Path planning problem Algorithms reviews

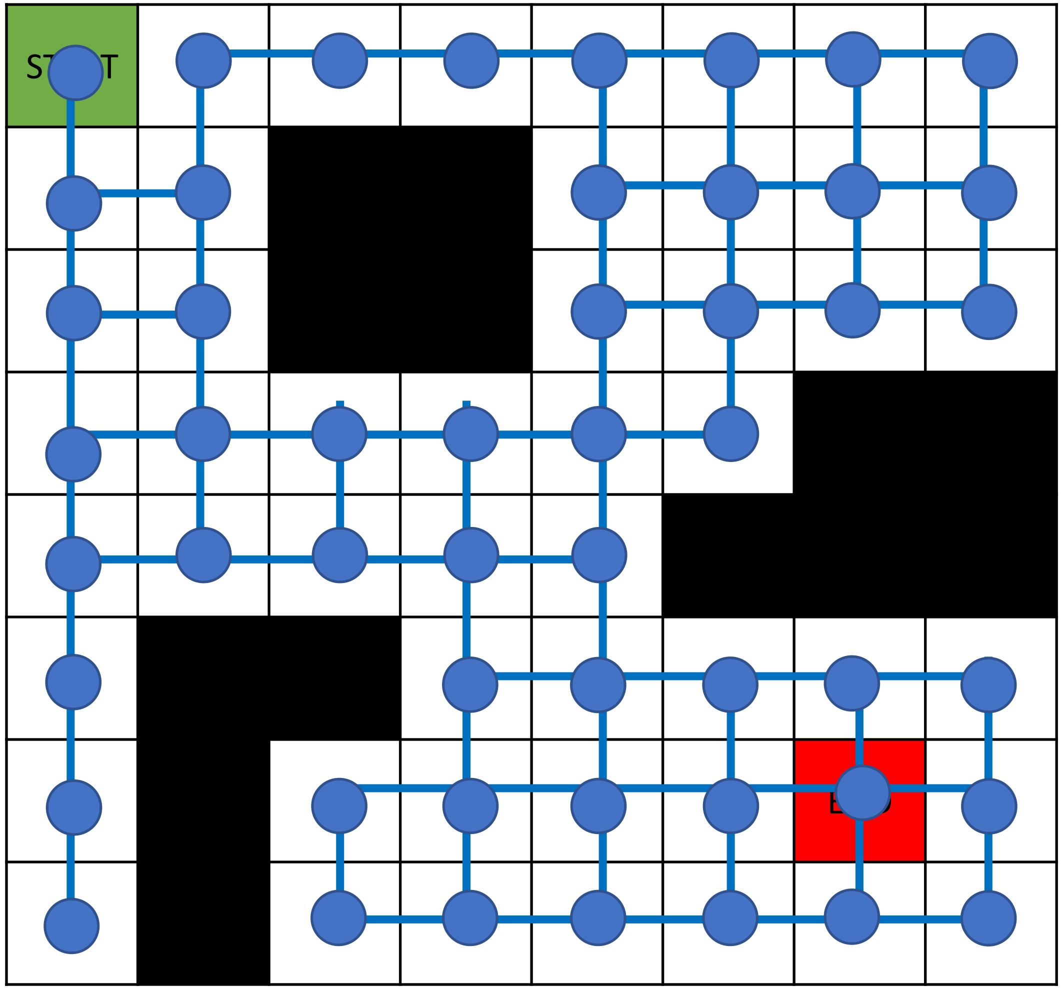
Amit Bouzaglo

Autonomous Navigation

- Goal
  - Develop techniques that would allow a robot to automatically decide how to move from position one to another.
- Steps:
  - Localization (GPS, INR, sensors)
  - Mapping (obstacles, roads, POI)
  - **Path-planning**
  - Path-following

