

# Formulas

## Formulas and Theorem

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### Expansion in Orthogonal Basis

If we have an Orthogonal Basis  $\{\vec{u}_1, \dots, \vec{v}_n\}$  in  $\mathbb{R}^n$  then for any  $\vec{w} \in \mathbb{R}^n$ ,

$$\vec{w} = c_1\vec{u}_1 + \dots + c_n\vec{v}_n$$

$C_q$  can be found using  $c_q = \frac{\vec{w} \cdot \vec{u}_q}{\vec{u}_q \cdot \vec{u}_q}$

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### Orthogonal Projection

Let non-zero  $\vec{u} \in \mathbb{R}^n$ , and  $\vec{y} \in \mathbb{R}^n$ . The orthogonal projection of  $\vec{y}$  onto  $\vec{u}$  is the vector in the span of  $\vec{u}$  that is closest to  $\vec{y}$ .

$$\text{proj}_{\vec{u}}\vec{y} = \frac{\vec{y} \cdot \vec{u}}{\vec{u} \cdot \vec{u}}\vec{u}$$

Also,  $\vec{y} = \hat{y} + \vec{z}$  and,  $\vec{z} \in W^\perp$

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