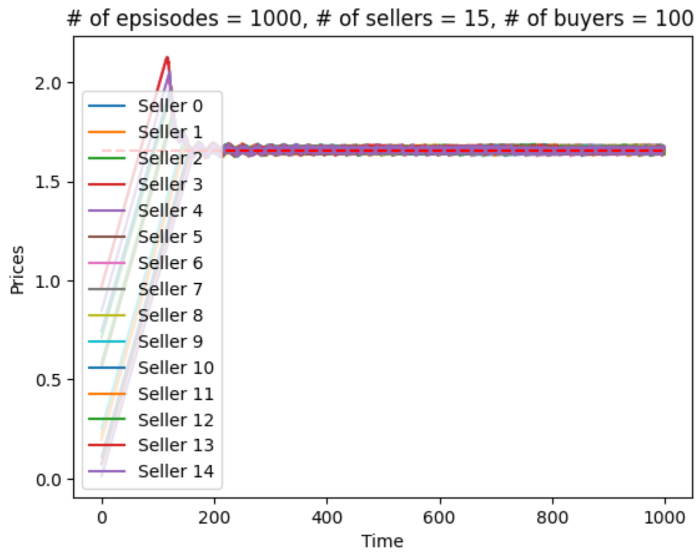


*Figure 8* ($\lambda = 0.02; \varphi = 0.01$)

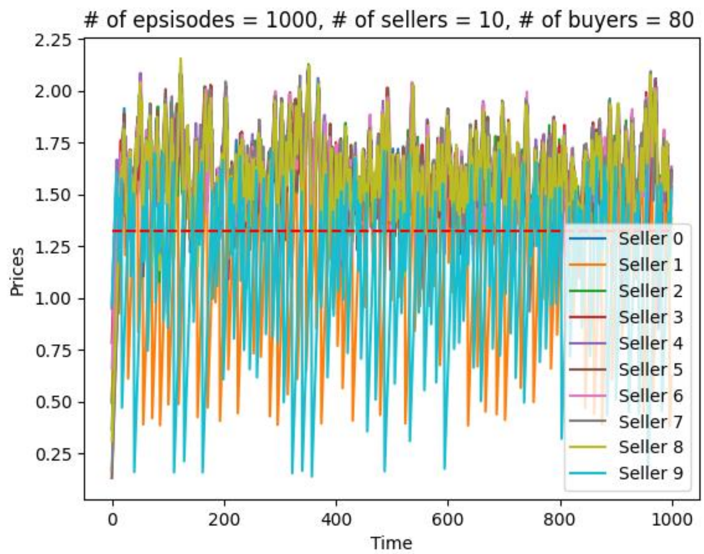


*Figure 9* ($\lambda = 0.1; \varphi = 0.08$)


*Figure 10* ($\lambda = 0.1; \varphi = 0.08$)

The previous simulations investigate the first simulation with varying parameters. To verify the existence of price convergence in our model, it is necessary to consider completely different parameters and observe the behaviour of prices under these conditions. Figures 9 and 10 depict a graph with 80 buyers and 10 sellers, $\lambda = 0.1$, $\varphi = 0.08$. All cost coefficients are new and different from those used in the previous simulations. As seen in the graphs, prices fluctuate around the clearing price. However, the price variances are considerably higher compared to the previous simulations, which is a result of higher competition due to a large number of sellers and a small number of buyers. Seller 1 exhibits the maximum price variance, even though seller 1 does not have the minimum cost coefficient among the sellers. However, seller 9 is second in price variance, while its cost coefficient is the minimum in the market. This indicates a correlation between price variance and the coefficient, although other factors can also influence price variance. The equilibrium price is significantly low due to the large number of sellers in the market. Although Figure 9 illustrates price convergence, the average market prices are higher than the equilibrium price, with a mean of the average prices being 1.492, while the equilibrium price is 1.324. This is particularly noticeable in Figure 10, which represents a zoomed-in section of Figure 9. However, this discrepancy may be attributed to the high variance of prices. None of the sellers goes bankrupt throughout the simulation.

