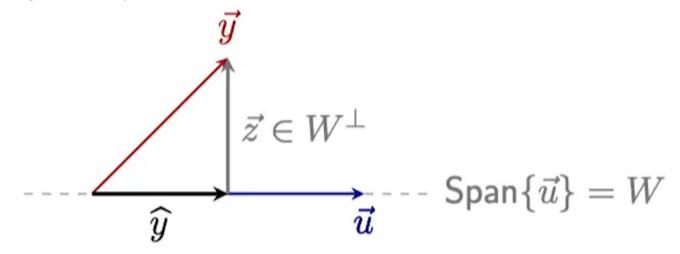
Projections

We have a subspace W (the span of \vec{u}), and we want to find the vector (\hat{y}) closest to \vec{y} in W.

We also want to find $\vec{z} \in W^\perp$ such that $\vec{y} = \hat{y} + \vec{z}$. Diagrammatically,



We know $\vec{z} \in W^{\perp}$, so:

$$\vec{z}\cdot\vec{u}=0$$

We also know $ec{y} = \hat{y} + ec{z}$ and $\hat{y} = k ec{u}$ $(k \in \mathbb{R})$, so:

$$egin{aligned} ec{z} &= ec{y} - k ec{u} \ 0 &= (ec{y} - k ec{u}) \cdot ec{u} \ &= ec{y} \cdot ec{u} - k ec{u} \cdot ec{u} \ k &= rac{ec{y} \cdot ec{u}}{ec{u} \cdot ec{u}}, \end{aligned} \qquad ec{u}
eq ec{0}$$

So finally, $\hat{y} = rac{ec{y} \cdot ec{u}}{ec{u} \cdot ec{u}} ec{u}$

Orthogonal Projection

Let non-zero $\vec{u} \in \mathbb{R}^n$, and $\vec{y} \in \mathbb{R}^n$. The orthogonal projection of \vec{y} onto \vec{u} is the vector in the span of \vec{u} that is closest to \vec{y} .

$$ext{proj}_{ec{u}}ec{y} = rac{ec{y}\cdotec{u}}{ec{u}\cdotec{u}}ec{u}$$