

# Gram-Schmidt Process

Given  $n$  [linearly independent](#) vectors that form the subspace  $W$ , you can use the Gram-Schmidt Process to find the orthogonal basis for  $W$ .

## 2 Vectors

We take one of the 2 vectors and find its 'z' using the other vector. Now this new vector and 'other' vector are the new orthogonal basis. As math:

Given the linearly independent vectors:  $\vec{v}, \vec{u}$  the orthogonal basis is  $\{\vec{v}, \vec{u} - \text{Proj}_{\vec{v}}\vec{u}\}$ .

More formally, given the linearly independent vectors:  $\vec{x}_1, \vec{x}_2$ ,

$$\begin{aligned}\vec{v}_1 &= \vec{x}_1 \\ \vec{v}_2 &= \vec{x}_2 - \frac{\vec{x}_2 \cdot \vec{x}_1}{\vec{x}_1 \cdot \vec{x}_1} \vec{x}_1\end{aligned}$$

Then the set  $\{\vec{v}_1, \vec{v}_2\}$  is the orthogonal basis.

## n-Vectors

Given the linearly independent vectors:  $\vec{x}_1, \vec{x}_2, \dots, \vec{x}_n$

$$\begin{array}{ll}\vec{v}_1 = \vec{x}_1 & W_1 = \text{Span}\{\vec{v}_1\} \\ \vec{v}_2 = \vec{x}_2 - \text{Proj}_{W_1}\vec{x}_2 & W_2 = \text{Span}\{\vec{v}_1, \vec{v}_2\} \\ \vec{v}_3 = \vec{x}_3 - \text{Proj}_{W_2}\vec{x}_3 & W_3 = \text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\} \\ \vdots & \vdots \\ \vec{v}_n = \vec{x}_n - \text{Proj}_{W_{n-1}}\vec{x}_n & W_n = \text{Span}\{\vec{v}_1, \dots, \vec{v}_n\}\end{array}$$

The orthogonal basis will be  $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$

We will use the Orthogonal Decomposition Theorem that can be found in [Formulas](#)