

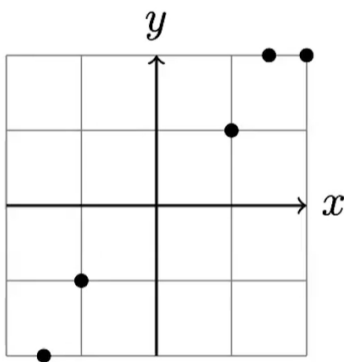
Least Squares

We want to find a line/curve that minimizes the sum of the square of the error caused due to deviation.

Line

Say we want to find a line $y = mx + b$ that is the best fit for the following points:

| | | | | | |
|---|------|----|---|-----|---|
| x | -1.5 | -1 | 1 | 1.5 | 2 |
| y | -2 | -1 | 1 | 2 | 2 |



We can create a list of linear equations using this:

$$m(-1.5) + b = -2$$

$$m(-1) + b = -1$$

$$m(1) + b = 1$$

$$m(1.5) + b = 2$$

$$m(2) + b = 2$$

We can turn this in to a matrix equation like so,

$$\begin{bmatrix} -1.5 & 1 \\ -1 & 1 \\ 1 & 1 \\ 1.5 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 1 \\ 2 \\ 2 \end{bmatrix}$$

Least squares

The least squares for $Ax = b$ is \hat{x} for which,

$$\|b - A\hat{x}\| \leq \|b - Ax\|$$

for all x