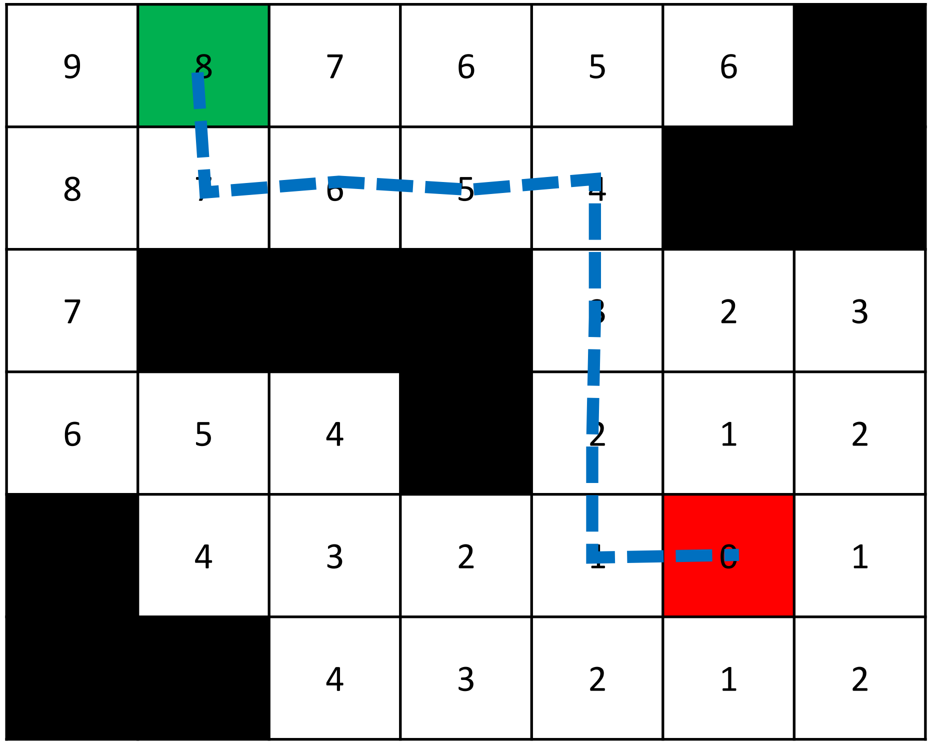
Mapping

- Grid to Graph
- Graph: $G(V, E)$
  - V – Vertexes (Nodes)
  - E – Edges



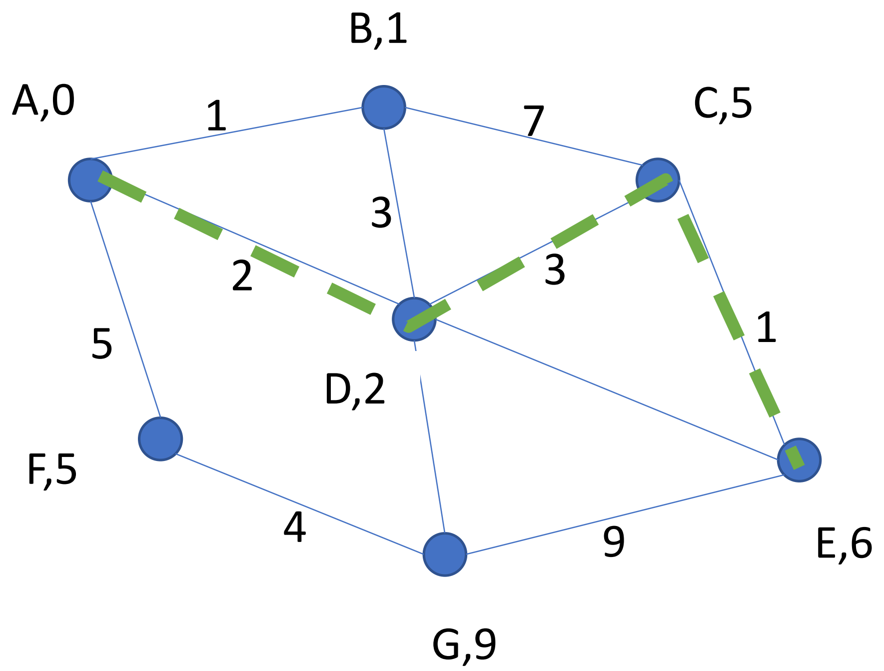
Grassfire Algorithm (BFS)

- For each node n in the graph:
  - $n.distance = \infty$
- Create empty list.
- $goal.distance = 0$, add goal to list.
- While list no empty:
  - Let current = first node in list.
  - Remove current from list.
  - For each node n, that is adjacent to current:
    - If $n.distance = \infty$:
      - $n.distance = current.distance + 1.$
      - Add n to the back of the list.
- **Trace a path:**
  - Start from Start.
  - Move to the neighbor with the smallest value.
  - Ties breaks arbitrarily.



