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Algebraic QFT: What is mass in a matrix world?

From fields and Feynman rules to spectra of D and $\Pi(\langle H\rangle)$

Key Insight. Textbook quantum field theory (QFT) usually introduces mass as a parameter in a Lagrangian density: a number m in a term $m\bar{\psi}\psi$ or $m^2\phi^2$. In a matrix-first, octonionic framework built around the transport equation $D\Psi = 0$, this is the wrong starting point. Here, masses are spectral data of operators: eigenvalues of D^2 and of the mass map $\Pi(\langle H\rangle)$ defined by a vacuum configuration $\langle H\rangle \in H_3(\mathbb{O})$. Different sectors (visible, neutrino-like, shadow) correspond to different spectral blocks of the same operator, not to separate sets of fields. "What is mass?" becomes a question about spectra, not about freely chosen coefficients in a scalar potential.

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The previous days assembled three pieces of the puzzle:

- On 16 December (gravity), the transport operator $D = \partial + A$ was identified as the common home for spacetime connection and internal gauge fields, with full quantum gravity still an open corner.
- On 17 December (spectral geometry), the spectrum of D entered centre stage: a single spectral action $S_{\text{spec}} = \operatorname{Tr} f(D^2/\Lambda^2)$ was seen to generate Einstein–Hilbert, cosmological constant and gauge dynamics in one stroke.
- On the 3rd Advent (14–15 December), the flavour atlas and the first numerical prototypes showed how fermion masses and mixings can be read from eigenvalues of compressor operators and of the mass map $\Pi(\langle H\rangle)$ in the Albert space $H_3(\mathbb{O})$.

What is still missing is a conceptual answer to a very simple question:

> In such a matrix-first, octonionic world, what *is* a mass?

Today we reformulate the textbook notion of mass in the language of operator spectra and algebraic QFT.

From Lagrangians to operator spectra

In introductory QFT, a field with mass m is defined by writing a term like

$$\mathcal{L} \supset -m\,\bar{\psi}\psi \quad \text{or} \quad \mathcal{L} \supset -\tfrac{1}{2}m^2\phi^2$$

in the Lagrangian. Mass appears as an *input parameter*: we choose m, quantise, and then compute scattering amplitudes and propagators with poles at $p^2 = m^2$.

In the octonionic, matrix-first picture, this order is reversed. The basic object is the transport operator

$$D = \partial + A$$

acting on an internal–spacetime state Ψ. The equation

$$D\Psi = 0$$

encodes dynamics; the spectrum of D (and of D^2) encodes masses. What we used to call "the mass m" becomes an eigenvalue of an operator built from geometry and vacuum data.

Mass from D^2 in a matrix-first frame

Already in standard relativistic quantum mechanics, the dispersion relation

$$E^2 = p^2 + m^2$$

tells us that a rest mass m is a spectral quantity: m^2 is the eigenvalue of $p^\mu p_\mu$ at zero spatial momentum. In the present framework, the square of the transport operator plays this role:

$$D^2\Psi = m^2\Psi$$

for suitable modes Ψ. Here D^2 contains both spacetime and internal contributions:

$$D^2 = (\partial + A)^2 = \partial^2 + [\partial, A] + A^2,$$

with gravitational and gauge parts of A as explained around 16 and 17 December.

The message is:

> Masses are eigenvalues of D^2, not coefficients we type into a Lagrangian by hand.

The mass map $\Pi(\langle H\rangle)$ on $H_3(\mathbb{O})$

Internally, the vacuum is encoded by a Jordan element $\langle H\rangle \in H_3(\mathbb{O})$. From this, the model builds a mass map

$$\Pi(\langle H\rangle)$$

acting on the internal state space. Its eigenvalues (after appropriate normalisation by Yukawa-like factors) are interpreted as fermion masses.

