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Weinberg angle and couplings from rotor norms

When α, α_s and $\sin^2\theta_W$ are geometric overlaps

Key Insight. In the Standard Model, the fine-structure constant α, the strong coupling α_s and the Weinberg angle θ_W are independent running parameters. In the octonionic rotor picture, they have a common geometric origin: they are read off from norms and mutual angles of commutators in the internal operator algebra. The electroweak mixing angle θ_W becomes a literal angle between two rotor directions that define hypercharge and weak isospin inside the exceptional stage.

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Rotor generators for internal forces

The internal symmetries of one generation are encoded by rotor-like operators G_a acting on the octonionic internal space. In this setting,

- color $SU(3)_C$ corresponds to one set of rotor directions,
- weak $SU(2)_L$ to another set,
- hypercharge $U(1)_Y$ to a particular combination of internal rotations.

The basic data are the commutators

$$[G_a, G_b],$$

whose norms and mutual angles in operator space reflect the structure constants and coupling strengths of the effective gauge theory.

Couplings as norms in operator space

Schematically, one defines an inner product on the space of rotor operators, for example via a trace on the internal Hilbert space:

$$\langle A, B\rangle := \mathrm{Tr}(A^\dagger B) \quad \text{(up to normalisation)}.$$

With this structure, the effective gauge couplings can be associated with the sizes of commutators:

- the electromagnetic coupling α with a suitable abelian combination of rotors,
- the strong coupling α_s with the norm of $SU(3)$ commutator directions,
- the weak coupling g with the norm of $SU(2)$ rotors.

Symbolically,

$$\alpha \sim \|\,[Q, Q']\,\|^2, \qquad \alpha_s \sim \|[T_a, T_b]\|^2,$$

where Q is an electromagnetic charge operator and T_a are color generators. The proportionality constants depend on normalisation conventions, but the qualitative statement is: *couplings measure the non-commutativity of appropriate rotor directions.*

Weinberg angle as an internal angle

Electroweak unification mixes weak isospin $SU(2)_L$ and hypercharge $U(1)_Y$ into the physical photon A_μ and Z_μ boson. In the octonionic rotor picture, this mixing is literally an angle in operator space.

Let G_W denote the (properly normalised) weak-isospin rotor in the relevant direction and G_Y the hypercharge rotor constructed from the internal algebra. Then one can define an angle θ by

$$\cos\theta = \frac{\langle G_W, G_Y\rangle}{\|G_W\|\,\|G_Y\|}.$$

Up to renormalisation effects, this geometric angle is identified with the Weinberg angle θ_W, and the usual electroweak relations

$$e = g\sin\theta_W, \qquad g' = g\tan\theta_W$$

are reinterpreted as relations between norms and inner products of rotor directions.

Relations among α, α_s and $\sin^2\theta_W$

Because all three quantities are read from the same operator space, they are not arbitrary:

- The relative normalisation of $SU(3)$, $SU(2)$ and $U(1)$ generators is fixed by the representation of the exceptional algebra.
- This fixes ratios of norms like $\|T_a\|^2 : \|G_W\|^2 : \|G_Y\|^2$.
- Consequently, at an appropriate reference scale, the couplings are correlated.

Schematisch:

$$\alpha : \alpha_s : \frac{1}{\sin^2\theta_W} \;\sim\; \|Q\|^2 : \|T_a\|^2 : \frac{\|G_W\|^2}{\|G_Y\|^2},$$

mit allen Größen aus demselben Operatorraum gelesen. Laufende mit der Energie kommt zusätzlich durch Renormierungsgruppeneffekte; die Ausgangswerte sind jedoch geometrisch eingeengt.

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Was das konzeptionell ändert

Die Weinberg-Winkel-Tag soll weniger eine neue Zahl liefern als den Blickwinkel ändern:

1. α, α_s und $\sin^2\theta_W$ sind keine völlig unabhängigen Parameter, sondern verschiedene Projektionen derselben internen Operatorgeometrie.
2. Der Weinberg-Winkel wird zu dem, was sein Name verspricht: einem *Winkel* zwischen zwei ausgezeichneten Rotorrichtungen.
3. Mögliche Relationen zwischen Kopplungen sind keine Zufälle, sondern Fingerabdrücke der Einbettung von $SU(3)_C \times SU(2)_L \times U(1)_Y$ in die Ausnahmegeometrie.

Wenn spätere Rechnungen zeigen, dass die experimentell gemessenen Werte von α, α_s und $\sin^2\theta_W$ sich gut als Rotor-Normen und -Winkel eines konkreten Oktaven-/Albert-Embeddings rekonstruieren lassen, wäre das ein starkes Indiz dafür, dass „innere Geometrie“ mehr ist als eine Metapher.

References

[1] S. Weinberg, “A model of leptons,” *Phys. Rev. Lett.* **19**, 1264–1266 (1967).

[2] C. Furey, “$SU(3)_C \times SU(2)_L \times U(1)_Y$ from division algebras,” *Phys. Lett. B* **785**, 84–89 (2018).

[3] [Internal notes on rotor norms and couplings: `arxiv-const.tex; appK_neu.tex`.]

In the rotor picture, couplings and the Weinberg angle are not arbitrary constants but norms and mutual angles of commutators in the exceptional internal operator space.

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