Complexity = $\boldsymbol{O(|V|) = O(n * m)}$

Dijkstra's Algorithm (BFS)

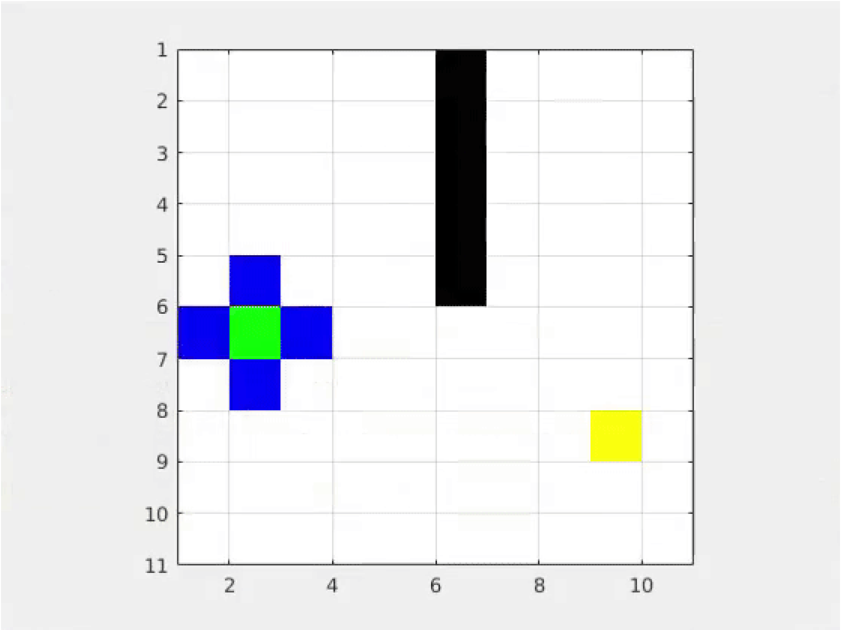


- For each node n in the graph:
  - $n.distance = \infty$
- Create an empty list.
- $start.distance = 0$, add start to list.
- While list no empty:
  - Let current = node n in the list with the smallest distance.
  - Remove current from list.
  - For each node n, that is adjacent to current:
    - If $n.distance > current.distance + length\ of\ edge\ from\ n\ to\ current$
      - $n.distance = current.distance + length\ of\ edge\ from\ n\ to\ current$
      - $n.parent = current$
      - Add n to list if it isn't there already
- **Trace a path:**
  - Start from destination.
  - Move from son to his parent.

Complexity = $\boldsymbol{O((|E| + |V|)log(|V|))}$

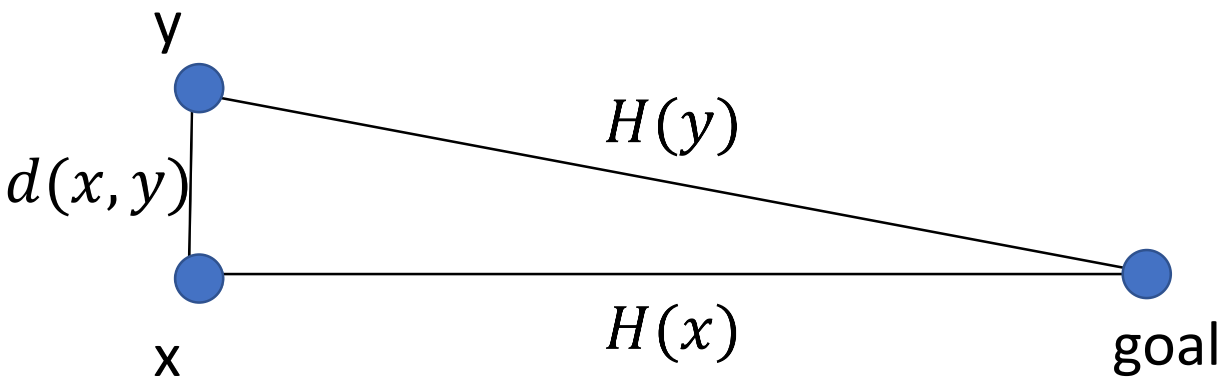
Quick summary

- Dijkstra/Grassfire Algorithm explores evenly in all directions until it finds the gold node.
- Could we do better?
  - YES!
  - Using the information we already have.
  - The destination…



Heuristic function H(n)

