# Basis

Refer to Vocabulary for definitions.

Say we wish to find the basis for $H = \left\{ \vec{x} \in \mathbb{R}^4 | x_1 - 3x_2 - 5x_3 + 7x_4 = 0 \right\}$ (Note: this is set builder notation)

We must:

1. Convert this into a $A\vec{x} = 0$ form
2. Convert this into Parametric Vector Form
3. Profit???

$$
\begin{gathered}
A\vec{x} = 0 \\
\begin{bmatrix} 1 & -3 & -5 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0 \\
\begin{bmatrix} 3x_2 + 5x_3 - 7x_4 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0 \\
x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -7 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 0 \\
\text{The basis for the null space of } H = \boxed{\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}}
\end{gathered}
$$

This is the basis of the null space of $H$ due to the equation being $x_1 - 3x_2 - 5x_3 + 7x_4 = \color{red}{0}$.

# Basis of the Null Space

Now say we wish to find the basis of the null space of $A$
We can:

1. Convert it into a $A\vec{x} = 0$ form.
2. Convert that into Parametric Vector Form
3. The set of the vectors in parametric vector form is the basis of the null space.

# Basis of the Column Space

Say we wish to find the basis of the column space of $A$
We could:

1. Find the pivot columns of $A$.
2. The set of the pivot columns of $A$ is the basis of the column space.