深蓝学院
shenlanxueyuan.com

3D Point Clouds

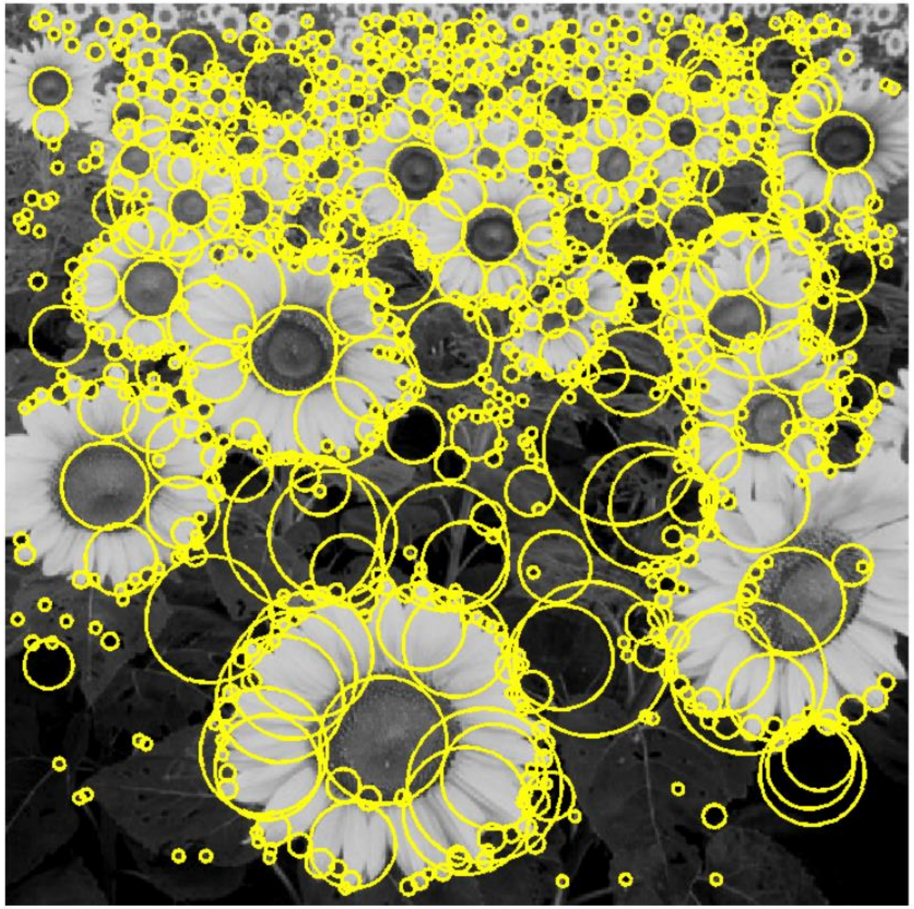
Lecture 7 – Feature Detection

主讲人 黎嘉信

Aptiv 自动驾驶
新加坡国立大学 博士
清华大学 本科

1. Introduction to Features/Keypoints
2. Detector – Classical and Modern
3. Descriptor – Classical and Modern

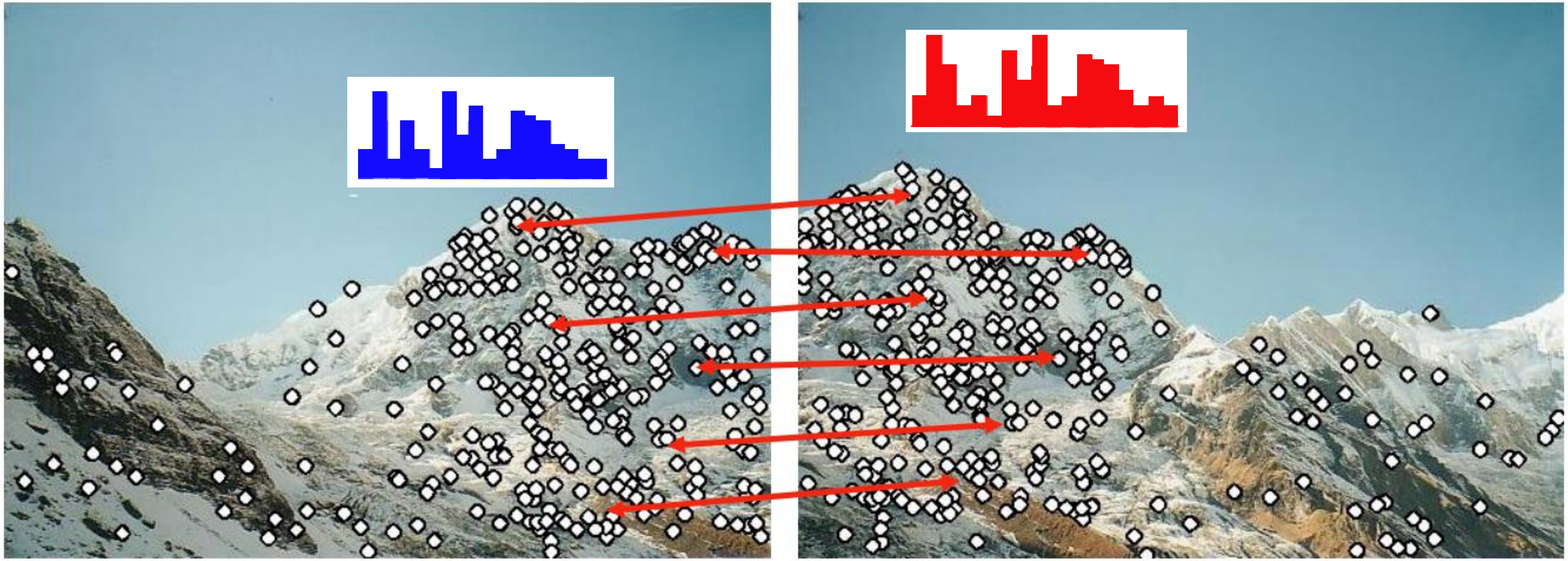
Image Features – Points / Blobs



Feature **Detection**: Identify the interest points

Source: Computer Vision, Raquel Urtasun

Image Features – Description and Matching

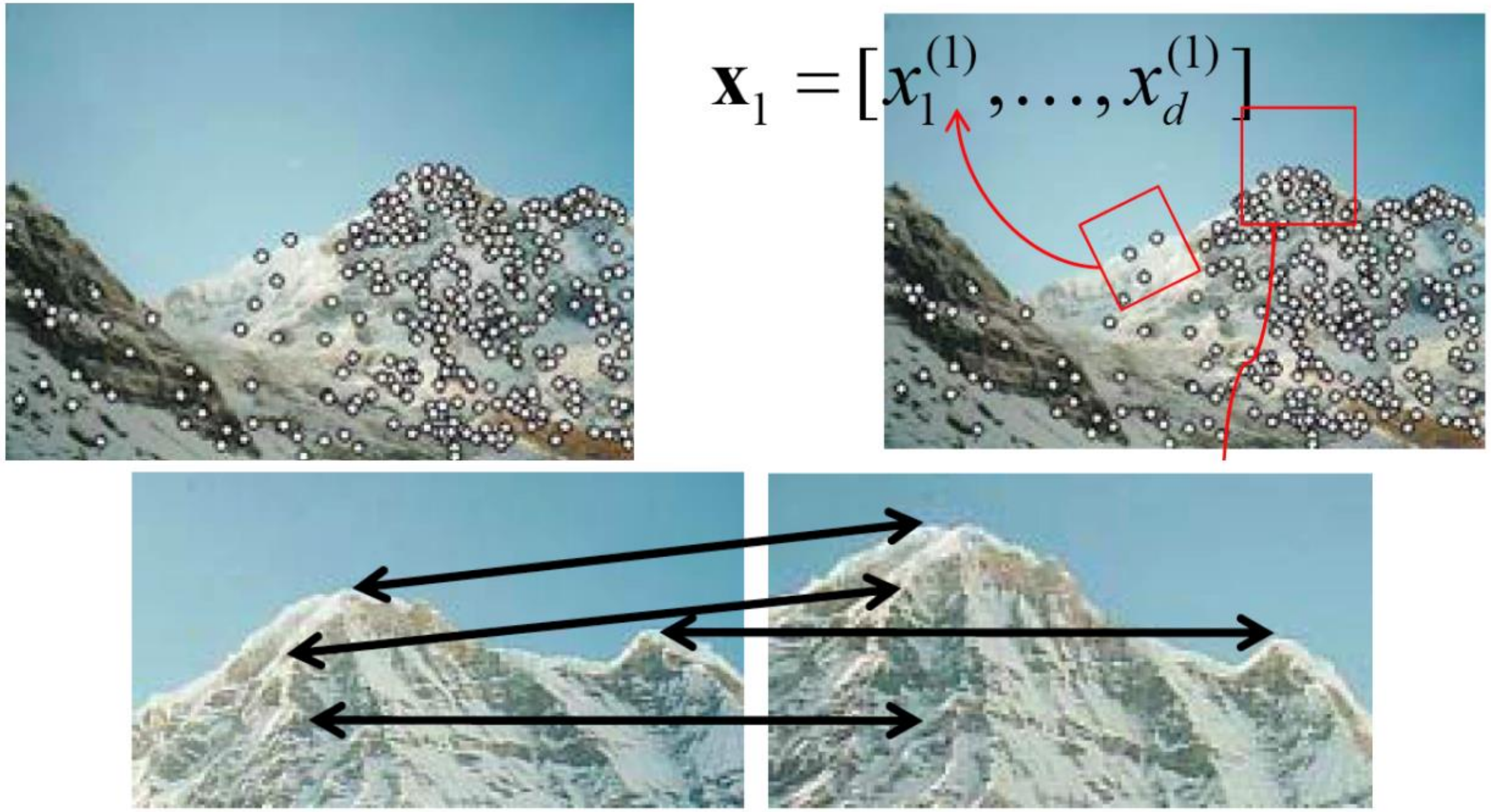


Feature **Description**: Extract a vector around the feature point to descript it.
Feature **Matching**: Determine correspondence between descriptors

Source: Computer Vision, Raquel Urtasun

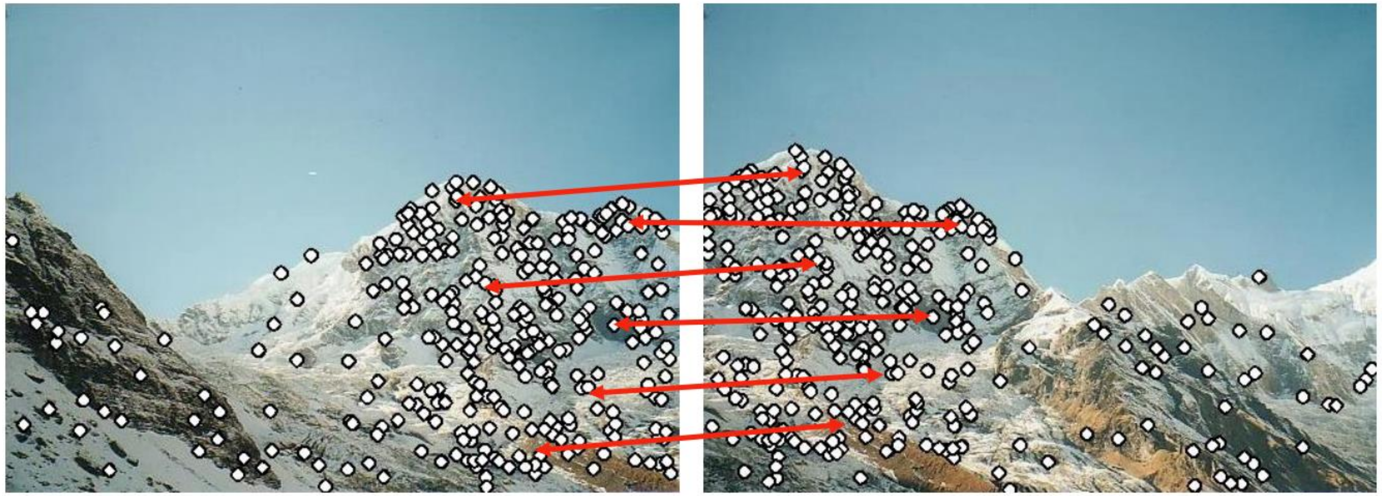
Image Features

Detection: Identity the interest points
Description: Extract a vector around the feature point to descript it.
Matching: Determine correspondence between descriptors



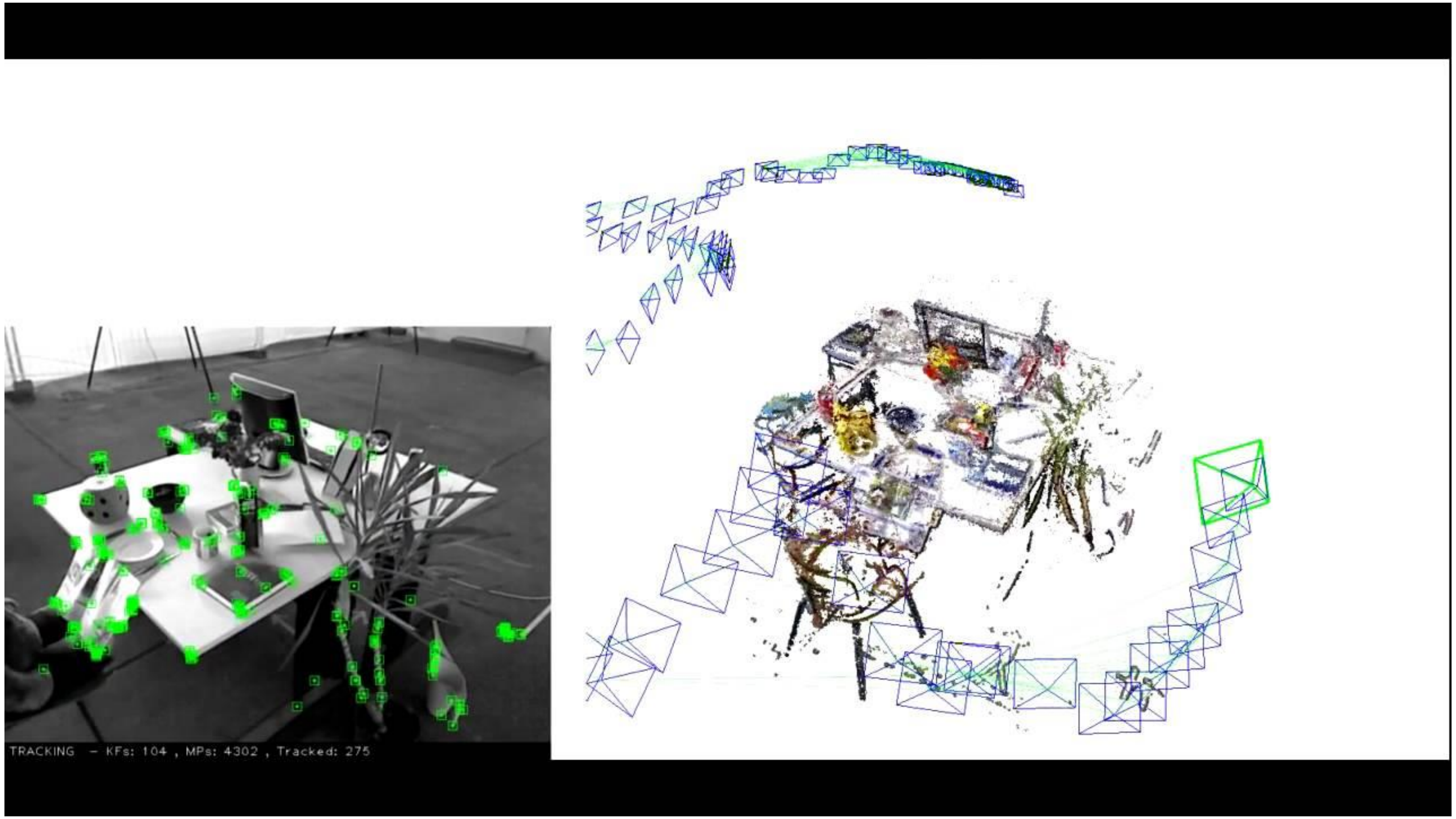
Source: Computer Vision, Raquel Urtasun

Image Features – Applications – Panorama



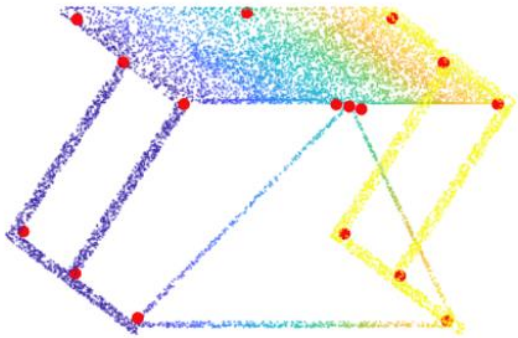
Source: Computer Vision, Raquel Urtasun

Image Features – Applications – SLAM (Simultaneous Localization and Mapping)

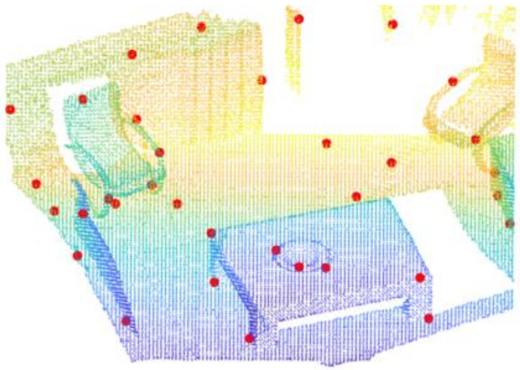


Source: ORB SLAM, https://webdiis.unizar.es/~raulmur/orbslam/

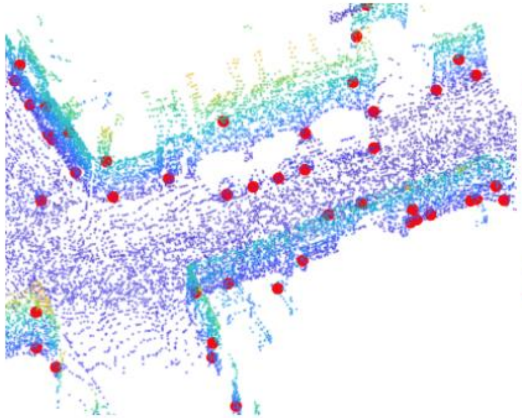
Point Cloud Features



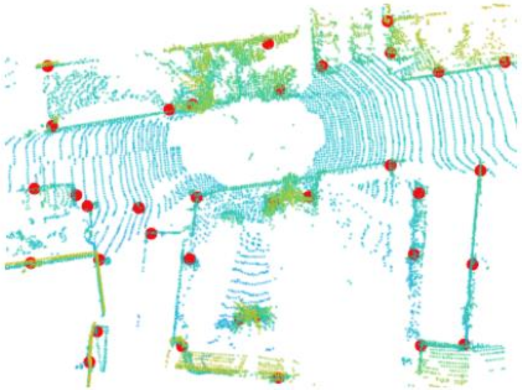
CAD Model



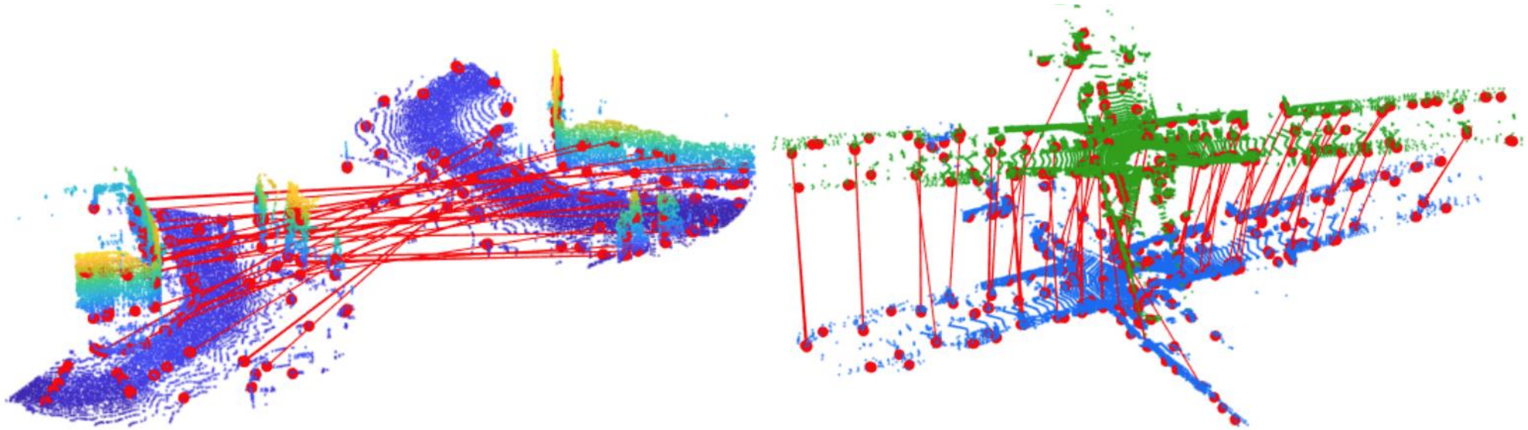
RGB-D



Oxford RobotCar



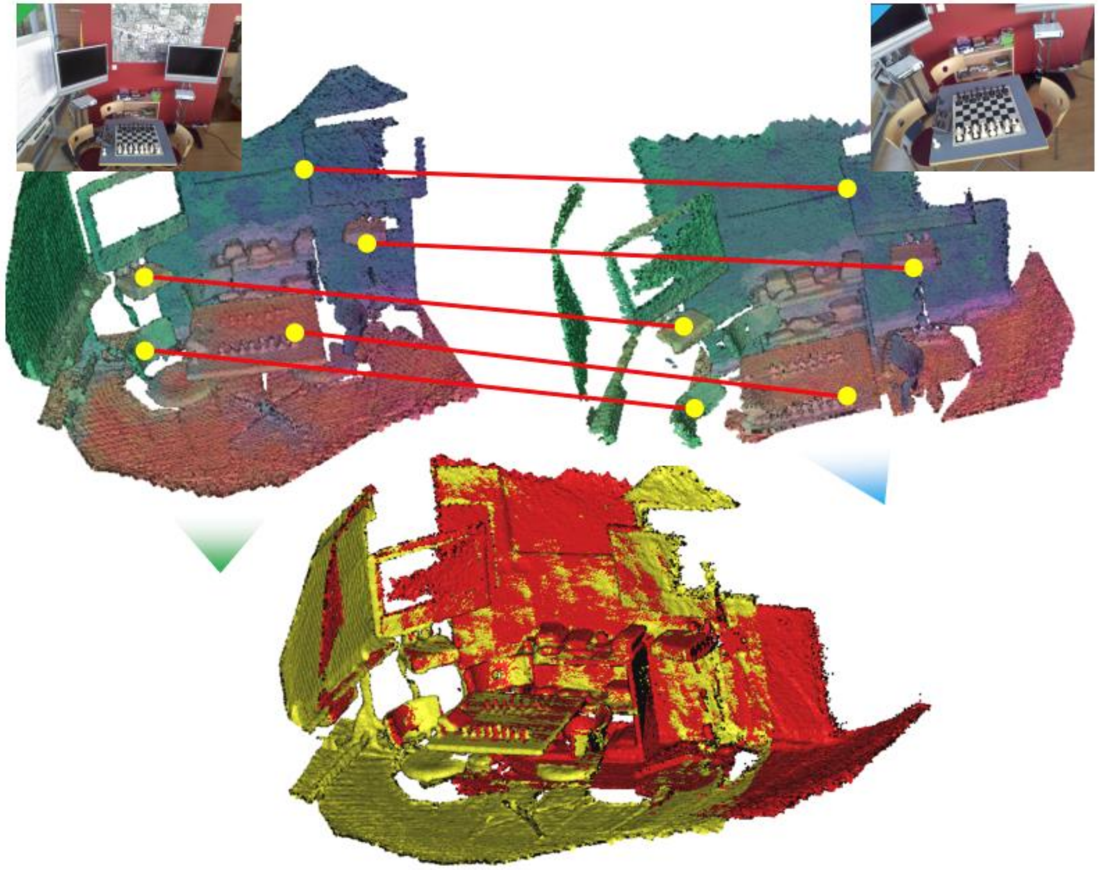
KITTI



Description and Matching

Source: USIP: Unsupervised Stable Interest Point Detection from 3D Point Clouds, Jiaxin Li and Gim Hee Lee