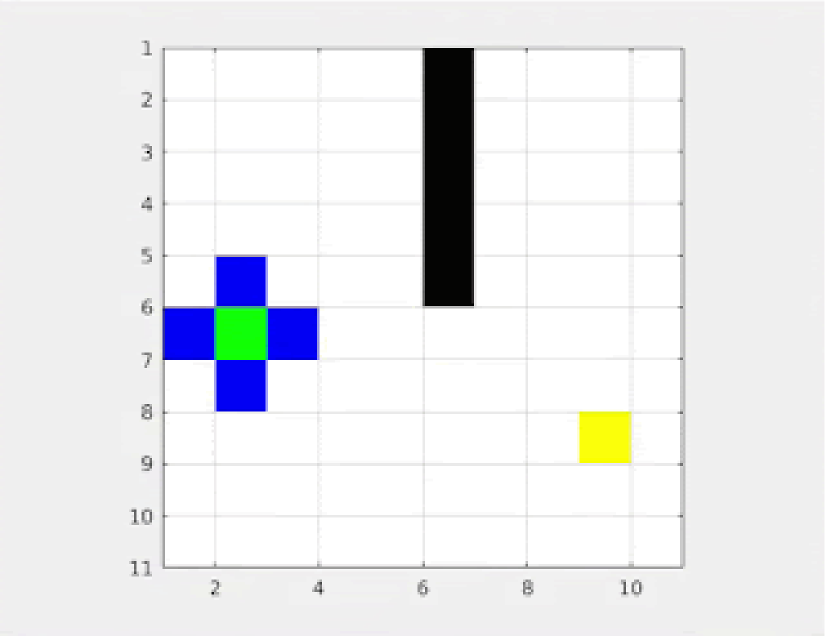
- $H(n) =$ a non-negative value that is indicative of the distance from that node to the goal.
  - $H(goal) = 0$
  - $H(x) \leq H(y) + d(x, y)$
  - $d(x, y) = length\ of\ the\ x - y\ edge$



- Example of H function:
  - Euclidean distance - $H(x_n, y_n) = \sqrt{(x_n - x_g)^2 + (y_v - y_g)^2}$
  - Manhattan distance - $H(x_n, y_n) = |x_n - x_g| + |y_v - y_g|$

A* Algorithm (Best-FS)

- For each node n in the graph:
  - $\mathrm{n.g} = \infty$ (the distance from n to start)
  - $n.f = \infty$ (g value + **estimate** distance to goal)
- Create an empty list.
- $start.g = 0, \mathrm{start.f} = \mathrm{H(start)}$. add start to list.
- While list no empty:
  - Let current = node n in the list with the smallest f value.
  - Remove current from list.
  - If (current == goal node) – return success.
  - For each node n, that is adjacent to current:
    - If $(n.g > current.g + cost\ of\ edge\ from\ n\ to\ current$.
      - $n.g = current.g + cost\ of\ edge\ from\ n\ to\ current.$
      - $n.f = n.g + H(n)$
      - $n.parent = current$
      - Add n to list if it isn't there already
- **Trace a path:**
  - Start from destination.
  - Move from son to his parent.



Complexity = Dijkstra's complexity (worst case)

D* Algorithm

- Suitable for use in a dynamic or complex environment
- RAISE-STATE:
- PROCESS-STATE (backward step):
  - Compute optimal path to the goal.
  - Initially set h(End) = 0 and insert it into the OPEN list.
  - Repeatedly called until the robot’s state Start is removed from the OPEN list.
- Determine optimal path by following gradient of h values. (forward step)
  - MODIFY-COST:
    - Immediately called, once the robot detects an error in the arc cost function (i.e. discover a new obstacle)
    - Change the arc cost function and enter affected states on the OPEN list.

Complexity = $\boldsymbol{O}\left(|\boldsymbol{E}|\boldsymbol{log}(|\boldsymbol{V}|)\right)$ (worst case)

