Gram-Schmidt Process

Given n linearly independent vectors that form the subspace W, you can use the Gram-Schmidt Process to find the orthogonal basis for W.

2 Vectors

We take one of the 2 vectors and find its 'z' using the other vector. Now this new vector and 'other' vector are the new orthogonal basis. As math:
Given the linearly independent vectors: $\vec{v}, \vec{u}$ the orthogonal basis is $\{\vec{v}, \vec{u} - \mathrm{Proj}_{\vec{v}}\vec{u}\}$.

More formally, given the linearly independent vectors: $\vec{x_1}, \vec{x_2}$,

$$\begin{aligned} \vec{v_1} &= \vec{x_1} \\ \vec{v_2} &= \vec{x_2} - \frac{\vec{x_2} \cdot \vec{x_1}}{\vec{x_1} \cdot \vec{x_1}} \vec{x_1} \end{aligned}$$

Then the set $\{\vec{v_1}, \vec{v_2}\}$ is the orthogonal basis.

n-Vectors

Given the linearly independent vectors: $\vec{x_1}, \vec{x_2}, \ldots, \vec{x_n}$

$$\begin{aligned} \vec{v_1} &= \vec{x_1} & W_1 &= \mathrm{Span}\{v_1\} \\ \vec{v_2} &= \vec{x_2} - \mathrm{Proj}_{W_1}\vec{x_2} & W_2 &= \mathrm{Span}\{v_1, v_2\} \\ \vec{v_3} &= \vec{x_3} - \mathrm{Proj}_{W_2}\vec{x_3} & W_3 &= \mathrm{Span}\{v_1, v_2, v_3\} \\ &\vdots & &\vdots \\ \vec{v_n} &= \vec{x_n} - \mathrm{Proj}_{W_{n-1}}\vec{x_n} & W_n &= \mathrm{Span}\{v_1, \ldots, v_n\} \end{aligned}$$

The orthogonal basis will be $\{\vec{v_1}, \vec{v_2}, \ldots \vec{v_n}\}$
We will use the Orthogonal Decomposition Theorem that can be found in Formulas