CS 2305 HW2

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1.5.4. Let $P(x,y)$ be the statement "Student x has taken class y," where the domain for x consists of all students in your class and for y consists of all computer science courses given at your school. Express each quantification as a simple English sentence.

a. $\exists x \exists y\, P(x,y)$
For some student x, that student has taken a CS class.

b. $\exists x \forall y\, P(x,y)$
For some student x, that student has taken every CS class.

c. $\forall x \exists y\, P(x,y)$
For each student, that student has taken a CS class.

d. $\exists y \forall x\, P(x,y)$
For some CS class y, that class has been taken by every student in your class.

e. $\forall y \exists x\, P(x,y)$
For each CS class, that class has been taken by a student in your class.

f. $\forall x \forall y\, P(x,y)$
For each student, that student has taken every CS class.

1.5.8. Let $Q(x,y)$ be the statement "student x has been a contestant on quiz show y." Express each of these sentences in terms of $Q(x,y)$, quantifiers, and logical connectives, where the domain for x consists of all students at your school and y consists of all quiz shows on television.

a. There is a student at your school who has been a contestant on a television quiz bowl.
$\exists x \exists y\, Q(x,y)$

b. No student at your school has ever been a contestant on a television quiz show.
$\neg \exists x \exists y\, Q(x,y)$

c. There is a student at your school who has been a contestant on "Jeopardy" and "Wheel of Fortune."
$\exists x (Q(x, \text{"Jeopardy"}) \land Q(x, \text{"Wheel of Fortune"}))$

d. Every television quiz show has had a student from your school as a contestant.
$\forall y \exists x\, Q(x,y)$

e. At least two students from your school have been contestants on "Jeopardy."
$\exists x_1\, Q(x_1, \text{"Jeopardy"}) \land \exists x_2\, Q(x_2, \text{"Jeopardy"}) \land x_1 \neq x_2$

1.5.32 Express the negations of these statements so that all negation symbols immediately precede predicates.

a. $\exists z \forall y \forall x\, T(x,y,z)$
$\forall z \exists y \exists x \neg T(x,y,z)$

b. $\exists x \exists y\, P(x,y) \land \forall x \forall y\, Q(x,y)$
$\forall x \forall y \neg P(x,y) \lor \exists x \exists y \neg Q(x,y)$

c. $\exists x \exists y\, (Q(x,y) \leftrightarrow Q(y,x))$
$\forall x \forall y\, ((Q(x,y) \land \neg Q(y,x)) \lor (\neg Q(x,y) \land Q(y,x)))$

d. $\forall y \exists x \exists z\, (T(x,y,z) \lor Q(x,y))$
$\exists y \forall x \forall z\, (\neg T(x,y,z) \land \neg Q(x,y))$

1.6.18. What's wrong with this argument? Let $S(x,y)$ be "x is shorter than y." Given the premise $\exists s\, S(s, Max)$, it follows that $S(Max, Max)$. Then by existential generalization it follows that $\exists x\, S(x,x)$, so that someone is shorter than himself.

The step "it follows that $S(Max, Max)$" is invalid. This does not follow because you can't infer that Max is the one value from set x that is shorter than Max.

1.6.24. Identify the error or errors in this argument that supposedly shows that if $\forall x\,(P(x) \vee Q(x))$ is true then $\forall x\, P(x) \vee \forall x\, Q(x)$ is true.

invalid simplification

1. $\forall x\,(P(x) \vee Q(x))$	Premise
2. $P(c) \vee Q(c)$	Universal Instantiation 1
☆ 3. $P(c)$	Simplification 2
4. $\forall x\, P(x)$	Universal Generalization 3
☆ 5. $Q(c)$	Simplification 2
6. $\forall x\, Q(x)$	Universal Generalization 5
7. $\forall x\, P(x) \vee \forall x\, Q(x)$	Conjunction 4, 6

1.6.28. Use rules of inference (or logical equivalences) to show that if $\forall x\,(P(x) \vee Q(x))$ and $\forall x\,((\neg P(x) \wedge Q(x)) \to R(x))$ are true, then $\forall x\,(\neg R(x) \to P(x))$ is also true, where the domains of all quantifiers are the same.

1. $\forall x\,(P(x) \vee Q(x))$	Premise
2. $P(c) \vee Q(c)$	Universal Instantiation 1
3. $\forall x\,((\neg P(x) \wedge Q(x)) \to R(x))$	Premise
4. $(\neg P(c) \wedge Q(c)) \to R(c)$	Universal Instantiation 3
5. $\neg(\neg P(c) \wedge Q(c)) \vee R(c)$	$p \to q \equiv \neg p \vee q$ 4
6. $(P(c) \vee \neg Q(c)) \vee R(c)$	DeMorgan's Rule 5
7. $\neg Q(c) \vee P(c) \vee R(c)$	Associative Law 6
8. $P(c) \vee P(c) \vee R(c)$	Resolution 2, 7
9. $P(c) \vee R(c)$	Idempotent Law 8
10. $R(c) \vee P(c)$	Associative Law 9
11. $\neg R(c) \to P(c)$	$p \to q \equiv \neg p \vee q$ 10
12. $\forall x\,(\neg R(x) \to P(x))$	Universal Generalization 11

1.7.2. Use a direct proof to show that the sum of two even integers is even.

1. Let x and y be even integers.	Premise
2. $x = 2k_1$	Definition of even int
3. $y = 2k_2$	Definition of even int
4. $x+y = 2k_1 + 2k_2$	Algebra 2,3
5. $x+y = 2(k_1 + k_2)$	Algebra 4
6. $k_1 + k_2$ is an integer	Closure on Integer Addition 5
7. $x+y$ is even	Definition of even 5, 6

1.7.10 Use a direct proof to show that the product of two rational numbers is rational.

1. Let x and y be rational.	Premise
2. $x = \frac{a}{b}$; a and b are integers; $b \neq 0$; a and b are in lowest terms.	Definition of a rational number
3. $y = \frac{c}{d}$; c and d are integers; $d \neq 0$; c and are in lowest terms	Definition of a rational number

4. $xy = \frac{ac}{bd}$	Algebra 2,3
5. ac is an integer	Closure on Integer multiplication
6. bd is an integer	Closure on Integer multiplication
7. $bd \neq 0$	Zero product property 2, 3, 4
8. xy is rational	Definition of rational number