Point Cloud Features - Registration

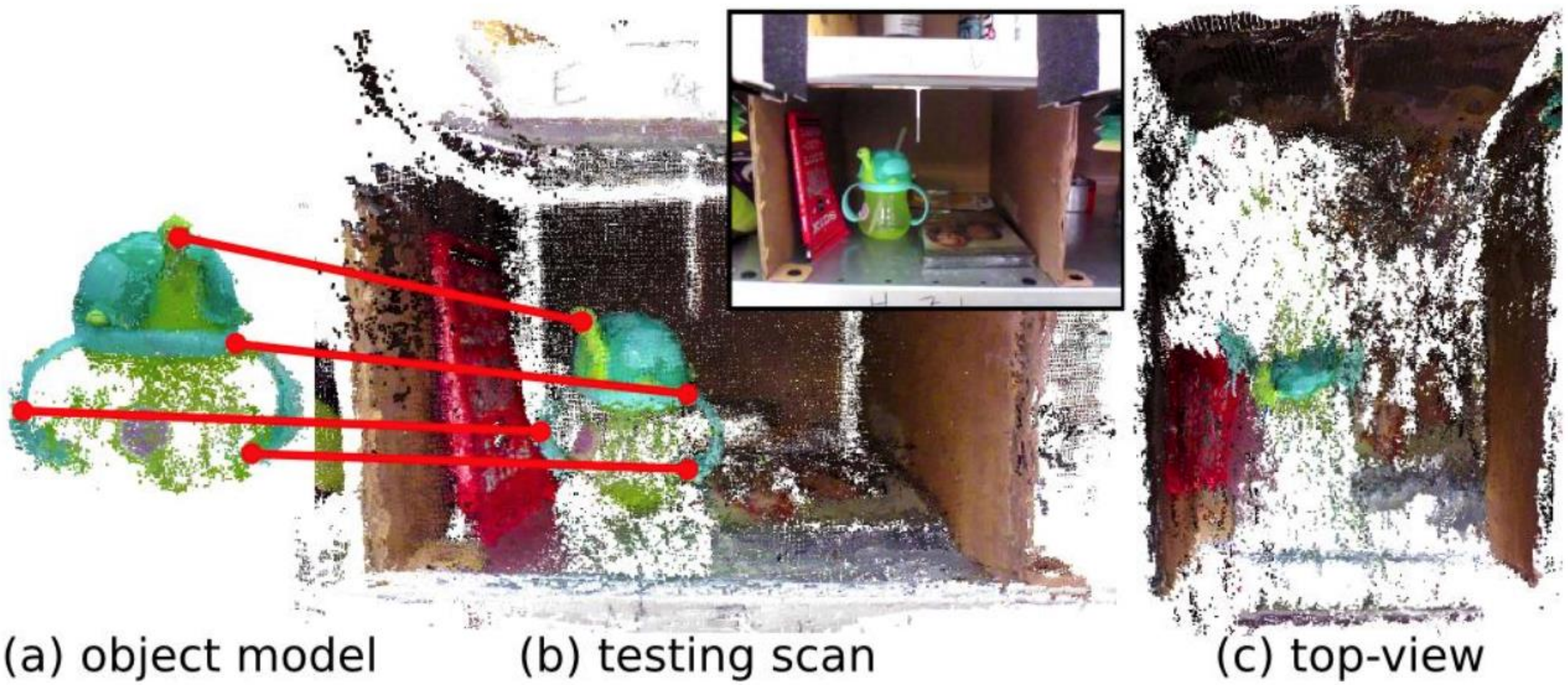


Registration

- Find a transform to align two point clouds
- A transform consists of
  - Rotation R
  - Translation t
- Method 1 - Iterative Closest Point (ICP)?
  - ICP requires proper initial guess
  - Low overlapping ratio
- Method 2 – Detect and match features
  - No initialization required
  - Works for low overlapping ratio

Source: PPFNet: Global Context Aware Local Features for Robust 3D Point Matching

Point Cloud Features – Object 6D Pose

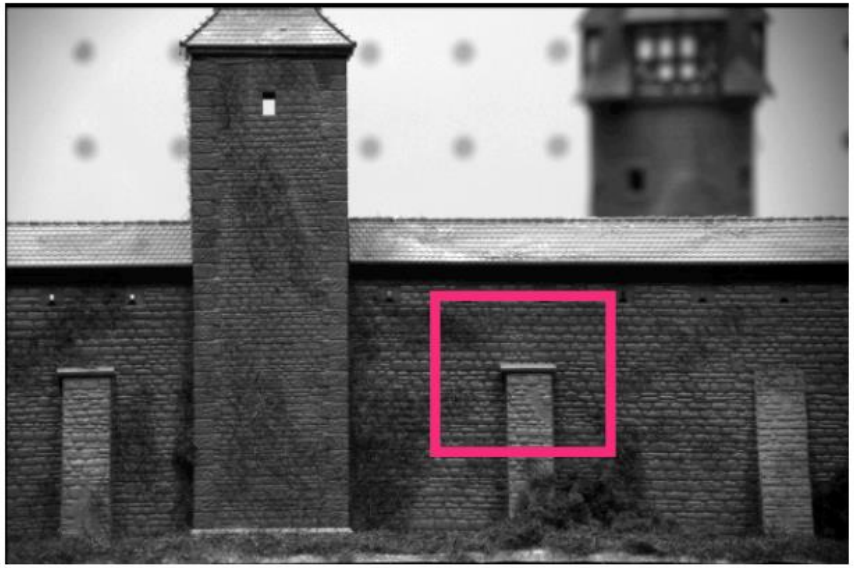


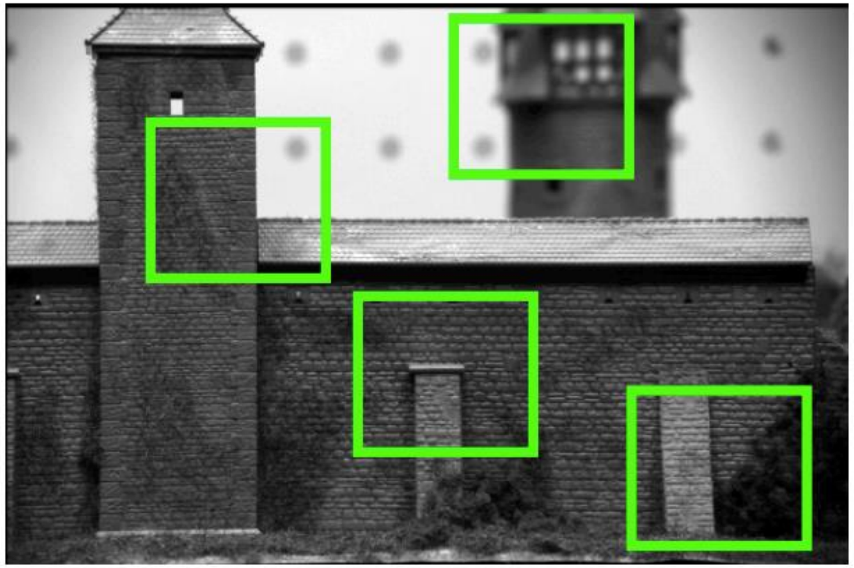
- In fact, it is still registration

Source: 3DMatch: Learning Local Geometric Descriptors from RGB-D Reconstructions

- Borrow ideas from image features
  - Harris
  - SUSAN (Small Univalue Segment Assimilating Nucleus)
  - SIFT (Scale-Invariant Feature Transform)
- Native in 3D geometry
  - ISS (Intrinsic Shape Signatures)
- Deep learning
  - Very few method available

Harris Detector – Patch Matching

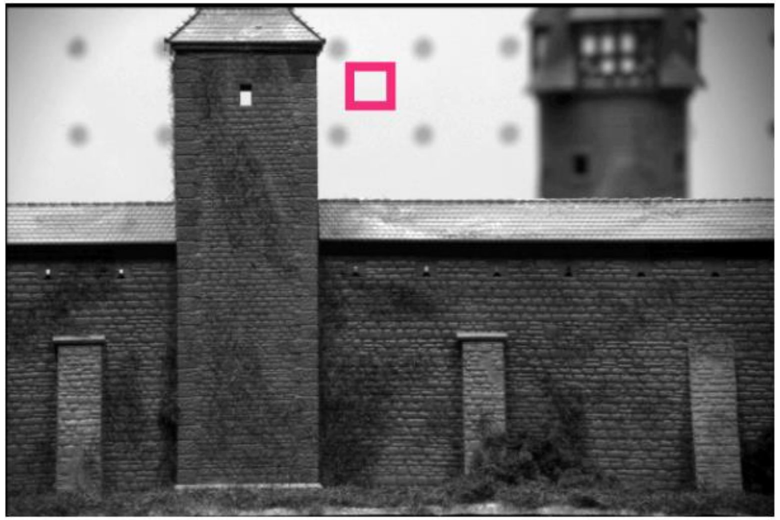




- Find the most similar patch in the second image.

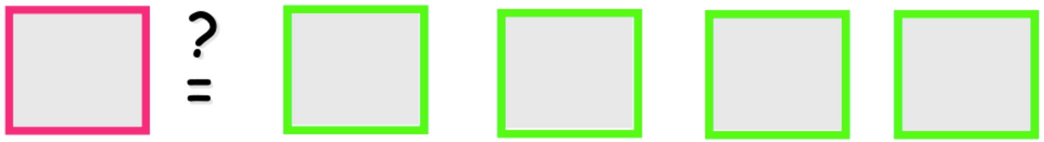
Source: CSE486, Penn State, Robert Collins

Harris Detector – Patch Matching



- Not all ~~patches~~ animals are created equal. Some are more equal than others.

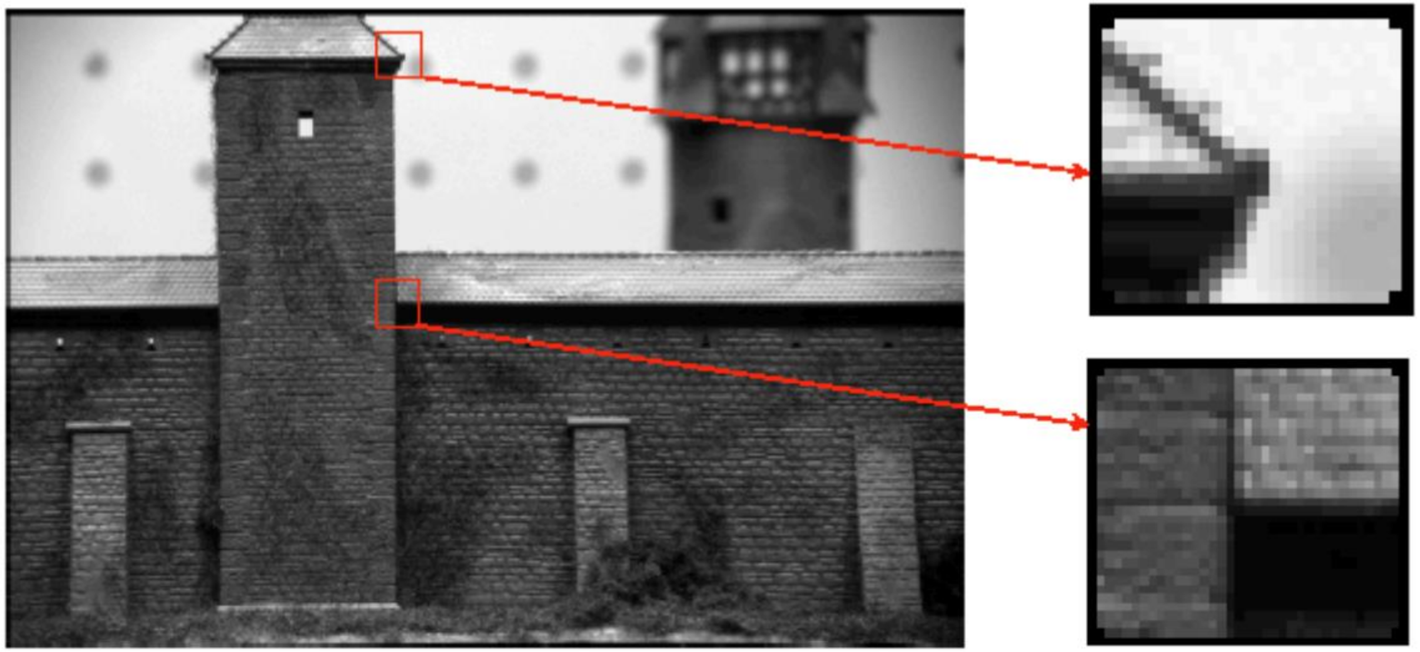
  -- Animal Farm, George Orwell



Source: CSE486, Penn State, Robert Collins

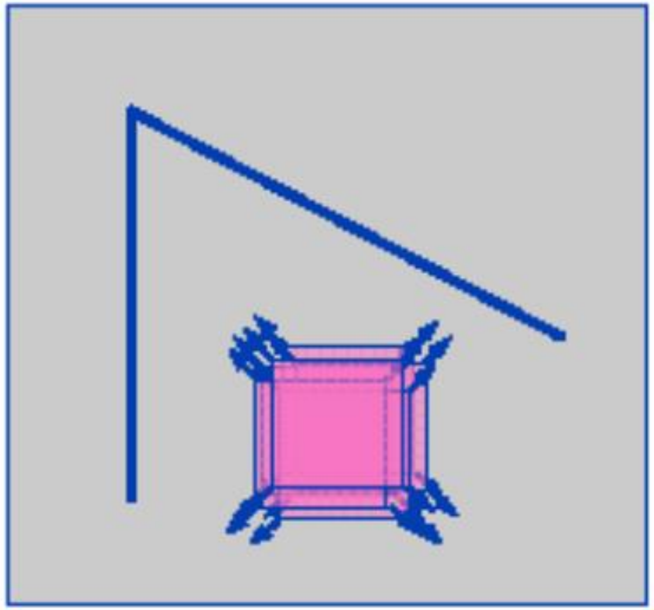
Harris Detector – Corner Points

- What are corners?
  - Junctions of contours
- Corners are distinctive.
  - Large variations in neighborhood
  - Stable to viewpoint change
  - Good features!



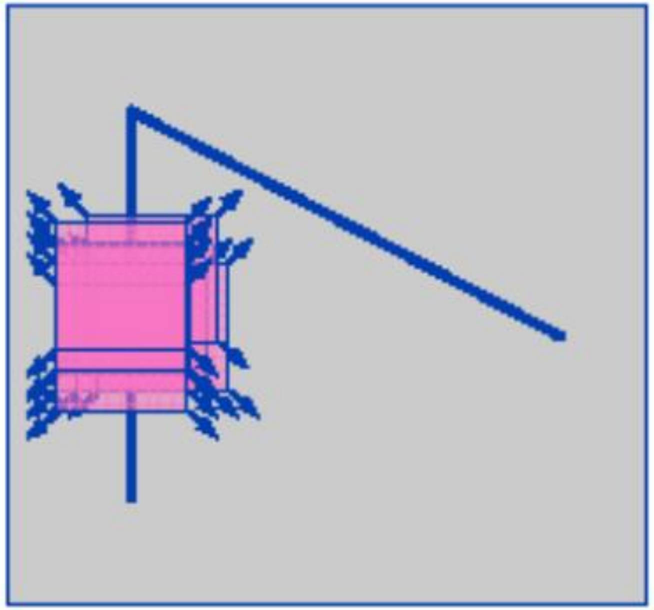
Source: CSE486, Penn State, Robert Collins

Corner – Basic Idea



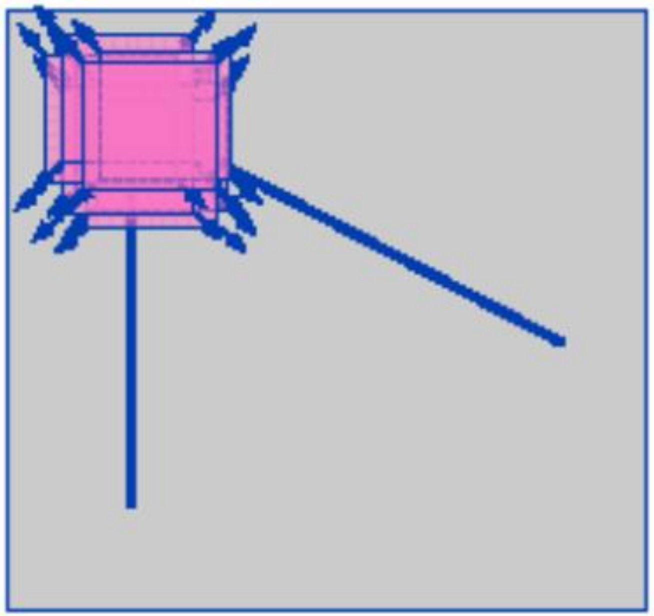
“flat”:

no change in all directions



“edge”:

no change along the edge direction



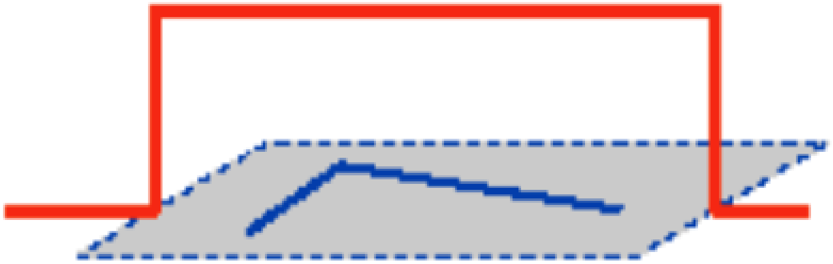
“corner”:

significant change in all directions

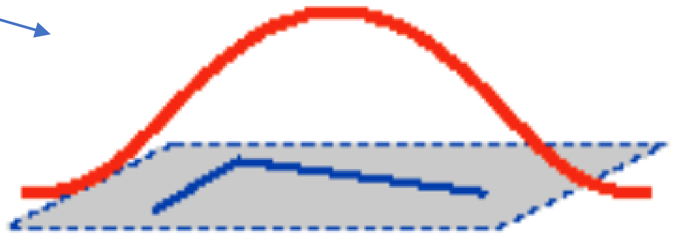
Source: CSE486, Penn State, Robert Collins

- Given an image I, consider a patch $x, y \in \Omega$
- Shift the patch by $[u, v]$, the intensity change is:

$$E(u,v) = \sum_{x,y\in\Omega} \underbrace{w(x,y)}_{\text{Windows function}}[\underbrace{I(x+u,y+v)}_{\text{Shifted intensity}} - \underbrace{I(x,y)}_{\text{Intensity}}]^2$$



1 in window, 0 outside



Gaussian

Source: CSE486, Penn State, Robert Collins

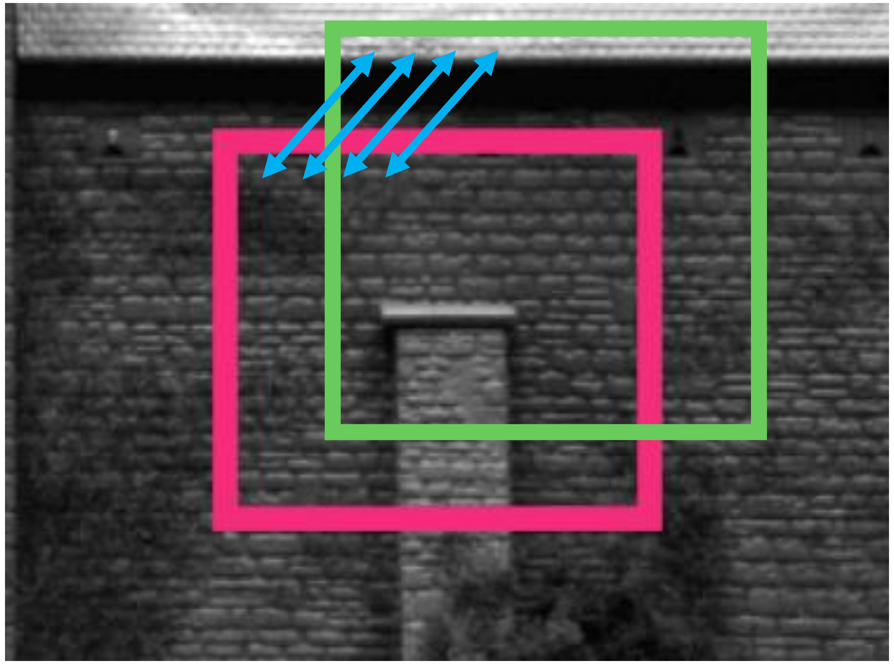
$$E(u,v) = \sum_{x,y \in \Omega} w(x,y)[I(x+u, y+v) - I(x,y)]^2$$

u, v is fixed

x, y is loop over the patch

How to select u, v?