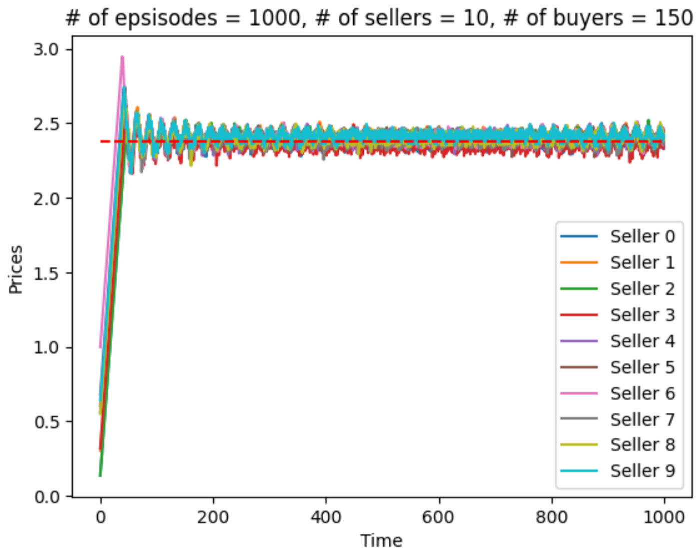


*Figure 11* ($\lambda = 0.02; \varphi = 0.05$)

The eighth simulation is depicted in Figure 11. The coefficients λ and φ are equal to 0.02 and 0.05, respectively, the number of buyers is 150, and the number of sellers is 10. The seller with the lowest price coefficient, seller 3, exhibits the highest price variance. Overall, the price variances of sellers have decreased significantly compared to the previous simulation due to reduced competition resulting from an increased number of buyers and lower values of $\lambda$ and $\varphi$ coefficients. The equilibrium price has also increased due to increase in number of bueyrs. The overall behaviour of prices demonstrates their convergence towards the clearing price. The average prices among sellers are much closer to the equilibrium price than in the previous simulation.

All previous simulations demonstrate a tendency for prices to converge to the clearing prices. However, the ninth simulation shows a result that completely contradicts the concept of price convergence, and we cannot explain it except for the model's limitations. As can be seen in Figure 12, the prices of the sellers after stabilisation differ significantly from each other and the equilibrium price. Their cost coefficients also vary greatly, with seller 0 having a price coefficient of 0.222 while seller 1 has a coefficient of 0.919. The price variances are large compared to the first simulation and almost equal among sellers, but seller 1 has a slightly higher price variance, despite having the greater cost coefficient. After stabilisation, the average profit for both firms is approximately 8 and 7 for sellers 0 and 1, respectively. The quantity of goods sold is also significantly different between the two firms, with seller 0 selling on average nearly twice as much as seller 1. Such price movements that contradict the idea of price convergence may be caused by limitations inherent in the model. However, additional research is needed to identify the actual cause of such price behaviour.

