Formulas

An $n \times n$ symmetric matrix A has the following properties,

- All eigenvalues of A are real.
- The eigenspaces are mutually orthogonal.
- A can be diagonalized as $A = PDP^T$, where D is diagonal and P is orthogonal.

The eigenvalues of A^TA are non-negative.

Proof: recall that $\vec{v}_j^T \vec{v}_j = \vec{v}_j \cdot \vec{v}_j = ||\vec{v}_j||^2 = 1$ because \vec{v}_j are unit eigenvectors of $A^T A$.

$$||A\vec{v}_{j}||^{2} = (A\vec{v}_{j})^{T}A\vec{v}_{j} = \vec{v}_{j}A^{T}A\vec{v}_{j} = \lambda_{j}\vec{v}_{j}^{T}\vec{v}_{j} = \lambda_{j} \geq 0.$$

Therefore:

- ullet the eigenvalues of A^TA must be real and non-negative
- ullet the singular values of A, which are the square roots of the eigenvalues, must also be real and non-negative