Orthogonal Matrices

An $m imes n, m \geq n$ matrix has orthonormal columns $\iff U^T U = I_n$ Notice, $U^T U = I_n$ implies that $U^T = U^{-1}$

Properties

- $||U\vec{x}|| = ||\vec{x}||$
- $(U\vec{x})\cdot(U\vec{y})=\vec{x}\cdot\vec{y}$
- $(U\vec{x})\cdot(U\vec{y})=0\iff \vec{x}\cdot\vec{y}=0$
- ullet $U^{-1}=U^T$
- $\det A = 1 \text{ or } -1$

$$U^T = U^{-1}$$