1

Corridor Ramp Metering Using Particle Filter-Model Predictive Control PF-MPC

Anush Badii, Vince Wong, Tae Kim, Harry Li, Natasha Balac

***Abstract*— To date, "Corridor Wide Ramp Metering" has remained a desirable concept rather than a methodology. Despite some early attempts by Caltrans (System Wide Adaptive Ramp Metering and its derivatives) to devise a methodology for ramp metering, Caltrans reverted back to the original traffic responsive methods; meaning that the meter rates are adjusted according to the locally measured occupancy or volume or both. The adjustment is basically linear and inversely proportional to either volume or occupancy and varies between the maximum and the minimum of the practical discharge rates for one car per green per lane or two cars per green per lane (see the Appendix 2 table of Practical Discharge Rates). Knowing that the freeway traffic flow is nonlinear and inevitably unstable, the injection of vehicles from onramps via this method is unlikely to affect the trajectory of its instability. As such, it results in the daily congested traffic that is experienced in all urban areas. In this paper we present a proposed methodology of ramp metering that uses the most current state of the art control for nonlinear systems, to control the trajectory of traffic density evolution, thus smoothing the speed and reducing the congestion costs including the greenhouse gases that presently is the most important issue in global warming.**

The proposed method is commonly known as Particle Filter-Model Predictive Control (PF-MPC) and it will be applied to all on-ramp discharge rates at all stations along the freeway so as to filter out the most suitable rate at each station at any time for any given condition of the freeway.

***Index Terms*— Highway, Ramp Metering, Particle Filter**

I. Introduction

Utilization of the highway systems as a main method of transportation has led to worsening congestion conditions. Traffic jams are estimated to lead to an additional 54 hours on the road per year for the average American commuter, costing $179 billion nationally on fuel consumption. Both the congestion conditions and the costs related to it are predicted to increase in the upcoming decades [1]. Most important yet is the increase of greenhouse gases due to congestion that studies have shown can be decreased by as much as 60% if the freeway traffic could be smoothed out [11]. The current method of managing traffic along the highway can be improved through better management of the freeway by controlling the injection rate of traffic throughout the corridor with an intended goal of linearizing the traffic density along the travel way. Traffic congestion tends to occur in the section of the highway between onramp and an exit ramp due to the excessive inflow of traffic that disrupts the stability of the system. This causes back propagation, leading to congestion.

In our findings, we demonstrate our conclusions through existing real-time field data from the publicly accessible California Department of Transportation Performance Measurement System (PeMS) [2].

II. Review of Current Literature

Many attempts to specify a relation among traffic flow, density, and speed have been made. A macroscopic traffic model to correlate them has formed the so-called fundamental diagrams of traffic flow. Greenshields [3] derived a parabolic fundamental diagram between flow and density. Lighthill, Whitham [4] and Richards [5] used Greenshields' hypothesis and a conservation law of vehicles to provide a concave fundamental diagram, which is called the first order LWR model.

Newel [6] proposed a triangular flow-density fundamental diagram as a simpler alternative to Greenshield and LWR models. Only two velocities characterize this model: a maximum free flow velocity in a free-flow regime and a propagation velocity for a congestion area. However, he did not explain the correlation between propagation velocity and driving patterns. Banks [7] considered the time-gap, which is the time required to travel the distance between the front-end of a vehicle to the back-end of its leading vehicle, and showed the relation between the time-gap and speed using real traffic data in the USA. He found that during congested times, the average time gap is relatively constant, while it diverges with large deviation during free-flow periods. However, he did not derive a fundamental diagram from the time-gap nor an analysis method to estimate the time-gap from raw traffic data. Cho, Cruz, Rao and Badii[12] derived an analytic equation for the speed of freeway as a function of density in the congested state consistent with Newell's diagram [13].

Density Evolution Model

The model we present can be simplified by tracking the evolution of main lane density between two consecutive freeway ramps having $\aleph$ lanes and differential milepost Δx, with measured time dependent quantities $Q_{ups}(t)$ for upstream Volume, and $Q_{ds}(T)$ and $OCC_{ds}(T)$ for downstream Volume and downstream Occupancy, having an onramp

1

injection rate $q_{ramp}(t)$and a downstream exit rate from the off ramp $q_{exit}(T)$. The delayed time T arises from the fact that there is a delay in detection at downstream location for the actions at upstream station.

