Basis

Refer to Vocabulary for definitions.

Say we wish to find the basis for $H = \left\{\vec{x} \in \mathbb{R}^4 | x_1 - 3x_2 - 5x_3 + 7x_4 = 0\right\}$ (Note: this is set builder notation)

We must:

1. Convert this into a $A\vec{x} = 0$ form
2. Convert this into Parametric Vector Form
3. Profit???

$$
\begin{aligned}
A\vec{x} &= 0 \\
\begin{bmatrix} 1 & -3 & -5 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} &= 0 \\
\begin{bmatrix} 3x_2 + 5x_3 - 7x_4 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} &= 0 \\
x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -7 \\ 0 \\ 0 \\ 1 \end{bmatrix} &= 0 \\
\text{The basis for the null space of } H &= \boxed{\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}}
\end{aligned}
$$

This is the basis of the null space of H due to the equation being $x_1 - 3x_2 - 5x_3 + 7x_4 = \boxed{0}$.

Basis of the Null Space

Now say we wish to find the basis of the null space of A
We can:

1. Convert it into a $A\vec{x} = 0$ form.
2. Convert that into Parametric Vector Form
3. The set of the vectors in parametric vector form is the basis of the null space.

Basis of the Column Space

Say we wish to find the basis of the column space of A
We could:

1. Find the pivot columns of A.
2. The set of the pivot columns of A is the basis of the column space.