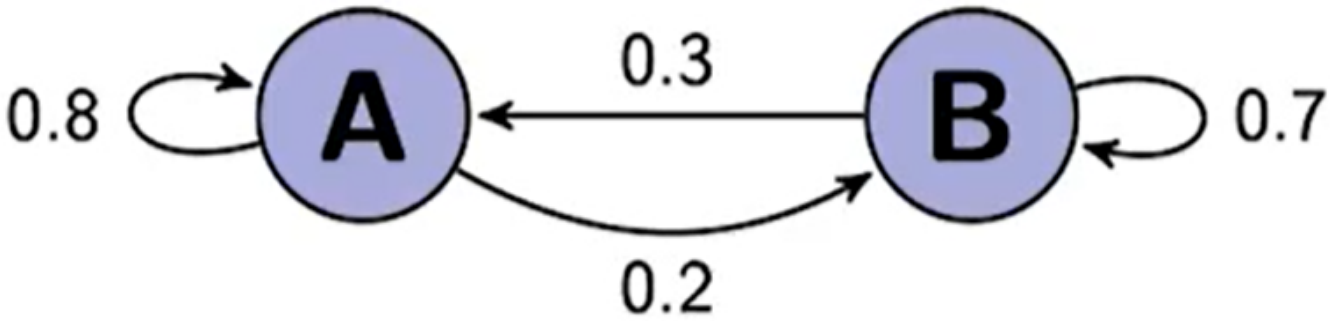
Markov Chain

Say, A and B are libraries with 1000 books.



In the beginning, $x_0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

Now, after 1 month, $x_1 = \begin{bmatrix} 0.8 \cdot 0.5 + 0.3 \cdot 0.5 \\ 0.2 \cdot 0.5 + 0.7 \cdot 0.5 \end{bmatrix} = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = Px_1$

$$\boxed{P = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}}$$

Now, after 2 month, $x_2 = Px_1 = Px_0 = P^2x_0$

$\vdots$

Now, after k months, $x_k = P^kx_0$

Steady State

Find the steady state of $P = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$.

$$
\begin{aligned}
P\vec{q} &= \vec{q} \\
P\vec{q} - \vec{q} &= 0 \\
P\vec{q} - I_n\vec{q} &= 0 \\
(P - I_n)\vec{q} &= 0 \\
\left(\begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\vec{q} &= 0 \\
\begin{bmatrix} 0.2 & -0.3 \\ -0.2 & 0.3 \end{bmatrix}\vec{q} &= 0 \\
\begin{cases} 2x_1 & -3x_2 & = 0 \\ -2x_1 & +3x_2 & = 0 \end{cases} & \\
\{x_1 = 3, x_2 = 2\} & \\
\frac{1}{3+2}\begin{bmatrix} 3 \\ 2 \end{bmatrix} &= \vec{q} \\
\vec{q} &= \boxed{\begin{bmatrix} \frac{3}{5} \\ \frac{2}{5} \end{bmatrix}}
\end{aligned}
$$