ASSIGNMENT-3

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PROBLEM 1:

Finally we see that $\sigma_2(i)=2$, ($\sigma_2 = \sigma'$ from the question)

$$\frac{\dfrac{}{\langle 1,\sigma\rangle \Downarrow 1}(13) \quad \dfrac{\langle i,\sigma\rangle \Downarrow \sigma(i)=1}{\langle i,\sigma\rangle \Downarrow 1}(14)}{\langle i+1,\sigma\rangle \Downarrow 2}(15)$$

$$\frac{\dfrac{\langle i,\sigma\rangle \Downarrow \sigma(i)=1}{\langle i,\sigma\rangle \Downarrow 1}(14) \quad \dfrac{\langle i,\sigma\rangle \Downarrow \sigma(i)=1}{\langle i,\sigma\rangle \Downarrow 1}(14)}{\langle i*i,\sigma\rangle \Downarrow 1 \; (1*1=1)}(17) \qquad \frac{\langle i,\sigma\rangle \Downarrow \sigma(i)=1}{\langle i,\sigma\rangle \Downarrow 1}(14)$$

$$\frac{\langle i*i,\sigma\rangle \Downarrow 1 \; (1*1=1) \quad \langle i,\sigma\rangle \Downarrow 1}{\langle i*i<=i,\sigma\rangle \Downarrow T \; (1<=1)}(9)$$

$$\frac{\langle i+1,\sigma\rangle \Downarrow 2}{\langle i:=i+1,\sigma\rangle \Downarrow \sigma[i\mapsto 2] = \sigma_2}(3) \qquad \frac{D_2}{\langle \text{while } i*i<=i \text{ do } i:=i+1, \sigma_2\rangle \Downarrow \sigma_2}(6)$$

$$\frac{\langle i:=i+1,\sigma\rangle \Downarrow \sigma[i\mapsto 2] = \sigma_2 \quad \langle \text{while } i*i<=i \text{ do } i:=i+1, \sigma_2\rangle \Downarrow \sigma_2}{\langle i:=i+1;\ \text{while } i*i<=i \text{ do } i:=i+1, \sigma\rangle \Downarrow \sigma_2}(2)$$

$D1 =$

$$\frac{\dfrac{\langle i*i<=i,\sigma\rangle \Downarrow T \; (1<=1) \quad \langle i:=i+1;\ \text{while } i*i<=i \text{ do } i:=i+1, \sigma\rangle \Downarrow \sigma_2}{\langle \text{if } i*i<=i \text{ then } (i:=i+1;\ \text{while } i*i<=i \text{ do } i:=i+1) \text{ else skip}, \sigma\rangle \Downarrow \sigma_2}(4)}{\langle \text{while } i*i<=i \text{ do } i:=i+1, \sigma\rangle \Downarrow \sigma_2}(6)$$

$$D2 = \dfrac{\dfrac{\dfrac{\dfrac{\langle i,\sigma_2\rangle \Downarrow \sigma_2(i)=2}{\langle i,\sigma_2\rangle \Downarrow 2}(14) \quad \dfrac{\langle i,\sigma_2\rangle \Downarrow \sigma_2(i)=2}{\langle i,\sigma_2\rangle \Downarrow 2}(14)}{\langle i*i,\sigma_2\rangle \Downarrow 4\ (2\times 2=4)}(17) \quad \dfrac{\langle i,\sigma_2\rangle \Downarrow \sigma_2(i)=2}{\langle i,\sigma_2\rangle \Downarrow 2}(14)}{\langle i*i<=i,\sigma_2\rangle \Downarrow F\ (4\leq 2)}(9) \quad \dfrac{}{\langle \text{skip},\sigma_2\rangle \Downarrow \sigma_2}(1)}{\langle \text{if } i*i<=i \text{ then } (i:=i+1;\ \text{while } i*i<=i \text{ do } i:=i+1) \text{ else skip}, \sigma_2\rangle \Downarrow \sigma_2}(5)$$

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PROBLEM 4

(a) Large step operational semantics

ARITHMETIC RULES:

(13) $(n,\sigma) \Downarrow n, \sigma$

(14) $(v,\sigma) \Downarrow \sigma(v), \sigma$

(15) $$\frac{\langle a_1,\sigma\rangle \Downarrow n_1, \sigma_2 \quad \langle a_2, \sigma_2\rangle \Downarrow n_2, \sigma'}{\langle a_1+a_2,\sigma\rangle \Downarrow n_1+n_2, \sigma'}$$

