December 2, 2025

Triality: three equal 8D representations

When space, matter and antimatter share the same dimension

Key Insight. Spin(8) has a unique feature: *triality*. It admits three equivalent 8-dimensional representations—one vector representation V_8 and two chiral spinor representations S_8^+ and S_8^-. This is not a decorative curiosity: it provides a geometric template for three kinds of "8-dimensional stuff" that can later be read as space, matter and antimatter. Today we meet triality as a symmetry between representations; later days will turn it into the organising principle behind three fermion generations.

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Three eights instead of one

In most Lie groups, vector and spinor representations look different and behave differently. Spin(8) is special: it has

$$V_8, \qquad S_8^+, \qquad S_8^-,$$

three irreducible 8-dimensional representations that are related by a nontrivial outer automorphism group of order 6. This automorphism group acts by permuting (V_8, S_8^+, S_8^-); its $\mathbb{Z}_3$-part is called *triality*:

$$S: \quad V_8 \longrightarrow S_8^+ \longrightarrow S_8^- \longrightarrow V_8.$$

Geometrically, the three representations are on equal footing: no one is more fundamental than the others.

Why triality matters for an octonionic model

Octonions $\mathbb{O}$ naturally support an 8-dimensional real representation. In the model described by this calendar, one uses:

- an 8D "vector-like" role associated with internal space directions,
- two 8D "spinor-like" roles associated with chiral matter and antimatter sectors.

Triality then becomes the statement that there is an underlying symmetry relating these three roles. It is the reason why it is natural to package internal degrees of freedom in blocks of size 8 and why it is not absurd to think of space-like and matter-like sectors as different faces of the same algebraic coin.

From three 8D reps to three generations

The XLS plan for the calendar reserves a later day (12 December) for the statement:

> "Three fermion generations mirror the three triality representations."

Today we prepare that statement conceptually:

- If there is a symmetry that permutes (V_8, S_8^+, S_8^-), it is natural to try to attach one 8D "copy" of the internal structure to each of them.
- In particle language, this suggests three families of fermions with identical gauge quantum numbers but different "triality label".
- The existence of *exactly* three such representations motivates the existence of *exactly* three generations, not one or four.

The details (how charges and masses are assigned in each block) are left to the later flavor and generation days. The conceptual point is simple:

> Three fermion generations are not an arbitrary count; they reflect a built-in threefold symmetry of Spin(8).

Link to octonions and G_2

Spin(8) and octonions are tightly linked:

- The group G_2 is the automorphism group of the octonions $\mathbb{O}$.
- Spin(8) acts on $\mathbb{O}$ in ways compatible with this G_2-structure.
- Triality is a statement about how vector and spinor actions can be interchanged without breaking the internal octonionic structure.

In the broader project, this becomes one ingredient in the *symmetry atlas*:

- G_2 as the minimal exceptional symmetry (tomorrow),
- F_4 as the automorphism group of the Albert algebra $H_3(\mathbb{O})$,
- triality as the bridge between space-like and spinor-like sectors.

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Conceptual gain from triality

What is gained by taking triality seriously?

1. **Symmetry-based multiplicity:** the multiplicity "three" is not an afterthought but a symmetry consequence.
2. **Unified treatment of sectors:** space-like, matter-like and antimatter-like sectors share the same dimension and are related by an actual group action.
3. **Constraints for model building:** any attempt to modify the number of generations must explain how triality is broken or extended.

Instead of asking "Why three generations?" as a bare phenomenological question, the model asks a more geometric one:

> How does Spin(8) triality appear inside the octonionic/Albert structure, and how does it force a threefold replication of internal degrees of freedom?

References

[1] E. Cartan, "Le principe de dualité et la théorie des groupes simples et semi-simples," *Bull. Sci. Math.* **49**, 361–374 (1925).

[2] J. C. Baez, "The octonions," *Bull. Amer. Math. Soc.* **39**, 145–205 (2002).

[3] [Internal notes on triality and its role in generation structure: `chap01_neu.tex; appA_neu.tex; appN_neu.tex`.]

Triality of Spin(8) provides three equal 8D representations. In an octonionic model, this becomes the natural algebraic source of the threefold replication of fermion content that we observe as "three generations".

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