Trade in a two-country two-good SOE model with incomplete financial markets

Pedro Amuchastegui

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1 Introduction

I build on the model proposed by Schmitt-Grohé and Uribe (2003) (section 3) and introduce some modifications in order to analyze the business-cycle behaviour of the economy under incomplete markets. In particular, I analyze two small open economies that trade goods at competitive international markets and borrow from incomplete international asset markets at a debt-elastic interest rate. My analysis deviates from the aforementioned article in that goods are not homogeneous in the sense that households have a stronger preference for domestically produced goods. I calibrate the model to Canada, for the period 1996-2017 and perform some experiments to explore the dynamics implied by the model.

2 The model

In this section I develop the optimization problem that households in country H face. Then, I take advantage of the symmetry of the setting and obtain similar results for country J. Finally I define a global competitive equilibrium and compute the steady state of this economy.

Households derive utility from the consumption of goods according to:

$$\sum_{t=0}^{\infty} \beta^t \, \mathbb{E}_0 \, U(C_{H,t}; C_{F,t}) \tag{1}$$

Where $C_{H,t}$ and $C_{F,t}$ are consumption of the goods produced in country H and F accordingly.The specific utility function is:

$$U(C_{H,t}; C_{F,t}) = \frac{(C_{H,t}^{\theta} C_{F,t}^{1-\theta})^{1-\gamma}}{1-\gamma}$$

Also, each household owns the only firm of its country. The only production factor, capital, is used to produce consumption goods according to the following technology:

$$y_t = A_t k_t^{\alpha} \tag{2}$$

where A_t is stochastic. A_t follows an AR(1) process which (log) form is:

$$a_t = \rho a_{t-1} + \epsilon_t$$

where $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$. Capital evolves according to a standard law of motion:

$$k_{t+1} = k_t(1-\delta) + s_t \tag{3}$$

where k_t is the stock of capital, s_t is gross investment and δ the depreciation rate. Households can borrow debt denominated in domestic good from international markets. The interest rate that households face R_t, is increasing in the aggregate level of debt $(\tilde{d})$:

$$R_t = 1 + r + p(\tilde{d})$$

where r is the world risk-free interest rate, and the risk premium $p(\tilde{d})$ is given by:

$$p(\tilde{d}) = \psi(e^{\tilde{d}-\bar{d}} - 1) \tag{4}$$

where ψ, $\bar{d}$ are constant parameters and $\bar{d}$ is the steady-state level of foreign debt. The timing is as follows: at time t households in country H obtain certain level of output y_t (which depends on the stock of capital it carries from $t-1$) and decide how much to invest s_t. Household must pay $d_{t-1} * R_t$ units of domestic good as debt from previous period and can borrow d_t units of domestic good. Finally, households can trade in the international goods market domestic goods $C_{H,t}$ and foreign goods $C_{F,t}$, at competitive prices $P_{H,t}$ and $P_{F,t}$. The budget constraint of the household in nominal terms is:

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \leq (y_t - s_t - d_{t-1}R_{t-1} + d_t)P_{H,t}$$ [1]

rearranging terms, the budget constraint can be expressed in a more convenient way:

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + d_{t-1}R_{t-1}P_{H,t} \leq (y_t - s_t - +d_t)P_{H,t} \tag{5}$$

Combining equations (2)-(5) yields:

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_{H,t}k_{t+1} + d_{t-1}R_{H,t-1}P_{H,t} \leq P_{H,t}A_t k_t^{\alpha} + P_{H,t}k_t(1-\delta) + P_{H,t}d_t \tag{6}$$

Terms of trade of a country are defined as the ratio between price of exports and price of imports. In this simple setting terms of trade of country H are:

$$TOT_{H,t} = \frac{P_{H,t}}{P_{F,t}}$$

since country H sells to the rest of the world good H and buys good F. Finally, the nominal value of the trade balance of country H is:

[1] I postpone price normalization until discussion of steady state in section XXX because the presence of both prices in the euler equations makes their economic meaning more explicit.

$$tb_{H,t} = P_{H,t}C^*_{H,t} - P_{F,t}C_{F,t}$$

where $C^*_{H,t}$ is the consumption of good H by country F.