(16) $$\frac{\langle a_1,\sigma\rangle \Downarrow n_1, \sigma_2 \quad \langle a_2, \sigma_2\rangle \Downarrow n_2, \sigma'}{\langle a_1-a_2,\sigma\rangle \Downarrow n_1-n_2, \sigma'}$$

(17) $$\frac{\langle a_1,\sigma\rangle \Downarrow n_1, \sigma_2 \quad \langle a_2, \sigma_2\rangle \Downarrow n_2, \sigma'}{\langle a_1 * a_2,\sigma\rangle \Downarrow n_1 n_2, \sigma'}$$

(18) $$\frac{\langle c,\sigma\rangle \Downarrow \sigma' \quad \langle a, \sigma'\rangle \Downarrow n, \sigma''}{\langle (c,a),\sigma\rangle \Downarrow n, \sigma''}$$

BOOLEAN RULES:

(7) $\langle true,\sigma\rangle \Downarrow T, \sigma$

(8) $\langle false,\sigma\rangle \Downarrow F, \sigma$

(9) $$\frac{\langle a_1,\sigma\rangle \Downarrow n_1, \sigma_2 \quad \langle a_2, \sigma_2\rangle \Downarrow n_2, \sigma'}{\langle a_1 \le a_2,\sigma\rangle \Downarrow n_1 \le n_2, \sigma'}$$

(10) $$\frac{\langle b1, \sigma\rangle \Downarrow p, \sigma_2 \quad \langle b2, \sigma_2\rangle \Downarrow q, \sigma'}{\langle b1 \&\& b2, \sigma\rangle \Downarrow p \wedge q, \sigma'}$$

(11) $$\frac{\langle b1, \sigma\rangle \Downarrow p, \sigma_2 \quad \langle b2, \sigma_2\rangle \Downarrow q, \sigma'}{\langle b1 \,\|\, b2, \sigma\rangle \Downarrow p \vee q, \sigma'}$$

(12) $$\frac{\langle b, \sigma\rangle \Downarrow p, \sigma'}{\langle !b, \sigma\rangle \Downarrow \neg p, \sigma'}$$

(b) small step operational semantics

(27) $$\frac{\langle c, \sigma\rangle \longrightarrow_1 \langle c', \sigma'\rangle}{\langle (c, a), \sigma\rangle \longrightarrow_1 \langle (c', a), \sigma'\rangle}$$

COLLABORATORS:

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SOURCES:

-class lecture notes, handouts

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PROBLEM 2:

given $W =$ (while true do $x := x+2$) and $\sigma = \{(x,0)\}$

Step 1: $\langle$while true do $x := x+2, \sigma\rangle \longrightarrow_1 \langle$if true then ($x := x+2$; while true do $x := x+2$) else skip, $\sigma\rangle$

(8)

Step 2: $\langle$if true then ($x := x+2$; while true do $x := x+2$) else skip, $\sigma\rangle$

(6) $\longrightarrow_1 \langle x := x+2$; while true do $x := x+2, \sigma\rangle$

Step 3:

(1)

$$\frac{\langle x := x+2, \sigma\rangle \longrightarrow_1 \langle (x := x+2)', \sigma'\rangle}{\langle x := x+2;\ \text{while true do } x := x+2, \sigma\rangle \longrightarrow_1 \langle (x := x+2)';\ \text{while true do } x := x+2, \sigma'\rangle}$$

Step 4:

(3)

$$\frac{\langle x+2, \sigma\rangle \longrightarrow_1 \langle (x+2)', \sigma'\rangle}{\langle x := x+2, \sigma\rangle \longrightarrow_1 \langle x := (x+2)', \sigma'\rangle}$$

Step 5

(23)

$$\frac{\langle x, \sigma\rangle \longrightarrow_1 \langle x', \sigma'\rangle}{\langle 2+x, \sigma\rangle \longrightarrow_1 \langle 2+x', \sigma'\rangle}$$

Step 6 $\langle x, \sigma \rangle \longrightarrow_1 \langle \sigma(x), \sigma \rangle$ (here $\sigma(x)$ would be 0)
(21)

PROBLEM 3:

Theorem: If judgement $\langle a, \sigma \rangle \Downarrow n$ holds then $|n| \leq 2^{3k+1}$

Proof: There exists some derivation D of judgement $\langle a, \sigma \rangle \Downarrow n$. We will prove the theorem by structural induction on D.

