

# Orthogonal Matrices

An  $m \times n, m \geq n$  matrix has orthonormal columns  $\iff U^T U = I_n$

Notice,  $U^T U = I_n$  implies that  $U^T = U^{-1}$

## Properties

- $\|U\vec{x}\| = \|\vec{x}\|$
- $(U\vec{x}) \cdot (U\vec{y}) = \vec{x} \cdot \vec{y}$
- $(U\vec{x}) \cdot (U\vec{y}) = 0 \iff \vec{x} \cdot \vec{y} = 0$
- $U^{-1} = U^T$
- $\det U = 1$  or  $-1$

$$U^T = U^{-1}$$