December 13, 2025

Top quark: heaviest fermion as vacuum anchor

Why $m_t \approx 173\,\mathbf{GeV}$ sits next to the electroweak scale

Key Insight. The top quark mass $m_t \approx 173\,\text{GeV}$ is not an outlier but the largest eigenvalue of the mass map $\Pi(\langle H\rangle)$ associated with the vacuum configuration $\langle H\rangle$ in the Albert algebra $H_3(\mathbb{O})$. It acts as an *anchor* for the electroweak scale: the same structure that fixes the vacuum also singles out the top as the heaviest fermion. Without such an anchor, the electroweak scale would be unstable.

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In the octonionic model, fermion masses are not free Yukawa coefficients but eigenvalues of a linear map $\Pi(\langle H\rangle)$ valuated at the vacuum configuration $\langle H\rangle \in H_3(\mathbb{O})$. The physical mass matrix is schematically $M = y \cdot \Pi(\langle H\rangle))$, with y a universal coupling. Its eigenvalues are the fermion masses. Among these, the largest eigenvalue is identified with the top quark mass m_t.

The spectrum of $\Pi(\langle H\rangle))$ is strongly constrained: $\langle H\rangle$ is not arbitrary; it minimises a Jordan potential $V_J(H)$ under symmetry constraints. Π respects the F_4 symmetry of the Albert algebra. The resulting eigenvalues come in structured patterns rather than random numbers.

In this setting, the top quark mass appears as the largest eigenvalue in the relevant sector. This is not just a numerical statement; it has a stability interpretation.

The electroweak scale Y_S is fixed by the minimum of an internal potential. The vacuum configuration $\langle H\rangle$ sits at this minimum, and $\Pi(\langle H\rangle))$ inherits that structure. The largest eigenvalue m_t then behaves as an anchor:

- If m_t were much smaller, the curvature of the potential near the minimum would change, destabilising the electroweak scale.
- If m_t were much larger, the same structure would shift the position of the minimum, again spoiling the observed scale.
- The observed value $m_t \approx 173$ GeV lies naturally next to the electroweak scale, reflecting the shared origin in $\langle H\rangle$.

Lighter fermions (electron, muon, tau; light quarks) correspond to smaller eigenvalues of the same map. Their smallness is explained by the geometry of $\langle H\rangle$ in the symmetry atlas: some directions in $H_3(\mathbb{O})$ generate large eigenvalues, associated with heavy fermions; other directions generate exponentially suppressed eigenvalues, associated with light fermions.

The top quark is simply the fermion whose eigenvector aligns best with the 'steep' direction of the potential at the vacuum point.

In the usual Standard Model narrative, Yukawa couplings are free parameters, and the large top Yukawa is a brute fact. In the octonionic picture:

1. The vacuum configuration $\langle H\rangle$ is fixed by internal geometry and potential minimisation.
2. The mass operator $\Pi(\langle H\rangle)$ is uniquely determined by this vacuum and the symmetry atlas.
3. The top quark emerges as the strongest-coupled mode to this vacuum, i.e. the maximal eigenvalue.

The hierarchy 'm_t heavy, others light' is no longer a collection of independent choices but a single structural statement about one operator evaluated at one point in the Albert algebra.

The top quark is not an accidental heavy outlier. It is the largest eigenvalue of the mass map $\Pi(\langle H\rangle))$ and thus an anchor of the electroweak vacuum. The same internal structure that fixes the electroweak scale also singles out the top as the heaviest fermion.

Masses from the mass map $\Pi(\langle H\rangle)$

Inthe octonionic model, fermion masses are not free Yukawa coefficients but eigenvalues of a linear map

$$\Pi : H_3(\mathbb{O}) \longrightarrow \mathrm{End}(\mathbb{R}^8),$$

evaluated at the vacuum configuration $\langle H\rangle \in H_3(\mathbb{O})$. The physical mass matrix is schematically

$$M = y\,\Pi(\langle H\rangle),$$

with y a universal coupling. Its eigenvalues

$$m_i = \lambda_i\big(\Pi(\langle H\rangle)\big)$$

are the fermion masses. Among these, the largest eigenvalue is identified with the top quark mass m_t.

The top as the maximal eigenvalue

The spectrum of $\Pi(\langle H\rangle)$ is strongly constrained:

- $\langle H\rangle$ is not arbitrary; it minimises a Jordan potential $V_J(H)$ under symmetry constraints.

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- Π respects the F_4 symmetry of the Albert algebra.
- The resulting eigenvalues come in structured patterns rather than random numbers.

In this setting, the top quark mass appears as

$$m_t = \max \operatorname{spec}\bigl(\Pi(\langle H \rangle)\bigr),$$

the largest eigenvalue in the relevant sector. This is not just a numerical statement; it has a stability interpretation.

Vacuum anchoring at the electroweak scale

The electroweak scale Y_S is fixed by the minimum of an internal potential. Schematically,

$$Y_S^2 = -\frac{\mu^2}{2(\lambda + \kappa c)},$$

with $\mu^2, \lambda, \kappa, c$ determined by the symmetry atlas. The vacuum configuration $\langle H \rangle$ sits at this minimum, and $\Pi(\langle H \rangle)$ inherits that structure.

The largest eigenvalue m_t then behaves as an *anchor*:

- If m_t were much smaller, the curvature of the potential near the minimum would change, destabilising the electroweak scale.
- If m_t were much larger, the same structure would shift the position of the minimum, again spoiling the observed scale.
- The observed value $m_t \approx 173\,\mathrm{GeV}$ lies naturally next to the electroweak scale, reflecting the shared origin in $\langle H \rangle$.

Top quark versus lighter fermions

Lighter fermions (electron, muon, tau; light quarks) correspond to smaller eigenvalues of the same map. Their smallness is explained by the geometry of $\langle H \rangle$ in the symmetry atlas:

- Some directions in $H_3(\mathbb{O})$ generate large eigenvalues, associated with heavy fermions.
- Other directions generate exponentially suppressed eigenvalues, associated with light fermions (as discussed for neutrinos on 15 December).

The top quark is simply the fermion whose eigenvector aligns best with the "steep" direction of the potential at the vacuum point.

Conceptual picture

In the usual Standard Model narrative, Yukawa couplings are free parameters, and the large top Yukawa is a brute fact. In the octonionic picture:

1. The vacuum configuration $\langle H \rangle$ is fixed by internal geometry and potential minimisation.
2. The mass operator $\Pi(\langle H \rangle)$ is uniquely determined by this vacuum and the symmetry atlas.
3. The top quark emerges as the strongest-coupled mode to this vacuum, i.e. the maximal eigenvalue.

The hierarchy "m_t heavy, others light" is no longer a collection of independent choices but a single structural statement about one operator evaluated at one point in the Albert algebra.

References

[1] CDF and D0 Collaborations, "Combination of CDF and D0 results on the mass of the top quark," *Phys. Rev. D* **89**, 072001 (2014).

[2] P. Jordan, J. von Neumann and E. Wigner, "On an algebraic generalization of the quantum mechanical formalism," *Ann. Math.* **35**, 29–64 (1934).

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