As small as possible → gradient



Source: CSE486, Penn State, Robert Collins

$$f(x+u, y+v) = f(x,y) + uf_x(x,y) + vf_y(x,y)$$

First order partial derivatives

$$+\frac{1}{2!}\left[u^2 f_{xx}(x,y) + uvf_{xy}(x,y) + v^2 f_{yy}(x,y)\right]$$

Second order partial derivatives

$$+\frac{1}{3!}\left[u^3 f_{xxx}(x,y) + u^2 v f_{xxy}(x,y) + uv^2 f_{xyy}(x,y) + v^3 f_{yyy}(x,y)\right]$$

Third order partial derivatives

$$+ \cdots (higher\ order\ terms)$$

First order approximation: $f(x+u, y+v) \approx f(x,y) + uf_x(x,y) + vf_y(x,y)$

- Intensity change for a patch when shifted $[u, v]$ is represented by

$$\sum_{x,y\in\Omega} w(x,y)[I(x+u,y+v)-I(x,y)]^2$$

- Let's assume $w(x,y)$ is the binary windows function, i.e, $w(x,y)=1, \forall x,y \in \Omega$

$$E(u,v)=\sum_{x,y\in\Omega}\big(I(x+u,y+v)-I(x,y)\big)^2$$

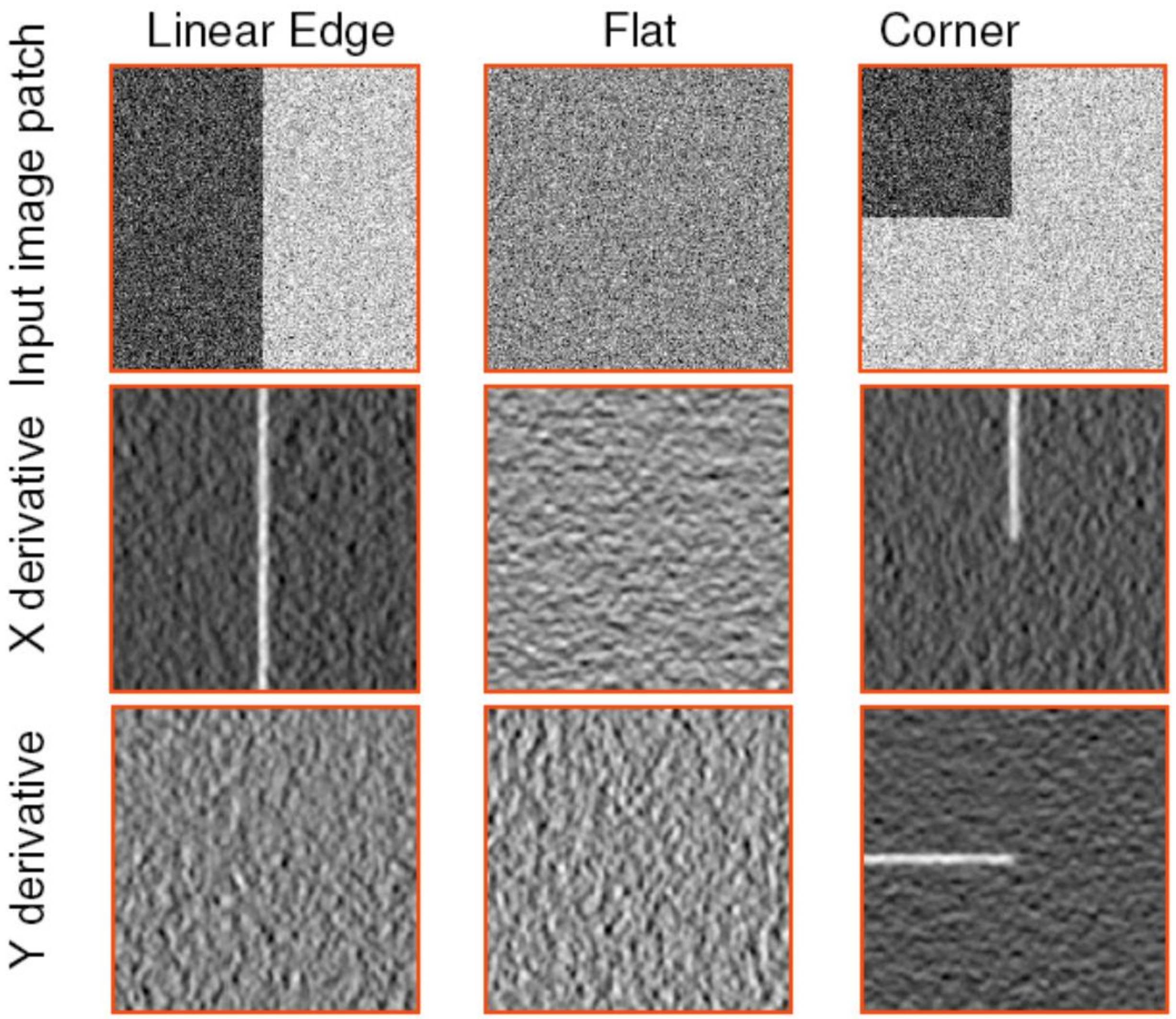
- Intensity change is given by

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}, \quad M = \sum_{x,y \in \Omega} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- M is the covariance matrix of the image gradient
  - Denote intensity gradient for location $[x_i, y_i]$ as $I_i = [I_{x_i}, I_{y_i}]^T \in \mathbb{R}^2$
  - Covariance matrix $M = \sum_i I_i I_i^T$
- However,
  - What does it actually mean?
  - How can we get feature points?

2D Derivatives

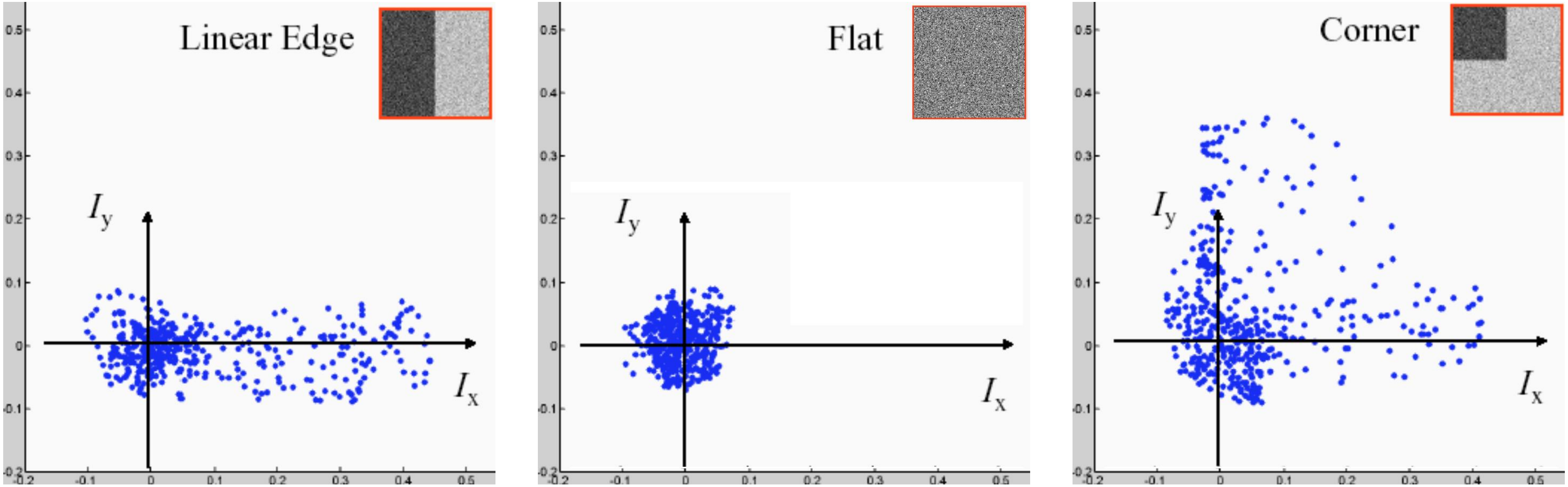
- Linear edge
    - $I_i = [I_{x_i}, 0]^T$
- Flat
    - $I_i = [0,0]^T$
- Corner
    - $I_i = [I_{x_i}, I_{y_i}]^T$



Source: CSE486, Penn State, Robert Collins

Gradient I_i as points

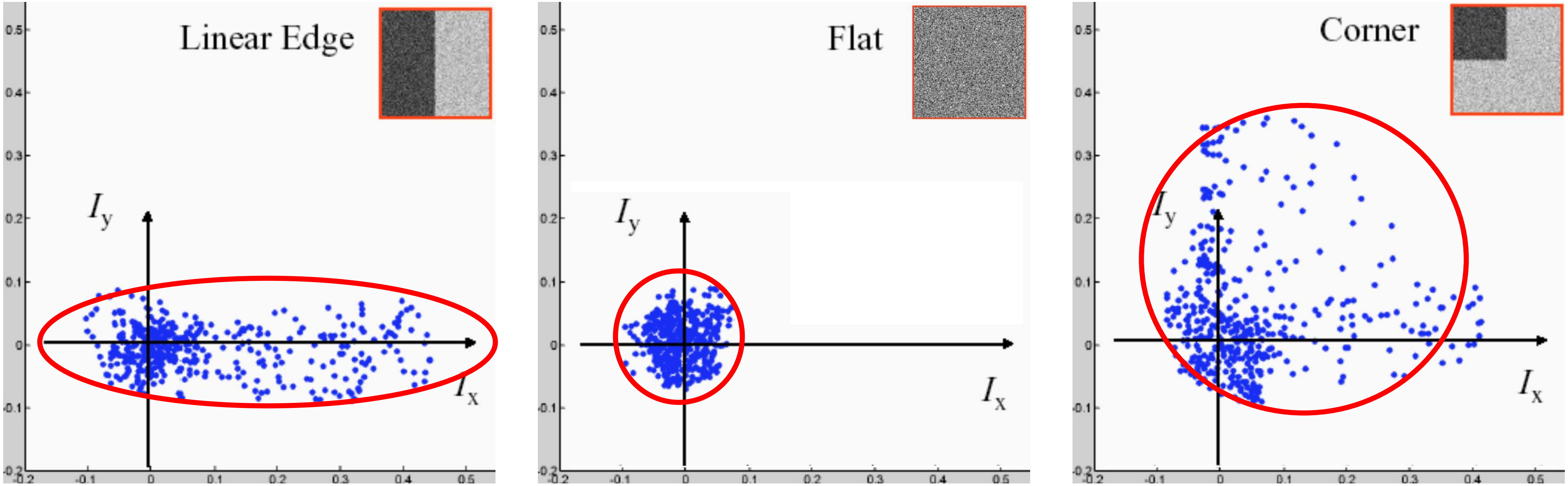
- Plot the gradient $I_i = \left[I_{x_i}, I_{y_i}\right]^T$ for three case: Linear Edge, Flat, Corner



Source: CSE486, Penn State, Robert Collins

Gradient I_i as points

- We can distinguish these 3 cases by covariance matrix M
- The eigenvalues of M are λ_1, λ_2



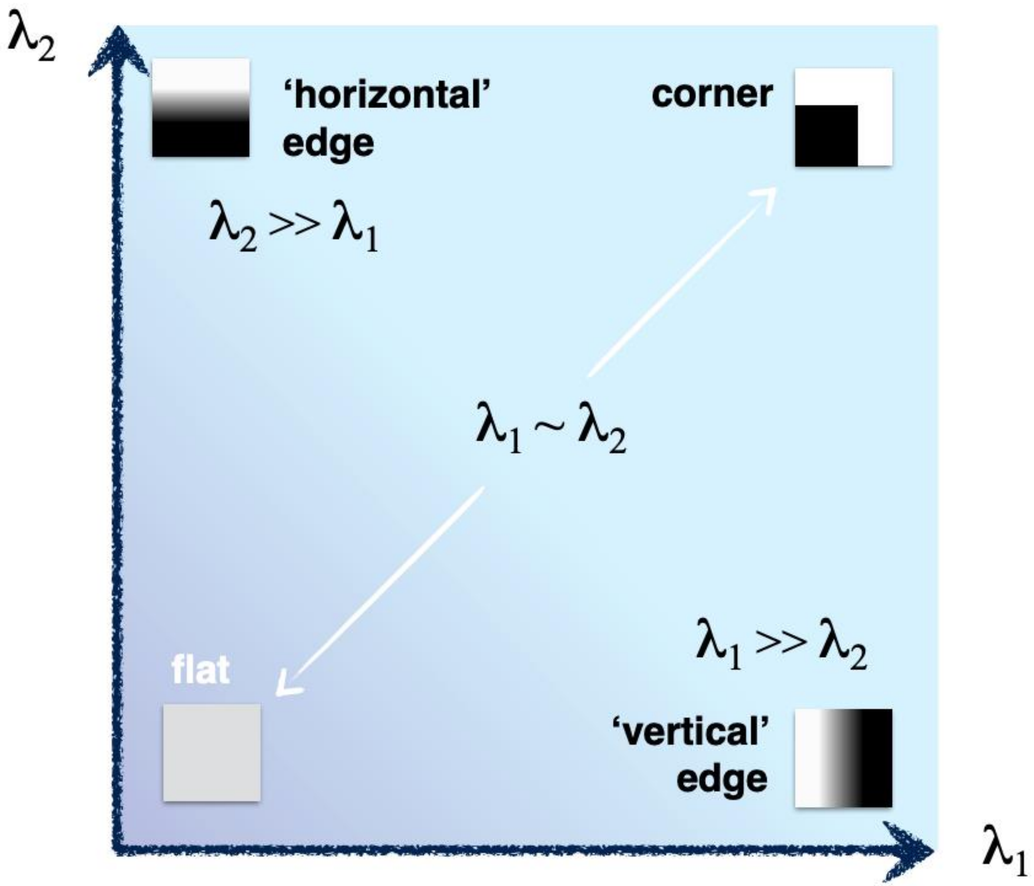
Source: CSE486, Penn State, Robert Collins

- The intensity change is

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}, \quad M = \sum_{x,y \in \Omega} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- We want corner points
  - Large gradient along x → I_i spreads alone x
  - Large gradient along y → I_i spreads along y

- Corner points – both λ_1, λ_2 are large



Source: 16-385 Computer Vision, CMU, Kris Kitani

Response Function R

- Harris & Stephens (1988): A Combined Corner and Edge Detector

$$R = \det M - k(trace\ M)^2,$$
$$\det M = \lambda_1 \lambda_2$$
$$trace\ M = \lambda_1 + \lambda_2$$
$$k \in [0.04, 0.06]\ (\text{empirically determined } constant)$$

- Kanade & Tomasi (1994): Good Features to Track

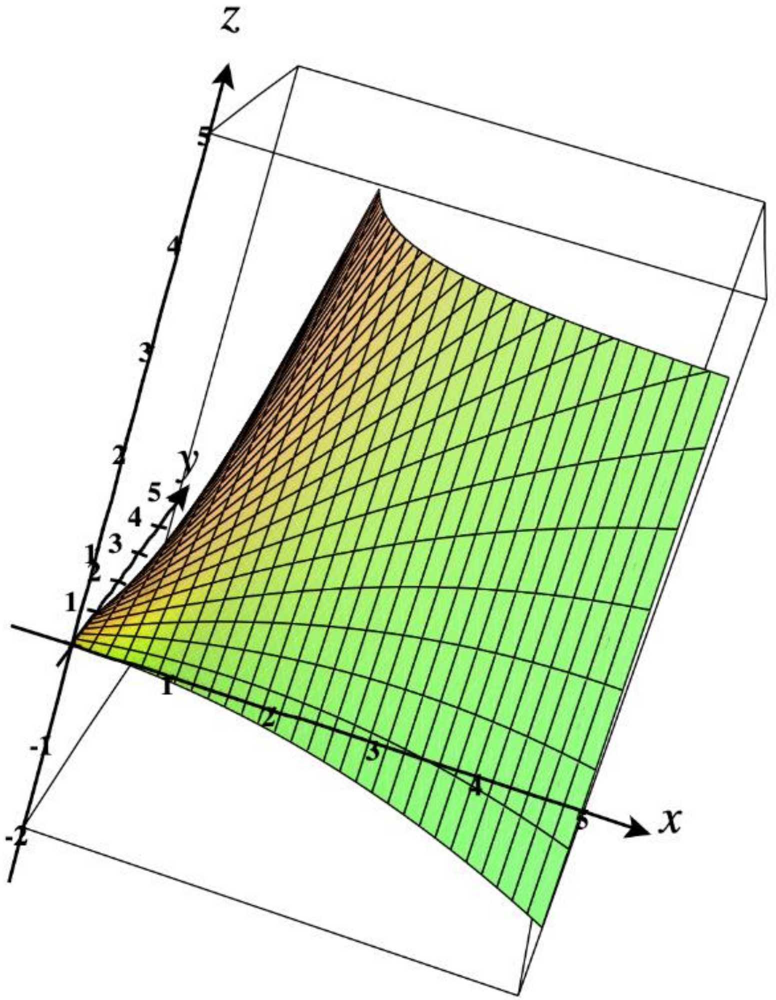
$$R = \min(\lambda_1, \lambda_2)$$

- Nobel (1998): Finding Corners.

$$R = \frac{\det M}{trace(M) + \epsilon}$$

Response Function R

- Set a threshold τ for R
  - Corner: $R \geq \tau$
- Harris Detector Algorithm
  - Compute R for every pixel in the image
  - Store result as a ”response image”
  - Normalized the “response image” to e.g. [0, 1]
  - Empirically set a τ
  - Non-Maximum Suppression (NMS)

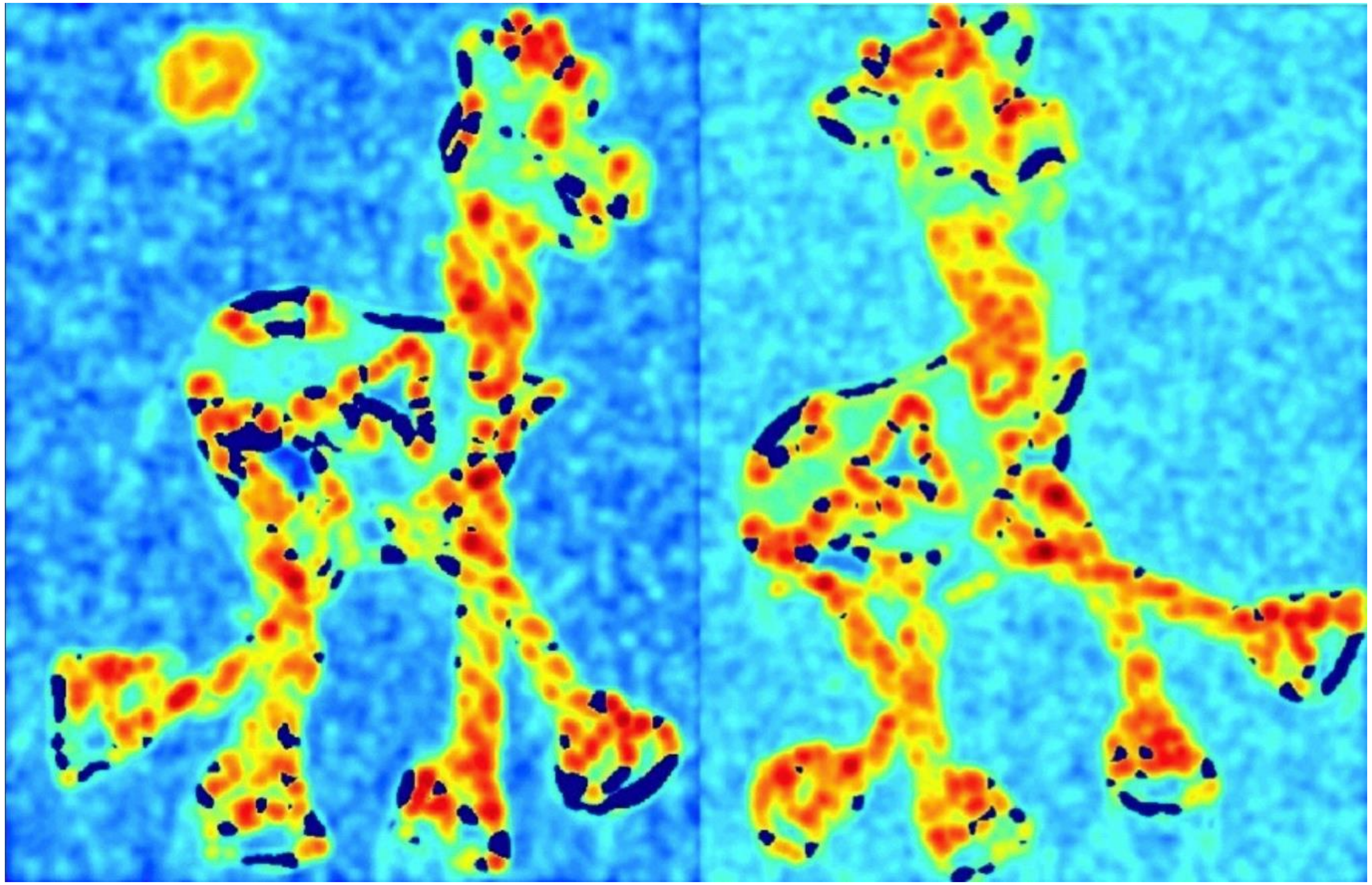


Plot of Harris response $R = \det M - k(trace\ M)^2$

Harris Corner Detector – Input Image

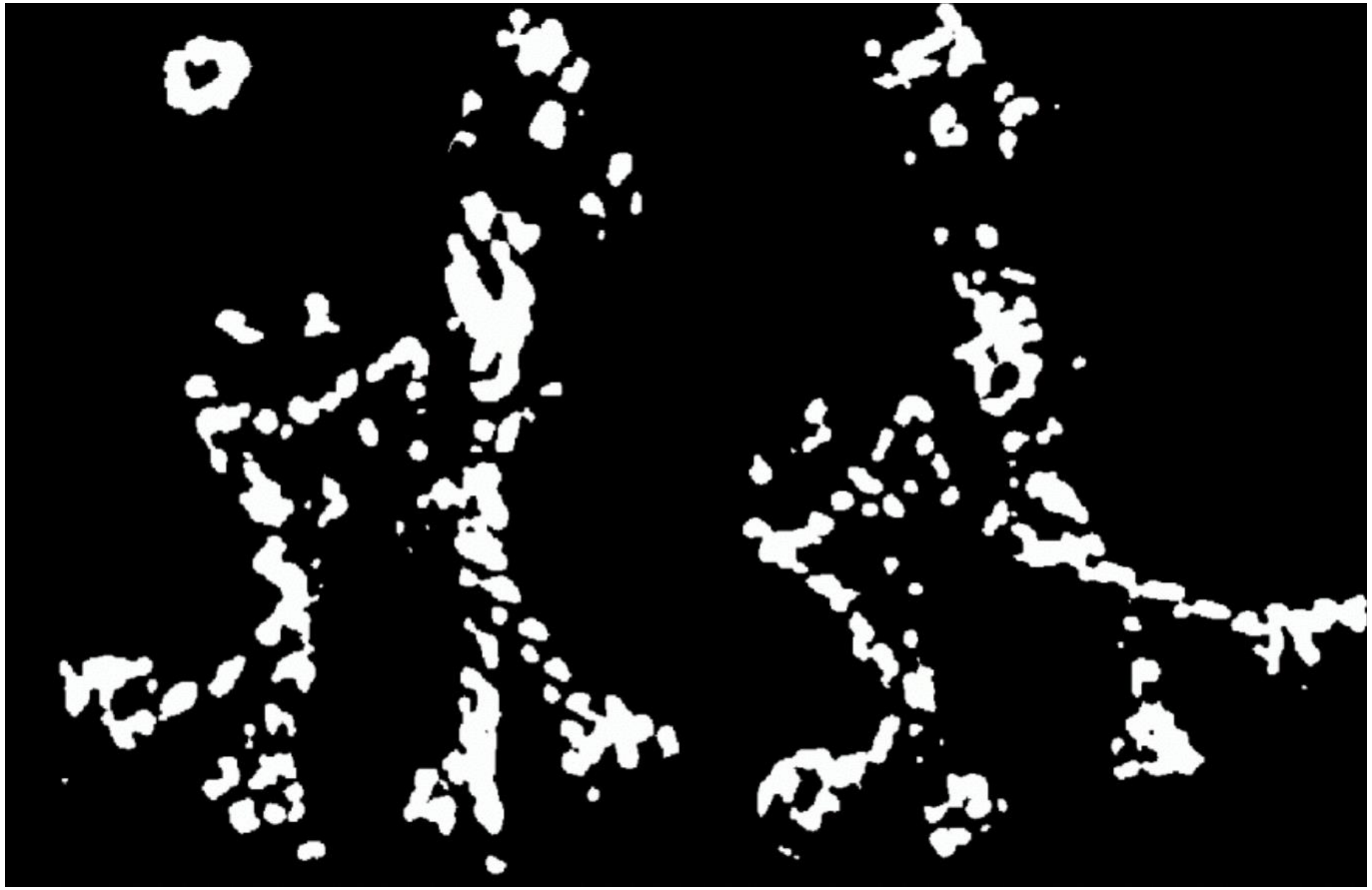
Source: 16-385 Computer Vision, CMU, Kris Kitani

