Dot Products and Length

Dot products are done like so,

$$\vec{u} \cdot \vec{v} = \vec{u}^T \vec{v} = \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = u_1 v_1 + u_2 v_2 + \cdots + u_n v_n$$

Properties

$(\vec{v} + \vec{w}) \cdot \vec{u} = \vec{v} \cdot \vec{u} + \vec{w} \cdot \vec{u}$
$(c\vec{v}) \cdot \vec{u} = c(\vec{v} \cdot \vec{u})$
$\vec{v} \cdot \vec{u} = \vec{u} \cdot \vec{v}$
$\vec{u} \cdot \vec{u} \geq 0$
$\vec{u} \cdot \vec{u} = 0 \iff \vec{u} = 0$

Length

$||\vec{u}|| = \sqrt{\vec{u} \cdot \vec{u}}$

Properties

$||c\vec{v}|| = |c| \cdot ||\vec{v}||$

Theorem

$\vec{a} \cdot \vec{b} = ||\vec{a}||\,||\vec{b}|| \cos\theta$. Thus, if $\vec{a} \cdot \vec{b} = 0$, then:

- $\vec{a}$ and/or $\vec{b}$ are zero vectors, or
- $\vec{a}$ are $\vec{b}$ are perpendicular to each other.