Households choose $\{C_{H,t}, C_{G,t}, k_{t+1}, d_t\}_{t=0}^{\infty}$ so as to maximize (1) subject to equation (6) and a transversality condition, taking as given $\{P_{H,t}, P_{G,t}, R_t\}_{t=0}^{\infty}$.

$$\begin{aligned}
&\underset{\{C_{H,t},C_{F,t},k_{t+1},d_t\}_{t=0}^{\infty}}{\text{Max}} \sum_{t=0}^{\infty} \beta^t \, \mathbb{E}_0 \, U(C_{H,t}; C_{F,t}) \\
&\text{st} \quad P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_{H,t}k_{t+1} + d_{t-1}R_{H,t-1}P_{H,t} \leq \\
&P_{H,t}A_t k_t^{\alpha} + P_{H,t}k_t(1-\delta) + P_{H,t}d_t \ \forall t
\end{aligned} \tag{7}$$

given k_0 and d_{-1}. Let λ_t be the lagrange multiplier on the budget constraint, the lagrangian associated to this problem is:

$$\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \, \mathbb{E}_0 \, U(C_{H,t}; C_{F,t}) + \sum_{t=0}^{\infty} \sum_{z_t} \lambda_t \, [P_{H,t}A_t k_t^{\alpha} + P_{H,t}k_t(1-\delta) + P_{H,t}d_t \\
- P_{H,t}C_{H,t} - P_{F,t}C_{F,t} - P_{H,t}k_{t+1} + d_{t-1}R_{H,t-1}P_{H,t}]
\end{aligned}$$

first order conditions of household's optimization problem are equation (6) holding with equality and:

$$\lambda_t = \beta^t \pi_t \frac{U_H(C_{H,t}; C_{F,t})}{P_{H,t}} \tag{8}$$

$$\lambda_t = \beta^t \pi_t \frac{U_F(C_{H,t}; C_{F,t})}{P_{F,t}} \tag{9}$$

$$\lambda_t P_{H,t} = \sum_{z_{t+1}} \lambda_{t+1} P_{H,t+1} \, [f_k(k_{t+1}) + 1 - \delta] \tag{10}$$

$$\lambda_t P_{H,t} = \sum_{z_{t+1}} \lambda_{t+1} P_{H,t+1} R_t \tag{11}$$

$$\tag{12}$$

Intra-temporal optimality requires:

$$\frac{U_H(C_{H,t}; C_{F,t})}{P_{H,t}} = \frac{U_F(C_{H,t}; C_{F,t})}{P_{F,t}} \tag{13}$$

and inter-temporal optimality requires:

$$U_H(C_{H,t}; C_{F,t}) = \beta \, \mathbb{E}_t \, [U_H(C_{H,t+1}; C_{F,t+1})(f_k(k_{t+1}) + 1 - \delta)] \tag{14}$$

$$U_F(C_{H,t}; C_{F,t}) \frac{P_{H,t}}{P_{F,t}} = \beta \, \mathbb{E}_t \left[U_F(C_{H,t+1}; C_{F,t+1}) \frac{P_{H,t+1}}{P_{F,t+1}} (f_k(k_{t+1}) + 1 - \delta) \right] \tag{15}$$

$$U_H(C_{H,t}; C_{F,t}) = \beta R_h \, \mathbb{E}_t \, [U_H(C_{H,t+1}; C_{F,t+1})] \tag{16}$$

$$U_F(C_{H,t}; C_{F,t}) \frac{P_{H,t}}{P_{F,t}} = \beta R_h \, \mathbb{E}_t \left[U_F(C_{H,t+1}; C_{F,t+1}) \frac{P_{H,t+1}}{P_{F,t+1}} \right] \tag{17}$$

Equation (13) implies that the marginal benefit of consuming C_H, at the optimal choice, must be equal to the marginal benefit of consuming C_F.

Equation (14) implies that the marginal cost of investing at time t in terms of good C_H (left-hand side), at the optimal choice, must be equal to the discounted expected marginal benefit of investing (right-hand side).

Equation (15) implies that the marginal cost of investing at time t in terms of good C_F (left-hand side), at the optimal choice, must be equal to the discounted expected marginal benefit of investing in terms of good C_F (right-hand side).

Equation (16) says that, at the optimal choice, the marginal benefit of increasing debt at period t in terms of consumption of good C_H (left-hand side) must be equal to its discounted expected marginal cost, which is given by the claims that will be faced next period.

