Quadratic Surfaces

Curves in $\mathbb{R}^2$

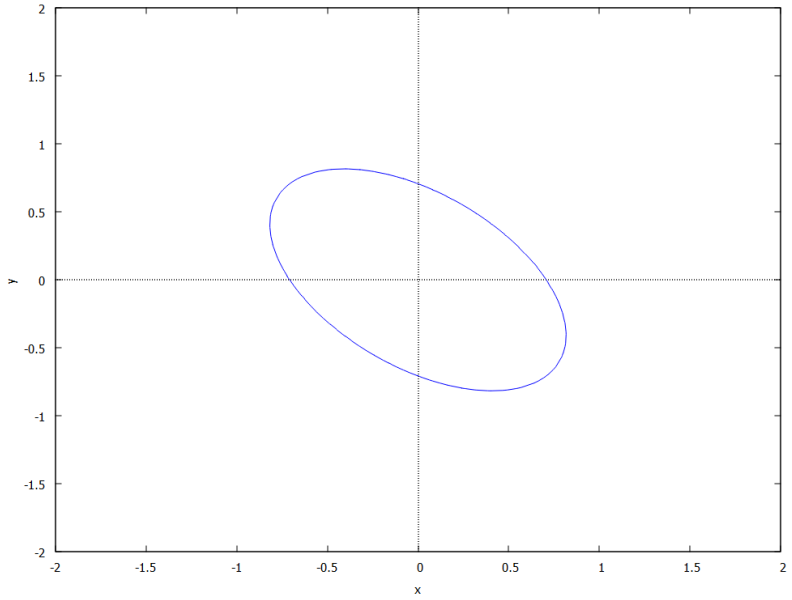
Say we have a constant C and $Q(x) = x^T Ax$ where $A \in \mathbb{R}^{2\times 2}$. Then,

$$C = x^T Ax$$

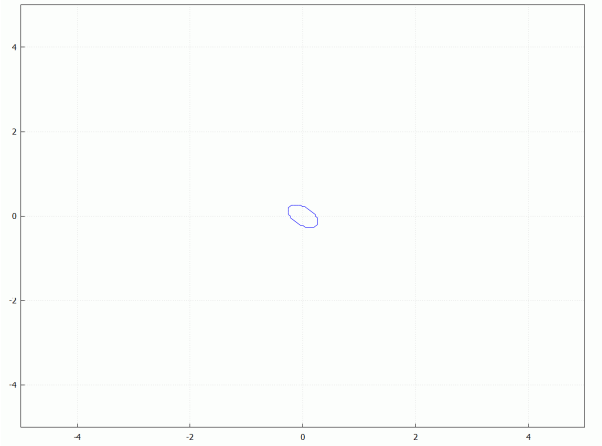
Is a curve in $\mathbb{R}^2$.

Say $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Then $Q(x) = 2x^2 + 2y^2 + 2xy = C$.

If $C = 1$ we can plot,



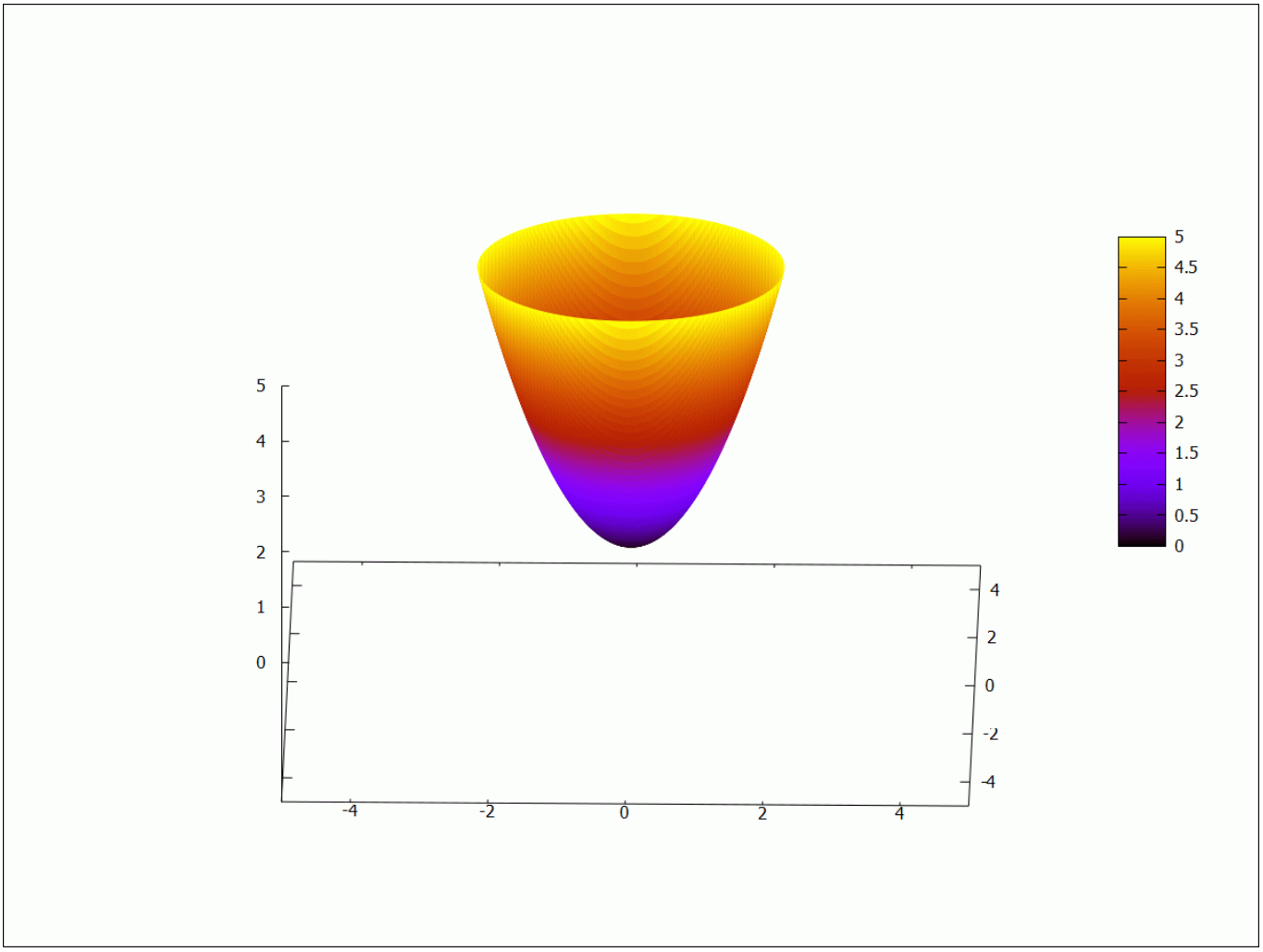
Here is it plotted when we changed C from 1 to 10.



Curves in $\mathbb{R}^3$

Say we have $z = Q(x)$, $z = x^2 + y^2 = x^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x$

We can graph this,



Definitions

Word	Definition	Eigenvalues
Positive Definite	If $Q > 0$ for all $\vec{x} \neq 0$	all eigenvalues are positive
Negative Definite	If $Q < 0$ for all $\vec{x} \neq 0$	all eigenvalues are negative
Positive Semidefinite	If $Q \geq 0$ for all $\vec{x}$	
Negative Semidefinite	If $Q \leq 0$ for all $\vec{x}$	
Indefinite	If Q takes on positive and negative values for $\vec{x} \neq 0$	at least one negative and one positive