December 21, 2025 (Fourth Advent Sunday)

AQFT variant and F_4 potential

Quantisation and electroweak scale equilibrium

Key Insight. An octonionic model of particle physics must pass two hard tests: it has to admit a mathematically clean quantisation, and it has to explain why the electroweak scale sits where it does. Both issues meet on this fourth Advent Sunday. An algebraic quantum field theory (AQFT) formulation replaces Hilbert-space dogma by local algebras on the octonionic stage, while an F_4-symmetric potential fixes the electroweak scale Y_S as a true equilibrium quantity:

$$Y_S^2 = -\frac{\mu^2}{2(\lambda + \kappa c)}.$$

* * *

QFT without Hilbert-space dogma

Standard quantum field theory is usually presented as a story about fields on spacetime, quantised on a Hilbert space and expanded in creation and annihilation operators. For highly nontrivial internal structures like octonions and the Albert algebra, this picture is more a hindrance than a help.

Algebraic quantum field theory (AQFT) offers a different starting point:

- The fundamental objects are *local *-algebras* $\mathcal{A}(\mathcal{O})$ assigned to spacetime regions $\mathcal{O}$.
- States are positive linear functionals $\omega : \mathcal{A}(\mathcal{O}) \to \mathbb{C}$, not vectors in a pre-chosen Hilbert space.
- Dynamics and symmetries act as automorphisms of the net $\mathcal{O} \mapsto \mathcal{A}(\mathcal{O})$.

This fits the octonionic framework naturally:

- The internal degrees of freedom live in nonassociative structures; their observables sit more comfortably in operator algebras than in a single global Hilbert basis.
- Locality and covariance are built into the net of algebras, not into a particular field representation.

Local algebras on an octonionic stage

In the present model, the internal octonionic/Albert structure augments ordinary spacetime. A minimal AQFT-compatible picture is:

- To each spacetime region $\mathcal{O}$ we assign a local algebra $\mathcal{A}(\mathcal{O})$ generated by
  - rotor operators (internal symmetry generators),
  - compressor-derived fields (mass/mixing structures),
  - and possibly additional scalar modes related to the Jordan potential.
- The net $\mathcal{O} \mapsto \mathcal{A}(\mathcal{O})$ satisfies the Haag–Kastler axioms: isotony, locality (commutativity at spacelike separation), covariance under the relevant spacetime symmetries.

Quantisation in this picture means: we specify the algebra and its relations, then look at representations (states) that realise this net on Hilbert spaces *a posteriori*. This reverses the usual logic:

- Instead of quantising classical fields, we start from an operator algebra informed by octonionic geometry.
- Hilbert spaces appear only as GNS completions of chosen states, not as primary input.

The F_4-symmetric potential

Beyond the kinematics of local algebras, we need dynamics and vacuum structure. The central object is a Jordan element $H \in H_3(\mathbb{O})$ and a potential $V_J(H)$ that is invariant under an F_4 symmetry acting on the Albert algebra.

A typical F_4-invariant potential has the schematic form

$$V_J(H) \;=\; \mu^2 \,\mathrm{Tr}(H^2) + \lambda \,\mathrm{Tr}(H^4) + \kappa \, c \,(\det H) + \cdots,$$

where the invariants (trace, determinant, higher Jordan invariants) are organised such that F_4 acts as the symmetry group of the potential.

The electroweak order parameter Y_S is then read as a particular component or invariant of the vacuum expectation value $\langle H \rangle$. Minimising V_J with respect to this degree of freedom leads to a condition of the form

$$\frac{\partial V_J}{\partial Y_S} = 0 \quad\Longrightarrow\quad Y_S^2 = -\frac{\mu^2}{2(\lambda + \kappa c)}.$$

Andreas Müller, Kempten University of Applied Sciences, andreas.mueller@hs-kempten.de

Electroweak scale as equilibrium quantity

The formula

$$Y_S^2 = -\frac{\mu^2}{2(\lambda + \kappa c)}$$

should be read structurally, not numerically. It expresses three key points:

1. The electroweak scale is not inserted by hand; it is a *minimum* of an F_4-symmetric potential.
2. The same invariants that control the structure of the operator toolbox (rotors, compressors, radius operator) also enter the coefficients $(\mu^2, \lambda, \kappa c)$.
3. Small changes in these coefficients lead to controlled shifts in Y_S, not to arbitrary values: the scale is a stable equilibrium of the internal geometry.

In other words, the 246 GeV scale is reinterpreted as an equilibrium quantity of the exceptional internal space, analogous to how a crystal lattice spacing is an equilibrium of an atomic potential, not an external parameter.

Connecting AQFT and the potential

The AQFT and potential pictures are not separate stories:

- The potential $V_J(H)$ determines the vacuum configuration $\langle H \rangle$, hence the compressor spectra and the operator content of the local algebras $\mathcal{A}(\mathcal{O})$.
- Conversely, the dynamics of fields in AQFT—encoded in the local algebras and their time evolution—are constrained by the same internal symmetries (G_2, F_4) that shaped the potential.

This dual view is typical of modern mathematical physics:

- On the *operator side*, we quantise via local algebras and states, avoiding early commitment to particular Fock spaces.
- On the *potential side*, we see vacuum structure and symmetry breaking as consequences of exceptional invariants.

What remains open and what is gained

The fourth Advent Sunday is intentionally honest about open questions:

- The gravitational sector is not yet fully encoded; the ratio $\kappa = m_p/m_P$ still enters as an external constant, to be addressed by a future spectral-gravity analysis.
- Renormalisation and nonperturbative effects in an octonionic AQFT remain technically challenging.

But we gain two important things:

1. A quantisation framework—AQFT—that is robust enough to house nonassociative internal structures.
2. A structural explanation of the electroweak scale as an equilibrium value of an F_4-symmetric potential, linked tightly to the rest of the operator toolbox.

References

[1] R. Haag, *Local Quantum Physics*, Springer, 1996.

[2] R. Haag and D. Kastler, "An algebraic approach to quantum field theory," *J. Math. Phys.* **5**, 848–861 (1964).

[3] P. G. O. Freund, *Introduction to Supersymmetry*, Cambridge University Press, 1985.

[4] A. Connes and A. H. Chamseddine, "The spectral action principle," *Commun. Math. Phys.* **186**, 731–750 (1997).

Octonionic AQFT and an F_4-symmetric potential turn the electroweak scale from a fitted number into an equilibrium of the internal exceptional geometry.

Andreas Müller, Kempten University of Applied Sciences, andreas.mueller@hs-kempten.de