TWO TAILED HYPOTHESIS TEST

A two-tailed hypothesis test helps us determine if there is a significant difference between a sample statistic and a population parameter, allowing for differences in either direction—whether the sample statistic is higher or lower than the parameter. It's called "two-tailed" because it checks for significant deviations in both directions from the null hypothesis.

Steps for a Two-Tailed Hypothesis Test

1. Formulate the Hypotheses:

- Null Hypothesis (H_0): This hypothesis suggests that there is no difference or effect. It assumes any observed difference is due to random chance.

- Alternative Hypothesis (H_1 or H_a): This hypothesis suggests that there is a difference or effect. It asserts that the observed difference is not due to chance.

2. Set the Significance Level (α):

- This is the threshold for deciding whether to reject the null hypothesis. Common choices are 0.05 or 0.01. For a two-tailed test, the α level is split into two parts (e.g., 0.025 in each tail).

3. Gather Data and Compute the Test Statistic:

- Use your sample data to calculate the test statistic (e.g., z-score or t-score).

4. Find the Critical Values or P-value:

- Look up the critical values from statistical tables corresponding to your α level, or calculate the p-value associated with your test statistic.

5. Make Your Decision:

- Compare your test statistic to the critical values or the p-value to the α level to decide whether to reject the null hypothesis.

6. Draw a Conclusion:

- Based on your comparison, determine if there is enough evidence to reject the null hypothesis and accept the alternative hypothesis.

Example

Imagine you're evaluating a new manufacturing process and want to check if it changes the average weight of a product from the known standard of 100 grams.

1. State the Hypotheses:

- Null Hypothesis (H_0): The average weight is still 100 grams ($\mu = 100$).

- Alternative Hypothesis (H_1):The average weight is not 100 grams ($\mu \neq 100$).

2. Set the Significance Level (α):

- Choose $\alpha = 0.05$. For a two-tailed test, this means 0.025 in each tail.

3. Gather Data and Compute the Test Statistic:

- Suppose you sample 30 products and find an average weight of 102 grams with a standard deviation of 5 grams. Calculate the t-score:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Where $\bar{x}$ = 102, μ = 100, s = 5, and n = 30:

$$t = \frac{102 - 100}{5/\sqrt{30}} \approx \frac{2}{0.912} \approx 2.19$$

4. **Find the Critical Values or P-value:**

- For a two-tailed test with α = 0.05 and 29 degrees of freedom:

1. **Split α for Two Tails:**
   - α/2 = 0.025
2. **Find the Cumulative Probability:**
   - Since the table often gives the area in one tail, the cumulative probability you are interested in is 1 - 0.025 = 0.975.
3. **Consult the T-Distribution Table:**
   - Look at the row for 29 degrees of freedom and find the value under the column for a cumulative probability of 0.975.
4. **Read the Critical t-Value:**
   - From the t-table, you find that the critical t-value for 29 degrees of freedom and a cumulative probability of 0.975 is approximately ±2.045. (using software or a calculator.)

5. Make Your Decision:

- Since your t-score (2.19) is greater than the critical value (2.045), you reject the null hypothesis.

6. Draw a Conclusion:

- The data suggests that the new manufacturing process significantly affects the mean weight of the product, indicating a difference from the standard weight of 100 grams.

ONE TAILED HYPOTHESIS TEST

A one-tailed hypothesis test is used to determine if there is a significant difference in a specific direction from what is stated in the null hypothesis. Unlike a two-tailed test, which checks for differences in both directions (higher or lower), a one-tailed test looks for a difference in only one direction—either higher or lower.

Steps for Conducting a One-Tailed Hypothesis Test

1. Formulate the Hypotheses:

- Null Hypothesis (H_0): This states that there is no effect or difference. It usually includes an equality statement (e.g., the mean is less than or equal to a certain value).

- Alternative Hypothesis (H_1 or H_a): This states that there is an effect or difference in a specific direction. It includes an inequality statement (e.g., the mean is greater than a certain value).

2. Set the Significance Level (α):

- This is the probability of incorrectly rejecting the null hypothesis. Common choices are 0.05 or 0.01. In a one-tailed test, the entire α level is applied to one end of the distribution.

3. Collect Data and Calculate the Test Statistic:

- Based on your sample data, calculate the test statistic (e.g., z-score or t-score).

4. Find the Critical Value or P-Value:

- Look up the critical value from the t-distribution table or calculate the p-value for your test statistic. For a one-tailed test with $\alpha = 0.05$, the critical value will be the value that corresponds to the top 5% of the distribution.

5. Make a Decision:

- Compare your test statistic to the critical value or p-value. If the test statistic exceeds the critical value or the p-value is less than α, reject the null hypothesis.

6. Draw a Conclusion:

- Decide if the data provides enough evidence to support the alternative hypothesis.

Example

Suppose you want to test whether a new drug reduces blood pressure more effectively than an existing drug. The known reduction from the existing drug is 10 mmHg.

1. State the Hypotheses:

- Null Hypothesis (H_0): The new drug reduces blood pressure by 10 mmHg or less ($\mu \leq 10$ mmHg).

- Alternative Hypothesis (H_1): The new drug reduces blood pressure by more than 10 mmHg ($\mu > 10$ mmHg).