Combining equations (14) and (16) and (15) and (17) accordingly yields two non-arbitrage condition:

$$\mathbb{E}_t\left[U_H(C_{H,t+1}; C_{F,t+1})(f_k(k_{t+1}) + 1 - \delta)\right] = R_h\, \mathbb{E}_t\left[U_H(C_{H,t+1}; C_{F,t+1})\right] \tag{18}$$

$$\mathbb{E}_t\left[U_F(C_{H,t+1}; C_{F,t+1})\frac{P_{H,t+1}}{P_{F,t+1}}(f_k(k_{t+1}) + 1 - \delta)\right] = \mathbb{E}_t\left[U_F(C_{H,t+1}; C_{F,t+1})\frac{P_{H,t+1}}{P_{F,t+1}}\right] \tag{19}$$

Because I assume that agents are identical, in equilibrium aggregate per-capita debt equals individual debt:

$$\tilde{d}_t = d_t$$

Households in country F face a symmetric problem which yields symmetric solutions.

A global competitive equilibrium is a set of sequences $\{C_{H,t}, C_{F,t}, s_t, y_t, d_t\}_{t=0}^{\infty}$, $\{C^*_{H,t}, C^*_{F,t}, s^*_t, y^*_t, d^*_t\}_{t=0}^{\infty}$ and $\{P^*_{H,t}, P^*_{F,t}, R_{H,t}, R_{F,t}\}_{t=0}^{\infty}$ (where star variables refer to country F) such that (i) households in both countries solve their optimization problems, taking as given $\{P^*_{H,t}, P^*_{F,t}, R_{H,t}, R_{F,t}\}_{t=0}^{\infty}$ and initial conditions k_0, k^*_0, d_{-1}, d^*_{-1}; (ii) goods market clear:

$$C_{H,t} + C^*_{H,t} + s_t = y_{H,t}$$

$$C_{F,t} + C^*_{F,t} + s^*_t = y_{F,t}$$

2.1 Steady State

In this section I illustrate the procedure I use to compute the steady-state in country H and exploit the symmetry of the setting to obtain the steady-state for country F.

In the steady state, all shocks are set to zero and constant values of choice variables must satisfy equations (6) and (8) to (11).

In steady state, foreign debt d_t is equal to $\bar{d}$. It follows from equation (4) that $R_t = 1 + r_t$. Then, from equation (14): $1/\beta = f_k(k_{ss}) + 1 - \delta$ which yields the steady-state stock of capital:

$$k_{ss} = \left(\frac{A\alpha}{\frac{1}{\beta} - 1 + \delta} \right)^{\frac{1}{1-\alpha}} \tag{20}$$

from which I obtain the steady-state levels of output and investment:

$$y_{ss} = f(k_{ss}) \quad \text{and} \quad s_{ss} = \delta k_{ss}$$

From equation (13) I express $C_{F,ss}$ as a function of $C_{H,ss}$:

$$C_{F,ss} = \frac{1-\theta}{\theta} C_{H,ss} \frac{P_H}{P_F} \tag{21}$$

By Walras' Law I make good $C_{F,t}$ the numeraire in time t's market and set $P_{F,t} = 1$. Also, since countries are symmetric, at equilibrium $P_{G,t} = P_{F,t} = 1$.[2] Then I substitute into the budget constraint (6) expressions for k_{ss}, y_{ss}, s_{ss} and $C_{F,ss}$ and solve it for $C_{H,ss}$:

$$C_{H,ss} = \theta(y_{ss} - k_{ss}\delta - d_{ss}r) \tag{22}$$

Combining equations (21) and (22) yields:

$$C_{F,ss} = (1-\theta)(y_{ss} - k_{ss}\delta - d_{ss}r) \tag{23}$$

2.2 Dynare

I use Dynare to compute a second order approximation to the steady state described above. In particular, I distinguish 3 types of equations: (i) within country equilibrium conditions; (ii) global equilibrium conditions; and (iii) accounting equations.