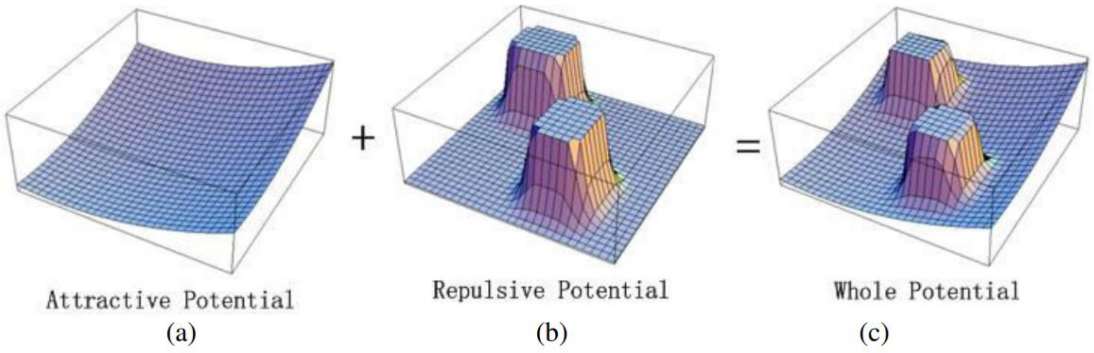
D* Lite Algorithm

- A* from End to Start (save the date!)
- Start moving from start according the path planed.
- When obstacle occurred:
  - Replan – only the necessary area.
- D*-Lite is considered much simpler than D*, and since it always runs at least as fast as D*, it has completely obsoleted D*.

Complexity = $\boldsymbol{O\left(|E|log(|V|)\right)}$ (worst case)

Potential Field Algorithm

- Create APT model
- Potential field calculation (gradient descent)
- Start movement at starting point
  - If (goal reached)
    - Stop navigation
  - Else:
    - If obstacle is detected:
      - APF model
      - Potential field calculation
      - Turning angel to find safe position
      - if: (goal eached):
        - Stop navigation
      - Else if (obstacle):
        - Repeat APF
    - Else:
      - Keep navigate till goal is reached

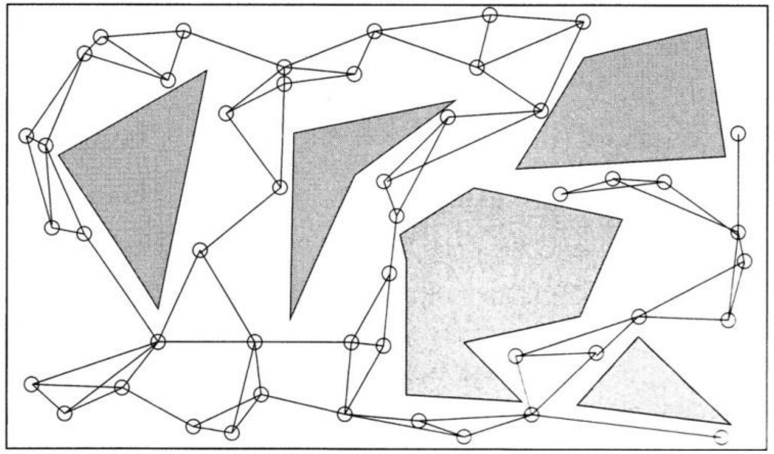


Complexity = $O(|V| * dimension)$

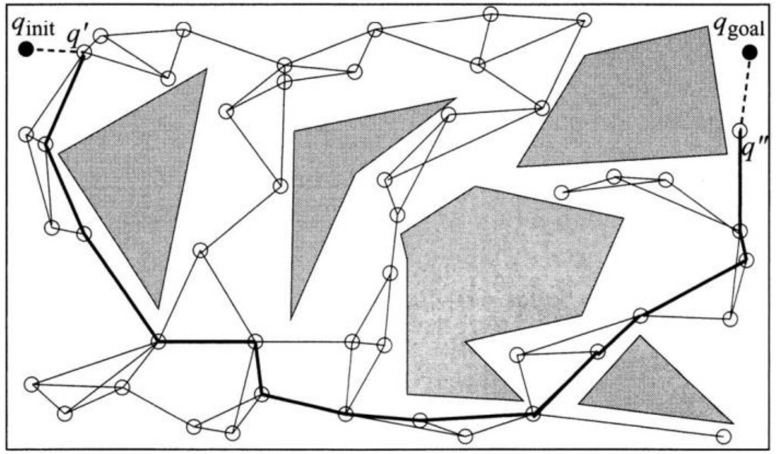
Random algorithms

Probabilistic Road-Map (PRM) Algorithm

- **Step 1** – learning the map:
    - Initially empty graph – G (V,E).
    - Chose q randomly.
    - If q is free (collision detection) add to G.
    - Repeat until N nodes chosen.
    - For each q’ select k closest neighbors:
        - For each neighbor – connect q to neighbors q’.
        - If connect successful – add edge (q,q’) to G.