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Concretely, for a simple diagonal vacuum with two distinct eigenvalues,

$$\langle H\rangle = \mathrm{diag}(\lambda_{\mathrm{low}}, \lambda_{\mathrm{low}}, \lambda_{\mathrm{high}}),$$

the spectrum of $\Pi(\langle H\rangle)$ organises the 8-dimensional state space into structured multiplets:

- light blocks with eigenvalues near λ_{low},
- heavy blocks near λ_{high},
- colour triplets and singlets grouped by their octonionic position.

In more realistic vacua, hierarchies and mixings appear. But throughout, "having a mass" means "sitting in a certain spectral slot of $\Pi(\langle H\rangle)$".

Visible, neutrino-like and shadow sectors

A key point is that different physical sectors correspond to different *blocks* of the same operator, not to independent fields living in disconnected Hilbert spaces. Schematically, the internal space splits as

$$\mathcal{H}_{\mathrm{int}} = \mathcal{H}_{\mathrm{vis}} \oplus \mathcal{H}_{\nu} \oplus \mathcal{H}_{\mathrm{shadow}}$$

with corresponding block decompositions

$$\Pi(\langle H\rangle) = \Pi_{\mathrm{vis}} \oplus \Pi_{\nu} \oplus \Pi_{\mathrm{shadow}}.$$

- On $\mathcal{H}_{\mathrm{vis}}$, the spectrum of $\Pi(\langle H\rangle)$ gives charged-lepton and quark masses and mixings, as discussed on the flavour days of 11–15 December.
- On $\mathcal{H}_{\nu}$, it gives neutrino-like spectra: tiny eigenvalues, sensitive to seesaw-like structures and to the radii (a_0, b_0, c_0).
- On $\mathcal{H}_{\mathrm{shadow}}$, eigenvalues describe states that do not couple to the visible charges Q, Y and colour; they are natural dark or shadow candidates (see 20 December).

All three sectors are parts of a single operator story: no extra "dark Lagrangian" or ad-hoc neutrino sector is needed.

Why algebraic QFT is the right home

Algebraic QFT (AQFT) frames this in a representation-free language. Instead of starting from a fixed Fock space and a list of field operators, it associates to each spacetime region $\mathcal{O}$ a local algebra

$$\mathcal{O} \longmapsto \mathcal{A}(\mathcal{O}),$$

with isotony, locality and covariance. States are positive linear functionals ω on the global algebra, not preferred vectors. The Hilbert space and its particle interpretation arise afterwards, via the GNS construction.

For the octonionic model, this is exactly the right setting:

- The internal nonassociative geometry resides in the construction of the operators (rotors, compressors, radius, Jordan elements), while the local algebras $\mathcal{A}(\mathcal{O})$ remain associative operator algebras.
- Different vacua $\langle H\rangle$ correspond to different states ω, and thus to different GNS representations, but the underlying operator content (and thus the definition of mass as spectral data of D^2 and $\Pi(\langle H\rangle)$) is common to all.

Where the numbers will enter

Today's message is conceptual: it clarifies what "mass" means in a matrix-first, octonionic QFT. The nearby days fill in the numbers:

- 17 December (spectral geometry) explains how D and its spectrum encode gravity and gauge structure.
- 19 December (scales as attractor fixed points) connects the eigenvalues (a_0, b_0, c_0) of the radius operator R to the three fundamental scales (Planck, electroweak, QCD).

In this block, "mass" is thus neither a free parameter nor a single number, but a whole pattern of eigenvalues tied to the same algebraic invariants that also fix couplings and scales.

References

[1] R. Haag and D. Kastler, "An algebraic approach to quantum field theory," *J. Math. Phys.* **5**, 848–861 (1964).

[2] R. Haag, *Local Quantum Physics*, Springer, 1996.

In a matrix-first, octonionic framework, masses are not input numbers in a Lagrangian but eigenvalues of operators: spectral data of D^2 and of the mass map $\Pi(\langle H\rangle)$. Algebraic QFT provides the natural language to formulate this without committing to a single Fock space or perturbative expansion.

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