The within country equilibrium equations are:

$$U_H(C_{H,t}; C_{F,t}) = \beta\, \mathbb{E}_t \left[U_H(C_{H,t+1}; C_{F,t+1})(f_k(k_{t+1}) + 1 - \delta) \right]$$

$$U_H(C_{H,t}; C_{F,t}) = \beta R_h\, \mathbb{E}_t \left[U_H(C_{H,t+1}; C_{F,t+1}) \right]$$

$$\frac{U_H(C_{H,t}; C_{F,t})}{P_{H,t}} = \frac{U_F(C_{H,t}; C_{F,t})}{P_{F,t}}$$

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_{H,t}k_{t+1} + d_{t-1}R_{H,t-1}P_{H,t} = P_{H,t}A_t k_t^{\alpha} + P_{H,t}k_t(1-\delta) + P_{H,t}d_t$$

all of them for both countries. Where R_h is given by equation (4) for both countries. By Walras' Law, global equilibrium requires:

$$C_{H,t} + C^*_{H,t} + s_t = y_{H,t}$$

These equations conform a system of 11 equations on 11 unknowns: $C_{H,t}$, $C_{F,t}$, k_{t+1}, d_t, $C^*_{H,t}$, $C^*_{F,t}$, k^*_{t+1}, d^*_t, R_h, R_f and P_H. Also, I include the following accounting equations:

[2] Note that this implies that $TOT_{H,t} = P_{H,t}$.

$$s_t = k_{t+1} - k_t(1-\delta)$$

$$R_t = 1 + r + \psi(e^{d_t - \bar{d}} - 1)$$

$$tb_t = p_{H,t}C^*_{H,t} - p_{F,t}C_{F,t}$$

$$ca_t = tb_t - R_t d_t$$

$$a_t = \omega a_{t-1} + \epsilon_t$$

3 Calibration

I calibrate the model at a yearly frequency, for Canada, for the period 1996 - 2017. The parameters I need to calibrate are A, α, δ, ω, β, θ, γ, r, $\bar{d}$ and ψ. I use 3 strategies. First I take some parameters from the literature. Second, I take some values from data. Third, I calibrate the remaining parameters to match a specific moment from the data.

I normalize total factor productivity (A) and set α and δ to 0.32 and 0.1 accordingly. These values are standard in the real-business-cycle literature.[3] I set $\theta = 0.65$ to allow a stronger preference for domestic-produced goods. As is standard in the literature, I set the preference discount factor equal to the world interest rate $\beta = 1/(1+r)$ and the risk aversion parameter $\gamma = 2$.

I choose the international risk-free interest rate r as the average interest rate on treasury bills for United States; and $\bar{d}$ as to match the steady state ratio debt to GDP to the average ratio International Private Debt Securities to GDP.[4] Finally, I set the risk premium factor ψ as to approximate the volatility of the ratio current account to GDP. Table 1 summarizes the calibration strategy.

Table 1: Calibration Strategy

Parameter		Value	Strategy
TFP	A	1	Normalization
Capital share	α	0.32	Literature
Depretiation	δ	0.1	Literature
Technology shocks Persistence	ω	0.806	Literature
Risk-free world interest rate	r	0.022	Average Int Rte Treasury Bills
Discount factor	β	0.9785	$= 1/(1+r)$
Risk aversion	γ	2	Literature
Steady-state debt	$\bar{d}$	0.3984	$d_{ss} = 0.2531 * y_{ss}$
Preference for domestic goods	θ	0.65	Stronger preference for domestic goods
Risk premium	ψ	0.001542	Match σ_{ca}

[3] I take these values from Mendoza (1991) who proposes a real business cycle model of a small open economy to analyze the postwar Canadian business fluctuations.

[4] I collected this data from FRED, Federal Reserve Bank of St. Louis.

4 Simulations

In this section I use the model developed in section 2 calibrated as explained in section 3 to perform an experiment in order to assess the behaviour of the relevant variables of the model. Since countries are symmetric, I perform this experiment only for country H

