1 Summary

The basic idea of this exercise is to write a simple solver for a system of ODEs,

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(t, \mathbf{x}),\ \mathbf{x}(t_0) = \mathbf{x}_0,\ \mathbf{x} \in \Re^n.$$

The particular system to be solved by the program is defined in a Fortran module, `equations.f90`, which implements the function $\mathbf{F}(t, \mathbf{x})$ and provides the initial state, $\mathbf{x}_0$. To begin with, the `equations.f90` module should be implemented to describe the Lorenz system,

$$\begin{aligned}
\frac{dx}{dt} &= \sigma(y - x), \\
\frac{dy}{dt} &= x(\rho - z) - y, \\
\frac{dz}{dt} &= xy - \beta z,
\end{aligned}$$

with $\rho = 28$, $\sigma = 10$, and $\beta = 8/3$. Plotting the integrated trajectory in the xz-plane should yield the butterfly-shaped Lorenz attractor.

The particular numerical method used by the solver may be any reasonable method, but all information about $\mathbf{F}(t, \mathbf{x})$ must be obtained through the `equations.f90` module (i.e., the solver should be written generically).

Integration of the system through time will yield a sequence of state vectors, $\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k$ valid at $t = 0, \Delta t, 2\Delta t, \ldots, k\Delta t$. So that a user can examine these states, the solver should write them to a file at every time step using the Fortran module `observer.f90`, whose job is to take a state vector, format it appropriately, and write it to a file. The interface to the `observer.f90` module should not be modified, and the initial implementation of the module should simply write vectors to a text file, with one vector per line and values in the vector written in a reasonable format.

2 Requirements

The solver should meet the following requirements.

1. The particular system to be integrated should be implemented using the `equations.f90` module, whose interfaces cannot change.
2. The size of the system's state vector must be obtained by the solver using the `get_system_size()` function in the `equations.f90` module.
3. Writing of the system state to a file must be accomplished via the `observer.f90` module, whose interfaces cannot change. The initial implementation should write all state vectors to the same text file, with one vector per line. System state should be written at every time step.
4. The time step, Δt, to be used by the solver, as well as the total number of time steps to take, must be provided at run-time by the user.
5. Code should be written in Fortran90/95.