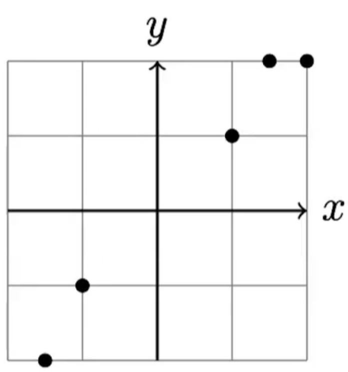
Least Squares

We want to find a line/curve that minimizes the sum of the square of the error caused due to deviation.

Line

Say we want to find a line $y = mx + b$ that is the best fit for the following points:

x	-1.5	-1	1	1.5	2
y	-2	-1	1	2	2



We can create a list of linear equations using this:

$$\begin{aligned} m(-1.5) + b &= -2 \\ m(-1) + b &= -1 \\ m(1) + b &= 1 \\ m(1.5) + b &= 2 \\ m(2) + b &= 2 \end{aligned}$$

We can turn this in to a matrix equation like so,

$$\begin{bmatrix} -1.5 & 1 \\ -1 & 1 \\ 1 & 1 \\ 1.5 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 1 \\ 2 \\ 2 \end{bmatrix}$$

Least squares

The least squares for $Ax = b$ is $\hat{x}$ for h=which,

$$||b - A\hat{x}|| \leq ||b - Ax||$$

for all x

Normal Equation

$$A^T A\hat{x} = A^T\vec{b}$$

Manipulating this we can get this,

$$\hat{x} = (A^T A)^{-1} A^T\vec{b}$$

Using QR Factorization

$$R\hat{x} = Q^T\vec{b}$$

Proof

$$\begin{aligned} A^T A\hat{x} &= A^T\vec{b} \\ (QR)^T QR\hat{x} &= (QR)^T\vec{b} \\ R^T Q^T QR\hat{x} &= R^T Q^T\vec{b} \\ R^T R\hat{x} &= R^T Q^T\vec{b} \\ R\hat{x} &= Q^T\vec{b} \end{aligned}$$