Harris Corner Detector – Corner Response



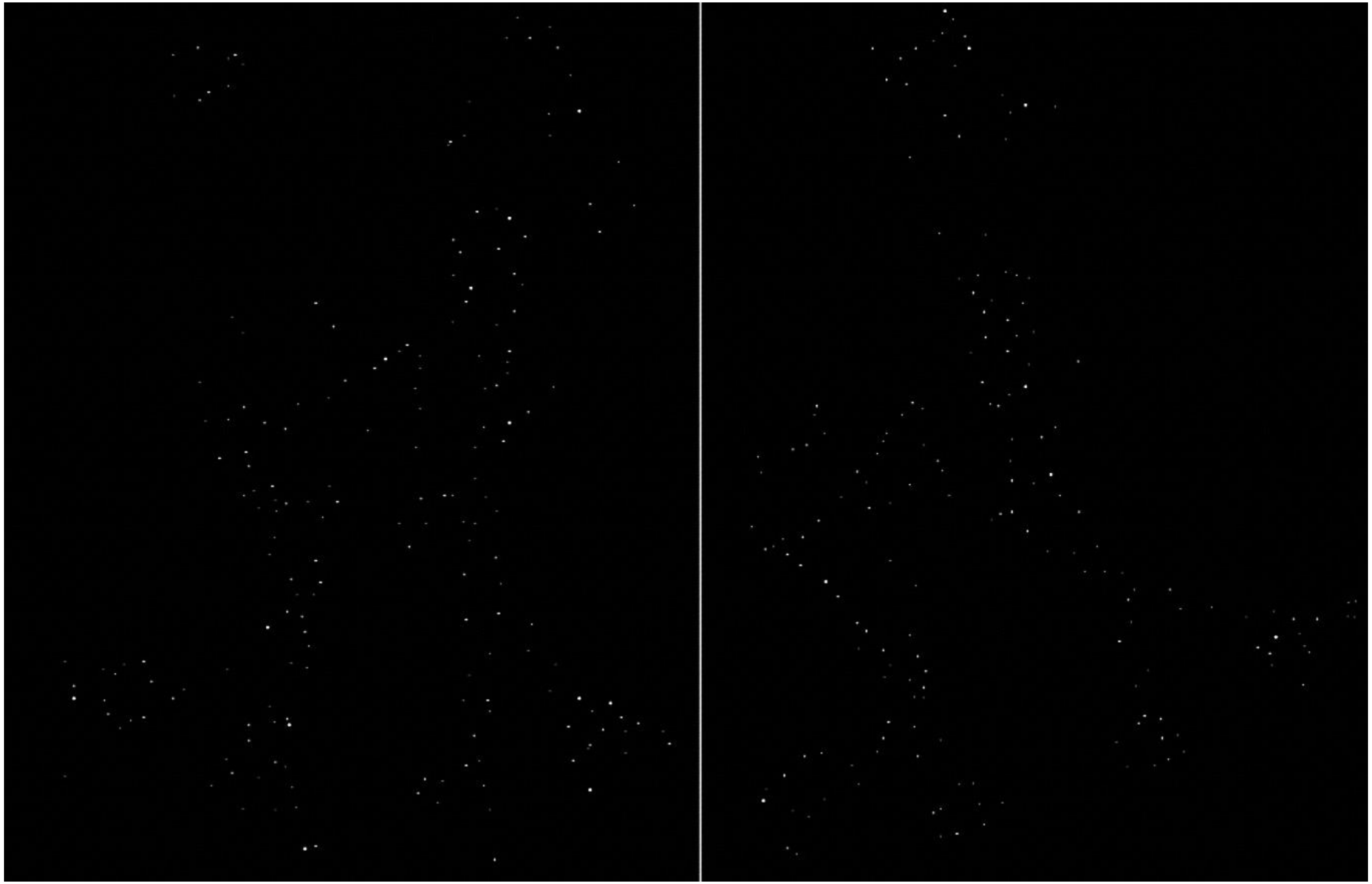
Source: 16-385 Computer Vision, CMU, Kris Kitani

Harris Corner Detector – Corner Response Thresholded



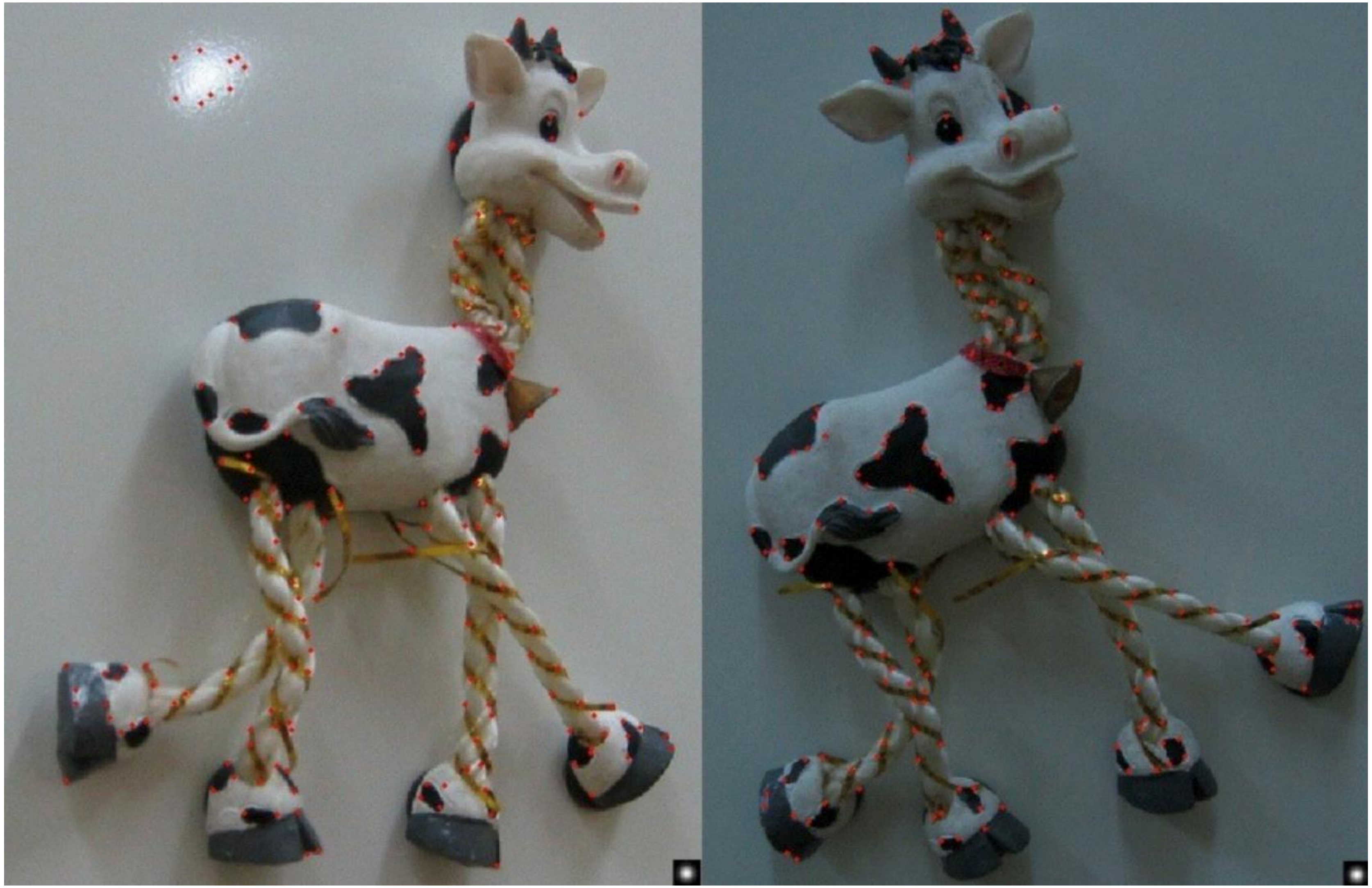
Source: 16-385 Computer Vision, CMU, Kris Kitani

Harris Corner Detector – NMS



Source: 16-385 Computer Vision, CMU, Kris Kitani

Harris Corner Detector – Detection Result



Source: 16-385 Computer Vision, CMU, Kris Kitani

- Harris Corner is based on intensity change of a patch
  - Move the patch, how much the intensity changes?

$$E(u,v) = \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}, \quad \boxed{M = \sum_{x,y \in \Omega} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}}$$

Covariance of image gradient

- Assume there is intensity in the point cloud, how can we formulate a cost function?
  - Points are discrete.
  - What is a “patch” in point cloud?
  - How to “move a patch”?

- For a local (small) region Ω over a point
- Assume intensity is a continuous function

$$I(x,y,z): \mathbb{R}^3 \rightarrow \mathbb{R}, [x,y,z] \in \Omega$$

- Assume the small "move" is $[u,v,w]$
- The intensity change is

$$E(u,v,w) = \sum_{x,y,z \in \Omega} [I(x+u, y+v, z+w) - I(x,y,z)]^2$$

- Cost function

$$E(u,v,w) = \sum_{x,y,z\in\Omega} [I(x+u,y+v,z+w) - I(x,y,z)]^2$$

- Apply first-order Taylor Series approximation, we have

$$E(u,v,w) = \begin{bmatrix} u & v & w \end{bmatrix} M \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad M = \sum_{x,y,z\in\Omega} \begin{bmatrix} I_x^2 & I_x I_y & I_x I_z \\ I_x I_y & I_y^2 & I_y I_z \\ I_x I_z & I_y I_z & I_z^2 \end{bmatrix}$$

M is the covariance matrix of intensity over the surface.

- How to compute I_x, I_y, I_z?

- Denote the point we are analyzing is $p = [p_x, p_y, p_z]^T$
- In the neighborhood Ω, there are points $\{\mathbf{x}_i = [x_i, y_i, z_i]\}$
- The intensity gradient around p is a vector $\mathbf{e} = [e_x, e_y, e_z]^T \in \mathbb{R}^3$
  - Direction of e is the direction of greatest intensity increase
  - Magnitude $||e||$ is the rate of intensity change
- Ideally, there will be:

$$
\begin{gathered}
(x_1 - p_x)e_x + (y_1 - p_y)e_y + (z_1 - p_z)e_z = I(x_1, y_1, z_1) - I(p_x, p_y, p_z) \\
\cdots = \cdots \\
(x_i - p_x)e_x + (y_i - p_y)e_y + (z_i - p_z)e_z = I(x_i, y_i, z_i) - I(p_x, p_y, p_z) \\
\cdots = \cdots
\end{gathered}
$$

- Scalar form

$$(x_i - p_x)e_x + (y_i - p_y)e_y + (z_i - p_z)e_z = I(x_i, y_i, z_i) - I(p_x, p_y, p_z)$$

- Vector form

$$\mathbf{x}_i'^T \mathbf{e} = \Delta I_i,$$
$$\mathbf{x}_i' = [x_i', y_i', z_i']^T = \left[x_i - p_x, y_i - p_y, z_i - p_z\right]^T$$

- Matrix form

$$A\mathbf{e} = \mathbf{b},\ \ A = \begin{bmatrix} x_1' & y_1' & z_1' \\ \vdots & \vdots & \vdots \\ x_i' & y_i' & z_i' \\ \vdots & \vdots & \vdots \end{bmatrix}, \mathbf{b} = \begin{bmatrix} \Delta I_1 \\ \vdots \\ \Delta I_i \\ \vdots \end{bmatrix}$$

- We want to find the intensity gradient $\mathbf{e}$ that satisfies

$$A\mathbf{e} = \mathbf{b},\ \ A = \begin{bmatrix} x'_1 & y'_1 & z'_1 \\ \vdots & \vdots & \vdots \\ x'_i & y'_i & z'_i \\ \vdots & \vdots & \vdots \end{bmatrix}, \mathbf{b} = \begin{bmatrix} \Delta I_1 \\ \vdots \\ \Delta I_i \\ \vdots \end{bmatrix}$$

- Which is impossible in when number of point > 3, so actually,

$$\min_{\mathbf{e}} ||A\mathbf{e} - \mathbf{b}||_2^2$$

- Solution given by $\mathbf{e} = (A^T A)^{-1} A^T \mathbf{b}$

- Come back to the Harris 3D with intensity

$$E(u,v,w) = \begin{bmatrix} u & v & w \end{bmatrix} M \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad M = \sum_{x,y,z\in\Omega} \begin{bmatrix} I_x^2 & I_x I_y & I_x I_z \\ I_x I_y & I_y^2 & I_y I_z \\ I_x I_z & I_y I_z & I_z^2 \end{bmatrix}$$

- Now we know $\mathbf{e} = [I_x, I_y, I_z]$

- One more thing – optionally we can project $\mathbf{e}$ onto local surface.

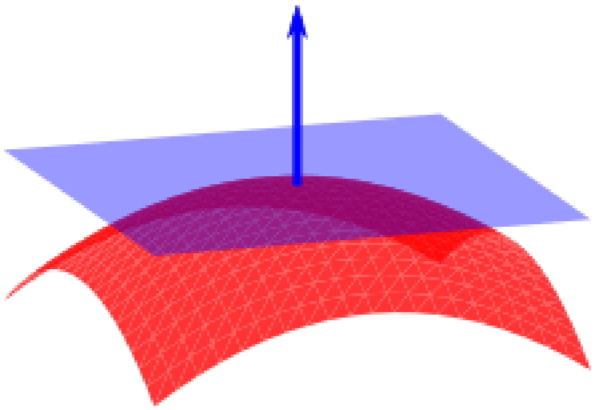
- In many cases, point cloud is from scanning, i.e., points are on the surface of the object/environment
- For every point p, we can fit a surface over the local (small) neighborhood.

$$f(x, y, z) = 0$$

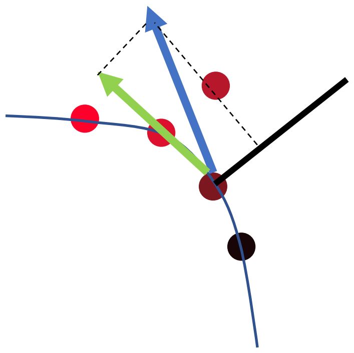
- Using first-order approximation, the surface becomes a plane

$$ax + by + cz + d = 0$$

Surface normal of p: $\mathbf{n} = [n_x, n_y, n_z]^T = \frac{[a, b, c]^T}{\|[a, b, c]^T\|_2}$



- Projecting intensity gradient $\mathbf{e}$ onto the surface may reduce the effect of noise
  - Look at the 2D example
  - Color represents intensity, red – large, black – small
  - Blue line - $\mathbf{e}$
  - **Green** line - $\mathbf{e}$ projected onto the surface (this case it is a curve) → e'
  - Black line – surface normal $\mathbf{n}$
- How to project $\boldsymbol{e}$ onto surface/curve?
  - $\mathbf{e}' = \mathbf{e} - \mathbf{n}(\mathbf{n}^T\mathbf{e}) = \mathbf{e} - \mathbf{n}\underbrace{(\mathbf{e}^T\mathbf{n})}_{\text{Projection onto the surface normal}}$



- The intensity change function

$$E(u,v,w) = \begin{bmatrix} u & v & w \end{bmatrix} M \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad M = \sum_{x,y,z \in \Omega} \begin{bmatrix} I_x^2 & I_x I_y & I_x I_z \\ I_x I_y & I_y^2 & I_y I_z \\ I_x I_z & I_y I_z & I_z^2 \end{bmatrix}$$

- Now, how to compute corner response?
- Kanade & Tomasi (1994): $R = \lambda_3$
  - assume eigenvalues are sorted from large to small
- However, $R = \lambda_2$ is also valid.
  - Intensity corner on a surface is still a corner.

- Harris on image

$$E(u,v) = \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}, \quad \boxed{M = \sum_{x,y \in \Omega} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}}$$

Covariance of image gradient

- Harris on point cloud with intensity

$$E(u,v,w) = \begin{bmatrix} u & v & w \end{bmatrix} M \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad \boxed{M = \sum_{x,y,z \in \Omega} \begin{bmatrix} I_x^2 & I_x I_y & I_x I_z \\ I_x I_y & I_y^2 & I_y I_z \\ I_x I_z & I_y I_z & I_z^2 \end{bmatrix}}$$

Covariance of intensity gradient

- How about point cloud without intensity? Still Covariance!

- A local surface around p is

$$f(x,y,z)=0$$

- Similarly, we construct a cost function

$$E(u,v,w)=\sum_{x,y,z\in\Omega}[f(x+u,y+v,z+w)-f(x,y,z)]^2$$

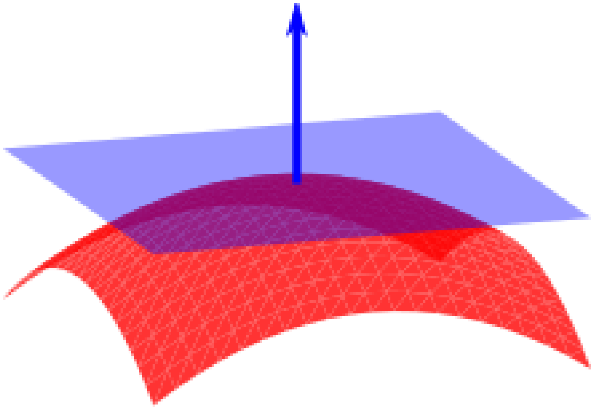
- But, what does it mean?
  - $f(x,y,z)=0 \rightarrow$ point $[x,y,z]$ on the surface
  - Move it by $[u,v,w]$, is it still on the surface?
    - Likely no.
    - Then, how far is it from the surface $f(x,y,z)=0$?

- Again, first-order approximation. A local surface around p becomes a plane.
  - The surface normal of p is $n = \left[n_x, n_y, n_z\right]^T$
  - $ax + by + cz + d = 0$, where $a = n_x, b = n_y, c = n_z$
- What is

$$ax' + by' + cz' + d,$$

$$x' = x + u, y' = y + v, z' = z + w$$

  - Distance of $[x', y', z']$ to the plane!
  - Why?
  - $dist(point, plane) = \frac{ax'+by'+cz'+d}{\sqrt{a^2+b^2+c^2}} = \frac{ax'+by'+cz'+d}{1}$

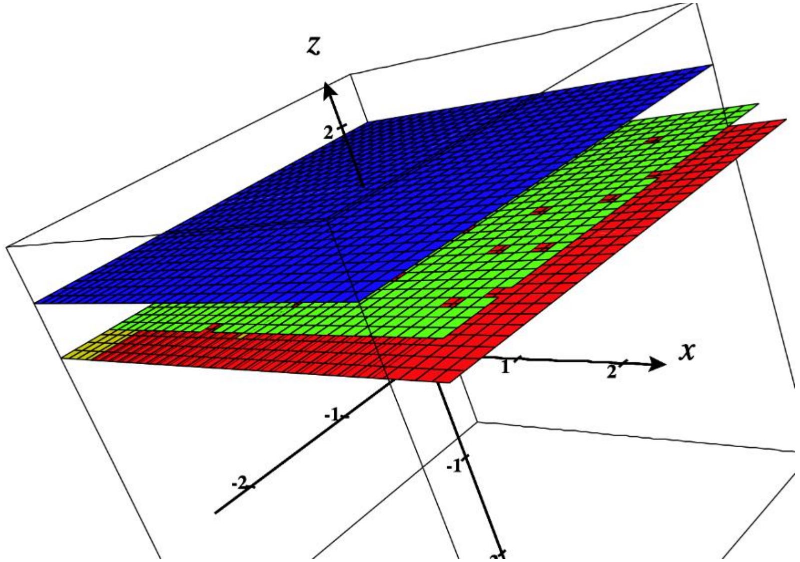


- Cost function

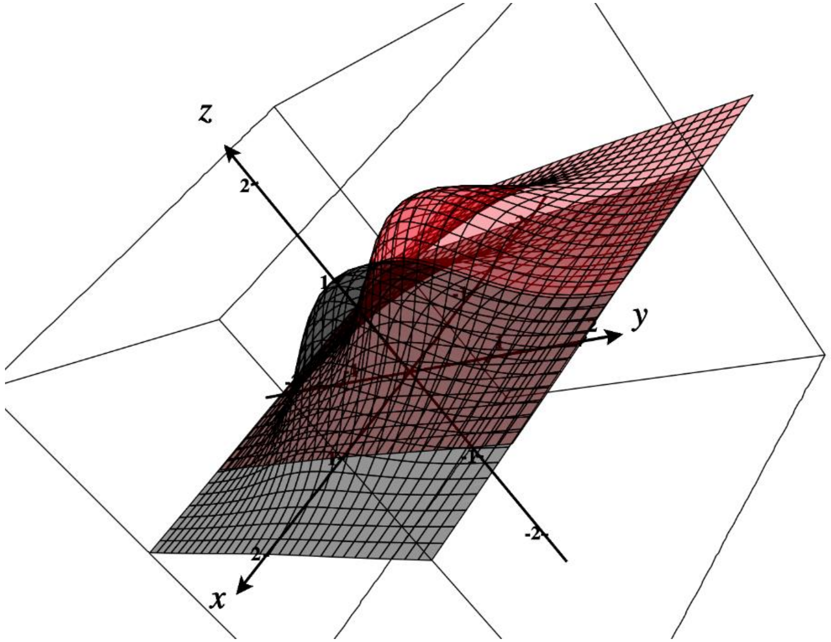
$$E(u,v,w) = \sum_{x,y,z \in \Omega} [f(x+u, y+v, z+w) - f(x,y,z)]^2$$

- Intuitively, what is that?
  - Move a point by $[u,v,w]$, how far is the new point to the surface?

- Example 1: points distributed over a x-y plane
  - Points in Ω change cost E only with w (along z direction)



- Example 2: points distributed over some shape
  - Points in Ω change cost E in various directions.



- Cost function

$$E(u,v,w) = \sum_{x,y,z\in\Omega} [f(x+u, y+v, z+w) - f(x,y,z)]^2$$

$$f(x,y,z) = ax + by + cz + d = n_x x + n_y y + n_z z + d$$

- Apply first-order Taylor Series approximation, we have

$$E(u,v,w) = \begin{bmatrix} u & v & w \end{bmatrix} M \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad M = \sum_{x,y,z\in\Omega} \begin{bmatrix} n_x^2 & n_x n_y & n_x n_z \\ n_x n_y & n_y^2 & n_y n_z \\ n_x n_z & n_y n_z & n_z^2 \end{bmatrix}$$

- M is the covariance matrix of surface normal over the surface.