2. Set the Significance Level (α):

- Choose $\alpha = 0.05$. For a one-tailed test, this means you will look at the top 5% of the distribution.

3. Collect Data and Calculate the Test Statistic:

- Suppose you test 50 patients and find an average reduction of 12 mmHg with a standard deviation of 4 mmHg. Using a t-test, you calculate the t-statistic as:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Here, $\bar{x} = 12$, $\mu = 10$, $s = 4$, and $n = 50$:

$$t = \frac{12 - 10}{4/\sqrt{50}} \approx \frac{2}{0.566} \approx 3.53$$

4. Find the Critical Value or P-Value:

- For $\alpha = 0.05$ and 49 degrees of freedom, the critical t-value is about 1.68. Your calculated t-value of 3.53 is higher than this critical value.

- Alternatively, the p-value for a t-value of 3.53 is less than 0.05, indicating significance.

5. Make a Decision:

- Since your t-value is greater than the critical value, or your p-value is less than 0.05, you reject the null hypothesis.

6. Draw a Conclusion:

- There is strong evidence that the new drug reduces blood pressure more effectively than the existing drug.

CHI-SQUARE TEST

The Chi-Square test is a statistical tool used to assess if there is a significant relationship between categorical variables or if observed data fits a particular expected pattern. It comes in two main forms:

1. Chi-Square Test of Independence:

- Purpose: To check if two categorical variables are related or independent from each other.

- How It Works: It compares the observed counts in each category of a contingency table with the counts we would expect if the variables were independent.

- Null Hypothesis (H_0): The two variables are independent (no relationship).

- Alternative Hypothesis (H_1): The two variables are dependent (there is a relationship).

2. Chi-Square Test of Goodness of Fit:

- Purpose: To see if the distribution of sample data fits a theoretical distribution.

-How It Works: It compares the observed counts of a single categorical variable with the counts expected under a specific hypothesis.

- Null Hypothesis (H_0): The sample data matches the expected distribution.

- Alternative Hypothesis (H_1): The sample data does not match the expected distribution.

Key Concepts

- Chi-Square Statistic: This value shows how much the observed counts differ from the expected counts. A higher value suggests a bigger difference.

- Degrees of Freedom (df):

- For independence: (number of rows−1) × (number of columns−1)

- For goodness of fit: number of categories−1

- P-Value: This is the probability of seeing a Chi-Square statistic as extreme as the one calculated, assuming the null hypothesis is true. A lower p-value means stronger evidence against the null hypothesis.

Example

1. Chi-Square Test of Independence

Scenario:

You want to test if there is a relationship between gender (Male, Female) and product preference (Likes, Dislikes) in a survey of 100 people. The results are summarized in the following contingency table:

	Likes Product	Dislikes Product	Total
Male	30	20	50
Female	25	25	50
Total	55	45	100

Steps:

1. Calculate Expected Frequencies:

For each cell, the expected frequency is calculated as:

$$\text{Expected frequency} = (\text{Row Total} \times \text{Column Total}) / \text{Grand Total}$$

For example, for males who like the product:

$$\text{Expected frequency} = (50 \times 55) / 100 = 27.5$$

Repeat this for all cells.

2. Calculate Chi-Square Statistic:

Use the formula:

$$\chi 2 = \sum(\text{ Observed} - \text{Expected})\text{^}2 \text{ / Expected Frequency}$$

For the cell with males who like the product:

$$\chi 2 = (30 - 27.5)\text{^}2 / 27.5 = 0.227$$

Sum these values for all cells to get the Chi-Square statistic.

3. Determine Degrees of Freedom (df):

$$(\text{number of rows} - 1) \times (\text{number of columns} - 1) = (2\text{-}1) \text{ x } (2\text{-}1) = 1$$

4. Find the P-Value:

Using the Chi-Square distribution with 1 degree of freedom, find the p-value associated with the calculated Chi-Square statistic.

5. Decision:

Compare the p-value to the significance level (e.g., $\alpha = 0.05$). If the p-value is less than α, reject the null hypothesis, indicating a significant relationship between gender and product preference.

2. Chi-Square Test of Goodness of Fit

Scenario:

You want to test if a six-sided die is fair. You roll the die 120 times and get the following frequencies for each face:

Face	Observed Frequency
1	18
2	22
3	20
4	19
5	21
6	20

For a fair die, the expected frequency for each face is:

Expected Frequency= Total Rolls / Number of Faces = 120/6 = 20

Steps:

1. Calculate Chi-Square Statistic:

χ2= ∑(Observed−Expected)^2 / Expected Frequency

For each face:

(18-20)^2 / 20 = 0.2

(22-20)^2 / 20 = 0.2

And so on for all faces.

Sum these values to get the Chi-Square statistic.

2. Determine Degrees of Freedom (df):

df = Number of categories−1=6−1=5

3. Find the P-Value:

Using the Chi-Square distribution with 5 degrees of freedom, find the p-value associated with the calculated Chi-Square statistic.

4. Decision:

Compare the p-value to the significance level (e.g., α = 0.05). If the p-value is less than α, reject the null hypothesis, suggesting that the die is not fair.