Diagonal Matrices

Finding powers of a diagonal matrix, is very simple.

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}^k = \begin{bmatrix} a^k & 0 \\ 0 & b^k \end{bmatrix}$$

Diagonalizable

If A is similar to diagonal matrix D ($A = PDP^{-1}$) then A is diagonalizable.

$$\begin{aligned} A &= PDP^{-1} \\ A &= [v_1 \quad v_2 \dots \quad v_n] \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_4 \end{bmatrix} [v_1 \quad v_2 \dots \quad v_n]^{-1} \end{aligned}$$

Important

$v_1 \dots v_n$ are eigenvectors
$\lambda_1 \dots \lambda_n$ are eigenvalue

Proof

We construct $P = (\vec{v}_1 \ \vec{v}_2 \ \dots \vec{v}_n)$. Then

$$\begin{aligned} AP &= A(\vec{v}_1 \ \vec{v}_2 \ \dots \vec{v}_n) \\ &= (A\vec{v}_1 \ A\vec{v}_2 \ \dots A\vec{v}_n) \\ &= (\lambda_1\vec{v}_1 \ \lambda_2\vec{v}_2 \ \dots \lambda_n\vec{v}_n) \\ AP &= (\vec{v}_1 \ \vec{v}_2 \ \dots \vec{v}_n) \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_n \end{pmatrix} \\ &= PD \end{aligned}$$

Or, $A = PDP^{-1}$.

A is diagonalizable if and only if A has n linearly independent eigenvectors.

Example

Diagonalize $A = \begin{bmatrix} 2 & 6 \\ 0 & -1 \end{bmatrix}$

Eigenvalue $= 2, -1$

Eigenvectors $= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$$\begin{aligned} A &= PDP^{-1} \\ &= \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \end{aligned}$$

Note

This is a special case and is not always true,

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$$

If A is $n \times n$ and has n distinct eigenvalues, then A is diagonalizable.

A does not **have to** have n distinct eigenvalues for A to be diagonalizable.