The density evolution model will follow the transformation of upstream values to downstream via a simple equation

$$n_{ds}(T) = \frac{1}{V_{ups}(t)}\left(Q_{ups}(t) + \frac{q_{ramp}(t)}{\aleph}\right) - \frac{1}{\aleph}\frac{q_{exit}(T}{V_{merg}} \quad 1$$

Where V_{merge} is given by

$$V_{merge} = min\{V_{ups}(t),\ \frac{5280\,\aleph V_{ups}(t)}{\tau c(\aleph Q_{ups}(t)+q_{ramp}(t))} - \frac{L}{\tau c}\} \quad 2$$

where τ is the minimum headway at 1.75 sec and L is a typical car length at 14.75 ft ,5280 is the number of feet in one mile and c is conversion from ft/sec to MPH.

The Downstream volume $Q_{ds}(T)$ and the downstream occupancy $Occ_{ds}(T)$ are readily calculated from the downstream density $n_{ds}(T)$ via the following formulations where l is the detector loop length (10,6,5,or 4 feet depending on the freeway)

$$Occ_{ds}(T) = \frac{n_{ds}(T).(L+l)}{5280+ln_{ds}(T)} \quad 3$$

were $T = \frac{\Delta x}{V_{merge}}$ is the time that takes for the downstream sensors to detect the action of the upstream ramp meter at the time t. The downstream Volume can be calculated via the formula for speed

$$V_{ds}(T) = \{\,V_{fast},\ \frac{5280-n_{ds}(T)L}{n_{ds}(T)\tau c}\,\} \quad 4$$

where V_{fast} is assumed to be speed limit or the prevailing high speeds as much as 75 MPH, leading to downstream volume of

$$Q_{ds}(T)= V_{ds}(T)n_{ds}(T) \quad 5$$

To achieve our objective of smoothing out the speed we will try to control the freeway density through the controlled injection of vehicles from the ramp to attain gradual increase in density. Assume that our ramp metering objectives will be a gradual increase in density (say no more than 15% increase) at every downstream station *i,* compared to its corresponding upstream station. We introduce a number of randomly selected rates (say 15) with values between the maximum and the minimum allowable discharge rates, for one car per green or two cars per green depending on the particular ramp. Since each randomly selected rate has an equal chance of being the chosen rate, we assign an initial equal weight of

$$w_k = \frac{1}{15} \quad k=1,2..15 \quad 6$$

to each rate representing a random rate code.

We put each rate value through our model (equations 1 and 2) using the predicted $q_{exit}(T)$ from the GMDH predictive time series or in the absence predictive data, the last known $q_{exit}(T-1)$ from the real time data and calculate n^i_{ds} and its corresponding $V^i_{ds}(T)$ *and* $Q^i_{ds}(T)$ were $T = \frac{\Delta x}{V^i_{merge}}$.

To find out which particle has produced an outcome nearest to the desired goal, we re-compute a new weight for each particle by

$$w_k = \frac{1}{\sqrt{2\pi R}}\, e^{\frac{-(n^i_{ds}-n_{goal})^2}{2R}} \quad 7$$

where R is an arbitrary variance. Then we would normalize the weights

$$w_{k\,normalized} = \frac{w_k}{\sum_{k=1}^{k=15} w_k} \quad 8$$

The rate corresponding to the largest normalized weight is considered the winning rate and qualifies for resampling. In this round calculation, we produce a new set of metering rates with values normally distributed (see appendix I) around the value of the winning rate and feed it through our model. The rate produced by the winner of the second round of calculations is then submitted to the controller.

A. Derivation of the model

The model we propose was developed by Cho, Cruz, Rao and Badii [12] , consistent with Newell's proposal. It stems from the geometry of two vehicles with a typical length L and a separation D both measured in feet, we define the density n as the number of vehicles in this configuration that will fit inside one mile measured in feet

$$n = \frac{5280}{L+D} \tag{9}$$

where 5280 is the number of feet in a mile. Solving for D and dividing both sides by v we arrive at the headway τmeasured in seconds

$$\tau = \frac{D}{v} = \frac{5280-nL}{nv} \tag{10}$$

or conversely:

$$V = \frac{D}{\tau \underline{c}} = \frac{5280-nL}{n\tau \underline{c}} \tag{11}$$

Since speed V in measured in MPH we have introduced a conversion factor $\underline{c} = \frac{5280}{3600}$ to convert ft/sec to MPHin the above equation.