Base case: suppose D only consists of 1 rule. Then, $D = \langle n, \sigma \rangle \Downarrow n$,

so here $k=0$ and $n=2$

Thus $|n| \leq 2^{3k+1}$ ie $2 \leq 2$

Inductive hypothesis: Assume that the theorem holds for all the derivations that are strictly smaller than D. That is, assume that if $\langle a_0, \sigma_0 \rangle \Downarrow n_0$ has a derivation strictly smaller than D, then $|n_0| \leq 2^{3k_0+1}$ where there are exactly k_0 '+' symbols, k_0 '-' symbols, k_0 '*' symbols and all integer constants are 2 in a_0

Inductive case: Suppose that D consists of more than one rule. In that case, D must end with one of the 3 derivation rules for arithmetic ie 15, 16, 17.

We therefore must consider the 3 cases

Case 1: Rule 17 Case 3: Rule 15

Case 2: Rule 16.

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Case 1: Suppose D ends with the Rule 17

$$D = \frac{\dfrac{D1}{\langle a_1, \sigma \rangle \Downarrow n_1} \qquad \dfrac{D2}{\langle a_2, \sigma \rangle \Downarrow n_2}}{\langle a_1 * a_2, \sigma \rangle \Downarrow n_1 n_2}$$

We know that D1 is strictly smaller than derivation D. If there are 'k_1' binary operators in the expression a_1, then we conclude by I.H that

$|n_1| \leq 2^{3k_1+1}$

Likewise, we know that D_2 is strictly smaller than derivation D. If there are 'k_2' binary operators in the expression a_2, then we conclude by I.H that

$|n_2| \leq 2^{3k_2+1}$

Thus, the total operators in D would be i.e. $k = k_1 + k_2 + 1$

$\therefore |n_1 n_2| \leq 2^{3(k_1+k_2+1)+1}$

$\Rightarrow \left| 2^{(3k_1+1)} \cdot 2^{(3k_2+1)} \right| \leq 2^{3k_1+3k_2+4}$

Thus; the relation holds ie $|n_1 n_2| \leq 2^{3k+1}$

Case 2: Suppose D ends with Rule 16.

$$D = \frac{\dfrac{D1}{\langle a_1, \sigma \rangle \Downarrow n_1} \qquad \dfrac{D2}{\langle a_2, \sigma \rangle \Downarrow n_2}}{\langle a_1 - a_2, \sigma \rangle \Downarrow n_1 - n_2}$$

We know that D1 is strictly smaller than derivation D. If there are 'k_1' binary operators in the expression a_1, then we conclude by I.H that

$|n_1| \leq 2^{3k_1+1}$

Likewise, we know that D2 is strictly smaller than derivation D. If there are 'k_2' binary operators in the expression a_2, then we conclude by I.H that

$|n_2| \leq 2^{3k_2+1}$

Thus, the total operators in D would be $k = k_1 + k_2 + 1$

$\therefore\ |n_1 - n_2| \leq 2^{3(k_1+k_2+1)+1}$

$\Rightarrow \left| 2^{(3k_1+1)} - 2^{(3k_2+1)} \right| \leq 2^{3k_1+3k_2+4}$

Thus, the relation holds ie $|n_1 - n_2| \leq 2^{3k+1}$

<u>Case 3</u>: Suppose D ends with Rule 15

$$D = \cfrac{\cfrac{D1}{\langle a_1, \sigma\rangle \Downarrow n_1} \qquad \cfrac{D2}{\langle a_2, \sigma\rangle \Downarrow n_2}}{\langle a_1 + a_2, \sigma\rangle \Downarrow n_1 + n_2}$$

We know that D1 is strictly smaller than derivation D. If there are 'k_1' binary operators in the expression a_1, then we conclude by I.H that

$|n_1| \leq 2^{3k_1+1}$

Likewise, we know that D2 is strictly smaller than derivation D. If there are 'k_2' binary operators in the expression a2, then we conclude by I.H that $|n_2| \leq 2^{3k_2+1}$

Thus, the total operators in D would be $k = k_1 + k_2 + 1$

$\therefore |n_1 + n_2| \leq 2^{3(k_1+k_2+1)+1}$

$\Rightarrow \left| 2^{(3k_1+1)} + 2^{(3k_2+1)} \right| \leq 2^{3k_1+3k_2+4}$

Thus, the relation holds ie $|n_1+n_2| \leq 2^{3k+1}$