PCP Decomposition

If we have a real 2×2 matrix A with eigenvalue $\lambda = a - bi$ (where $b \neq 0$) and eigenvector $\vec{v}$, we can construct,

$$A = PCP^{-1}$$

Where

$$P = \begin{bmatrix} \text{Re}(\vec{v}) & \text{Im}(\vec{v}) \end{bmatrix}$$

$$C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

C is a rotation dilation matrix

> **Important**
>
> We are looking at the eigenvalue $a - bi$ **not** $a \pm bi$