We contend that although τ can assume any number, but it has a prevailing minimum depending on the region. In Southern California, $\tau = 1.75\ sec$ is an acceptable norm. Also, the formula for the speed V has no cap and can become very large as $n \rightarrow 0$, thus we introduce V_{fast} as the speed limit or prevailing acceptable speeds and rewrite the equation as

$$V = min\left[V_{fast}, \frac{5280-nL}{n\tau \underline{c}}\right] \tag{12}$$

subject to

$$\frac{\partial v}{\partial n} = 0,\ V = V_f \quad \forall\, n < n_c \tag{13}$$

which imposes the legal speed limit for values of density below a critical density and

$$\frac{\partial v}{\partial n} \neq 0,\ V = \frac{D}{\tau \underline{c}} = \frac{5280 - nL}{n\tau c} \quad \forall\, n > n_c \tag{14}$$

which imposes safe driving headway at constant τ. In the above equation V_{fast} is the forward speed and may be interpreted as a legally allowable speed limit. Systematic measurements of the headway τ, affirms that the value of τ very rapidly converges to a constant.

The only unknown in the above equations is n_c which will be derived from intersecting the line $q = nv_{fast}$, where q is the flow volume measured in vehicles per hour with the line $q = n[\frac{5280-nL}{n\tau \underline{c}}]$, and solving for n.

$$n_c = \frac{5280}{L+V_{fast}\tau \underline{c}} \tag{15}$$

The maximum capacity of the freeway per lane will then be $q_{max} = n_c V_{fast}$. A more important observation may be made that the downward slope of $q = n\left[\frac{5280-nL}{n\tau \underline{c}}\right]$ has dimension of speed and computes to be $[-\frac{l}{\tau \underline{c}}]$.

It is noteworthy that this derivation is universal and applies to any roadway regardless of condition, and since the roadway capacity is a function v_{fast} and τ, it can dramatically vary, for example in a situation like NASCAR whereV_{fast} is very large and τ is very small, the capacity is more than 20,000 veh/hr. Also the freeway has been observed to operate above the capacity; this is due to a temporary smaller $\tau < 1.75\ sec$ among a group of vehicles and the lack of V_{fast}enforcement. Regardless of the reasons, the time gap based model is inherently unstable (in the Lyoponov sense of stability) and temporary, and it will inevitably collapse to the congested region of Newell diagram. It is also noteworthy that the reason that NASCAR is able to retain stability is due to the closed loop course. Perry Y. Li and Ankur Shrivastava [8], while studying a policy for automated cruise control investigated the stability of time gap based models and concluded that it was unstable on an open course and stable on a closed loop.

B. Relation to other Models

Applying the assertions first developed by (Cho, Cruz, Rao and Badii) to the LWR wave equation, we derive the LWR equation and assert our model.

In LWR model, one may write the number of vehicles N in the system between two measuring stations A at upstream and B at downstream may be written as

$$\frac{dN(t)}{dt} = q_B(B,t) - q_A(A,t) \tag{16}$$

On the other hand we may express N in terms of density

$$N(t) = \int_A^B n(x,t)dt \quad (17)$$

Substituting

$$\frac{dN(t)}{dt} = \frac{d}{dt}\int_A^B n(x,t)dt = q_B(B,t) - q_A(A,t) \quad (18)$$

by Leibniz axiom we may write $q_B(B,t) - q_A(A,t)$as

$$\int_A^B \frac{\partial v}{\partial n} q(x,t)dx \quad (19)$$

Substituting

$$\frac{d}{dt}\int_A^B n(x,t)dt = \int_A^B \frac{\partial v}{\partial n} q(x,t)dx \quad (20)$$

