Orthogonal Matrices

An $m \times n, m \geq n$ matrix has orthonormal columns $\iff U^T U = I_n$

Notice, $U^T U = I_n$ implies that $U^T = U^{-1}$

Properties

- $||U\vec{x}|| = ||\vec{x}||$
- $(U\vec{x}) \cdot (U\vec{y}) = \vec{x} \cdot \vec{y}$
- $(U\vec{x}) \cdot (U\vec{y}) = 0 \iff \vec{x} \cdot \vec{y} = 0$
- $U^{-1} = U^T$
- $\det A = 1 \text{ or } -1$

$$U^T = U^{-1}$$