Formulas

Credit Aarush Magic

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 + \ldots + a_n b_n$$

$$\begin{aligned} \vec{a} \times \vec{b} &= \left| \vec{i} \quad \vec{j} \quad \vec{k} a_1 \quad a_2 \quad a_3 b_1 \quad b_2 \quad b_3 \right| \\ &= (a_2 b_3 - b_2 a_3)\vec{i} - (a_1 b_3 + b_1 a_3)\vec{j} + (a_1 b_2 + b_1 a_2)\vec{k} \end{aligned}$$

Properties:

$$\begin{aligned} \vec{u} \cdot (\vec{v} \times \vec{w}) &= \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \\ \vec{a} \cdot \vec{b} &= \vec{b} \cdot \vec{a} \\ \vec{a} \times \vec{b} &= -\vec{b} \times \vec{a} \\ \vec{a} \cdot \vec{b} &= |\vec{a}| \cdot |\vec{b}| \cos(\theta) \\ |\vec{a} \times \vec{b}| &= |\vec{a}| \cdot |\vec{b}| \sin(\theta) \\ \vec{a} \cdot (\vec{b} + \vec{c}) &= \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} \\ \vec{a} \times (\vec{b} + \vec{c}) &= \vec{a} \times \vec{b} + \vec{a} \times \vec{c} \\ \vec{a} \cdot \vec{0} &= 0 \\ \vec{a} \times \vec{0} &= \vec{0} \\ (c\vec{a}) \cdot \vec{b} &= \vec{a} \cdot (c\vec{b}) \\ (c\vec{a}) \times \vec{b} &= \vec{a} \times (c\vec{b}) \\ \vec{a} \cdot \vec{a} &= ||\vec{a}||^2 \\ \vec{a} \times \vec{a} &= \vec{0} \\ \text{If } \vec{a} \perp \vec{b} \text{ then } \vec{a} \cdot \vec{b} &= 0 \\ \text{If } \vec{a} \parallel \vec{b} \text{ then } \vec{a} \times \vec{b} &= \vec{0} \\ \vec{u} \times (\vec{v} \times \vec{w}) &= (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w} \qquad nbsp; \end{aligned}$$

Graphs

$$\text{Cylinder: } ax^n + by^m = c, ax^n + bz^m = c, ay^n + bz^m = c$$

$$\text{Elliptical Paraboloid: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$

$$\text{Elliptical Cone: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$

$$\text{Ellipsoid: } \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\text{Hyperboloid of 1 sheet: } \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\text{Hyperboloid of 2 sheets: } -\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\text{Hyperbolic Paraboloid: } -\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}, c > 0$$

$$\text{Line through } P(p_0, p_1, p_2) \text{ and parallel to } \vec{v} = a\vec{i} + b\vec{j} + c\vec{k} :$$

$$x = at + p_0, y = bt + p_1, z = ct + p_2$$

$$\langle at + p_0, bt + p_1, ct + p_3 \rangle = \langle a, b, c \rangle t + \langle p_0, p_1, p_2 \rangle, -\infty < t < \infty$$

$$\text{Distance between line and point } Q : d = \frac{||\vec{PQ} \times \vec{v}||}{||\vec{v}||}$$

$$\text{Line through } P(p_0, p_1, p_2) \text{ and perpendicular to } \vec{n} = a\vec{i} + b\vec{j} + c\vec{k} :$$

$$a(x - p_0) + b(y - p_1) + c(z - p_3) = 0$$

$$\text{Angle between planes: } \theta = \cos^{-1}\left(\left|\frac{\vec{n_1} \cdot \vec{n_2}}{||\vec{n_1}|| \cdot ||\vec{n_2}||}\right|\right)$$

$$\text{Distance between Point } S \text{ and plane} : d = \left|\vec{PS} \cdot \frac{\vec{n}}{||\vec{n}||}\right|$$

$$
\begin{aligned}
\left\| \int_a^b \vec{f}(t)dt \right\| &\leq \int_a^b \|\vec{f}(t)\|dt \\
L &= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt \\
&= \int_a^b ||\vec{r}'(t)||dt \\
s(t) &= \int_{t_0}^t \sqrt{\left(\frac{dx}{d\tau}\right)^2 + \left(\frac{dy}{d\tau}\right)^2 + \left(\frac{dz}{d\tau}\right)^2} d\tau \\
&= \int_{t_0}^t ||\vec{r}'(\tau)||d\tau \\
\frac{ds}{dt} &= ||\vec{v}(t)|| \\
\vec{T}(t) &= \frac{\vec{r}'(t)}{||\vec{r}'(t)||} = \frac{\vec{v}(t)}{||\vec{v}(t)||} \\
\kappa &= \left\| \frac{d\vec{T}}{ds} \right\| = \frac{||\vec{T}'(t)||}{||\vec{r}'(t)||} = \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3} \\
p &= \frac{1}{\kappa} \\
\vec{N}(t) &= \frac{\vec{T}'(t)}{||\vec{T}'(t)||} \\
\vec{B}(t) &= \vec{T}(t) \times \vec{N}(t) \\
\vec{a} &= a_T\vec{T} + a_N\vec{N} \\
a_t &= \frac{d^2s}{dt^2} \\
&= \frac{d}{dt}||\vec{r}'(t)|| \\
a_N &= ||\vec{T}'(t)|| \cdot \frac{ds}{dt} \\
&= \kappa\left(\frac{ds}{dt}\right)^2 \\
&= \kappa||\vec{r}'(t)||^2 \\
&= \sqrt{||\vec{a}||^2 - a_T^2} \\
||\vec{a}||^2 &= a_T^2 + a_N^2 \\
\tau &= \frac{-d\vec{B}}{ds} \cdot \vec{N}'(t) \\
&= \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \dddot{x} & \dddot{y} & \dddot{z} \end{vmatrix}}{||\vec{r}'(t) \times \vec{r}''(t)||^2}
\end{aligned}
$$

$$= \frac{\vec{r}'(t) \cdot (\vec{r}''(t) \times \vec{r}'''(t))}{||\vec{r}'(t) \times \vec{r}''(t)||^2}$$

Projectile Motion:

$$\text{MaxHeight} = \frac{(v_0 \sin(\theta))^2}{2g}$$

$$\text{Range} = \frac{v_0^2 \sin(2\theta)}{g}$$

$$\text{Flighttime} = \frac{2v_0 \sin(\theta)}{g}$$

Polarandcylindricalequations:

$$\vec{u_r} = \cos\theta\vec{i} + \sin\theta\vec{j}$$

$$\vec{u_\theta} = -\sin\theta\vec{i} + \cos\theta\vec{j}$$

$$\vec{r}(t) = r\vec{u_r}$$

$$\vec{r}'(t) = \dot{r}\vec{u_r} + r\dot{\theta}\vec{u_\theta}$$

$$\vec{r}''(t) = (\ddot{r} - r\dot{\theta}^2)\vec{u_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u_\theta}$$

$$\vec{r}(t) = r\vec{u_r} + z\vec{k}$$

$$\vec{r}'(t) = \dot{r}\vec{u_r} + r\dot{\theta}\vec{u_\theta} + \dot{z}\vec{k}$$

$$\vec{r}''(t) = (\ddot{r} - r\dot{\theta}^2)\vec{u_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u_\theta} + \ddot{z}\vec{k}$$