Dropping the integrals we may write above as

$$\frac{\partial n(x,t)}{\partial t} - \frac{\partial q(x,t)}{\partial x} = 0 \quad (21)$$

but $q(x,t) = n(x,t)v(n)$with $v$itself having dependence on n i.e. $v = v(n(x,t))$

$$\frac{\partial n(x,t)}{\partial t} - v(n)\frac{\partial n(x,t)}{\partial x} - n(x,t)\frac{\partial v(n)}{\partial x} = 0 \quad (22)$$

Expanding the last term

$$\frac{\partial n(x,t)}{\partial t} - v(n)\frac{\partial n(x,t)}{\partial x} - n(x,t)\frac{\partial v(n)}{\partial x}\frac{\partial n(x,t)}{\partial x} = 0 \quad (23)$$

Substituting

$$v(n) = max[V_{fast}, \frac{5280-nL}{n\tau}] \quad (24)$$

And

$$\frac{\partial v(n)}{\partial x} = min[0, \frac{5280}{n^2\tau}] \quad (25)$$

Applying the two flow regime concepts

$$\frac{\partial v}{\partial n} = 0 \; and \; v = v_{fast} \; \forall \, n < n_c \quad (26)$$

We end up with a wave equation of the form

$$\frac{\partial n(x,t)}{\partial t} - [v_{fast}]\frac{\partial n(x,t)}{\partial x} = 0 \quad \forall \, n < n_c \quad (27)$$

which describes a density wave travelling in forward direction with the speed $[v_{fast}]$

And as for the case where

$$v = \frac{5280-nL}{n\tau} \quad \forall \, n > n_c \quad (28)$$

We may write

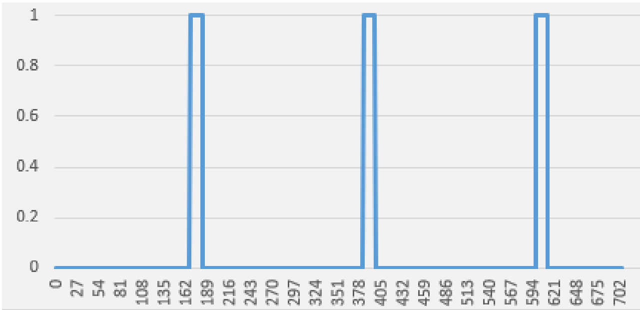
$$\frac{\partial n}{\partial t} - [\frac{5280-nL}{n\tau} - \frac{5280}{n^2 t}n]\frac{\partial n}{\partial x} = 0 \quad (29)$$

Leading to

$$\frac{\partial n(x,t)}{\partial t} + \frac{L}{\tau}\frac{\partial n(x,t)}{\partial x} = 0 \quad \forall \, n > n_c \quad (30)$$

which describes a wave travelling at speed of $[\frac{L}{\tau}]$backward, consistent with observations of scientists at the Nagoya University in Japan [9]. The appearance of $[\frac{L}{\tau}]$unifies the Newell model with the LWR fluid model consistent with Cho, Cruz, Rao and Badii .

For the solution of LWR equations we propose a box function made of two Heaviside function of the form $H(x - x_0 - vt) - H(x - (x_0 + L) - vt)$ The former raises the function to the value of 1 and the latter brings down the function back to zero at a distance L in effect representing a vehicle traveling at speed$v$. Since all the solutions to a differential equation are additive we may repeat the same construction at a distance D in effect making a chain of boxes each having a length L and a distance D between them all travelling at the speed v.



Similarly for the backward travelling wave we employ the Heaviside function form $H\left(x + x_0 + \frac{L}{\tau}t\right) - H\left(x + (x_0 + L\right) + \frac{L}{\tau}t)$ representing a backward travelling wave as it would be viewed by a stationary observer.

Although traffic waves travelling forward and backward are interesting as a phenomenon, they are of little value to practitioners of traffic engineering who deal on a day to day basis with congestion. However we can take advantage of the constancy of impact time $\tau = \frac{D}{v}$ and provide a new methodology for day to day traffic engineering problems that have confounded traffic engineers since the dawn of congestion some eighty years ago.