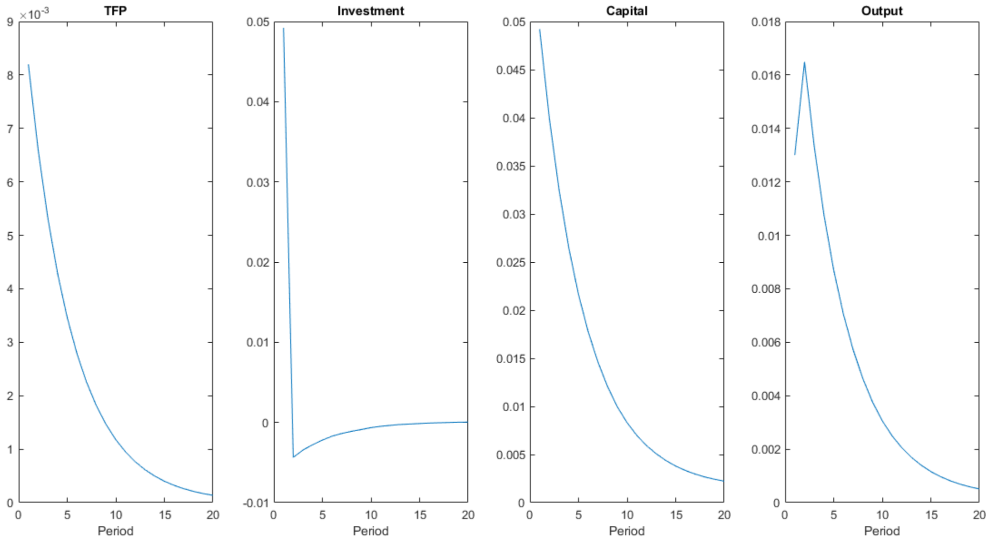

Figure 1: Impulse response to a one standard deviation shock on TFP.

Figure 1 shows the response of investment, capital and output to a shock of one standard deviation on TFP. When the shock hits the economy, investment increases on impact because households want to take advantage of the *transitory* good times. After this first adjustment on investment, households' optimal response is to simply let capital depreciate until it reaches its steady state value. Output increases on impact as a consequence of the major productivity, and after that it follows the path of capital.

Figure 2 displays the response of consumption of the two goods of country H and the terms of trade to a shock of one standard deviation to Total Factor Productivity. There are two different effects in consumption of both good. The first effect occurs when the shock hits the economy and it drives consumption of the domestic (foreign) good below (above) its steady state value. The second effect has the opposite sign: consumption of domestic (foreign) good goes above (below) the steady state. The rationale behind this response is the following: when the shock hits the economy, the marginal benefit of investment in country H increases, which in turn raises the opportunity cost of consumption of good H. However, once the adjustment in the stock of capital has taken place, this effect is reversed and the cost of producing goods in country H is smaller than in country F. This rationale also accounts for the behaviour of the terms of trade of country H.

Figure 3 illustrate the responses of the trade balance, foreign debt and the risk-premium charged on debt. It shows how households use the trade balance (foreign debt) to anticipate consumption taking advantage of their higher life-time wealth.[5] When the shock hits the economy, households optimal response is to incur

[5]Note that at the steady state, the trade balance is zero.

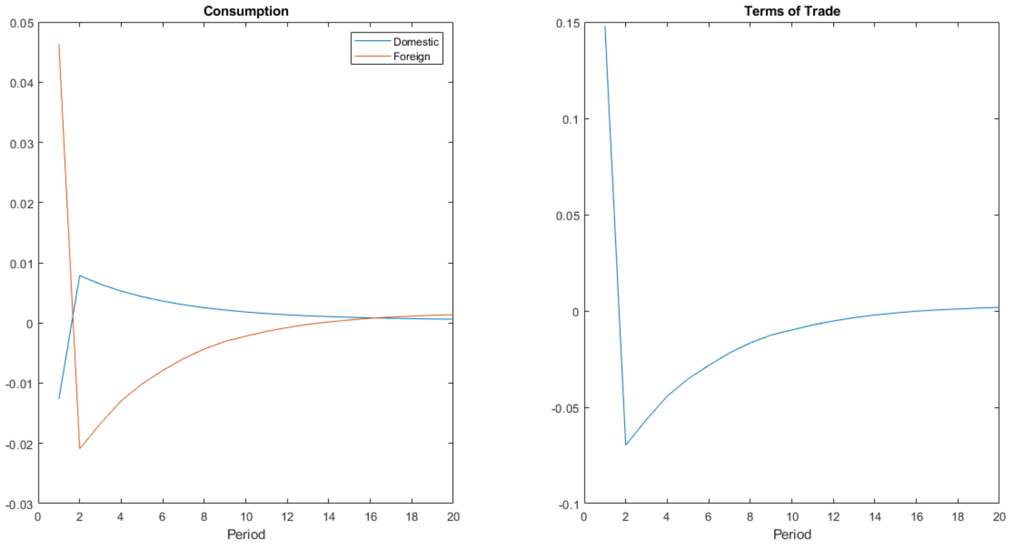

Figure 2: Impulse response to a one standard deviation shock on TFP.

