December 17, 2025

Spectral geometry: a bridge to gravity

When a Dirac spectrum encodes both spacetime and octonions

Key Insight. To pull gravity into the octonionic story, one needs a framework in which spacetime geometry and internal exceptional structure are encoded in a single object. Spectral geometry provides exactly that: a Dirac operator D whose spectrum knows about curvature, gauge fields and matter. A spectral action

$$S_{\text{spec}} = \operatorname{Tr} f(D^2/\Lambda^2)$$

then generates the Einstein–Hilbert term, cosmological constant and gauge dynamics in one stroke. This is the natural bridge between the octonionic operator toolbox and gravitational physics.

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From metrics to spectra

In Riemannian geometry, a metric $g_{\mu\nu}$ defines distances and curvature. Spectral geometry starts from a different object: a Dirac operator D acting on spinors. Remarkably, much of the geometric information can be recovered from the spectrum of D:

- The eigenvalues of D encode the volume and curvature of the manifold.
- Heat-kernel coefficients of D^2 reproduce scalar curvature, cosmological constant terms and higher curvature invariants.

The slogan is:

> "Geometry is what you can hear from the spectrum of D."

The spectral action principle

Chamseddine and Connes proposed to build the fundamental action of physics from the spectrum of D:

$$S_{\text{spec}} = \operatorname{Tr} f(D^2/\Lambda^2),$$

where f is a positive cut-off function and Λ a large mass scale.

The heat-kernel expansion of this trace yields schematically

$$S_{\text{spec}} \sim \int d^4x \sqrt{|g|}\big(\Lambda^4 a_0 + \Lambda^2 a_2 R + a_4(R^2, F_{\mu\nu}F^{\mu\nu}, \ldots) + \cdots\big),$$

where R is the scalar curvature, $F_{\mu\nu}$ are gauge field strengths and the a_n are Seeley–DeWitt coefficients built from the geometry and internal data.

Thus, from a single spectral expression one recovers:

- the cosmological constant term,
- the Einstein–Hilbert term,
- gauge kinetic terms,
- and higher-order corrections.

Adding internal structure

In noncommutative geometry, one extends the usual spinor space by an internal finite-dimensional Hilbert space $\mathcal{H}_F$ carrying the internal degrees of freedom (gauge charges, generations, etc.). The full Hilbert space becomes

$$\mathcal{H} = L^2(\text{spinors on } M) \otimes \mathcal{H}_F,$$

and the Dirac operator factorises as

$$D = D_M \otimes \mathbf{1} + \gamma^5 \otimes D_F,$$

where D_M is the spacetime Dirac operator and D_F encodes the internal structure.

For the Standard Model, a specific choice of D_F reproduces the known gauge group and particle content via the spectral action. For the octonionic model, D_F should encode:

- the octonionic/Albert structure $H_3(\mathbb{O})$,
- the rotor/compressor operator toolbox,
- triality and G_2/F_4 symmetries.

How octonions could enter D

From the perspective of this Advent calendar, we already have the ingredients for D_F:

- Compressors and rotors acting on the internal space of one generation.
- The mass map $\Pi(\langle H\rangle)$ as a candidate for the fermionic mass block in D_F.
- Gauge couplings read from rotor commutators, suggestive of the gauge-covariant part of D.

The spectral-geometry programme for the octonionic model can be stated in one sentence:

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Build a finite Dirac operator D_F from the existing octonionic operator toolbox, then feed it into the spectral action and see what gravitational and gauge sector emerges.

If successful, this would:

- Derive gravitational couplings (and possibly $\kappa = m_p/m_P$) from the same internal data that set masses and couplings.
- Turn the "missing gravity corner" of the previous day into a computed part of the spectrum of D.

What is realistic to expect

The spectral-geometry approach is ambitious but not magic. Realistically, one might hope for:

- **Structural results:** identification of which combinations of internal invariants control the effective Newton constant and cosmological constant in the spectral action.
- **Order-of-magnitude estimates:** showing that the same operator scales that fix internal hierarchies also produce a plausible separation between proton, electroweak and Planck scales.
- **Consistency checks:** demonstrating that the octonionic D_F reproduces the known gauge content and anomaly structure in the low-energy limit.

A full, detailed computation of all spectral coefficients is a long-term project, but the direction is clear.

Why this is the natural next step

The spectral-day message is simple:

1. We already think in terms of operators (rotors, compressors, mass maps) on an exceptional internal space.
2. Spectral geometry says that the *spectrum* of a combined Dirac operator captures both internal and spacetime geometry.
3. The spectral action then turns this spectrum into an effective action with gravity, gauge fields and matter.

In other words, spectral geometry is not an add-on but the natural language in which the octonionic model can talk to gravity. It is the mathematical framework in which yesterday's "missing corner" can, in principle, be filled.

References

[1] A. Connes and A. H. Chamseddine, "The spectral action principle," *Commun. Math. Phys.* **186**, 731–750 (1997).

[2] A. H. Chamseddine, A. Connes and M. Marcolli, "Gravity and the standard model with neutrino mixing," *Adv. Theor. Math. Phys.* **11**, 991–1089 (2007).

[3] [Internal notes on spectral geometry and gravity: `appE_neu.tex; chap20_neu.tex`.]

Spectral geometry offers exactly what the octonionic model needs next: a Dirac operator whose spectrum unifies internal exceptional structure with spacetime curvature and gravity.

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