Echelon Form

This matrix below in a form referred to as **row reduced echelon form**.

$$\left(\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 7 \end{array}\right)$$

A rectangular matrix is in echelon form if:

- All zero rows (if any are present) are at the bottom.
- The first non-zero entry (or leading entry) of a row is to the right of any leading entries in the row above it (if any).
- All entries below a leading entry (if any) are zero.

Matrix A is in echelon form. B is not in echelon form.

$$A = \begin{pmatrix} 2 & 0 & 1 & 1 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

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$\blacksquare$ = non-zero number, $\quad *$ = any number

$$\begin{pmatrix} 0 & \blacksquare & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & \blacksquare & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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A matrix in echelon form is in row reduced echelon form if:

- It is in echelon form
- All leading entries, if any, are equal to 1.

- Leading entries are the only nonzero entry in their respective column.

Matrix A is in RREF. B is not in RREF.

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 6 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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Each matrix is row equivalent to one and only one reduced echelon matrix.