December 31, 2025

From octonions to a research programme

A fireworks display of results at the end of the calendar

Key Insight. Over twenty-seven days, this calendar has assembled a remarkable collection of results. Quaternions and octonions, Jordan matrices, mass ladders, charge patterns, shadow sectors, a combined Dirac operator, and a structured vacuum have appeared as pieces of one story. Today we celebrate what has been achieved and look ahead to the work that remains.

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THE starting point was deliberately modest: treat quaternions and octonions not as exotic curiosities but as concrete building blocks for matrix algebras and operator structures. From there, the calendar built up a rich structure:

- quaternionic and octonionic matrices provided new ways to encode spin and internal degrees of freedom;
- the Jordan algebra $H_3(\mathbb{O})$ supplied a finite, highly structured internal space;
- simple operators on this space — rotations and compressors — generated rich spectra.

Along the way, we saw how non-associativity and triality introduce new possibilities for encoding families and interactions.

A fireworks display of results

By the end of the calendar, a remarkable number of concrete results had emerged:

One octonionic Dirac equation

The conceptual peak was reached on Christmas Eve: a single octonionic Dirac equation

$$\nabla_8 \Psi = 0$$

acting on spacetime spinors tensored with a finite internal space, with

$$\nabla_8 = D_M \otimes \mathbf{1}_{H_F} \; + \; \gamma^5 \otimes D_F.$$

This operator unifies:

- **Geometric sector:** The part of ∇_8 acting on spacetime encodes geometry and curvature through its large-scale spectral properties, reproducing the Planck scale $M_{\mathrm{Pl}} \sim 10^{19}$ GeV.
- **Internal sector:** The finite part D_F acts on the octonionic internal space and encodes charges, generations, masses and possible shadow states.
- **Spectral action:** Spectral functionals of ∇_8^2 naturally produce terms that look like actions for the metric and for internal symmetry fields.

A complete mass ladder

The octonionic mass operators R, L and their compressors organise observable and shadow scales into a ladder:

- **QCD scale:** $\Lambda_{\mathrm{QCD}} \sim 200\,\mathrm{MeV}$, where the strong force confines quarks into hadrons.
- **Electroweak scale:** $v \approx 246$ GeV, where the weak force becomes short-ranged and fermions acquire visible masses. This scale appears as a near-critical rung, where symmetric and broken phases balance.
- **Intermediate compressor scales:** $M_{C1} \sim 10^9$–10^{11} GeV and $M_{C2} \sim 10^{12}$–10^{14} GeV, organising the ladder and possibly associated with heavy shadow states.
- **High octonionic scale:** $M_8 \sim 10^{16}$–10^{18} GeV, the scale at which the full octonionic structure becomes manifest.

Concrete charge assignments

The internal Dirac operator D_F fixes quantum numbers as eigenvalues:

- **Electron:** $Q_e = -1$, colour singlet, weak doublet (left) or singlet (right).
- **Neutrino:** $Q_\nu = 0$, colour singlet, weak doublet (left) or singlet (right).
- **Up quark:** $Q_u = +\frac{2}{3}$, colour triplet, weak doublet (left) or singlet (right).
- **Down quark:** $Q_d = -\frac{1}{3}$, colour triplet, weak doublet (left) or singlet (right).

These values are not inputs but consequences of how the internal generators act on different subspaces of the $H_3(\mathbb{O})$-based internal space. The fact that quarks carry fractional charges while leptons carry integer charges, and that the specific fractions are $\pm\frac{1}{3}$ and $\pm\frac{2}{3}$, follows from the internal geometry.

Three generations from triality

The octonionic structure naturally accommodates three and only three generations of fermions, a direct consequence of triality. The same charge pattern repeats for all three generations:

- **First generation:** electron, neutrino, up, down.
- **Second generation:** muon, muon-neutrino, charm, strange.
- **Third generation:** tau, tau-neutrino, top, bottom.