- With intensity. Tomasi Response $R = \lambda_2$

Covariance of intensity gradient

$$E(u,v,w) = \begin{bmatrix} u & v & w \end{bmatrix} M \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad \boxed{M = \sum_{x,y,z \in \Omega} \begin{bmatrix} I_x^2 & I_x I_y & I_x I_z \\ I_x I_y & I_y^2 & I_y I_z \\ I_x I_z & I_y I_z & I_z^2 \end{bmatrix}}$$

- Without intensity. Tomasi Response $R = \lambda_3$

Covariance of surface normal

$$E(u,v,w) = \begin{bmatrix} u & v & w \end{bmatrix} M \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad \boxed{M = \sum_{x,y,z \in \Omega} \begin{bmatrix} n_x^2 & n_x n_y & n_x n_z \\ n_x n_y & n_y^2 & n_y n_z \\ n_x n_z & n_y n_z & n_z^2 \end{bmatrix}}$$

- Covariance matrix of $[I_x, I_y, I_z, n_x, n_y, n_z]$

$$M = \sum_{x,y,z \in \Omega} \begin{bmatrix} I_x^2 & I_xI_y & I_xI_z & I_xn_x & I_xn_y & I_xn_z \\ I_xI_y & I_y^2 & I_yI_z & I_yn_x & I_yn_y & I_yn_z \\ I_xI_z & I_yI_z & I_z^2 & I_zn_x & I_zn_y & I_zn_z \\ n_xI_x & n_xI_y & n_xI_z & n_x^2 & n_xn_y & n_xn_z \\ n_yI_x & n_yI_y & n_yI_z & n_xn_y & n_y^2 & n_yn_z \\ n_zI_x & n_zI_y & n_zI_z & n_xn_z & n_yn_z & n_z^2 \end{bmatrix}$$

- Tomasi Response $R = \lambda_4$
  - $R = \lambda_3$ is too loose
    - E.g., Harris 6D is a superset of Harris 3D without intensity when $R = \lambda_3$
  - $R = \lambda_5$ is too strict
    - E.g., A corner has to be a geometric corner AND intensity corner at the same time.

Harris Family Summary

	Input	Covariance Matrix	Criteria	Intuition
Harris Image	Image	Intensity gradient	λ_2 is small	Intensity corners
Harris 3D	PC with Intensity	Intensity gradient	λ_2 is small	Intensity corners in local surface
Harris 3D	PC	Surface normals	λ_3 is small	Corners in 3D space
Harris 6D	PC with Intensity	Intensity gradient + surface normals	λ_4 is small	Corners in either 3D space / local surface intensity

Native Method - ISS

- Harris Corners 3D/6D are extensions of image based Harris corners.
- How about native methods for point cloud?

- Intrinsic Shape Signatures (ISS): A Shape Descriptor for 3D Object Recognition
  - Keypoints are those have large 3D point variations in their neighborhood
  - Simple, Principle Component Analysis (PCA)
  - The smallest eigenvalue of the covariance matrix should be large.

ISS

- Given a point $p_i \in \mathbb{R}^3$, compute its weighted covariance matrix over a radius r
  - Points at sparse regions contribute more than those at dense regions

1. The weight of any point p_j is

   - $w_j = \frac{1}{|\{p_k : \|p_k - p_j\|_2 < r\}|}$
   - Inversely related to number of points in its neighborhood within r

2. Weighted covariance matrix:

$$Cov(p_i) = \frac{\sum_{\|p_j - p_i\|_2 < r} w_j (p_j - p_i)(p_j - p_i)^T}{\sum_{\|p_j - p_i\|_2 < r} w_j}$$

4. Compute eigenvalues of $Cov(p_i)$ as $\lambda_i^1, \lambda_i^2, \lambda_i^3$, in the order decreasing magnitude
5. p_i is a keypoint if

$$\frac{\lambda_i^2}{\lambda_i^1} < \gamma_{21} \;\; and \;\; \frac{\lambda_i^3}{\lambda_i^2} < \gamma_{32}$$

- A flat surface can be $\lambda_i^1 = \lambda_i^2 > \lambda_i^3$
- A line can be $\lambda_i^1 > \lambda_i^2 = \lambda_i^3$
- So we have to ensure $\lambda_i^1 > \lambda_i^2 > \lambda_i^3$

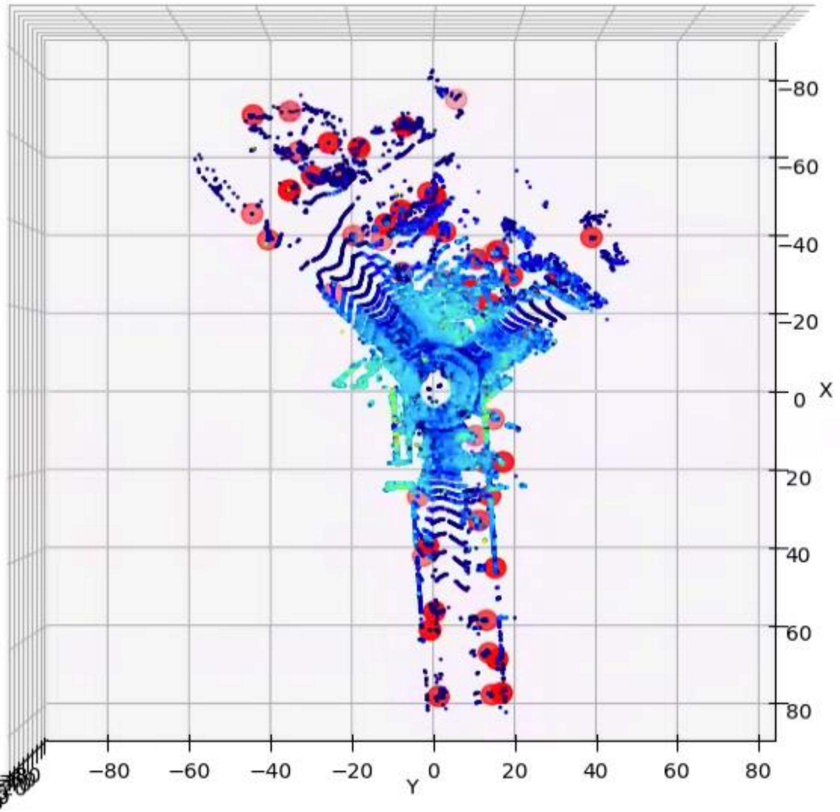
6. Non-Maximum Suppression (NMS) with λ_i^3

Classical Methods Summary

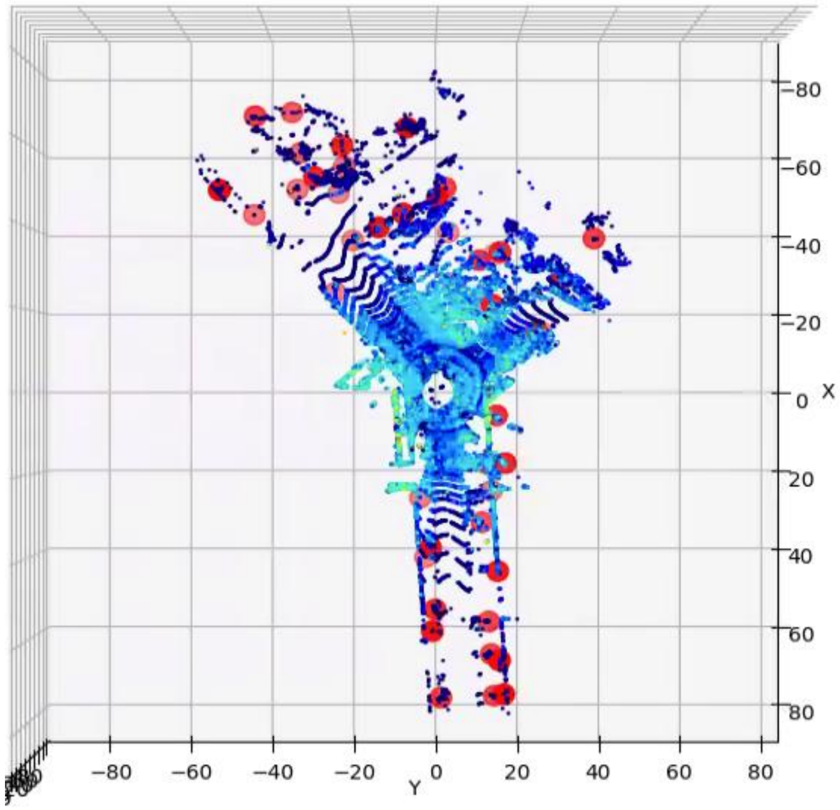
	Input	Covariance Matrix	Criteria	Intuition
Harris Image	Image	Intensity gradient	λ_2 is small	Intensity corners
Harris 3D	PC with Intensity	Intensity gradient	λ_2 is small	Intensity corners in local surface
Harris 3D	PC	Surface normals	λ_3 is small	Corners in 3D space
Harris 6D	PC with Intensity	Intensity gradient + surface normals	λ_4 is small	Corners in either 3D space / local surface intensity
ISS	PC	Weight point coordinates	$\frac{\lambda_i^2}{\lambda_i^1} < \gamma_{21}, \frac{\lambda_i^3}{\lambda_i^2} < \gamma_{32}$	Point distribution is different in 3 dimensions

Examples - KITTI

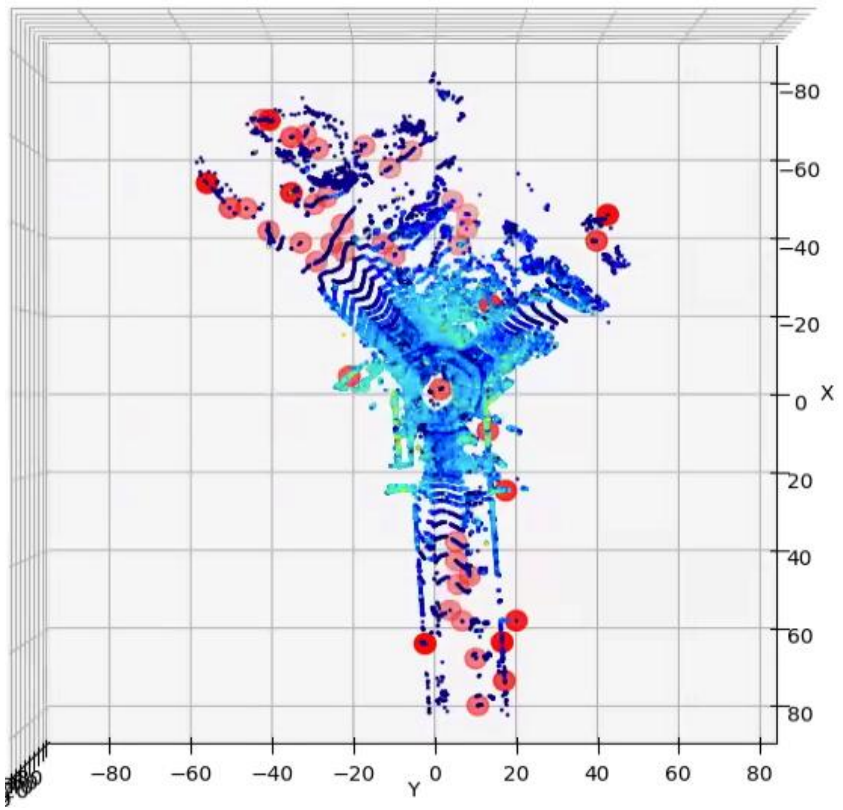
Harris 3D
without intensity



Harris 6D
with intensity

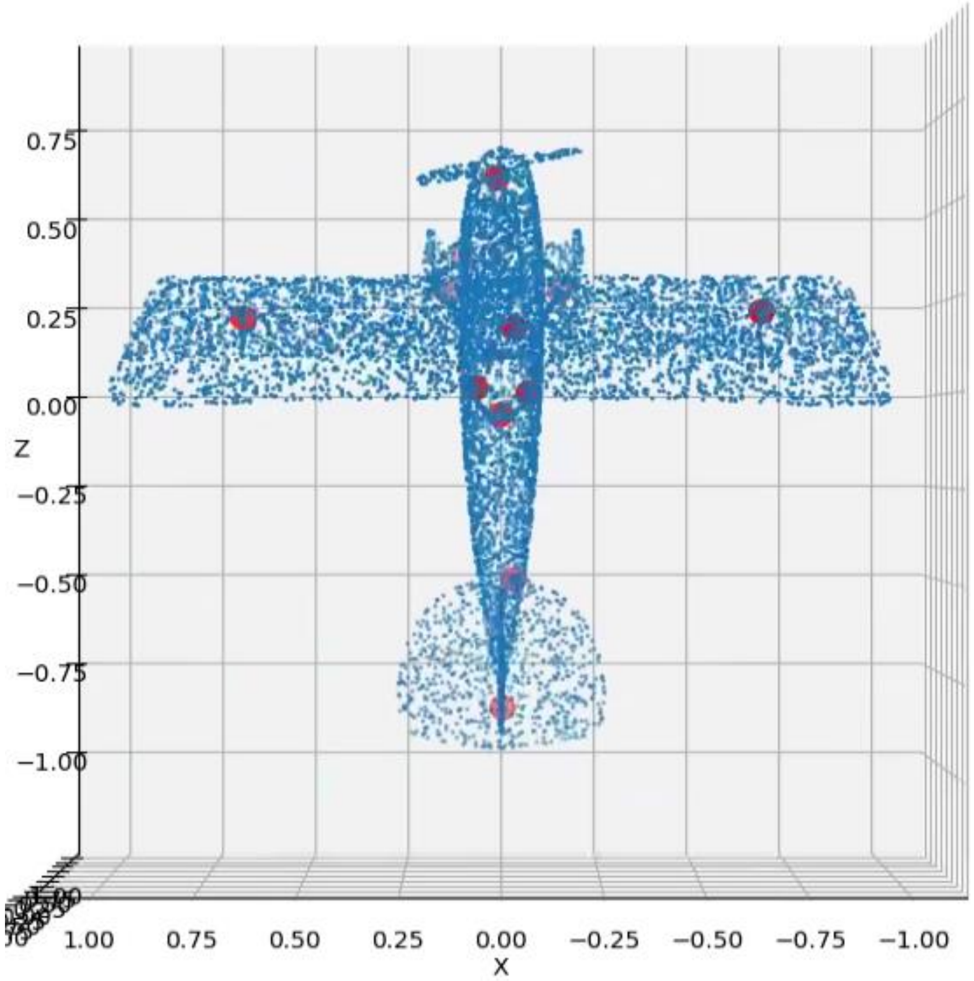


ISS

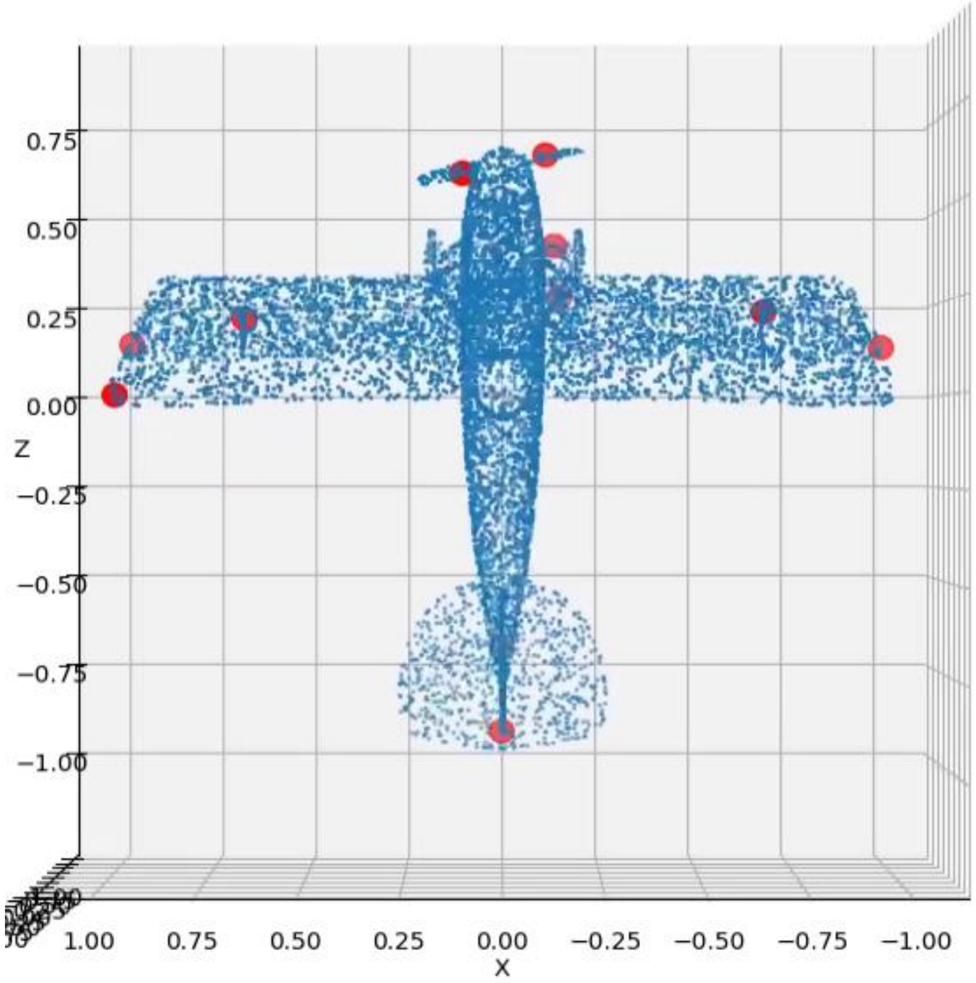


Examples – ModelNet

Harris 3D
without intensity

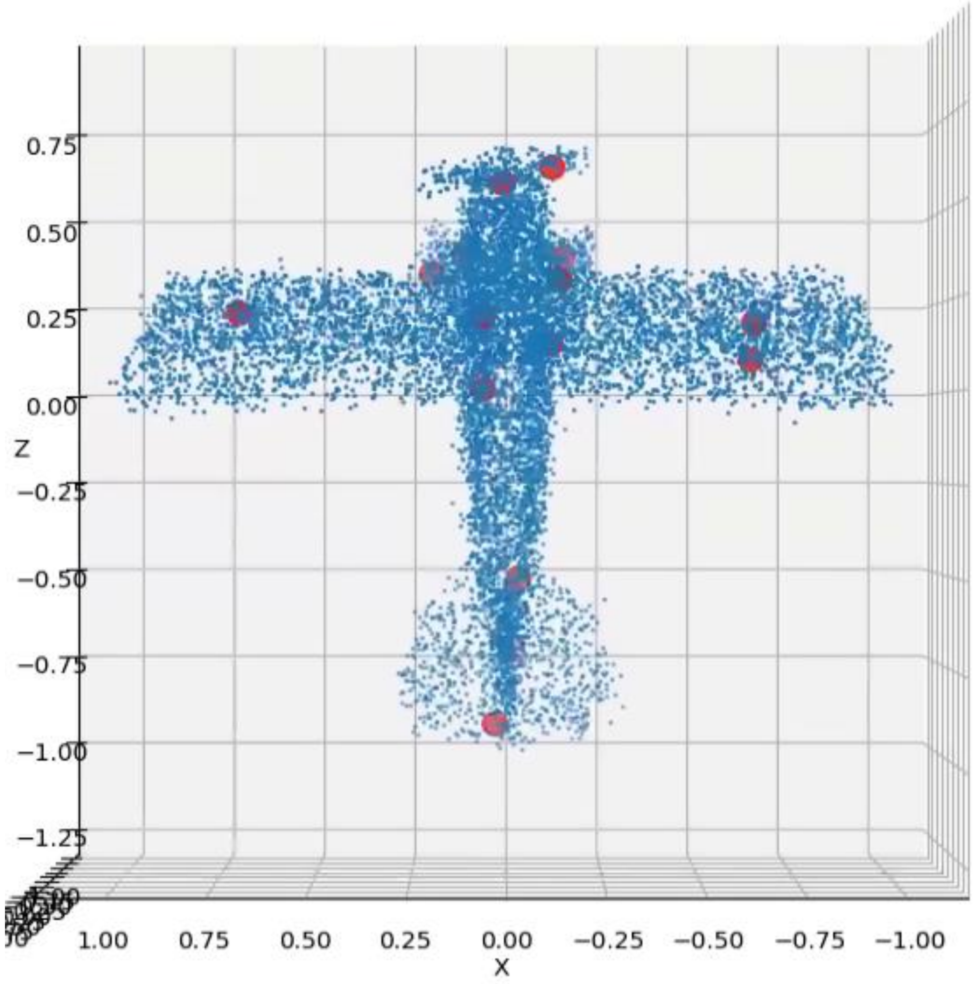


ISS

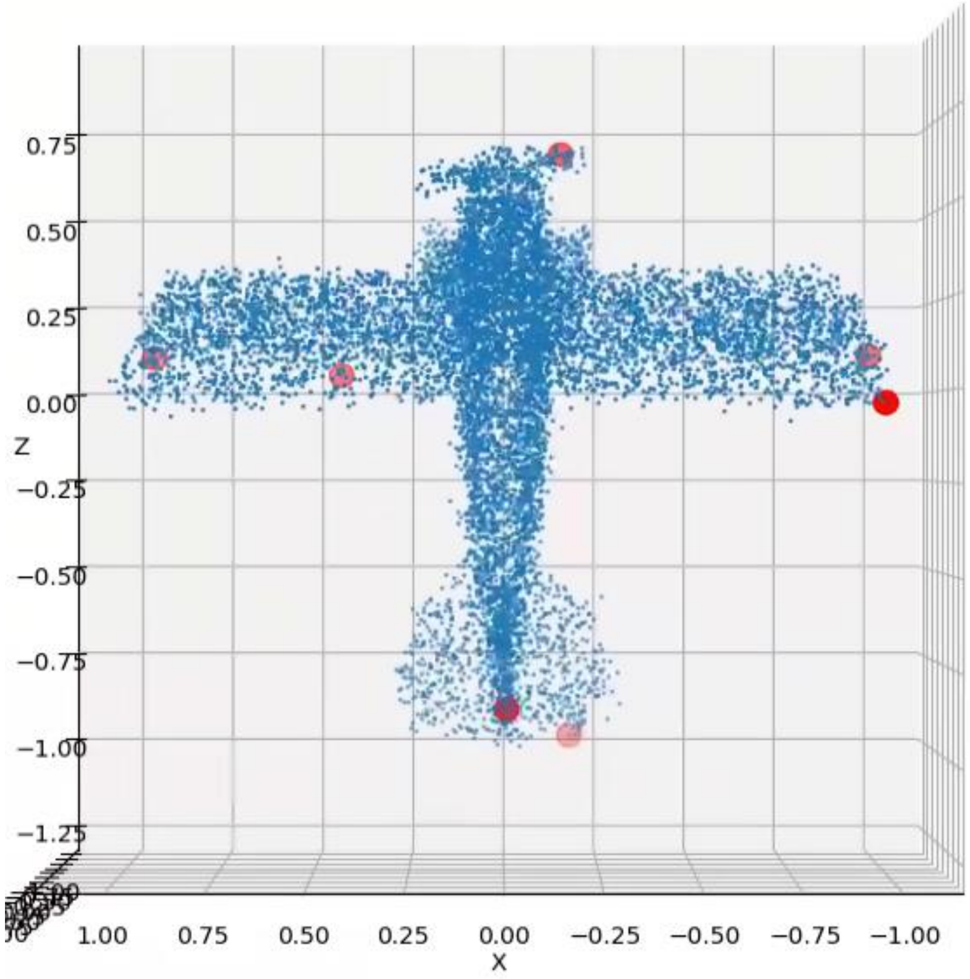


Examples – ModelNet with Noise

Harris 3D
without intensity



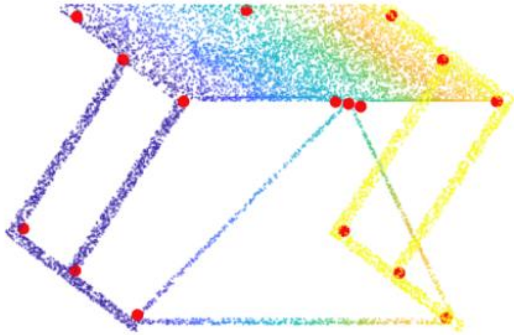
ISS



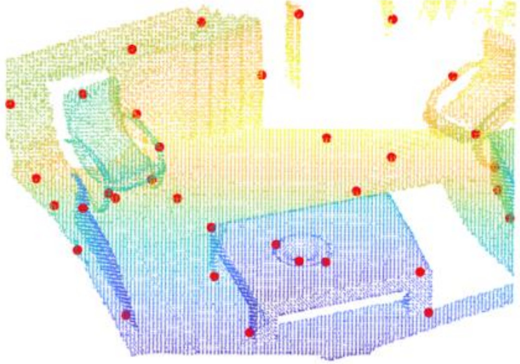
- Very few deep learning method for feature detection in point cloud
  - Definition of “feature” is unclear.
  - No annotation/dataset available.
  - Other difficulties of point cloud processing
    - Rotation equivariance.
    - Sparsity
    - … …

