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Constants as equilibrium values of exceptional geometry

From “input parameters” to attractors and minima

Key Insight. In the textbook Standard Model, parameters like the fine-structure constant α, the strong coupling α_s, the Weinberg angle θ_W, fermion masses and mixing angles appear as independent inputs. In the octonionic/Albert framework, many of them can be read as *equilibrium values*: minima of an F_4-symmetric potential, attractor radii of an internal flow, or angles between preferred rotor directions. Physical constants become what the internal exceptional geometry *settles into*, not arbitrary dials.

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Constants vs. equilibrium values

In everyday language, a “constant of nature” is a number that does not change over time. But for theory building there is a deeper question:

> Is the constant a fundamental input, or is it the value that some dynamical or geometric system relaxes to?

In statistical mechanics, equilibrium temperatures and densities are not fundamental; they are fixed points of microscopic dynamics. The octonionic model applies this logic to parts of high-energy physics:

- Couplings and angles become norms and overlaps in rotor space.
- Mass scales become attractor radii of an internal flow.
- Order parameters become minima of an F_4-invariant potential.

Electroweak scale as a potential minimum

A central example from the 4th Advent (21 December) is the electroweak scale Y_S. A Jordan potential

$$V_J(H) = \mu^2\,\mathrm{Tr}(H^2) + \lambda\,\mathrm{Tr}(H^4) + \kappa\, c\,\det H + \cdots$$

on $H_3(\mathbb{O})$, invariant under F_4, has a minimum at some vacuum $\langle H\rangle$. Reading Y_S as a suitable component or invariant of $\langle H\rangle$ and minimising V_J yields a relation of the type

$$Y_S^2 = -\frac{\mu^2}{2(\lambda+\kappa c)}.$$

Here Y_S is not an arbitrary input; it is the equilibrium value of the order parameter in an exceptional potential. The observed electroweak scale is then analogous to a lattice spacing in a crystal: an equilibrium distance, not a fundamental length.

Angles and couplings as geometric overlaps

From 10 December we remember: couplings and the Weinberg angle can be read from rotor geometry. With an inner product

$$\langle A, B\rangle = \mathrm{Tr}(A^\dagger B),$$

and rotor directions G_W (weak isospin) and G_Y (hypercharge), one defines

$$\cos\theta_W = \frac{\langle G_W, G_Y\rangle}{\|G_W\|\,\|G_Y\|}.$$

Similarly, α and α_s track the norms of appropriate commutators in the internal operator space. Once the embedding of $SU(3)_C \times SU(2)_L \times U(1)_Y$ into the exceptional algebra is fixed, the *relative* normalisation of these norms is fixed as well. Running with energy complicates the picture, but the low-energy values are no longer independent dials; they are correlated by the underlying geometry.

In this sense, θ_W and the ratios between α and α_s are *geometric equilibrium values*: the angles and norms selected by a particular embedding of the Standard Model group into the exceptional symmetry.

Mass hierarchies as attractor radii

The radius operator and attractor mechanism from earlier days provide another route to equilibrium values. Internal flows in the space of configurations—driven by renormalisation-group or effective potential gradients—develop attractors at characteristic radii

$$R \sim R_{*,1},\ R_{*,2},\ \dots$$

corresponding to:

- low-energy QCD scale,
- electroweak scale,
- higher intermediate scales.

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Fermion masses then fall into bands or shells around these radii, with the precise eigenvalues determined by the compressor spectra. The huge hierarchies between these radii (e.g. Planck vs. electroweak) can be traced back to nonassociativity (8 December): strong deviations from associativity produce exponential separations between attractor radii.

Thus, mass ratios are not "inserted by hand"; they are equilibrium outcomes of flows on an exceptional internal manifold.

What remains external

The equilibrium perspective is powerful but not unlimited. In the current state of the model:

- The gravitational coupling—equivalently the Planck mass m_P—is still external (16 December).
- The ratio $\kappa = m_p/m_P$ is acknowledged as an input, awaiting a spectral-geometry derivation.
- The cosmological constant is not yet computed from an octonionic spectral action.

So the slogan "constants are equilibrium values" is true for many, but not yet for all of them. The calendar is explicit about this boundary.

Conceptual shift: from parameter lists to geometry

The main conceptual gain of this day is not a new formula but a new classification:

1. **Equilibrium constants:** values fixed by minima of potentials, attractors of flows, or fixed geometric overlaps (e.g. Y_S, many fermion masses, θ_W, relative couplings).
2. **Spectral constants (future work):** values to be derived from the spectrum of a combined Dirac operator in spectral geometry (e.g. G, cosmological constant).
3. **Still-external constants:** numbers not yet tied to the exceptional structure in the present model.

This is a clear upgrade over a flat parameter list: we know which constants are already inside the exceptional story, which should be pulled in via spectral geometry, and which ones remain open.

Why this matters for physics, not just for aesthetics

From a pragmatic viewpoint, one might say: as long as the model matches data, who cares whether a parameter is an input or an equilibrium value? The answer is twofold:

- **Predictivity:** Equilibrium values are constrained: small changes in the microscopic setup lead to calculable shifts, not arbitrary variations. This opens the door to quantitative tests.
- **Stability:** Equilibrium interpretation explains why certain values are robust under perturbations, decoupling of heavy fields, or environmental changes.

If, step by step, more of the Standard Model parameter list can be turned into equilibrium values of an exceptional geometry, the notorious "fine-tuning" problems change character: we no longer tune against nothing; we characterise which equilibria of which internal structures are realised in our Universe.

References

[1] A. Connes and A. H. Chamseddine, "The spectral action principle," *Commun. Math. Phys.* **186**, 731–750 (1997).

[2] F. Gürsey and H. C. Tze, *On the Role of Division, Jordan and Related Algebras in Particle Physics*, World Scientific, 1996.

[3] [Internal notes on attractors, rotor norms and constants as equilibria: `arxiv-const.tex; appK_neu.tex; appE_neu.tex`.]

In the exceptional picture, many "constants" are not arbitrary inputs but equilibrium values: minima of F_4-symmetric potentials, attractor radii and geometric angles in an octonionic internal space.