Orthogonal Sets

If for set $\{\vec{u}_1, \ldots, \vec{v}_n\}$ for $j \neq k$, $\vec{u}_j \perp \vec{u}_k$.

If Set S is orthogonal, the vectors of S are linearly independent.

Expansion in Orthogonal Basis

If we have an Orthogonal Basis $\{\vec{u}_1, \ldots, \vec{v}_n\}$ in $\mathbb{R}^n$ then for any $\vec{w} \in \mathbb{R}^n$,

$$\vec{w} = c_1\vec{u}_1 + \cdots + c_n\vec{v}_n$$

C_q can be found using $c_q = \frac{\vec{w} \cdot \vec{u}_q}{\vec{u}_q \cdot \vec{u}_q}$

Length of a Vector in the basis of an orthogonal set

If $\vec{w} = c_1\vec{u}_1 + \cdots + c_n\vec{v}_n = (\vec{w} \cdot \vec{u}_1)\vec{u}_1 + \cdots + (\vec{w} \cdot \vec{u}_n)\vec{v}_n$

$$||\vec{w}|| = \sqrt{(\vec{w} \cdot \vec{u}_1)^2 + \cdots + (\vec{w} \cdot \vec{u}_n)^2}$$