- Probably there are only 2 methods before 2020.
  - USIP: Unsupervised Stable Interest Point Detection from 3D Point Clouds
  - 3DFeat-Net: Weakly Supervised Local 3D Features for Point Cloud Registration

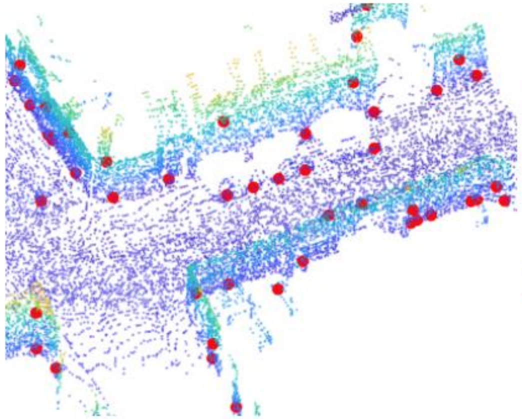
USIP Features



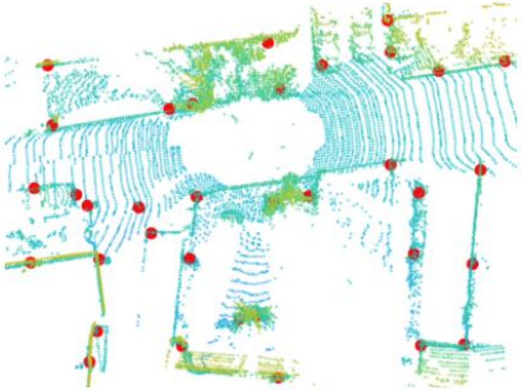
CAD Model



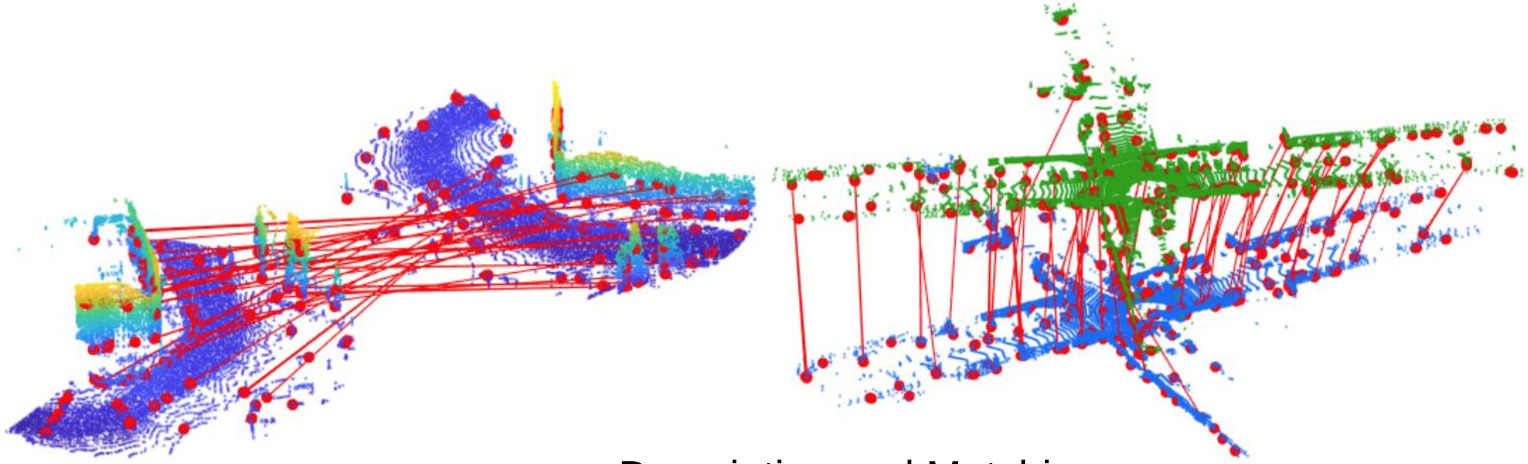
RGBD



Oxford Robotcar



KITTI



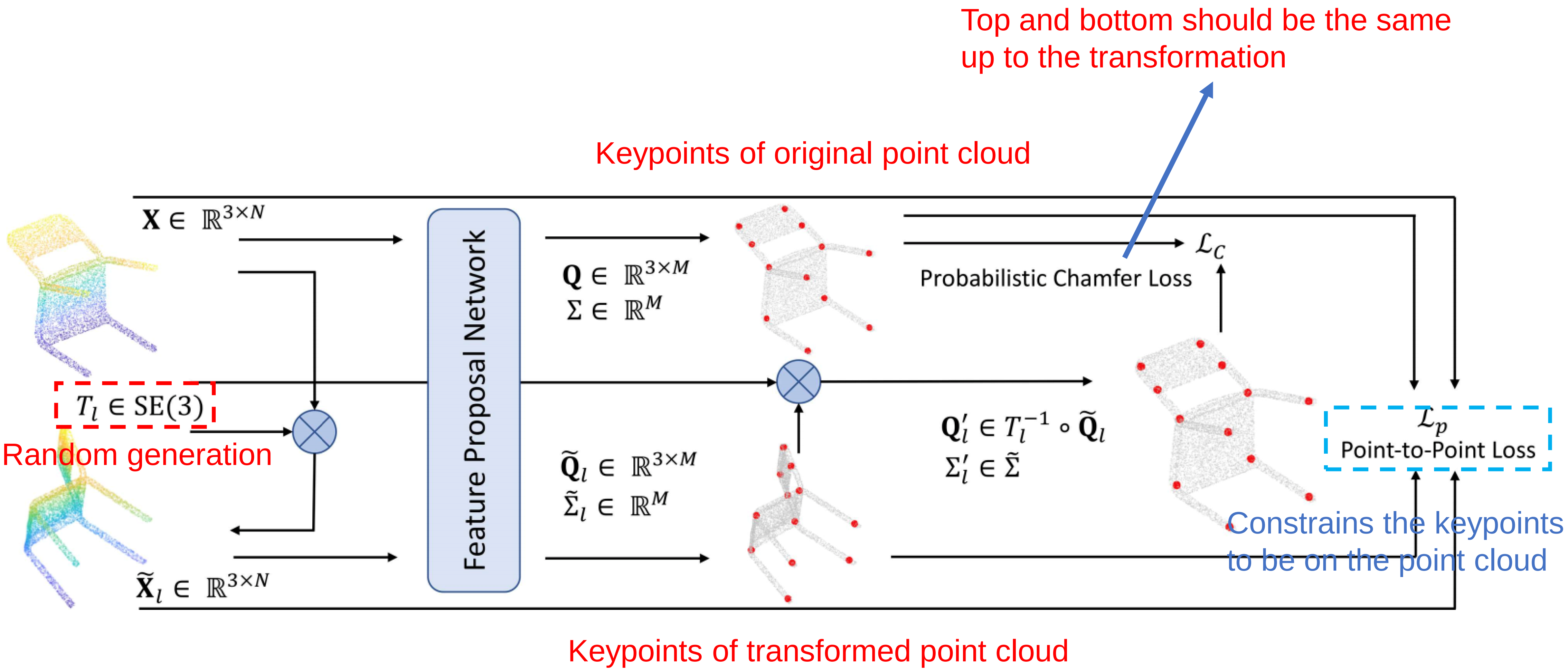
Description and Matching

Source: USIP: Unsupervised Stable Interest Point Detection from 3D Point Clouds, Jiaxin Li and Gim Hee Lee

- Unsupervised – Let the network decide what is a keypoint

- Intuition
  - A keypoint is still a keypoint no matter how we transform the object.
  - The concept “keypoint” depends on scale / level-of-detail
    - The center of an object is a keypoint, if we look at a large scale
    - The textures of the tire are keypoints if we look at the surface of the tire, not keypoints if we are at the scale of the vehicle

USIP – Architecture

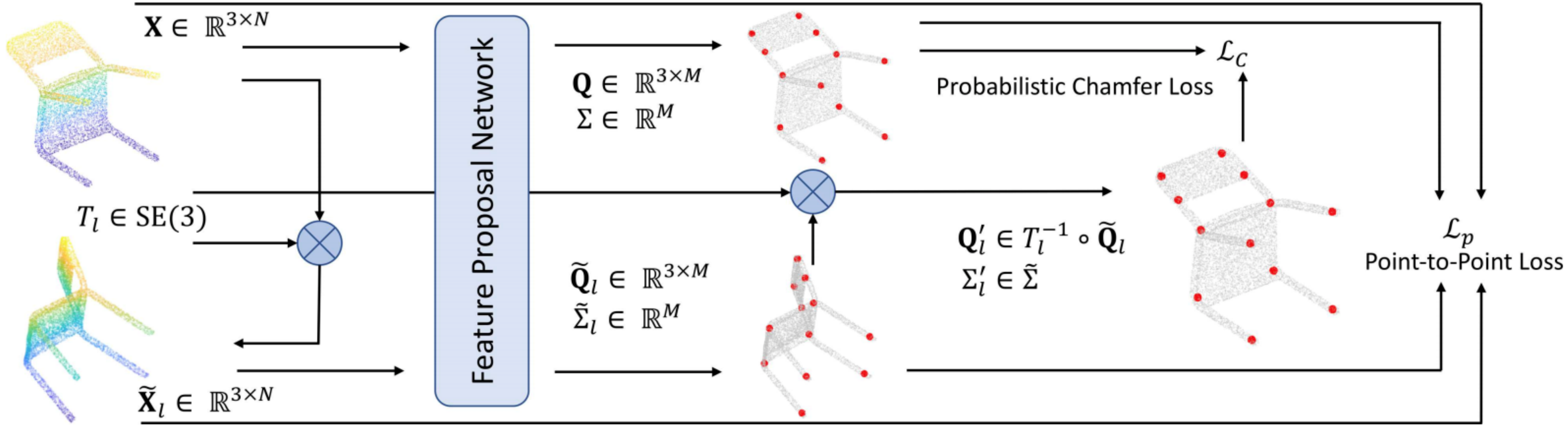


USIP – Architecture

- Notations
  - Point cloud $X = [X_1, \cdots, X_N] \in \mathbb{R}^{3\times N}$
  - Transformation matrix $T \in SE(3)$, i.e., $T = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{4\times 4}$
- Input
  - A point cloud: X
- Output
  - M Keypoints $Q = \{Q_1, \cdots, Q_M\}, Q_i \in \mathbb{R}^{3\times M}$
  - The uncertainties $\Sigma = \{\sigma_1, \cdots, \sigma_M\}, \sigma_i \in \mathbb{R}^+$

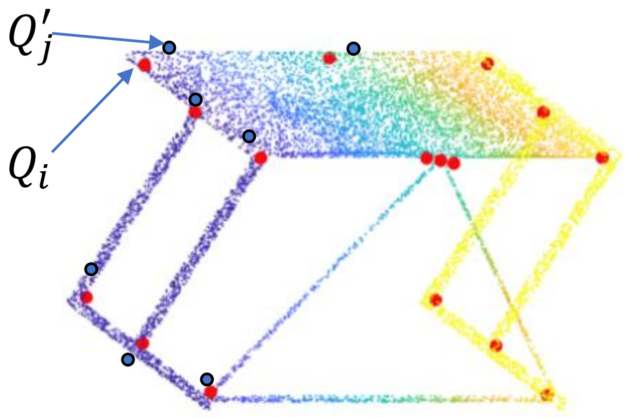
USIP – Training Procedures

1. Feed X into FPN (Feature Proposal Network), get Q, Σ
2. Randomly generate $T_l \in SE(3)$
3. Transform X into $\tilde{X}_l = T_l \circ X$, where $\tilde{X}_l = T_l \circ X = RX + t$
4. Feed $\tilde{X}_l$ into FPN, get $\tilde{Q}_l, \tilde{\Sigma}_l$
5. Transform $\tilde{Q}_l, \tilde{\Sigma}_l$ back into $Q_l' = T_l^{-1} \circ \tilde{Q}_l, \quad \Sigma_l' = \tilde{\Sigma}_l$
6. Loss function: make $\{Q, \Sigma\} = \{Q_l', \Sigma_l'\}$

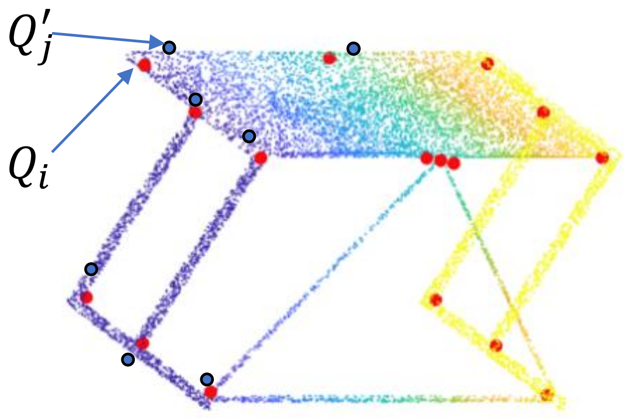


- Standard Chamfer Loss

$$\sum_{i=1}^{M} \min_{Q'_j \in \mathbf{Q}'} \|Q_i - Q'_j\|_2 + \sum_{j=1}^{M} \min_{Q_i \in \mathbf{Q}} \|Q_i - Q'_j\|_2$$



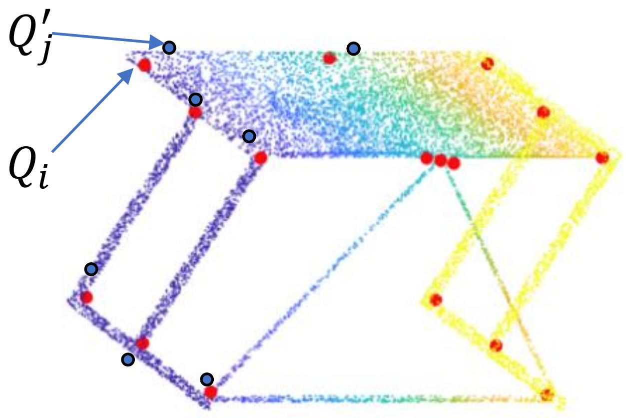
- Problems:
  - Not all keypoints are equal.
  - The receptive field (explained later) of the FPN is limited
    - Some keypoint Q_i locates on smooth surface
    - These highly uncertain keypoints hurts the loss function
- Solution → Probabilistic Chamfer Loss
  - Add weighting according the the predicted uncertainty Σ



- How to convert the uncertainty $\sigma_i \in \mathbb{R}^+$ into probability?
  - Consider a pair of keypoints $\{Q_i, \sigma_i\}$ and $\{Q_j', \sigma_j'\}$
  - Distance $d_{ij} = \left\|Q_i - Q_j'\right\|_2$
- Model the probability of Q_i, Q_j' being the same keypoint (they are matched) as,

$$p(d_{ij} \mid \sigma_{ij}) = \frac{1}{\sigma_{ij}} \exp\left(-\frac{d_{ij}}{\sigma_{ij}}\right), \quad \sigma_{ij} = \frac{\sigma_i + \sigma_j'}{2} > 0, \quad d_{ij} = \min_{Q_j' \in \mathbf{Q}'} \|Q_i - Q_j'\|_2 \geq 0.$$

- This is an exponential distribution
  - d_{ij} is the random variable
  - σ_{ij} is the parameter of the distribution, which is given by the FPN



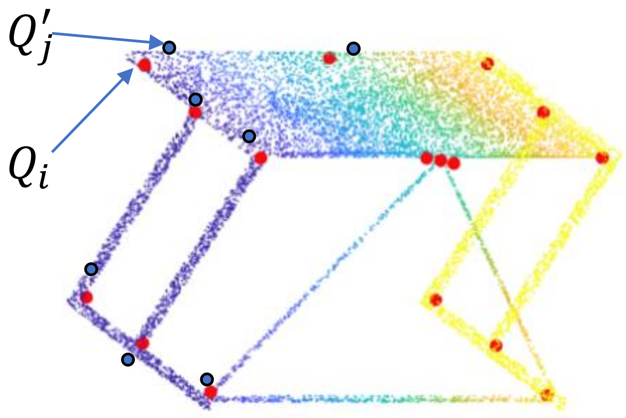
- For Q_i, how do I know which $\{Q'_j, \sigma'_j\}$ is the matched one?
  - Same as standard Chamfer loss
  - For each Q_i, find the nearest keypoint in Q' as the matched
  - For each Q'_j, find the nearest keypoint in Q as the matched
- Now, for the blue / green part, we have

$$p(D_{ij} \mid \Sigma_{ij}) = \prod_{i=1}^{M} p(d_{ij} \mid \sigma_{ij})$$

$$p(D_{ji} \mid \Sigma_{ji}) = \prod_{j=1}^{M} p(d_{ji} \mid \sigma_{ji})$$

$$\sigma_{ij} = \frac{\sigma_i + \sigma'_j}{2} > 0, \quad d_{ij} = \min_{Q'_j \in \mathbf{Q}'} \|Q_i - Q'_j\|_2 \geq 0$$

$$\sigma_{ji} = \frac{\sigma'_j + \sigma_i}{2} > 0, \quad d_{ji} = \min_{Q_i \in \mathbf{Q}} \|Q_i - Q'_j\|_2 \geq 0$$



- Final loss, maximize the joint likelihood → minimize the log likelihood

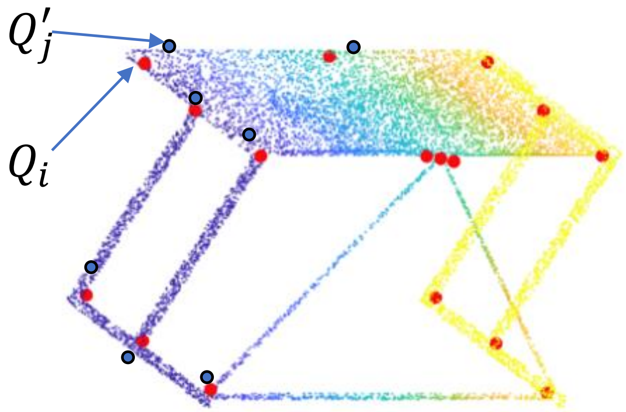
$$\mathcal{L}_c = -\ln p(D_{ji}|\Sigma_{ji}) - \ln p(D_{ji}|\Sigma_{ji})$$

$$\mathcal{L}_c = \sum_{i=1}^{M} -\ln p(d_{ij} \mid \sigma_{ij}) + \sum_{j=1}^{M} -\ln p(d_{ji} \mid \sigma_{ji})$$

$$= \sum_{i=1}^{M} \left(\ln \sigma_{ij} + \frac{d_{ij}}{\sigma_{ij}} \right) + \sum_{j=1}^{M} \left(\ln \sigma_{ji} + \frac{d_{ji}}{\sigma_{ji}} \right)$$

- In fact, according to our Probabilistic Chamfer Loss, uncertainty σ has physical meanings!

