Determinants

A general way to compute it is,
If If A is an $n \times n$ matrix where $n = 1$,

$$\det(A) = a_{1,1}$$

If A is an $n \times n$ matrix where $n > 1$,

$$\det(A) = a_{1,1} \det\left(A_{1,1}\right) - a_{1,2} \det\left(A_{1,2}\right) + \cdots + (-1)^{n+1} a_{1,n} \det\left(A_{1,n}\right)$$

$a_{i,j}$ means the element at the i^{th} row and the j^{th} column.
$A_{i,j}$ means the Matrix if you drop (get rid of) the i^{th} row and the j^{th} column.

Deriving it for a 2×2
Say we have $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{aligned} \det(A) &= (a_{1,1})(\det\left(A_{1,1}\right)) - (a_{1,2})(\det\left(A_{1,2}\right)) \\ \det(A) &= (a)\,(d) - (b)\,(c) \\ \det(A) &= ad - bc \end{aligned}$$

Using a Cofactor

The determinant of a matrix A can be computed down any row or column of the matrix.

For example, down the j^{th} column the determinant is:

$$\det(A) = a_{1,j} \det\left(A_{1,j}\right) - a_{2,j} \det\left(A_{2,j}\right) + \cdots + (-1)^{n+1} a_{n,j} \det\left(A_{n,j}\right)$$

This would be useful for a matrix with a few 0's.
Say $A = \begin{bmatrix} 5 & 4 & 3 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 3 \end{bmatrix}$ find $\det(A)$
We will use the first column due to the 3 zeros.

$$
\begin{aligned}
\det(A) &= 5C_{1,1} + 0C_{2,1} + 0C_{3,1} + 0C_{4,1} \\
&= 5 \cdot (-1)^{1+1} \cdot \det\left(\begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}\right) \\
3^{\text{rd}} \text{ column} & \\
&= 5 \cdot (0C_{1,3} + 0C_{2,3} + 3C_{3,3}) \\
&= 5 \cdot \left(3 \cdot (-1)^{3+3} \det\left(\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\right)\right) \\
\text{Formula} & \\
&= 5(3(1 \times 1 - 2 \times -1)) \\
&= \boxed{45}
\end{aligned}
$$

Triangular Matrices

The determinant of a triangular matrix is the product of the entries on the main diagonal.