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CKM and PMNS from an Exceptional Atlas

Mixing matrices as transition charts on $H_3(\mathbb{O})$

Key Insight. Quark and lepton mixings are usually encoded in two unitary matrices, CKM and PMNS, introduced as phenomenological parameters. In the octonionic model, these matrices are reinterpreted as transition maps between different eigenbases of operators on the Albert algebra $H_3(\mathbb{O})$. The pattern of mixings is constrained by how the mass map, flavor projectors and compressor operators fit together in the exceptional geometry.

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WHY do quarks mix so little, while neutrinos mix so much? The Standard Model encodes these patterns in two unitary matrices—CKM for quarks, PMNS for leptons—but offers no explanation. The numbers are simply fitted to data.

The octonionic model offers a geometric answer. Mixing matrices are not arbitrary parameters, but *transition maps* between different preferred bases in the exceptional internal space $H_3(\mathbb{O})$. One basis is aligned with gauge interactions (flavor eigenstates), the other with the mass map $\Pi(H)$ built from the vacuum $\langle H \rangle$ (mass eigenstates). The misalignment between these bases is what we call mixing.

For quarks, the flavor projectors and the mass map almost commute within $H_3(\mathbb{O})$—the two bases are nearly aligned, so mixing is small. For leptons, especially neutrinos, nonassociative chains in the octonionic structure frustrate this alignment—the bases are more misaligned, so mixing is large.

The mystery of mixing becomes a question of internal geometry: not "why these numbers?" but "why do these operators almost commute in one sector and fail to commute in another?" The exceptional structure of $H_3(\mathbb{O})$ constrains the answer.

Two mysterious unitary matrices

The Standard Model describes quark and lepton mixings by two unitary matrices:

$$V_{\mathrm{CKM}} \quad \text{for quarks,} \qquad U_{\mathrm{PMNS}} \quad \text{for leptons.}$$

Empirically, V_{CKM} is close to the identity with small off-diagonal elements, while U_{PMNS} shows large mixings between neutrino flavors. In the usual formulation, both matrices are introduced by hand when diagonalising Yukawa matrices. They are indispensable in phenomenology, but their structure appears accidental.

The octonionic model offers a different viewpoint. It treats mixing angles and phases not as arbitrary complex numbers, but as geometric data attached to how different preferred bases in the exceptional internal space intersect.

The internal atlas picture

The internal degrees of freedom live in the Albert algebra $H_3(\mathbb{O})$, with automorphism group F_4. On this space, several distinguished operators are defined:

- The *mass map* $\Pi(H)$, built from a vacuum configuration $\langle H \rangle$, whose eigenvectors encode mass eigenstates.
- *Flavor projectors* which single out quark-like and lepton-like subspaces.
- *Compressor* and *rotor* operators that encode hierarchies and couplings.

Different physical questions prefer different eigenbases:

- Gauge interactions "see" eigenbases aligned with flavor projectors.
- Mass measurements "see" eigenbases of $\Pi(H)$.

The resulting misalignment between these bases is what phenomenologists call "mixing". In the geometric language, it is nothing but a transition map between charts on the internal space.

CKM as a transition map in the quark sector

Focus first on the quark sector. Consider two orthonormal bases of the relevant internal subspace:

- $\{d_i\}$, $i = 1, 2, 3$, adapted to flavor (interaction) eigenstates: these diagonalise the couplings to $W^{\pm}$ and gluons.
- $\{d'_i\}$, $i = 1, 2, 3$, adapted to mass eigenstates: these diagonalise the restriction of $\Pi(H)$ to the down-type quark sector.

By definition, there exists a unitary matrix V_{CKM} such that

$$d_i \;=\; \sum_j (V_{\mathrm{CKM}})_{ij}\, d'_j.$$

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In the octonionic setting, both bases arise from eigenproblems for operators that live in the same exceptional algebra and are constrained by its structure. As a result:

- The allowed pattern of overlaps $(V_{\mathrm{CKM}})_{ij}$ is not arbitrary; some entries are naturally suppressed or enhanced.
- Small quark mixing angles become a consequence of how the quark flavor projectors and the mass map almost commute within the relevant subspace of $H_3(\mathbb{O})$.

The hierarchical structure of V_{CKM} thus reflects an almost compatible choice of charts in the internal atlas.

PMNS and the special role of neutrinos

For leptons, the situation is different. The PMNS matrix connects charged lepton mass eigenstates to neutrino flavor eigenstates. In many models, neutrinos are Majorana particles with a mass matrix that has a different origin from the charged lepton masses.

In the octonionic model, this difference is encoded in the way neutrino directions sit inside $H_3(\mathbb{O})$:

- Neutrino-like internal directions may be associated with subspaces that are more strongly affected by nonassociative chains in the octonionic structure.
- The corresponding projectors onto neutrino subspaces can fail to commute more strongly with the mass map and with certain rotor operators.

As a result, the overlap matrix U_{PMNS} between flavor and mass eigenbases for leptons can naturally have large off-diagonal entries: the relevant operators pick eigenbases that are more misaligned than in the quark case.

Qualitative patterns from exceptional constraints

The goal of the octonionic approach is not to predict every mixing angle and phase numerically from first principles in a single stroke. Instead, it aims to explain why certain *patterns* appear robust:

1. **Quark mixings are small:** quark flavor projectors and the quark part of $\Pi(H)$ are nearly simultaneously diagonalisable within the exceptional algebra, leading to small mixing angles.
2. **Lepton mixings are large:** leptonic projectors, especially in the neutrino sector, have a more frustrated relation to the mass map and rotor operators, leading to larger natural mixings.
3. **Correlations between sectors:** because all these operators live in the same $H_3(\mathbb{O})$ and share the same vacuum configuration, patterns in the CKM and PMNS matrices are not independent: a change in the internal geometry affects both.

This is reminiscent of how coordinate changes on a curved manifold cannot be chosen independently in overlapping charts.

Towards numerical explorations

Once a concrete vacuum configuration $\langle H \rangle$ and explicit forms for the mass map, projectors and compressors are chosen, the model becomes numerical:

- One can diagonalise the relevant operators in $H_3(\mathbb{O})$ and compute the induced mixing matrices.
- One can study how these matrices change under deformations of $\langle H \rangle$ or of the internal rotor structure.
- One can search for attractor configurations that yield mixing patterns close to the observed CKM and PMNS matrices.

In this sense, the exceptional geometry provides a *parameter space* for mixings that is much more constrained than a generic set of complex 3×3 unitary matrices.

References

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CKM and PMNS are read as transition maps between preferred bases in an exceptional internal space, not as arbitrary phenomenological matrices.

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