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Algebraic QFT: beyond Feynman diagrams and Hilbert-space dogma

Local operator algebras on an octonionic stage

Key Insight. Most physicists meet quantum field theory (QFT) through path integrals and Feynman diagrams on a fixed Hilbert space. For an octonionic model with a rich internal operator toolbox, this language is too rigid. Algebraic QFT (AQFT) replaces fields by nets of local algebras $\mathcal{O} \mapsto \mathcal{A}(\mathcal{O})$ and treats states as positive linear functionals, not as privileged vectors. This is the natural QFT framework for a nonassociative internal geometry.

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From fields to nets of algebras

In the textbook picture, a quantum field theory is given by a Lagrangian, a path integral and a set of Feynman rules. Hilbert space, canonical commutation relations and perturbative expansions are baked in from the start.

Algebraic QFT (Haag–Kastler) takes a different route. The basic object is a net of C*- or von Neumann algebras

$$\mathcal{O} \longmapsto \mathcal{A}(\mathcal{O}),$$

where $\mathcal{O}$ runs over open regions of spacetime. This net obeys:

- **Isotony:** $\mathcal{O}_1 \subset \mathcal{O}_2 \Rightarrow \mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)$.
- **Locality:** spacelike separated regions commute, $[\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2)] = 0$.
- **Covariance:** a symmetry group G acts by automorphisms α_g with $\alpha_g(\mathcal{A}(\mathcal{O})) = \mathcal{A}(g\mathcal{O})$.

States are positive linear functionals $\omega : \mathcal{A} \to \mathbb{C}$, not a priori elements of a given Fock space. A Hilbert-space representation arises afterwards via the GNS construction.

Why this fits the octonionic toolbox

The octonionic model organises internal physics in terms of operators:

- rotors (internal symmetry generators),
- compressors (mass/mixing operators),
- the radius operator and attractor structures,
- Jordan elements $H \in H_3(\mathbb{O})$ and their potentials.

These are naturally interpreted as elements of local algebras rather than as components of a single canonical field on a fixed Hilbert space. AQFT offers exactly the right viewpoint:

- The internal nonassociative structure influences which operators can be multiplied and localised, but the algebras themselves remain associative operator algebras.
- Different vacuum choices (different ω) lead to different Hilbert-space representations, without changing the underlying net $\mathcal{A}(\mathcal{O})$.

Octonionic local algebras

In an octonionic context, one can sketch the following assignment:

- To each spacetime region $\mathcal{O}$ we assign an algebra $\mathcal{A}(\mathcal{O})$ generated by:
  - localised rotor operators (internal symmetries),
  - compressor fields encoding mass/mixing effects,
  - fluctuation modes of the Jordan element H.
- These generators satisfy commutation relations constrained by the exceptional symmetry (G_2, F_4) and by locality.

The internal nonassociativity shows up in how these generators combine and in the structure of their spectra, not in the C*-algebra axioms themselves (which remain associative).

AQFT vs. Feynman-diagram intuition

The algebraic language does not forbid Feynman diagrams; it puts them in perspective. In simple regimes and near-Gaussian states, one can still expand correlation functions in diagrams. But:

- The existence of a Fock space and a perturbative expansion is no longer an axiom, but a property of certain states.
- Nonperturbative aspects (phase structure, superselection sectors, thermal states) can be discussed directly in terms of algebras and states.

Andreas Müller, Kempten University of Applied Sciences, andreas.mueller@hs-kempten.de

- The nonassociative internal geometry is built into the operator content, not into a particular path integral measure.

For an exceptional, nonassociative internal space, this is conceptually safer than starting from a single classical Lagrangian and hoping that perturbation theory can see everything.

Connection to spectral geometry and 21 December

AQFT and spectral geometry address complementary aspects:

- **Spectral geometry** (yesterday) tells us how to build an action from a Dirac spectrum that includes gravity and gauge fields.
- **AQFT** (today) tells us how to formulate the quantum theory of the resulting fields in a representation-free way.

On December 21, the fourth Advent Sunday, these threads meet: an octonionic version of AQFT is combined with an F_4-symmetric Jordan potential. The electroweak scale then appears as an equilibrium quantity of the internal geometry, and the local algebras inherit this scale through their operator content.

Why this day matters conceptually

The AQFT day is less about new numbers and more about cleaning up the foundations:

1. It frees the octonionic model from the straightjacket of a single Fock space and a fixed perturbative expansion.
2. It highlights that what matters fundamentally are *operators* and their algebraic relations, not a particular graphical technique.
3. It prepares the ground for rigorous discussions of locality, causality and phases in a world with an exceptional internal structure.

One hundred years after Heisenberg's operator approach, AQFT can be read as a modern continuation of his original spirit—now extended to an octonionic internal world that he could not have imagined.

References

[1] R. Haag and D. Kastler, "An algebraic approach to quantum field theory," *J. Math. Phys.* **5**, 848–861 (1964).

[2] R. Haag, *Local Quantum Physics*, Springer, 1996.

[3] [Internal notes on AQFT formulation and octonionic local algebras: `chap18_neu.tex; appF_neu.tex`.]

Algebraic QFT replaces Feynman-diagram dogma by nets of local algebras and states—exactly the language needed to house an octonionic internal geometry in a consistent quantum field theory.

Andreas Müller, Kempten University of Applied Sciences, andreas.mueller@hs-kempten.de