December 7, 2025 (Second Advent Sunday)

Heptagon, Radii and Attractor

The rotor/compressor toolbox for all observables

Key Insight. In the octonionic model, all observable quantities—couplings, masses, mixings and scales—are traced back to invariants of two operator families on the internal space: antisymmetric *rotors* (forces) and symmetric *compressors* (masses and mixings). The heptagon operator encodes the seven imaginary octonion directions; the radius operator R with spectrum (a_0, b_0, c_0) defines an attractor mechanism for fundamental scales. Together they form a minimal but sufficient operator toolbox: nothing else is added by hand.

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From algebra to operators

OCTONIONS and their automorphism group G_2 give us a rigid internal stage. But physical predictions are not read directly from the multiplication table; they arise from *operators* acting on the internal space. In this model, two operator families play the central role:

- **Rotors** — antisymmetric operators G_a generating internal rotations: they encode forces and couplings.
- **Compressors** — symmetric operators C with real spectra: they encode masses and mixing patterns.

Once this toolbox is in place, every later sheet becomes a story about eigenvalues, eigenvectors, commutators and norms of these operators.

The heptagon operator: seven directions, three invariants

The seven imaginary octonion units are arranged on the Fano-plane heptagon. Instead of handling them one by one, the model packages them into a single *heptagon operator* H_7 acting on the internal space:

$$H_7 \;=\; \sum_{i=1}^{7} c_i E_i,$$

where the E_i encode the seven directions and the c_i are fixed by the vacuum and G_2-symmetry.

Despite being built from seven directions, H_7 has only three independent eigenvalues,

$$\mathrm{Spec}(H_7) = (\alpha, \beta, \gamma),$$

which are invariants of the G_2-orbit of H_7. These three numbers will reappear as seeds for:

- gauge couplings (fine-structure, strong coupling, weak mixing),
- relative positions of flavour sectors,
- and parts of the hierarchy structure.

The radius operator and three fundamental scales

From the heptagon structure one constructs a *radius operator* R. It measures how far internal directions sit from preferred axes. Its spectrum is

$$\mathrm{Spec}(R) = (a_0, b_0, c_0), \quad a_0 > b_0 > c_0.$$

Exponentials of these radii define three characteristic energy scales:

$$E_{\text{Planck}} \sim e^{a_0}, \qquad E_{\text{EW}} \sim e^{b_0}, \qquad E_{\text{QCD}} \sim e^{c_0}.$$

Thus, the familiar hierarchy of Planck, electroweak and QCD scales is encoded in a few geometric invariants of R rather than inserted as three independent inputs.

Rotors: forces from commutator norms

Rotors are antisymmetric operators G_a generating continuous internal symmetries. Their commutators measure how two internal directions fail to commute. The squared norm of a commutator,

$$\big\|[G_a, G_b]\big\|^2,$$

acts as the prototype for a coupling constant. Symbolically,

$$\alpha \sim \big\|[G_{\text{em}}, G_{\text{ref}}]\big\|^2, \quad \alpha_s \sim \big\|[G_{\text{color}}, G_{\text{ref}}]\big\|^2, \quad \sin^2\theta_W \sim \big\|[G_{\text{weak}}, G$$

Choosing different rotor pairs recovers different interactions. In this view, a "strong" force is literally a large commutator norm in the internal algebra; a "weak" one corresponds to nearly commuting rotors.

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Compressors: masses and mixings as spectra

Compressors are symmetric operators with real eigenvalues. The most important example is the mass map $\Pi(\langle H\rangle)$ constructed from a vacuum configuration $\langle H\rangle \in H_3(\mathbb{O})$:

$$\Pi(\langle H\rangle)\Psi = m\,\Psi.$$

Its eigenvalues m provide prototype fermion masses; its eigenvectors define the associated mass eigenstates. Additional compressors act in flavour subspaces; misalignment between their eigenbases produces mixing matrices:

- down- vs. up-quark compressors $\Rightarrow$ CKM matrix,
- charged-lepton vs. neutrino compressors $\Rightarrow$ PMNS matrix.

Masses and mixings thus share a common origin: they are different ways of reading the same symmetric operators.

Attractor behaviour of scales and hierarchies

The combination of radius operator and compressors suggests an *attractor* picture:

- The spectrum (a_0, b_0, c_0) singles out preferred scales.
- Renormalisation-group flow tends to pull effective parameters towards these scales.
- Small deformations of $\langle H\rangle$ move eigenvalues, but certain patterns (like the large gap between Planck and EW) remain stable.

Hierarchies thus become *fixed points* of a dynamical process in operator space, not arbitrary distances between hand-picked numbers.

Why this toolbox is minimal and sufficient

What makes the toolbox attractive is its balance between simplicity and power:

- It is *minimal*: no independent Yukawa matrices, no unmotivated extra symmetries, no long list of unrelated constants.
- It is *sufficient*: in principle, all quantities that enter phenomenology—masses, mixing angles, couplings, scales—can be expressed in terms of the invariants of these operators.

The remaining Advent days flesh out this claim with concrete examples: from specific couplings (fine-structure, strong, Weinberg angle) to numerical mass prototypes and flavour structures.

References

[1] T. Dray and C. A. Manogue, *The Geometry of the Octonions*, World Scientific, 1999.

[2] G. M. Dixon, *Division Algebras: Octonions, Quaternions, Complex Numbers and the Algebraic Design of Physics*, Kluwer, 1994.

[3] A. H. Chamseddine, A. Connes and M. Marcolli, "Gravity and the standard model with neutrino mixing," *Adv. Theor. Math. Phys.* **11**, 991–1089 (2007).

One toolbox, two operator families: rotors and compressors turn octonionic geometry into concrete numbers for couplings, masses and scales.

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