Probabilistic Chamfer Loss



- Look at the basic element of the loss function

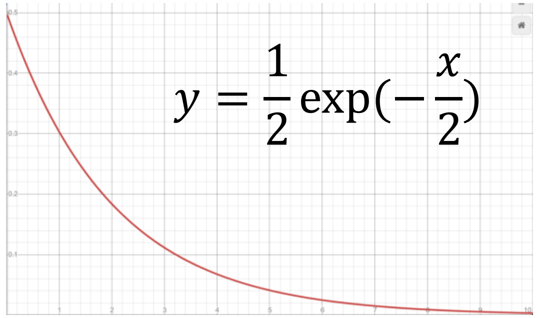
$$p(d_{ij} \mid \sigma_{ij}) = \frac{1}{\sigma_{ij}} \exp\left(- \frac{d_{ij}}{\sigma_{ij}} \right)$$

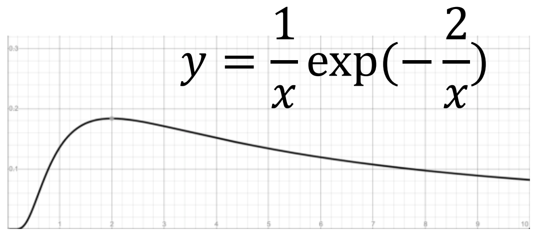
- Compute its first derivative over σ_{ij}

$$\frac{\partial p(d_{ij} \mid \sigma_{ij})}{\partial \sigma_{ij}} = \frac{d_{ij} \exp(-d_{ij}/\sigma_{ij})}{\sigma_{ij}^3} - \frac{\exp(-d_{ij}/\sigma_{ij})}{\sigma_{ij}^2}$$

- Solve for the stationary point:

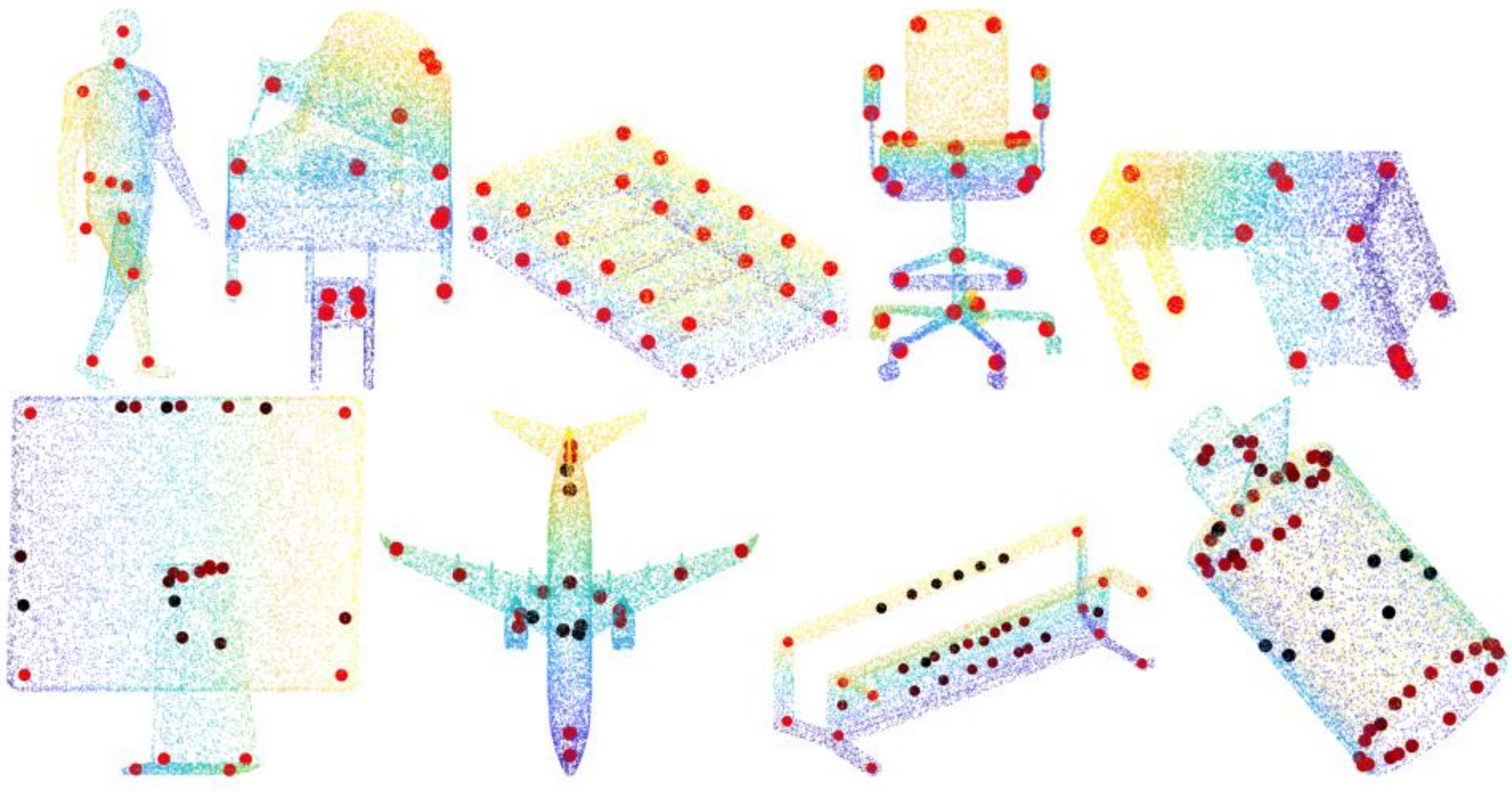
$$\frac{\partial p(d_{ij} \mid \sigma_{ij})}{\partial \sigma_{ij}} = 0 \Rightarrow \sigma_{ij} = d_{ij}$$





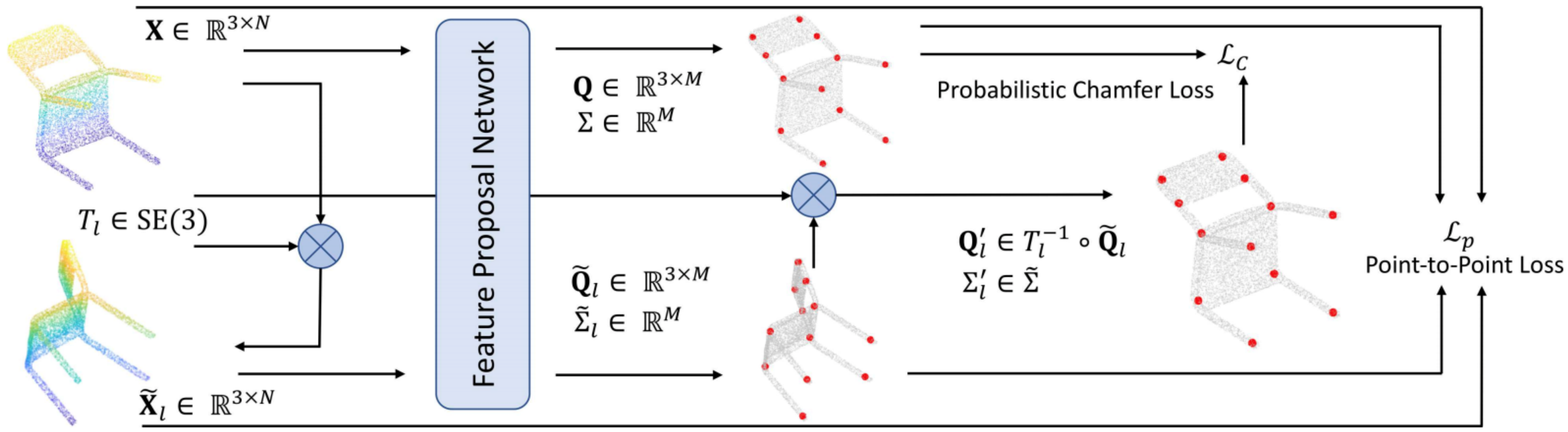
- It means the best σ_{ij} equals to d_{ij}. Network is predicting the matching distance!

Probabilistic Chamfer Loss



USIP keypoints on ModelNet40.

Red - σ is small, low uncertainty. Black - σ is large, high uncertainty



- What if the network predicts the object center as the keypoint?
  - $Q_1 = Q_2 = \cdots = Q_M = object\ center = \tilde{Q}_i = Q'_i$
  - $L_c = 0$, perfect!
  - But this is not what we want!

Lemma 1. *$f(\mathbf{Y}') \equiv Rf(\mathbf{Y}) \oplus t$ when $f(.)$ outputs the* ***centroid*** *of the input point cloud,* i.e., $f(\mathbf{Y}) = \frac{1}{N}\sum_n Y_n$ *and* $f(\mathbf{Y}') = \frac{1}{N}\sum_n Y'_n$.

Degeneracy and Receptive Field

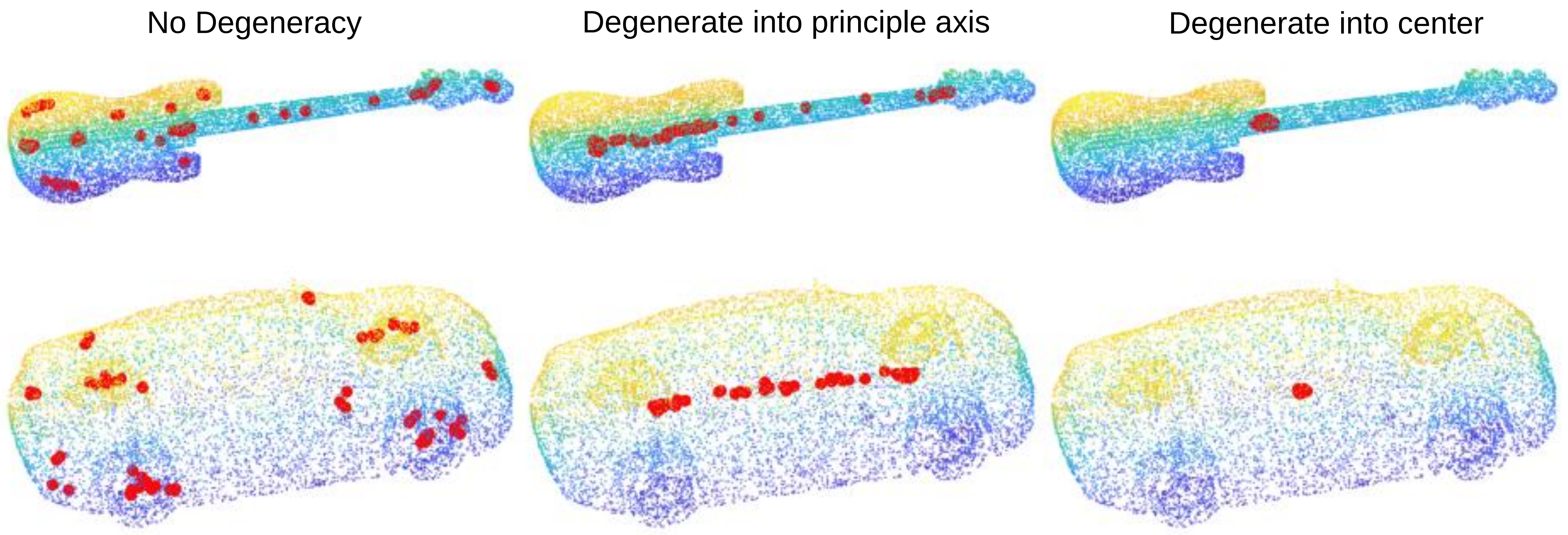
- Further more, the loss L_c will be small if
  - Network outputs points on the principle axis
- Intuition:
  - principle axis doesn't change no matter what is the transform T_l
  - Similar to object center
- Mathematically, it is Lemma 2
- How to proof?
  - Similar to the proof of PCA
  - Here we just need to understand the intuition

Lemma 2. *$f(\mathbf{Y}') \equiv R f(\mathbf{Y}) \oplus t$ when $f(.)$ is translational equivariant,* i.e., *$f(\cdot) \oplus t = f(\cdot \oplus t)$, and outputs points that are in the linear subspace of any **principal axis** from the input point cloud denoted as $\mathbf{U} = [U_1, U_2, U_3] \in \mathbb{R}^{3\times 3}$,* i.e., *$f(\mathbf{Y}) = [c_1 U_i^T, \cdots, c_M U_i^T]^T$ and*

$$\begin{aligned} f(\mathbf{Y}') &= f(R\mathbf{Y} \oplus t) \\ &= f(R\mathbf{Y}) \oplus t \qquad \textit{(translation equivariance)} \qquad (10) \\ &= [c_1 U_i'^T, \cdots, c_M U_i'^T]^T \oplus t, \end{aligned}$$

where U_i can be any principal axis in $\mathbf{U}$ and $c_1, \cdots, c_M$ are scalar coefficients in $\mathbb{R}$.

Degeneracy and Receptive Field

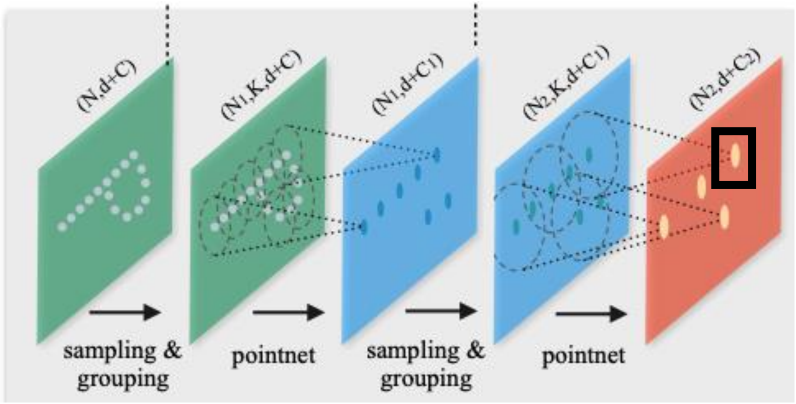


- When does these two “degeneracy” happen?
  - The receptive field is large enough
  - The network can predict where is the center / principle

Degeneracy and Receptive Field

- There are two degeneracy cases:
  - Object center
  - Principle axis
- How to prevent them?
  - Design the receptive field of the FPN carefully.
- A keypoint's Receptive Field
  - What are the points that are visible to the FPN when predicting this keypoint?
  - If the FPN only sees a small part of the object, it can't predict where is the center / principle axis

- FPN can be any network, with one requirement
  - The receptive field of each keypoint is controllable.
- For example, we can use PointNet++ as the FPN.



Each point has limited receptive field.

- In fact, USIP uses SO-Net as FPN
  - SO-Net: Self-Organizing Network for Point Cloud Analysis
  - Better performance, adaptive to various point density

- The Feature Proposal Network (FPN) doesn't ensure keypoints on the object
  - Simply add a constraint by standard Chamfer Loss

$$\mathcal{L}_{\text{point}} = \sum_{i=1}^{M} \min_{X_j \in \mathbf{X}} \|Q_i - X_j\|_2^2 + \sum_{i=1}^{M} \min_{\tilde{X}_j \in \tilde{\mathbf{X}}} \|\tilde{Q}_i - \tilde{X}_j\|_2^2$$

- Final Loss function

$$L = L_c + \lambda L_{point}$$

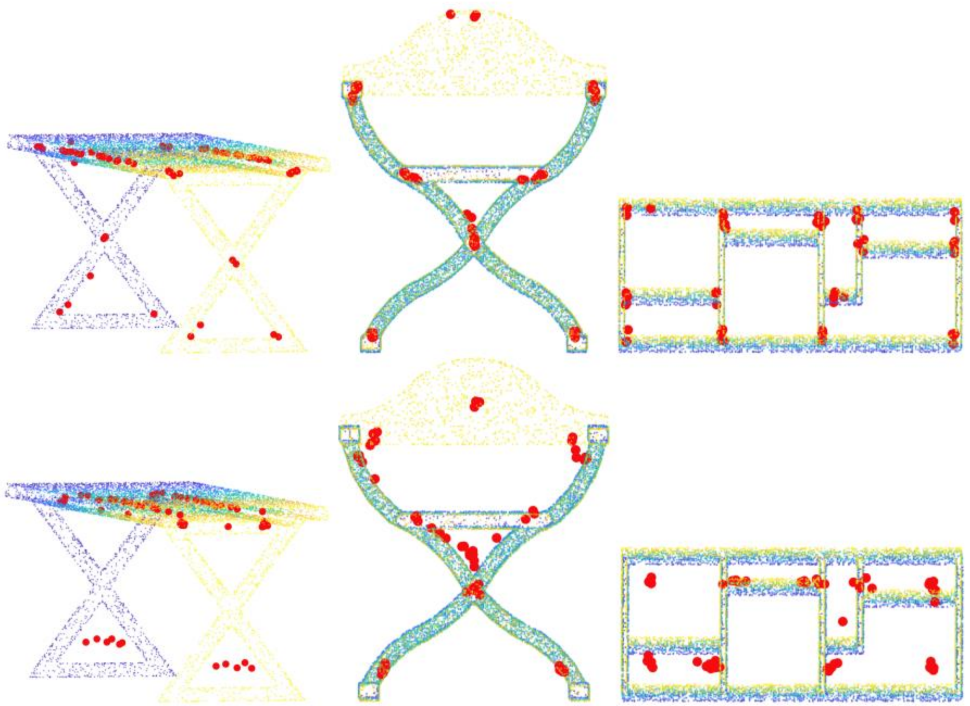
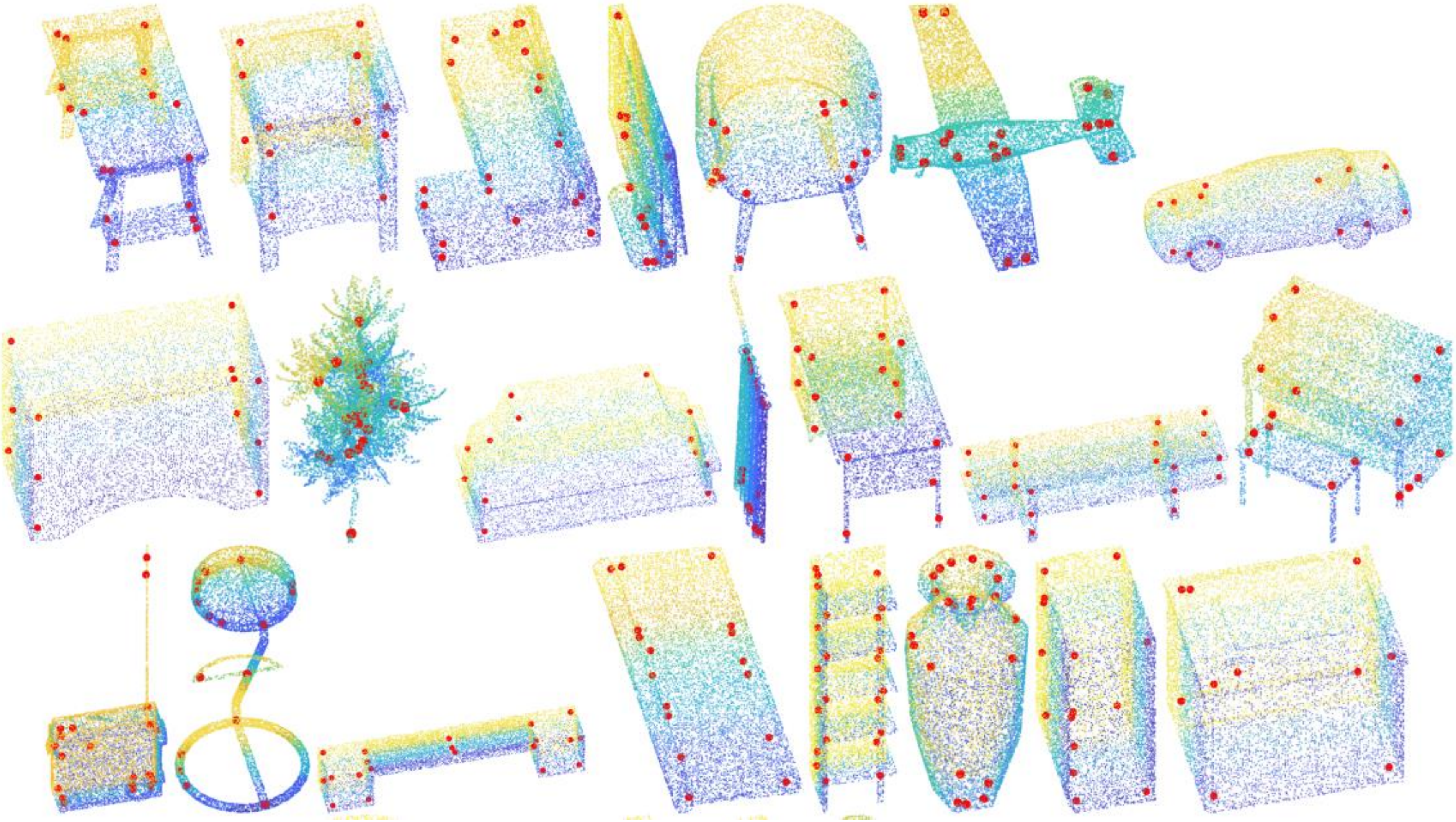
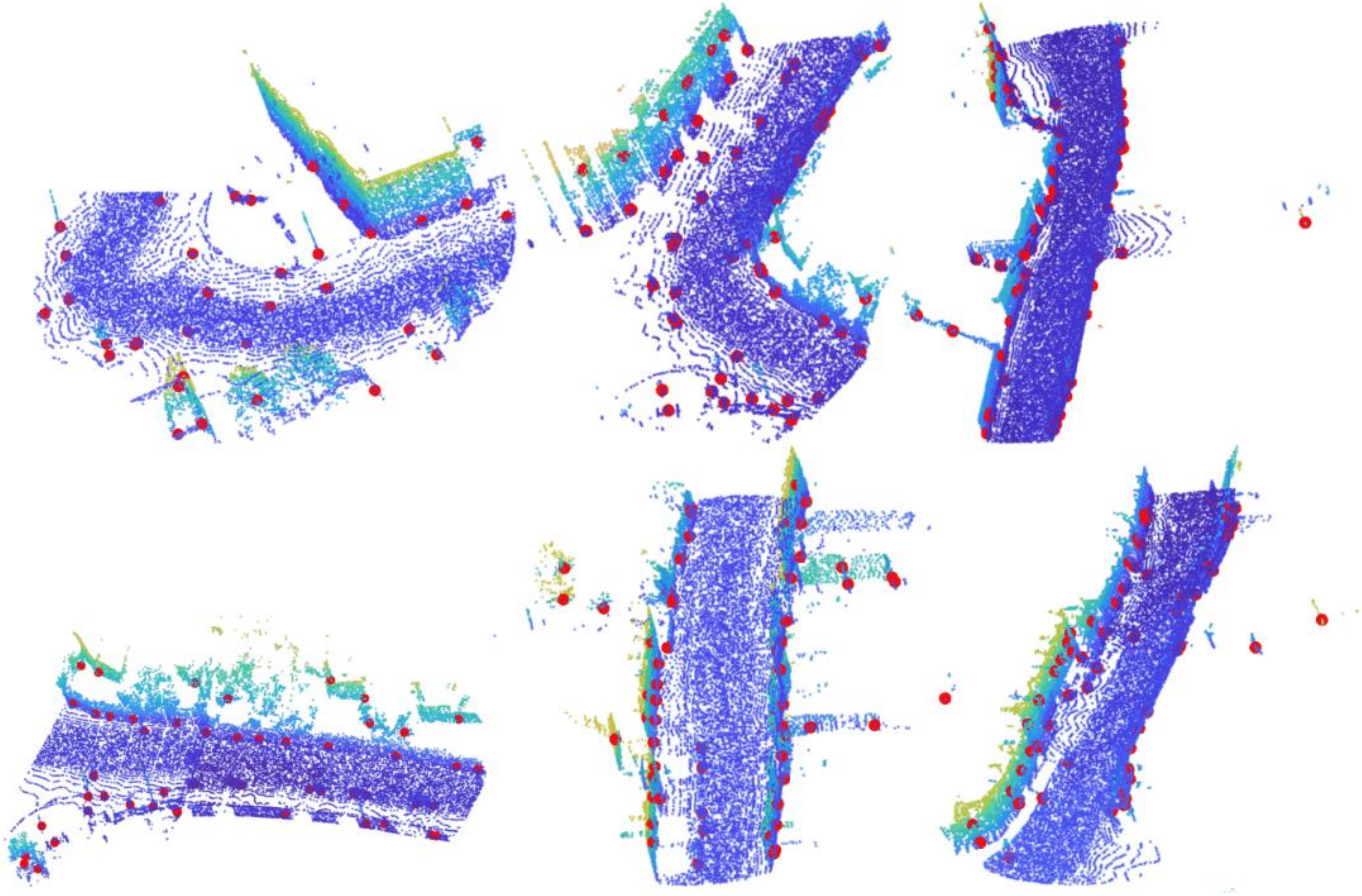


Figure 9. Visualization of USIP keypoints with different λ in Point-to-Point loss. First row $\lambda = 6$, second row $\lambda = 0$.

