Similar Matrices

If A and B similar, then they have the same characteristic polynomial.

$$A - \lambda I = PBP^{-1} - \lambda I$$

$$= PBP^{-1} - \lambda PP^{-1}$$

$$= (PB - \lambda P)P^{-1}$$

$$= P(B - \lambda I)P^{-1}$$

$$\det(A - \lambda I) = \det(P(B - \lambda I)P^{-1})$$

$$\det(A - \lambda I) = \det(P)\det(B - \lambda I)\det(P^{-1})$$

$$\det(A - \lambda I) = \det(P)\det(P^{-1})\det(B - \lambda I)$$

$$\det(A - \lambda I) = \det(P)\det(P^{-1})$$

The converse is not always true.