December 6, 2025 (Saint Nicholas Day)

One Single Equation $D\Psi = 0$

Matrix transport instead of a three-theory zoo

Key Insight. Instead of writing three separate sets of field equations for matter (Dirac), gauge fields (Yang–Mills) and gravity (Einstein), the octonionic model starts from a *single* matrix transport equation on $\mathbb{R}^8$: $D\Psi = 0$ with $D = \partial + A$, where the connection $A_\mu \in \mathfrak{so}(8)$ contains both the spin connection (gravity) and all internal gauge fields. The familiar equations reappear as projections of this one master equation.

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Modern physics usually thinks of three separate theories: There's the Dirac equation, which describes how matter particles like electrons and quarks move and interact. There are the Yang-Mills equations, which govern the gauge bosons — the photon, gluons, and W and Z bosons that carry forces. And there's Einstein's general relativity, which describes gravity through the curvature of spacetime.

These three theories work beautifully in their respective domains, but they feel logically separate. We write them down as distinct sets of equations, with different mathematical structures and different physical interpretations. It's a bit like having three separate instruction manuals for a single machine — each manual is useful, but you can't help wondering if there's a unified blueprint underneath.

What if there is? What if all three theories — matter, gauge forces, and gravity — are not independent starting points but different projections of a single, deeper equation?

This is the idea behind today's sheet. In the octonionic model, there is only one fundamental transport equation, written as $D\Psi = 0$, where $D = \partial + A$ is a covariant derivative built from an $\mathfrak{so}(8)$-valued connection A_μ (more precisely, acting via a Spin(8) representation) on an eight-component field Ψ. The connection A_μ contains both the spin connection (which encodes gravity) and the internal gauge fields (which encode the Standard Model forces). The field Ψ carries both gravitational and internal quantum numbers.

The familiar equations we know — Dirac, Yang-Mills, Einstein — are not separate laws. They are different ways of looking at this single master equation:

- The **Dirac equation** is what you see when you project $D\Psi = 0$ onto the fermionic components of Ψ and treat the connection as a fixed background.

- The **Yang-Mills equations** emerge as consistency conditions—the requirement that the connection A be compatible with itself, expressed through its curvature $F_{\mu\nu} = [D_\mu, D_\nu]$.

- The **Einstein equations** (or their effective counterpart) arise from the curvature of the spin connection part of A_μ.

Why does this matter? Because it suggests that the apparent diversity of fundamental physics — matter, forces, gravity — is not a collection of unrelated phenomena. It's a single geometric structure, viewed from different angles. The octonionic internal space provides the stage where all these angles fit together naturally.

Think of today's sheet as a unification statement: not in the sense of a "theory of everything" that predicts every detail, but in the sense of a single mathematical framework that contains all three classical theories as special cases. The equation $D\Psi = 0$ is not just compact notation — it's a statement about the underlying unity of the octonionic geometry.

Let us see how one equation replaces three separate theories.

From three theories to one equation

The split of these three theories is historically grown and pragmatically useful, but it hides the fact that all three are, at heart, transport equations for some kind of "spinor data" along some kind of connection.

The octonionic model takes this observation seriously and elevates it to a principle: *there is only one transport equation*, written purely in terms of a matrix-valued connection on an 8-dimensional real vector space. Everything else — matter dynamics, gauge-field dynamics, and effective gravitational dynamics — is obtained by projecting this single equation onto different sectors.

The central structure

The starting point is an $\mathfrak{so}(8)$-valued connection A_μ on $\mathbb{R}^8$, acting on an 8-component field Ψ:

$$D\Psi \;=\; 0, \qquad D \;=\; \partial + A, \qquad A_\mu \in \mathfrak{so}(8).$$

Here:

- The *kinematic* part ∂ encodes flat $\mathbb{R}^8$ as the basic stage on which everything lives.
- The *connection* A_μ decomposes into

$$A_\mu \;=\; \Gamma_\mu \,\oplus\, A_\mu^{\text{int}},$$

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where Γ_μ is recognized as a spin connection (gravity) and A_μ^{int} as the internal gauge fields associated with $SU(3) \times SU(2) \times U(1)$, embedded in the octonionic geometry.

- The field Ψ carries both gravitational and internal quantum numbers; its components are organized according to the octonionic/Spin(8) representation structure.

In this picture:

- The **Dirac equation** is the projection of $D\Psi = 0$ onto the fermionic component of Ψ, in a sector where A_μ is treated as a fixed background.
- The **Yang–Mills equations** arise as the integrability (compatibility) conditions for $D\Psi = 0$, expressed as differential constraints on the curvature $F_{\mu\nu} = [D_\mu, D_\nu]$ in the internal directions.
- The **Einstein equations** (or their effective counterpart) arise from the curvature components of Γ_μ and from the spectral action built from D^2 in the gravitational sector, in analogy with Connes' noncommutative geometry approach.

Physical meaning

From the perspective of the octonionic model, $D\Psi = 0$ is not "just another compact notation" for the Standard Model + gravity. It is a statement about the *underlying current* in the octonionic/Albert geometry:

- The operator D encodes both the geometric background (through Γ_μ) and the internal symmetry structure (through A_μ^{int}).
- The field Ψ encodes the "state of the universe" as a section of a bundle whose fibers are built from octonionic representations.
- The equation $D\Psi = 0$ enforces that this state is covariantly constant along all directions in $\mathbb{R}^8$ with respect to A.

This has several conceptual consequences:

1. **Unification of kinematics and interactions.** There is no kinematics "without" a connection: as soon as we write $D = \partial + A$, gravitational and gauge information are hard-wired into the basic notion of a derivative on $\mathbb{R}^8$.
2. **Operators before fields.** The primary object is the operator D, not a list of classical fields. Matter, gauge bosons and even gravitation appear as different faces of D and its curvature, in line with spectral geometry.
3. **Natural home for the octonionic structure.** The choice $A_\mu \in \mathfrak{so}(8)$, with its Spin(8) triality and G_2-compatible substructures, is not an accident: it is the unique stage where the octonionic and Albert-algebra data fit together into a single transport equation.

References

[1] A. Connes, Noncommutative Geometry, Academic Press, 1994.

[2] A. H. Chamseddine, A. Connes, The Spectral Action Principle, *Commun. Math. Phys.* **186**, 731–750 (1997).

[3] I. Todorov, Octonion Internal Space Algebra for the Standard Model, *Universe* **9**, 222 (2023).

One single matrix equation replaces three classical theories as independent starting points.

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