The mass hierarchy and mixing patterns (CKM and PMNS matrices) are determined by the eigenvalue structure of D_F and the alignment of generation-sensitive directions controlled by the mass operators R and L.

A structured shadow sector

The same operators that organise visible masses also predict shadow states:

- **Light shadow states:** in the GeV–TeV range, neutral under Standard Model charges, potentially relevant for dark matter.
- **Heavy shadow states:** near M_{C2} or M_8, coupling primarily gravitationally, possibly relevant for early-universe cosmology.

These are not ad-hoc additions but natural consequences of the spectrum of D_F.

A highly structured vacuum

The vacuum is not "empty space" but a special state of ∇_8 in which all the scales, charges and patterns above are already encoded. It is a highly ordered ground state in which certain octonionic directions are preferred and some symmetries are spontaneously broken. Phase transitions in the early universe can be seen as changes in the vacuum state of ∇_8.

Falsifiable predictions

The framework makes concrete, falsifiable predictions:

- approximate mass ratios for fermions across generations;
- mixing angles in the CKM and PMNS matrices;
- the existence and properties of right-handed neutrinos;
- the existence and mass windows of shadow states;
- the near-critical nature of the electroweak scale.

What remains to be done

From a research perspective, the next steps are now unusually well-defined:

- **Specify the internal part of ∇_8:** Choose and analyse explicit forms of the internal Dirac operator D_F compatible with the octonionic structure and with the empirical content of the Standard Model.
- **Compute spectra:** Determine the eigenvalues and eigenvectors of ∇_8 (and in particular of D_F) and extract approximate predictions for mass ratios, mixing angles and shadow-sector properties.
- **Confront data:** Compare these predictions with flavour physics, neutrino experiments, precision tests, dark matter searches and cosmological constraints.
- **Explore dynamics:** Study the phase structure of the theory, including different vacuum choices, phase transitions and the behaviour of the model under renormalisation-group flow.

Each of these tasks is substantial, and none can be completed in a few days. But they are concrete enough to turn a metaphor — "one operator to encode it all" — into a series of calculations that can succeed or fail.

A New Year's outlook

At the end of this calendar, the octonionic framework stands as a proposal for how to reorganise our understanding of particle physics and geometry:

- by making the internal space finite and algebraically rigid,
- by reading masses, charges and scales as spectral data,
- by treating the vacuum as a structured state of a single operator,
- by unifying geometry, gauge forces and matter in one octonionic Dirac equation.

The calendar has delivered:

- a complete mass ladder from $\Lambda_{\mathrm{QCD}} \sim 200$ MeV to $M_8 \sim 10^{16}$–10^{18} GeV;
- concrete charge assignments for all fermions;
- a natural explanation for three generations;
- a structured shadow sector;
- a unified operator ∇_8 that encodes all of this in one equation.

Whether this proposal ultimately survives detailed confrontation with experiment is an open question. But it has one rare virtue in high-energy theory: it is falsifiable at a deep, structural level. If no choice of octonionic internal space and ∇_8 can reproduce the measured pattern of our world, the model will have earned the right to be discarded. If, on the other hand, concrete spectra derived from this framework begin to match what experiments report, the idea that "numbers come from one operator" will move from speculation to explanation.

As we enter the new year, the octonionic programme is no longer a loose speculation. It is a concrete research programme with a clear question, a rich structure, and a long list of testable predictions. The work ahead is demanding, but the path is clear. This is the fireworks display at the end of the calendar: a celebration of what has been achieved, and an invitation to the work that lies ahead.

References

[1] A. Connes, *Noncommutative Geometry*, Academic Press, 1994.

[2] A. H. Chamseddine and A. Connes, *The Spectral Action Principle*, Commun. Math. Phys. **186**, 731–750 (1997).

The calendar delivers a complete package: a mass ladder from Λ_{QCD} to M_8, concrete charge assignments, three generations from triality, a structured shadow sector, and a unified octonionic Dirac equation $\nabla_8\Psi = 0$. This is not a loose speculation but a concrete research programme with testable predictions. The fireworks are lit; the work begins.