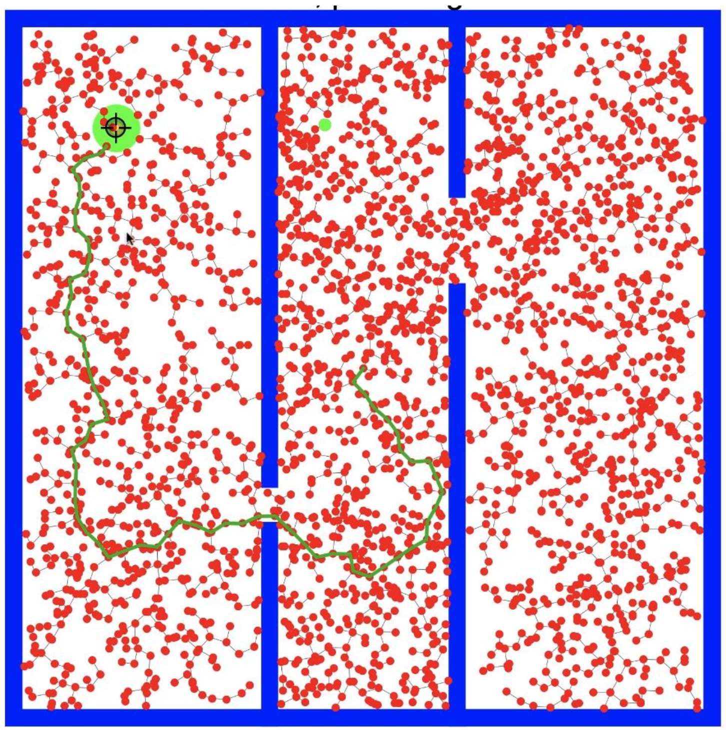
- **Step 2** – Finding a Path
    - Connect start and goal to exist G:
        - Find k nearest neighbors of <u>start</u> and <u>goal</u> in roadmap.
        - Connect start and goal to some exist nodes.
    - Use Dijkstra algorithm.



Complexity = Dijkstra’s complexity (worst case)

Rapidly-Exploring Random Trees (RRT)

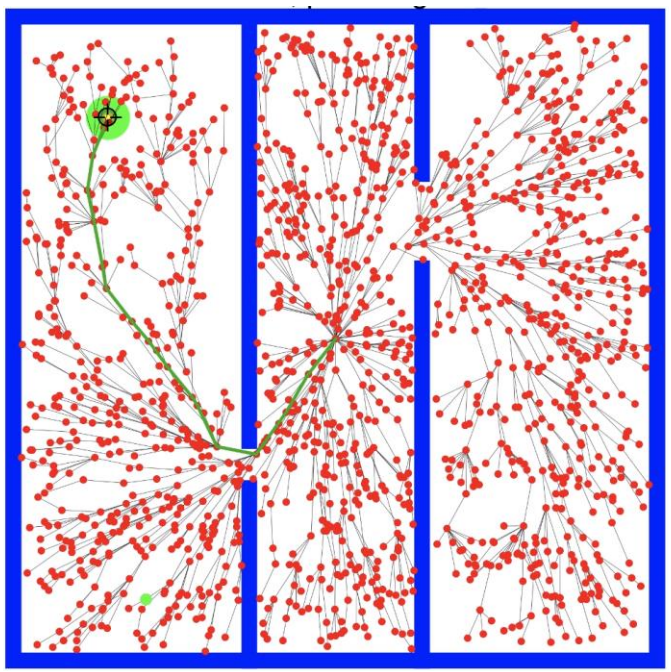
- Code:
    - Start, End
    - Counter = 0 - keeps track of iterations
    - lim - number of iterations algorithm should run for
    - G(V,E)
    - While counter < lim:
        - Xnew = Choose random position (using collision detection)
        - Xnearest = Select the nearest node this this position.
        - New_edge = Edge(Xnew, Xnearest)
        - G.append(new_edge)
        - if Xnew == end (or close):
            - Return G
    - Return G
- Find path (if exist) – but **not** optimal



Complexity = $\boldsymbol{O(nlog(n))}$

RRT* Algorithm

- Rad = r
- G(V,E)
- For i in range(0…n):
    - Xnew = = Choose random position (using collision detection)
    - Xnearest = Select the nearest node this this position.
    - Cost(Xnew) = Distance(Xnew, Xnearest)
    - Xbest, Xneighbors = findNeighbors(Xnew, Rad)
    - Link = Chain(Xnew, Xbest)
    - For x in Xneighbors
        - If Cost(Xnew) + Distance(Xnew,x) < Cost(x)
            - Cost(x) = Cost(Xnew)+Distance(Xnew,x)
            - Parent(x) = Xnew
            - G += {Xnew,x}
        - G += Link
- Return G



Complexity = $\boldsymbol{O(n\log(n))}$