The densities of the onramp and the exit ramp are derived from their respective flow q_{ramp} and q_{exit}

$$n_{ramp} = \frac{q_{ramp}}{V_{upstream}} \qquad 31$$

And

$$n_{exit} = \frac{q_{exit}}{V_{merge}} \qquad 32$$

realizing that the densities and not the flows are additive we arrive

$$n_{merge} = n_{upstream} + \frac{n_{ramp}}{\aleph} \qquad 33$$

And

$$n_{ds} = n_{merge} - \frac{n_{exit}}{\aleph} \qquad 34$$

Where $\aleph$ is the number of main lanes.

Combining equations 33 and 34 followed by insertion of 31 and 32

$$n_{ds} = \frac{Q_{upstream}}{V_{upstream}} + \frac{q_{ramp}}{\aleph V_{upstream}} - \frac{1}{\aleph}\frac{q_{exit}}{V_{merge}} \qquad 35$$

Which will lead to the model formulations presented in equations 1 of this paper.

$$n_{ds} = \frac{1}{V_{upstream}}\left(Q_{upstream} + \frac{q_{ramp}}{\aleph}\right) - \frac{1}{\aleph}\frac{q_{exit}}{V_{merg}} \qquad \text{Q.E.D } 36$$

Similarly, equation 2 of this model may be derived by algebraic manipulation of equation 12 reproduced here

$$V = min\left[V_{fast}, \frac{5280 - nL}{n\tau c}\right] \qquad 37$$

In the above equation if there is no change in density due to injection from on ramp $V_{fast} = V_{upstream}$ replacing the value of density with n_{merge} from equation 33

$$V = min\left[V_{upstream}, \frac{5280}{\left(n_{upstream} + \frac{n_{ramp}}{\aleph}\right)\tau c} - \frac{L}{\tau c}\right] \qquad 38$$

Substituting from 31

$$V = min\left[V_{upstream}, \frac{5280}{\left(\frac{Q_{upstream}}{V_{upstream}} + \frac{1}{\aleph}\frac{q_{ramp}}{V_{upstream}}\right)\tau c} - \frac{L}{\tau c}\right] \qquad 36$$

$$V = min\left[V_{upstream}, \frac{5280\, \aleph V_{upstream}}{\tau c(Q_{upstream}\aleph + q_{ramp})} - \frac{L}{\tau c}\right] \qquad \text{Q.E.D } 37$$

C. Particle Filter Results

In this paper we used the discharge rates as our particles and through selective weighting we chose the best rate that produced a result as close as possible to our desired goal which was a gradual 15% density increase at every downstream station *j*, compared to its corresponding upstream station *i*.

D. Verification of the Model with Neural Network

TensorFlow was trained with twenty four hours of thirty seconds traffic data and after the training session it was exposed to test data with resulting output at 95% accuracy. To test the density evolution model we exposed the trained TensorFlow matrix to a single upstream volume $Q_{ups}(t)$ and a single upstream $V_{ups}(t)$and a single q_{exit} and fifteen separate q_{ramp} discharge rates. We compared the TensorFlow output for downstream densities to the output density of our model. The TensorFlow output of the downstream density values had close correspondence to the density evolution model repeated here for convenience.

$$n_{ds}(T) = \frac{1}{V_{ups}(t)}\left(Q_{ups}(t) + \frac{q_{ramp}(t)}{\aleph}\right) - \frac{1}{\aleph}\frac{q_{exit}(T)}{V_{merge}} \quad 37$$

III. Conclusion

We derived the Newell diagram from a simple tenet of human behavior (τ has a minimum value). Applying our results we were able to compute the capacity of the roadway. Also we derived the LWR equation for both regions of the Newell diagram consistent with observation, we also derived the speed of the backward density waves consistent with observation. We provided a simple computational method for day to day traffic engineering calculations that is new and consistent with traffic observations. We have shown that through the use of particle filter model predictive flow we can control the evolution of highway density and smooth out the speed curve thus achieving the desired goals of reduction in congestion costs including the cost of congestion related greenhouse gas emissions. Finally we were able to validate our model using neural networks.

