Orthogonality Constraints

A continuation of Constrained Optimization

Orthogonality Constrained Optimization

If $Q = x^T Ax$, A is a real $n \times n$ symmetric matrix, with eigenvalues

$$\lambda_1 \geq \lambda_2 \cdots \geq \lambda_n$$

and associated normalized eigenvectors

$$u_1, u_2, \ldots, u_n$$

Also, $||x|| = 1$ AND $x \cdot u_1 = 0$
Then max value of $Q(x) = \lambda_2$ attained at u_2
Then min value of $Q(x) = \lambda_n$ attained at u_n