

# Basis

Refer to [Vocabulary](#) for definitions.

Say we wish to find the basis for  $H = \{\vec{x} \in \mathbb{R}^4 | x_1 - 3x_2 - 5x_3 + 7x_4 = 0\}$  (Note: this is [set builder notation](#))

We must:

1. Convert this into a  $A\vec{x} = 0$  form
2. Convert this into [Parametric Vector Form](#)
3. Profit???

$$\begin{aligned}
 & A\vec{x} = 0 \\
 & \begin{bmatrix} 1 & -3 & -5 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0 \\
 & \begin{bmatrix} 3x_2 + 5x_3 - 7x_4 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0 \\
 & x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -7 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 0
 \end{aligned}$$

$$\text{The basis for the null space of } H = \left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

This is the basis of the null space of  $H$  due to the equation being  $x_1 - 3x_2 - 5x_3 + 7x_4 = 0$ .

## Basis of the Null Space

Now say we wish to find the basis of the null space of  $A$

We can:

1. Convert it into a  $A\vec{x} = 0$  form.
2. Convert that into [Parametric Vector Form](#)
3. The set of the vectors in parametric vector form is the basis of the null space.

# Basis of the Column Space

Say we wish to find the basis of the column space of  $A$

We could:

1. Find the pivot columns of  $A$ .
2. The set of the pivot columns of  $A$  is the basis of the column space.