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Why $\alpha \approx 1/137$?

Fine-structure from rotor norms

Key Insight. The fine-structure constant α does not enter as a free input parameter. In the octonionic model, α is understood as the squared norm of a specific rotor commutator in the internal geometry. The same mechanism also fixes the strong coupling and the weak mixing angle. The famous number $1/137$ becomes a geometric shadow of how internal directions are arranged in the exceptional algebra.

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From mysterious constant to geometric norm

Few numbers in physics have attracted as much fascination as the fine-structure constant,

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137.035999\ldots}.$$

Historically, α enters quantum electrodynamics as a dimensionless coupling: it controls the strength of electromagnetic interactions and the convergence of perturbation theory. For decades, it has been treated as a parameter to be measured, not derived.

In the octonionic model, α is no longer a free parameter. It appears as the squared norm of a commutator built from internal *rotors* in the exceptional geometry. The value of α is tied to how certain planes inside the octonionic/Albert algebra are oriented with respect to each other.

Internal rotors and commutators

The internal space of the model is not a simple Lie algebra like $\mathfrak{su}(2)$ or $\mathfrak{su}(3)$ in isolation. Instead, it sits inside the exceptional Lie algebra associated with the Albert algebra $H_3(\mathbb{O})$ and its symmetry group F_4.

Within this setting, one considers special operators R_a, R_b, $\ldots$, which act as *rotors* on selected internal subspaces. Schematically, one writes

$$R_a \sim \exp(\theta_a X_a), \qquad R_b \sim \exp(\theta_b X_b),$$

where X_a, X_b are generators associated with particular internal directions (for instance, those tied to weak isospin and hypercharge).

The key object is the commutator

$$[R_a, R_b] \approx \theta_a\theta_b[X_a, X_b]$$

in an appropriate small-angle limit. The *norm* of this commutator defines an effective coupling:

$$\alpha \propto \big\|[R_a, R_b]\big\|^2.$$

Choosing R_a and R_b according to the embedding of $U(1)_{\text{em}}$ inside the internal algebra singles out the electromagnetic coupling.

From abstract norm to a concrete number

The statement that α is given by a squared norm would be empty if the norm could be tuned at will. The nontrivial part is that in the octonionic/Albert setting the relevant rotors are highly constrained:

- Triality and G_2-compatibility fix how vector and spinor directions in $\mathbb{R}^8$ are related.
- The Jordan structure of $H_3(\mathbb{O})$ restricts which combinations of internal directions can appear as "legal" rotors.
- The vacuum configuration $\langle H\rangle$ selects preferred eigen-directions and thereby preferred planes in which the rotors act.

Under these constraints, the norm $\big\|[R_a, R_b]\big\|^2$ is not a continuously tunable parameter. It falls into discrete bands determined by the eigenvalues of certain internal projectors and compressor operators. One of these bands lands numerically in the vicinity of $1/137$, and this is identified with the observed fine-structure constant at a particular reference scale.

Relations to strong and weak couplings

The same construction can be repeated for other choices of rotors:

- Rotors associated with the $SU(3)$ color directions lead to an effective strong coupling α_s.
- Rotors associated with the weak isospin and hypercharge mixture lead to $\sin^2\theta_W$, the weak mixing angle.

The guiding principle is that *all* gauge couplings arise as norms of commutators of rotors in the same internal algebra. This leads to relations of the symbolic form

$$\alpha \sim \|[R_{\text{em}}, R_{\text{ref}}]\|^2, \qquad \alpha_s \sim \|[R_{\text{color}}, R_{\text{ref}}]\|^2, \qquad \sin^2\theta_W \sim \|[$$

where R_{ref} is a common reference rotor set by the vacuum configuration in $H_3(\mathbb{O})$.

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In this picture, the observed pattern of couplings is a fingerprint of how the vacuum sits inside the exceptional algebra, not a list of independent constants.

Scaling and running

Of course, α is not a rigid number: in quantum field theory it runs with energy scale. The octonionic model does not deny this; instead it separates two roles:

1. The *bare geometric value*, determined by the internal rotor norm at a natural reference scale.
2. The *renormalized value* seen in experiments at a given energy, obtained by standard running from that reference scale.

The contribution of the geometry is to fix the starting point and the relative pattern of couplings. The quantum field theoretic machinery of running and thresholds then dresses these values to the ones we measure.

A different attitude towards constants

If α comes from an internal rotor norm, then the traditional question "Why $1/137$?" shifts its focus. Instead of asking for a closed-form expression in terms of π and e alone, we ask:

> Why does the vacuum select exactly this arrangement of internal directions in the exceptional algebra?

In other words, the mystery moves from "a magic number" to "a specific geometric configuration". This may or may not be more satisfying philosophically, but it is technically more tractable: one can compute norms of commutators, study their spectra, and connect them to the attractor dynamics discussed later in the Advent series.

References

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The famous $1/137$ is read as a norm in an exceptional internal geometry, not as an inexplicable magic number.