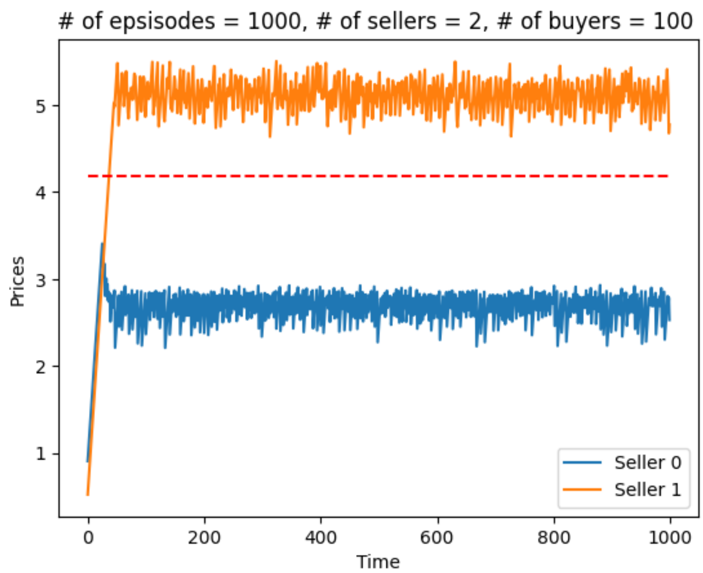


*Figure 12* $(\lambda = 0.5; \varphi = 0.1)$

## Conclusion

This research investigates the concept of price convergence, initially studied by Walras. According to the Walrasian theorem, there exists an equilibrium price on the market, and all other prices converge to this equilibrium price under certain assumptions. However, some researchers have raised doubts about the realism of the assumptions from the Walrasian theorem. Therefore, we developed a market model that is more realistic than the one used by Walras. This model is used to simulate a production closed homogeneous market to analyse the results and address the central question of this research: do prices converge to the clearing price?

The obtained results have indeed demonstrated that in the majority of cases, prices converge to the clearing price. The results have also revealed certain relationships between price variance and the $\lambda$, $\varphi$ coefficients, or the number of sellers and buyers in the market. High competition on the market, caused by a small number of buyers or a large number of sellers, leads to an increase in the average price variance for all sellers. When considering each firm individually, a low cost coefficient can contribute to an increase in price variance. The clearing price is also partially influenced by market competition, precisely the sellers-to-buyers ratio. A smaller number of sellers results in a higher equilibrium price, assuming the number of buyers remains constant. However, changes in the clearing price do not always proportionally correspond to changes in the number of sellers on the market, indicating the influence of other factors.

Nonetheless, an unexpected result in the last simulation contradicts the concept of price convergence. Prices in the final simulation have significantly different average values and do not tend towards the clearing price. Such price movements could be attributed to limitations or assumptions of the model. Nevertheless, the actual reasons for such price behaviour can not be identified.

All the obtained results point to the potential existence of price convergence in real markets and various factors that can influence both the equilibrium price and the tendency of prices to converge.

However, the model presented in this research still has several limitations, as it represents only a closed homogeneous market with a constant number of sellers and buyers. Therefore, a definitive answer to the research question cannot be provided. To gain a better understanding of price behaviour in real markets, further research is needed. Additional studies can help us determine if price convergence truly exists and provide more insight into the reasons and patterns behind price movements.

## References:


BRONFMAN, C., K. MCCABE, D. PORTER, S. RASSENTI, AND V. SMITH. (1996): “An Experimental Examination of the Walrasian tâtonnement Mechanism,” *The Rand journal of economics*, 27. RAND Journal of Economics, 681–99.

D’AGATA, A. (2022): “Walrasian equilibrium without homogeneity and Walras’ Law,” *Mathematical social sciences*, 119, 68–75.

GINTIS, H. (2007): “Dynamics of General Equilibrium,” *The Economic journal (London)*, 117, 1280–1309.

JOYCE, P. (1984): “The Walrasian tâtonnement Mechanism and Information,” *The Rand journal of economics*, 15. RAND Journal of Economics, 416–425.

KIRMAN, A. P., AND N. J. VRIEND. (2001): “Evolving market structure: An ACE model of price dispersion and loyalty,” *Journal of economic dynamics & control*, 25. Journal of Economic Dynamics and Control, 459–502.

LAHKAR, R. (2020): “Convergence to Walrasian equilibrium with minimal information,” *Journal of economic interaction and coordination*, 15, 553–78.

PINTO, A. A., M. FERREIRA, B. F. FINKENSTÄDT, B. OLIVEIRA, AND A. N. YANNACOPOULOS. (2012): “On the convergence to Walrasian prices in random matching Edgeworthian economies,” *Central European journal of operations research*, 20, 485–95.

SCHINKEL, M. P. (2006): “Disequilibrium Dynamics and Aggregate Excess Demand: On a Homunculus Fallacy in Economic Theory,” *History of political economy*, 38, 189–212.

WALRAS, L. (1954): “Elements of Pure Economics,” *Allen and Unwin*