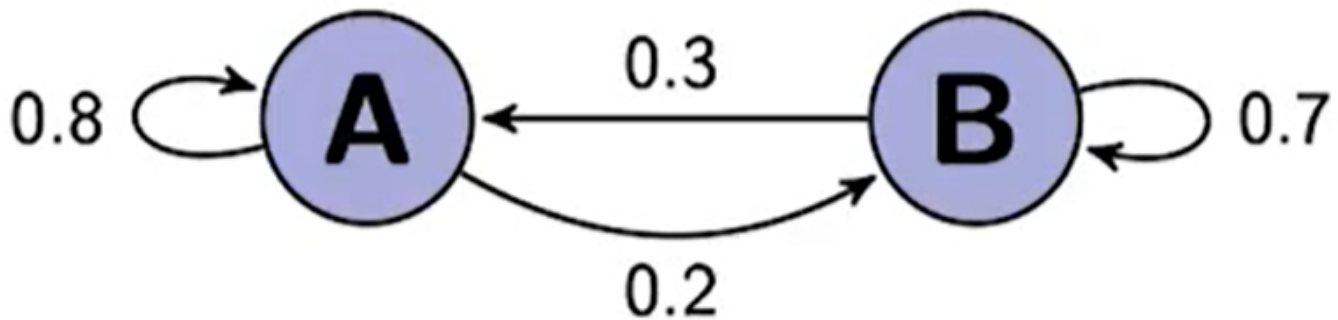


# Markov Chain

Say,  $A$  and  $B$  are libraries with 1000 books.



In the beginning,  $x_0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

Now, after 1 month,  $x_1 = \begin{bmatrix} 0.8 \cdot 0.5 + 0.3 \cdot 0.5 \\ 0.2 \cdot 0.5 + 0.7 \cdot 0.5 \end{bmatrix} = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = Px_1$

$$P = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$$

Now, after 2 month,  $x_2 = Px_1 = Px_0 = P^2x_0$

$\vdots$

Now, after  $k$  months,  $x_k = P^kx_0$

## Steady State

Find the steady state of  $P = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$ .

$$\begin{aligned}
P\vec{q} &= \vec{q} \\
P\vec{q} - \vec{q} &= 0 \\
P\vec{q} - I_n\vec{q} &= 0 \\
(P - I_n)\vec{q} &= 0 \\
\left( \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \vec{q} &= 0 \\
\begin{bmatrix} 0.2 & -0.3 \\ -0.2 & 0.3 \end{bmatrix} \vec{q} &= 0 \\
\begin{cases} 2x_1 & -3x_2 & = 0 \\ -2x_1 & +3x_2 & = 0 \end{cases} \\
\{x_1 = 3, x_2 = 2\} \\
\frac{1}{3+2} \begin{bmatrix} 3 \\ 2 \end{bmatrix} &= \vec{q} \\
\vec{q} &= \begin{bmatrix} \frac{3}{5} \\ \frac{2}{5} \end{bmatrix}
\end{aligned}$$

## Convergence

We have  $x_1, x_2, x_3, \dots, x_k$ . We want to know if while  $k \rightarrow \infty$   $x_k$  will converge to a [steady state](#).

If  $P$  is a regular stochastic matrix ([vocabulary](#)), then  $P$  has a unique steady-state vector  $\vec{q}$ , and  $\vec{x}_{k+1} = P\vec{x}_k$  converges to  $\vec{q}$  as  $k \rightarrow \infty$ .