Dot Products and Length

Dot products are done like so,

$$ec{u}\cdotec{v}=ec{u}^Tec{v}=egin{bmatrix}u_1 & u_2 & \cdots & u_n\end{bmatrix}egin{bmatrix}v_1\v_2\dots\v_n\end{bmatrix}=u_1v_1+u_2v_2+\cdots+u_nv_n$$

Properties

$$egin{aligned} (ec{v}+ec{w})\cdotec{u} &= ec{v}\cdotec{u} + ec{w}\cdotec{u} \ (cec{v})\cdotec{u} &= c(ec{v}\cdotec{u}) \ ec{v}\cdotec{u} &= ec{u}\cdotec{v} \ ec{u}\cdotec{u} &\geq 0 \ ec{u}\cdotec{u} &= 0 \iff ec{u} &= 0 \end{aligned}$$

Length

$$||ec{u}|| = \sqrt{ec{u} \cdot ec{u}}$$

Properties

$$||cec{v}||| = |c|\cdot||ec{v}||$$

Theorem

 $\overrightarrow{a} \cdot \overrightarrow{b} = ||\overrightarrow{a}|| \, ||\overrightarrow{b}|| \, \cos \theta$. Thus, if $\overrightarrow{a} \cdot \overrightarrow{b} = 0$, then: • \overrightarrow{a} and/or \overrightarrow{b} are zero vectors, or • \overrightarrow{a} are \overrightarrow{b} are perpendicular to each other.