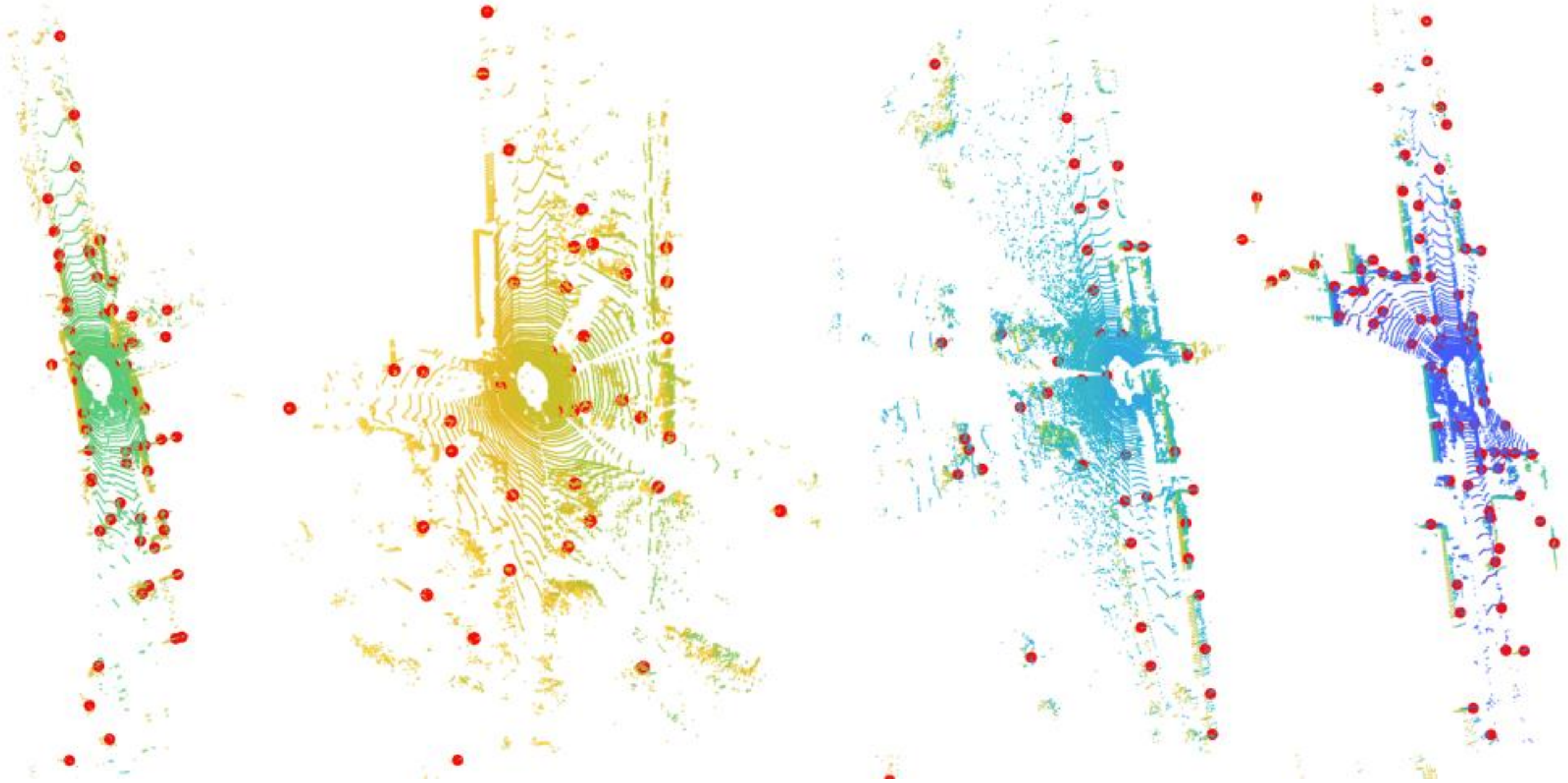
USIP Results – ModelNet40



USIP Results – Oxford RobotCar



USIP Results – KITTI



USIP Summary

- Ideas
  - Predicts keypoint locations and uncertainties
  - Unsupervised training by Probability Chamfer Loss
  - Controllable receptive field to prevent degeneracy

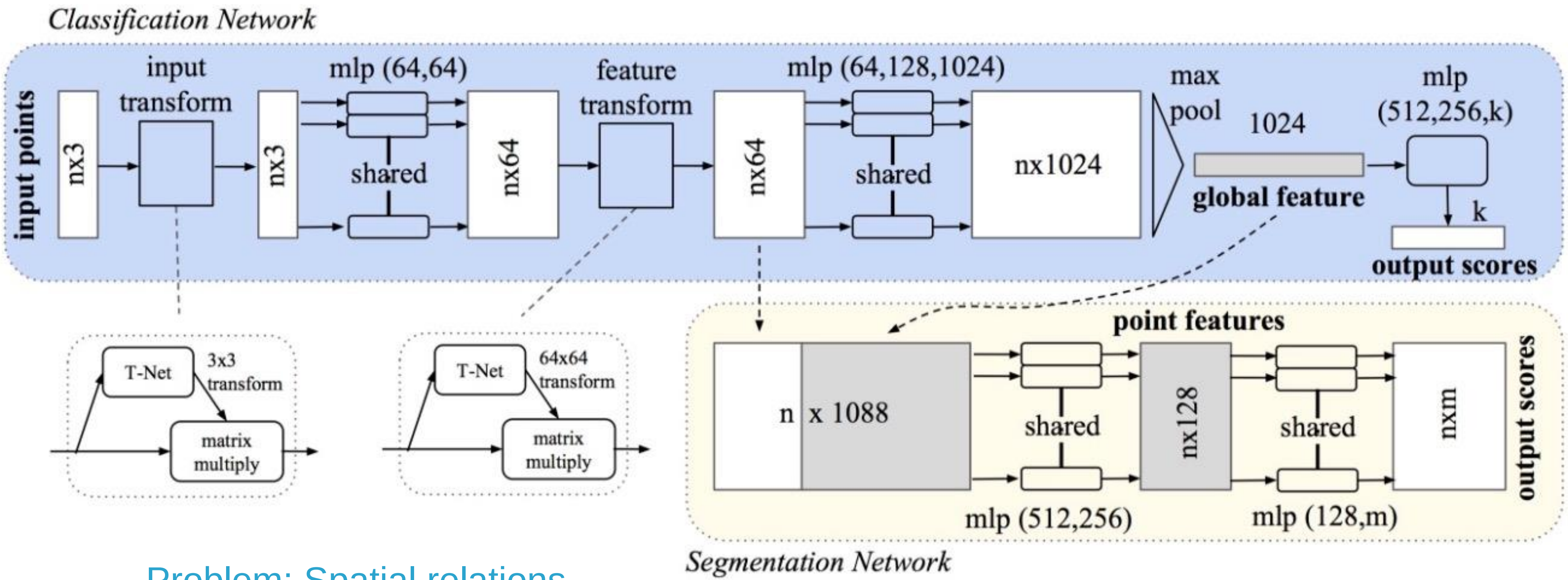
- Advantages
  - Data driven
  - Robust to noise / sparsity
  - Very stable & repeatable

Point Cloud Keypoints Summary

- Handcrafted – Not stable, sensitive to noise. But doesn't need training.
  - Harris family
    - Harris 3D with/without intensity
    - Harris 6D with intensity
  - ISS
- Deep learning – Stable and robust. But needs training (unsupervised).
  - USIP

- Implement your own ISS keypoint detection.
  - Apply your own ISS on ModelNet40 (choose 3 objects from different categories)
  - Visualize the object and the keypoints together.
  - Submit your code and the visualize screenshots
- Implement feature descriptors.
  - Homework of Lecture 8.
- Register two point clouds by feature detection and description
  - Homework of Lecture 9.

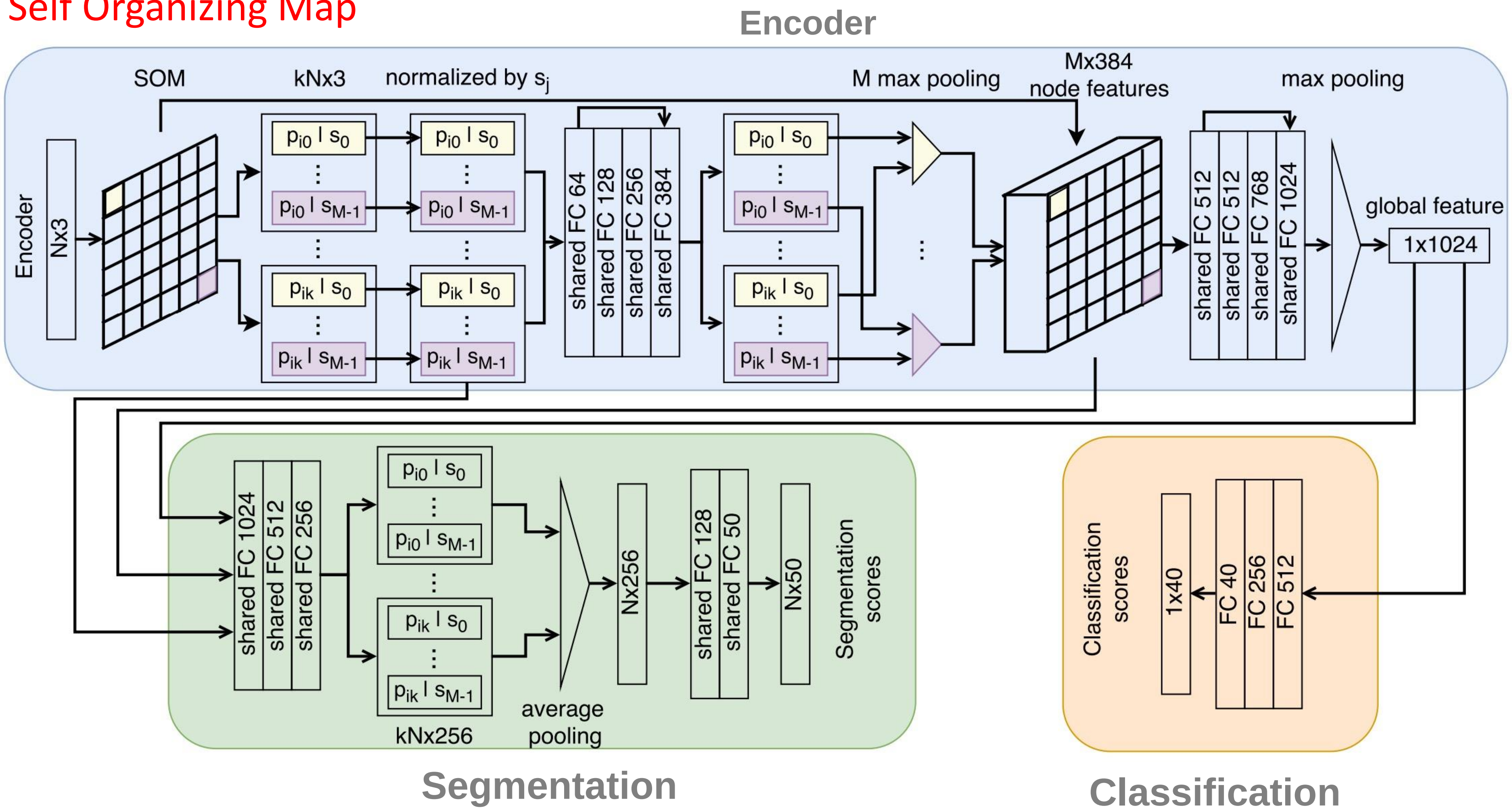
SO-Net – Prior Works



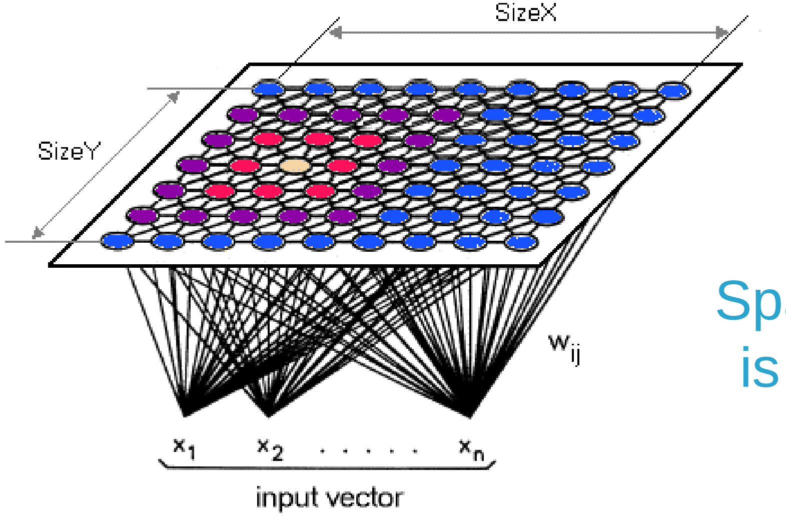
Problem: Spatial relations between points are ignored!

SO-Net – Model the Space with SOM

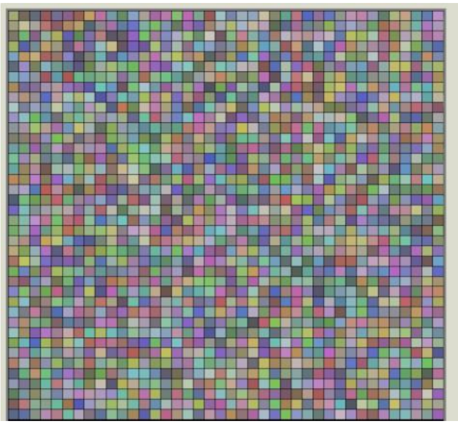
Self Organizing Map



Self-Organizing Map

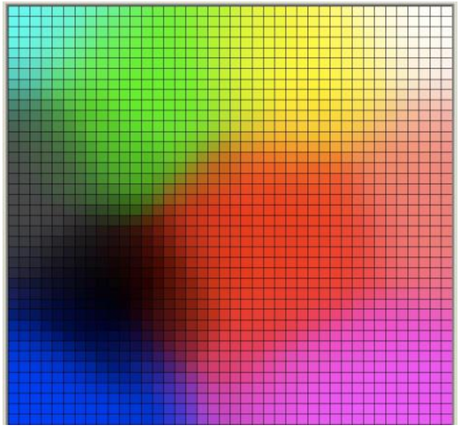


Spatial relation
is preserved!



SOM

Clustering effect



$$P = \{p_i \in \mathbb{R}^3, i = 1, \cdots, N\}$$
$$S_M = \{s_j \in \mathbb{R}^3, j = 1, \cdots, M\}$$

Competition
Given p_i, find the winning node s_j

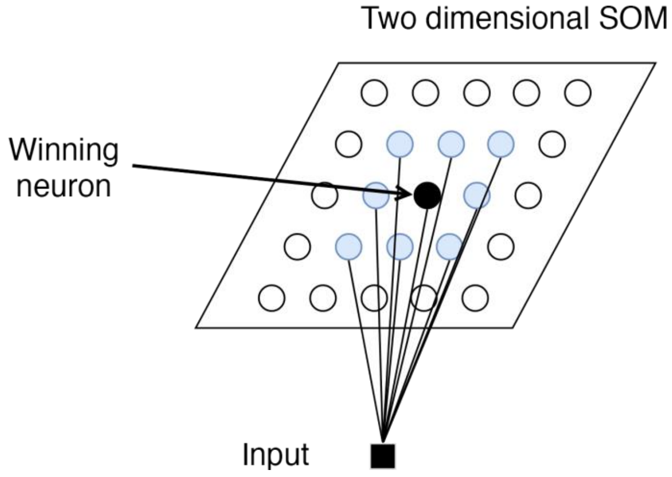
Cooperation
Find the topological neighborhood around node s_j

Adaptation
Update node s_j and its neighbors

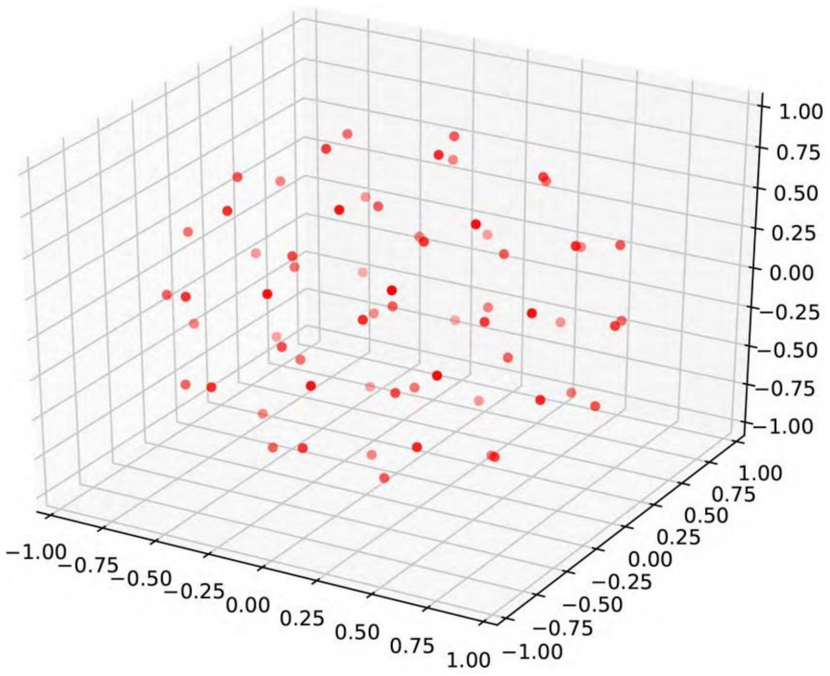
$$s_{j_k} = s_{j_k} + \lambda w_k (input - s_{j_k})$$

Learning rate

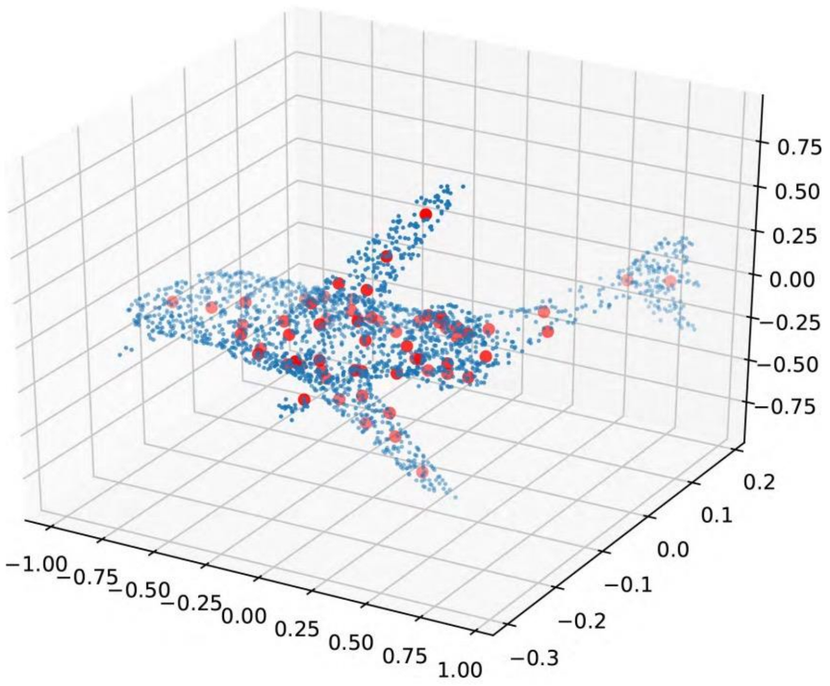
Weighting of k-th neighbor of s_j



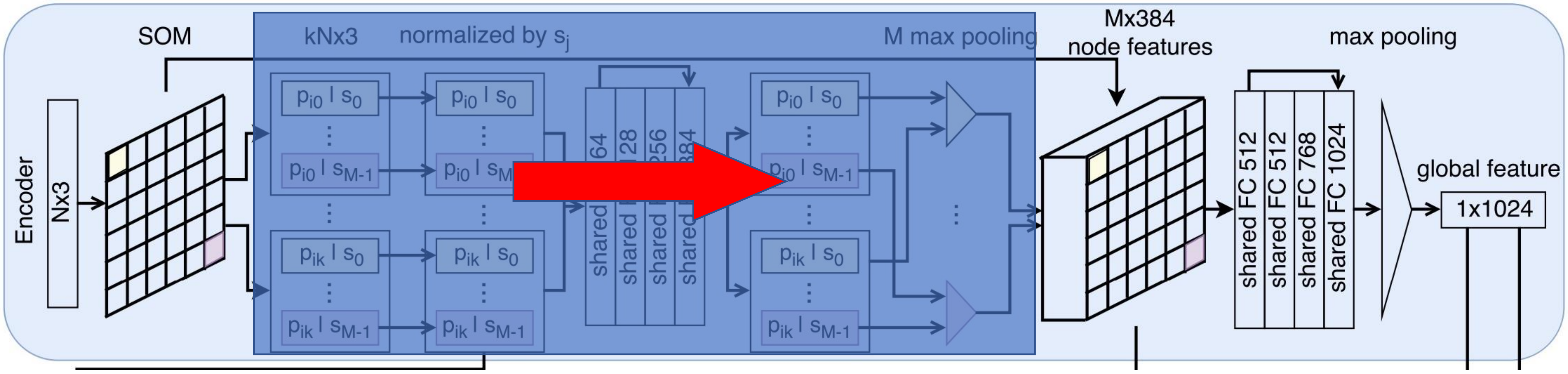
Initial nodes in a unit ball



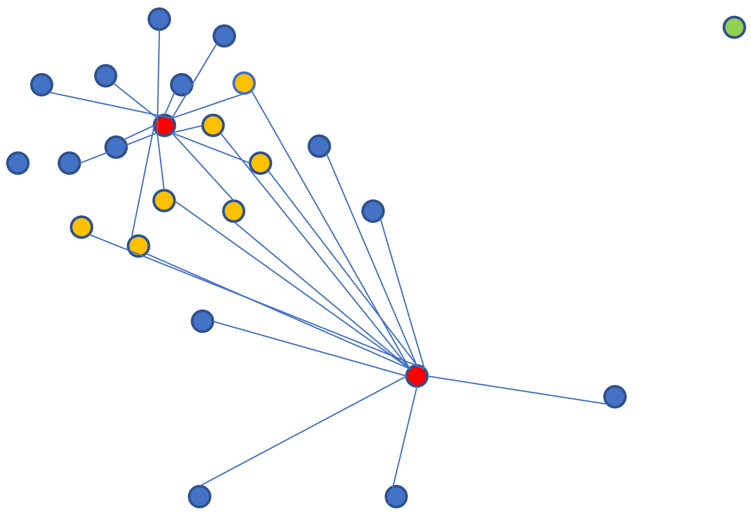
Weights after SOM



- Assume we have the SOM now
- How to convert SOM into "SOM Feature Map"

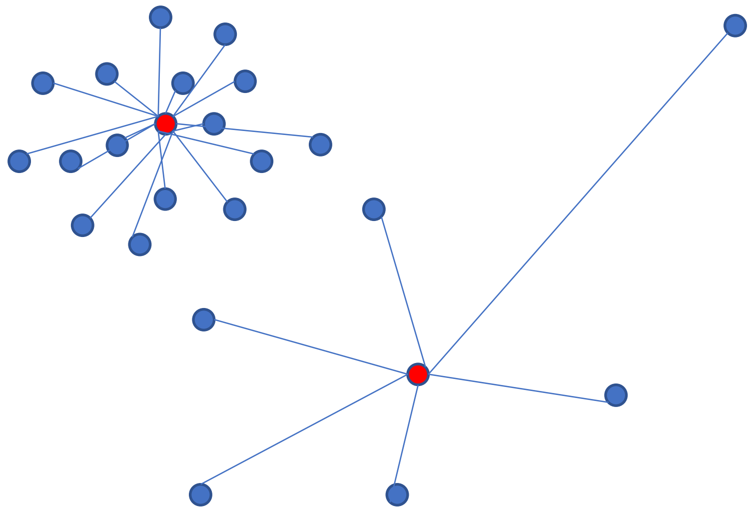


- Method 1, Node-to-Point (PointNet++ style)
  - Not adaptive to scale and density
  - Difficult to control receptive field
    - Some points will never be connected
    - Some points will be connected multiple times.



Node-to-Point kNN, k=13

- Method 1, Point-to-Node (SO-Net)
  - Adaptive to scale and density
  - Precise control of receptive field
    - Each point is connected k times.



Point-to-Node kNN, k=1

SO-Net – Grouping & PointNet & Maxpool

Point-to-Node Grouping

PointNet

Maxpool

SOM Feature Map