CS 2305 Homework #1

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1. Construct a truth table for the compound proposition $(p \to r) \vee \neg(p \wedge q)$. Which of tautology, contradiction or contingency is it?

p	r	q	$p \to r$	$p \wedge q$	$\neg(p \wedge q)$	$(p \to r) \vee \neg(p \wedge q)$
T	T	T	T	T	F	T
T	T	F	T	F	T	T
T	F	T	F	T	F	F
T	F	F	F	F	T	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

This is a contingency because there are some truth values and some false.

1.1.12 Let p, q, and r be the propositions
p: You have the flu
q: You miss the final examination
r: You pass the course
Express each of these propositions in English.

a. $p \to q$
If you have the flu, then you miss the final exam.

b. $\neg q \leftrightarrow r$
You pass the course if and only if you don't miss the final exam.

c. $q \to \neg r$
If you miss the final examination, then you will not pass the course.

d. $p \vee q \vee r$
You have the flu or you miss the final examination or you pass the course.

e. $(p \to \neg r) \vee (q \to \neg r)$
If you have the flu or if you miss the final examination, then you don't pass the course.

f. $(p \wedge q) \vee (\neg q \wedge r)$
You have the flu and you miss the final examination, or you don't miss the final examination and you pass the course

1.1.14 Let p, q, and r be the propositions
p: You get an A on the final exam
q: You do every exercise in this book
r: You get an A in this class
Write these propositions using p, q, and r and logical connectives.

a. $r \wedge \neg q$
b. $p \wedge q \wedge r$
c. $r \leftrightarrow p$
d. $(p \wedge \neg q) \wedge r$
e. $(p \wedge q) \to r$
f. $(r \leftrightarrow p) \vee (r \leftrightarrow q)$

1.1.16 Determine whether these biconditionals are true or false.

a. $2+2=4$ if and only if $1+1=2$.
True

b. $1+1=2$ if and only if $2+3=4$.
False

c. $1+1=3$ if and only if monkeys can fly.
True

d. $0>1$ if and only if $2>1$.
False

1.1.18 Determine whether each of these conditional statements is true or false

a. If $1+1=3$, then unicorns exist.
True

b. If $1+1=3$, then dogs can fly.
True

c. If $1+1=2$, then dogs can fly.
False

d. If $2+2=4$, then $1+2=3$.
True

1.3.10 Show that the expression is a tautology using truth tables.

b. $[(p\to q)\land(q\to r)]\to(p\to r)$

p	q	r	$p\to q$	$q\to r$	$(p\to q)\land(q\to r)$	$p\to r$	$[(p\to q)\land(q\to r)]\to(p\to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Show that the expression is a tautology using logical equivalences.

$[(p\to q)\land(q\to r)]\to(p\to r)\equiv$

$\equiv[(\neg p\lor q)\land(\neg q\lor r)]\to(\neg p\lor r)$ — $p\to q\equiv\neg p\lor q$

$\equiv[\neg(p\land\neg q)\land\neg(q\land\neg r)]\to(\neg p\lor r)$ — DeMorgan's

$\equiv\neg[(p\land\neg q)\lor(q\land\neg r)]\to(\neg p\lor r)$ — DeMorgan's

$\equiv[(p\land\neg q)\lor(q\land\neg r)]\lor(\neg p\lor r)$ — $p\to q\equiv\neg p\lor q$

$\equiv(\neg p\lor(p\land\neg q))\lor(r\lor(q\land\neg r))$ — Distributive

$\equiv[(\neg p\lor p)\land(\neg p\lor\neg q)]\lor[(r\lor q)\land(r\land\neg r)]$ — Distributive

$\equiv[T\land(\neg p\lor\neg q)]\lor[(r\lor q)\land T]$ — Negation

$\equiv(\neg p\lor\neg q)\lor(r\lor q)$ — Identity

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1.3.26 Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.

$\neg p \to (q \to r)$

$\equiv p \vee (\neg q \vee r)$ Important identity

$\equiv \neg q \vee p \vee r$ Associative Law

$\equiv q \to (p \vee r)$ Important identity

1.4.8. Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.

a. $\forall x (R(x) \to H(x))$
For every animal, if that animal is a rabbit then that animal hops.

b. $\forall x (R(x) \wedge H(x))$
All animals are rabbits and hop.

c. $\exists x (R(x) \to H(x))$
For some animal, if it is a rabbit then it hops.

d. $\exists x (R(x) \wedge H(x))$
For some animal, it is both a rabbit and hops.

1.4.10 Let $C(x)$ be the statement "x has a cat" let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. The domain consists of all students in the class.

a. A student in your class has a cat, a dog, and a ferret.
$\exists x (C(x) \wedge D(x) \wedge F(x))$

b. All students in your class have a cat, a dog, or a ferret.
$\forall x (C(x) \vee D(x) \vee F(x))$

c. Some student in your class has a cat and a ferret, but not a dog.
$\exists x (C(x) \wedge F(x) \wedge \neg D(x))$

d. No student in your class has a cat, a dog, and a ferret.
$\neg \exists x (C(x) \wedge D(x) \wedge F(x))$

e. For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this pet.
$\exists x (C(x)) \wedge \exists x (D(x)) \wedge \exists x (F(x))$

1.3.10 b CONT: $\equiv \neg p \vee r \vee \neg q \vee q$ Associative

$\equiv \neg p \vee r \vee T$ Negation

$\equiv T$ Domination