Eigen Things

If $A \in \mathbb{R}^{n \times n}$, and there is a $\vec{v} \neq 0 \in \mathbb{R}^n$, and

$$A\vec{v} = \lambda\vec{v}$$

Then $\vec{v}$ is an **eigenvectors** for A, and λ is the **eigenvalue**.

Eigenspaces

The span of the eigenvector of A is the eigenspace of A. It spans a subspace of $\mathbb{R}^n$ called the λ -eigenspace of A.

The λ-eigenspace of A is $\mathrm{Nul}(A - \lambda I)$

$$\begin{aligned} A\vec{v} &= \lambda\vec{v} \\ A\vec{v} - \lambda\vec{v} &= 0 \\ (A - \lambda I)\vec{v} &= 0 \end{aligned}$$

Theorems

- The diagonal elements of a triangular matrix are its eigenvalues.
- A not invertible $\Longleftrightarrow$ 0 is an eigenvalue of A.
- Stochastic matrices have an eigenvalue equal to 1.
- Eigenvectors with distinct eigenvalues are linearly independent vectors.

Compute Eigenvalues

We know that $(A - \lambda I)$ is non invertible, so $\det(A - \lambda I) = 0$.
We can solve $\det(A - \lambda I) = 0$ for λ.

$\det(A - \lambda I)$ is the characteristic polynomial
$\det(A - \lambda I) = 0$ is the characteristic equation
The **trace** of a matrix is the sum of its diagonal elements.
The sum of the Eigenvalues of A = the trace.

Algebraic and Geometric Multiplicities

- a_i is the algebraic multiplicity
- g_i is the geometric multiplicity
- $1 \leq a_i \leq n$

- $1 \le g_i \le n$