

# Eigen Things

If  $A \in \mathbb{R}^{n \times n}$ , and there is a  $\vec{v} \neq 0 \in \mathbb{R}^n$ , and

$$A\vec{v} = \lambda\vec{v}$$

Then  $\vec{v}$  is an **eigenvectors** for  $A$ , and  $\lambda$  is the **eigenvalue**.

## Eigenspaces

The span of the eigenvector of  $A$  is the eigenspace of  $A$ . It spans a subspace of  $\mathbb{R}^n$  called the  $\lambda$ -eigenspace of  $A$ .

The  $\lambda$ -eigenspace of  $A$  is  $\text{Nul}(A - \lambda I)$

$$\begin{aligned} A\vec{v} &= \lambda\vec{v} \\ A\vec{v} - \lambda\vec{v} &= 0 \\ (A - \lambda I)\vec{v} &= 0 \end{aligned}$$

## Theorems

- The diagonal elements of a triangular matrix are its eigenvalues.
- $A$  not invertible  $\iff 0$  is an eigenvalue of  $A$ .
- Stochastic matrices have an eigenvalue equal to 1.
- Eigenvectors with distinct eigenvalues are linearly independent vectors.

## Compute Eigenvalues

We know that  $(A - \lambda I)$  is non invertible, so  $\det(A - \lambda I) = 0$ .

We can solve  $\det(A - \lambda I) = 0$  for  $\lambda$ .

$\det(A - \lambda I)$  is the characteristic polynomial

$\det(A - \lambda I) = 0$  is the characteristic equation

The **trace** of a matrix is the sum of its diagonal elements.

The sum of the Eigenvalues of  $A$  = The trace.

## Algebraic and Geometric Multiplicities

- $a_i$  is the algebraic multiplicity

- $g_i$  is the geometric multiplicity
- $1 \leq a_i \leq n$
- $1 \leq g_i \leq n$
- $g_i \leq a_i$