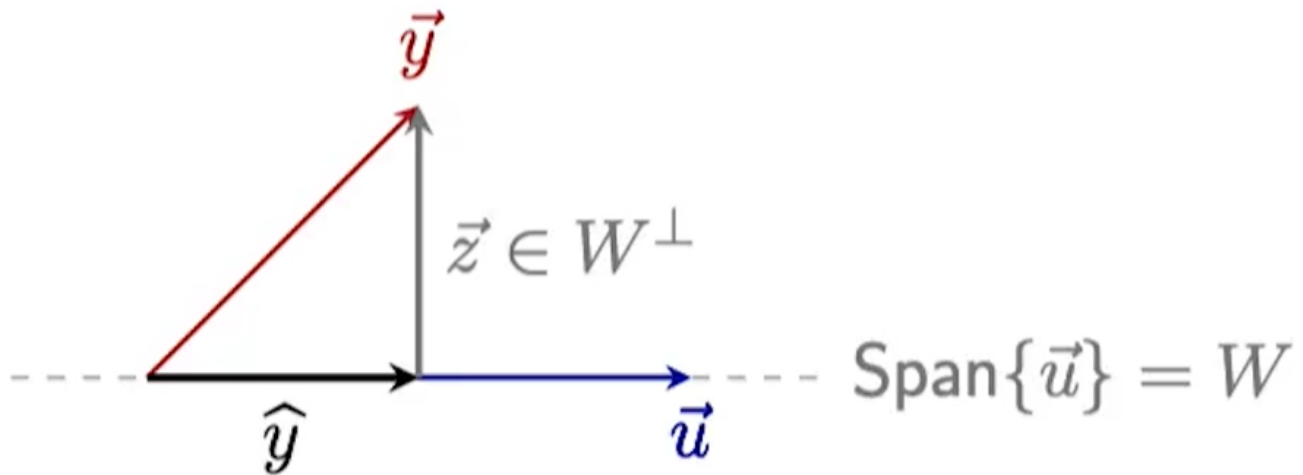


Projections

We have a subspace W (the span of \vec{u}), and we want to find the vector (\hat{y}) closest to \vec{y} in W .

We also want to find $\vec{z} \in W^\perp$ such that $\vec{y} = \hat{y} + \vec{z}$.

Diagrammatically,



We know $\vec{z} \in W^\perp$, so:

$$\vec{z} \cdot \vec{u} = 0$$

We also know $\vec{y} = \hat{y} + \vec{z}$ and $\hat{y} = k\vec{u}$ ($k \in \mathbb{R}$), so:

$$\begin{aligned}\vec{z} &= \vec{y} - k\vec{u} \\ 0 &= (\vec{y} - k\vec{u}) \cdot \vec{u} \\ &= \vec{y} \cdot \vec{u} - k\vec{u} \cdot \vec{u} \\ k &= \frac{\vec{y} \cdot \vec{u}}{\vec{u} \cdot \vec{u}}, \quad \vec{u} \neq \vec{0}\end{aligned}$$

So finally, $\hat{y} = \frac{\vec{y} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \vec{u}$

Orthogonal Projection

Let non-zero $\vec{u} \in \mathbb{R}^n$, and $\vec{y} \in \mathbb{R}^n$. The orthogonal projection of \vec{y} onto \vec{u} is the vector in the span of \vec{u} that is closest to \vec{y} .

$$\text{proj}_{\vec{u}} \vec{y} = \frac{\vec{y} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \vec{u}$$

Also, $\vec{y} = \hat{y} + \vec{z}$ and, $\vec{z} \in W^\perp$