The Invertible Matrix Theorem

1. A is invertible.
2. A is row equivalent to I_n.
3. A has n pivotal columns (all columns are pivotal).
4. $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
5. The columns of A are linearly independent.
6. The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all $b \in \mathbb{R}^n$
7. The columns of A span $\mathbb{R}^n$
8. There is a $n \times n$ matrix C so that $CA = I_n$ (A has a left inverse.)
9. There is a $n \times n$ matrix D so that $AD = I_n$ (A has a right inverse.)
10. A^T is invertible

Example:

Is $\begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ 0 & -1 & -1 \end{bmatrix}$ invertible?

$\mathrm{RREF}\left(\begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ 0 & -1 & -1 \end{bmatrix}\right)$ = I_3

Every column is pivotal.

So, $\begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ 0 & -1 & -1 \end{bmatrix}$ is **invertible**!