Appendix I

A Gaussian distribution with mean zero and standard deviation one, often known as a “standard normal” distribution, has the probability density function (PDF):

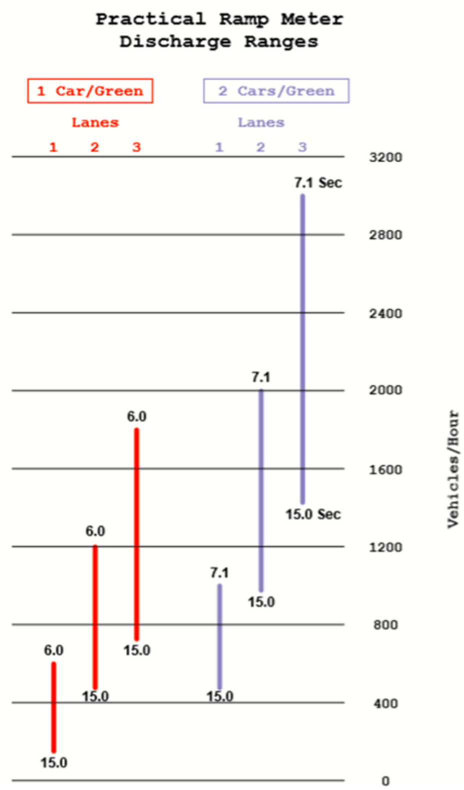
$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \frac{e^{-x^2}}{2}$$

A plot of φ(x) versus x gives the familiar bell-curve shape, but does not directly indicate the probability of occurrence of any particular range of values of x. Integrating the PDF from $-\infty$ to x gives the cumulative distribution function (CDF):

$$\emptyset(x) = \int_{-\infty}^{x} \varphi(x)dx = \frac{1}{2}[1 + erf(\frac{x}{\sqrt{2}})]$$

The CDF, $\emptyset(x)$ gives the probability that a random sample from the Gaussian distribution will have a value less than x. The CDF can be used to calculate the probability of values occurring within a given range, for example, the probability of a number between a and b (where a < b) is $\emptyset(b) - \emptyset(a)$. There is no closed-form solution for $\emptyset$, or for the related function erf , so it must be calculated numerically.

Appendix II



References

[1]]D. Schrank, B. Eisele and T. Lomax, "2019 Urban Mobility Report", The Texas A&M Transportation Institute, 2019, pp. 7-8.

[2] PeMS Performance Measurement System: http://pems.dot.ca.gov

[3] B. Greenshields, A study of highway capacity. Proceedings of the 14th Annual Meeting Highway Research Board. (PP448-477) 1935. Ohio State Highway Department.

[4] J. Lighthill, J.B. Whitham, On kinetic waves, II: A theory of traffic flow on long crowded roads. Proc. Royal Society A229 (PP 281-345) 1955.

[5] P.I. Richards, Shock waves on the highway. Operations Research 4 (PP 42-51) 1956.

[6] G. F. Newell, A review of traffic flow theory. Berkeley 1965. http://web.pdx.edu/~bertini/courses/559/newell_lecture_65.pdf

[7] J. H. Banks, Average time gaps in congested freeway flow. Transportation Research, Part A vol. 37, PP 539- 554, 2003

[8] P. Y. Li, Ankur. Shrivastava, Traffic flow stability Traffic flow stability induced by constant time headway policy for adaptive cruise control vehicles. Transportation Research Part C: Emerging Technologies. 2002;10(4):275-301 Department of Mechanical Engineering, University of Minnesota, Minneapolis, MN.

[9] Shockwave traffic jams recreated for the first time. Footage courtesy mathematical Society of traffic flow in Japan. https://www.youtube.com/watch?v=Suugn-p5C1M

[10] [NEW YORK DAILY NEWS, Tuesday, November 5, 2013, 10:39 AM. Congestion cured by cruise control? MIT researcher claims traffic woes could be cut without

[11] Mathew Bart et al Acces Magazine traffic congestion greenhouse gases https://www.accessmagazine.org/fall-2009/traffic-congestion-greenhouse-gases/

[12] S. Cho, R. Cruz, R. Rao and A. Badii, "Time-Gap Based Traffic Model for Vehicular Traffic Flow," *2014 IEEE 79th Vehicular Technology Conference (VTC Spring)*, Seoul, Korea (South), 2014, pp. 1-5, doi: 10.1109/VTCSpring.2014.7023125.

[13] G. F. Newell, "A Simplified Theory of Kinematic Waves in Highway Traffic Part I: General Theory", *Transportation Research Part B*, vol. 27, pp. 281-287, 1993.