in a negative trade balance today (accumulate claims) and then slowly return to the steady state level of debt running a positive trade balance (de-accumulate claims). The risk premium in turn, follows the path of foreign debt: is positive when debt is larger than its steady state and turns negative when debt is below its steady state.

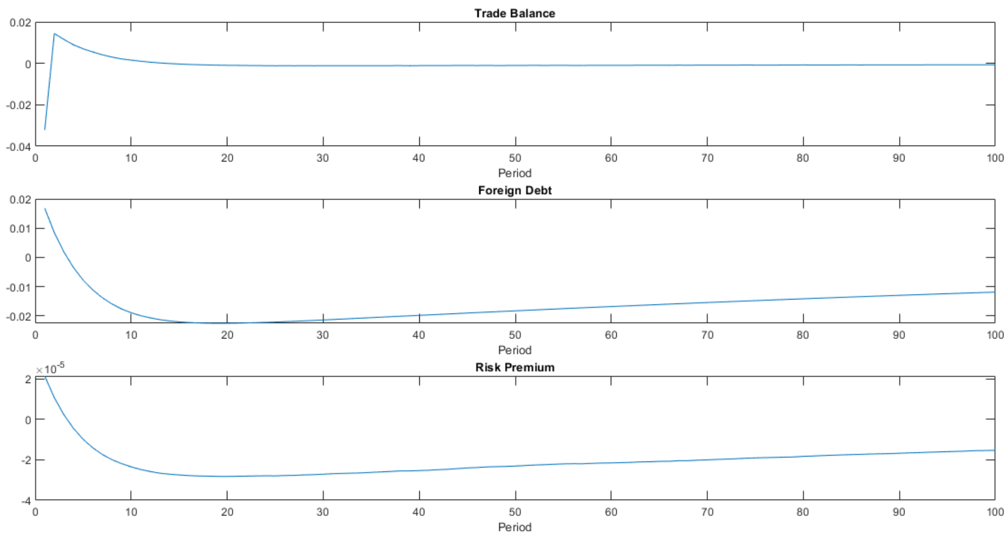

Figure 3: Impulse response to a one standard deviation shock on TFP.

Finally, table 2 shows some unconditional second moments implied by the model. As in Mendoza (1991) and Schmitt-Grohé and Uribe (2003), volatilities, in ascending order are: consumption of domestic goods, output, investment. Remarkably, consumption of foreign goods presents a high volatility compared to consumption of domestic goods, output and even investment. This high volatility in consumption of foreign goods drives the volatility in the trade balance. Also, volatility of foreign debt is only smaller than the volatility of the current account, suggesting that optimal response implies using it as a shock absorber.

Table 2: Country H - Implied second moments

Variable	C_H	C_F	d	y	s	tb	ca	$\frac{ca}{y}$
Std. Dev.	1.55	6.17	15.27	3.08	4.80	4.82	16.10	10.25

Legend: C_H, C_F Consumption of domestic and foreign goods; d International debt; y Output; s Investment; tb Trade Balance; ca Current Account.

5 Conclusion

This paper extends the theory of real business cycles of a small open economy by incorporating two-countries, two-goods and a debt-elastic interest rate in a framework of incomplete international asset markets. I calibrate the model following standard practice in the literature and use it to simulate the dynamic adjustment of the economy to a shock in Total Factor Productivity. The model replicates the standard results in the literature that agents use foreign debt as a shock absorber. It also attains second order moments that are similar to those reported by the literature.

References

Mendoza, E. G. (1991). Real business cycles in a small open economy. *The American Economic Review*, pages 797–818.

Schmitt-Grohé, S. and Uribe, M. (2003). Closing small open economy models. *Journal of international Economics*, 61(1):163–185.

A Appendix 1

In the following link, you can find a GitHub repository with the code-files I wrote: Repository