Formulas-1

Credit Aarush Magic

$$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3 + \cdots + a_nb_n$$

$$\begin{aligned}\vec{a}\times\vec{b} &= \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}\\ &= \begin{bmatrix}u_2 & u_3\\ v_2 & v_3\end{bmatrix}\mathbf{i} - \begin{bmatrix}u_1 & u_3\\ v_1 & v_3\end{bmatrix}\mathbf{j} + \begin{bmatrix}u_1 & u_2\\ v_1 & v_2\end{bmatrix}\mathbf{k}\\ &= (a_2b_3 - b_2a_3)\vec{i} - (a_1b_3 + b_1a_3)\vec{j} + (a_1b_2 + b_1a_2)\vec{k}\end{aligned}$$

Properties:

$$\begin{aligned}\vec{u}\cdot(\vec{v}\times\vec{w}) &= \begin{vmatrix}u_1 & u_2 & u_3\\ v_1 & v_2 & v_3\\ w_1 & w_2 & w_3\end{vmatrix}\\ \vec{a}\cdot\vec{b} &= \vec{b}\cdot\vec{a}\\ \vec{a}\times\vec{b} &= -\vec{b}\times\vec{a}\\ \vec{a}\cdot\vec{b} &= |\vec{a}|\cdot|\vec{b}|\cos(\theta)\\ |\vec{a}\times\vec{b}| &= |\vec{a}|\cdot|\vec{b}|\sin(\theta)\\ \vec{a}\cdot(\vec{b}+\vec{c}) &= \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}\\ \vec{a}\times(\vec{b}+\vec{c}) &= \vec{a}\times\vec{b} + \vec{a}\times\vec{c}\\ \vec{a}\cdot\vec{0} &= 0\\ \vec{a}\times\vec{0} &= \vec{0}\\ (c\vec{a})\cdot\vec{b} &= \vec{a}\cdot(c\vec{b})\\ (c\vec{a})\times\vec{b} &= \vec{a}\times(c\vec{b})\\ \vec{a}\cdot\vec{a} &= ||\vec{a}||^2\\ \vec{a}\times\vec{a} &= \vec{0}\\ \text{If } \vec{a}\perp\vec{b} \text{ then } \vec{a}\cdot\vec{b} &= 0\\ \text{If } \vec{a}\parallel\vec{b} \text{ then } \vec{a}\times\vec{b} &= \vec{0}\\ \vec{u}\times(\vec{v}\times\vec{w}) &= (\vec{u}\cdot\vec{w})\vec{v} - (\vec{u}\cdot\vec{v})\vec{w} \qquad nbsp;\end{aligned}$$

Graphs

Cylinder: $ax^n + by^m = c, ax^n + bz^m = c, ay^n + bz^m = c$
Elliptical Paraboloid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$
Elliptical Cone: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$
Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Hyperboloid of 1 sheet: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$
Hyperboloid of 2 sheets: $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
Hyperbolic Paraboloid: $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}, c>0$

Other Formulas

Line through $P(p_1,p_2,p_3)$ and parallel to $\vec{v}=a\vec{i}+b\vec{j}+c\vec{k}$:

$$x=at+p_1 \quad y=bt+p_2 \quad z=ct+p_3$$

$$\langle at+p_1, bt+p_2, ct+p_3\rangle = \langle a,b,c\rangle t + \langle p_1,p_2,p_3\rangle \ \forall\, t\in\mathbb{R}$$

Distance between line and point:

$$d=\frac{||\vec{PQ}\times\vec{v}||}{||\vec{v}||}$$

Distance from a Point to a Plane,

$$d=\left|\vec{PS}\cdot\frac{n}{||n||}\right|$$

Projection,

$$\text{proj}_b a=\left(\frac{a\cdot b}{||b||}\right)\frac{b}{||b||}$$

Line through $P(p_1,p_2,p_3)$ and perpendicular to $\vec{n}=a\vec{i}+b\vec{j}+c\vec{k}$:

$$a(x-p_1)+b(y-p_2)+c(z-p_3)=0$$

Angle between planes:

$$\theta=\cos^{-1}\left(\left|\frac{\vec{n_1}\cdot\vec{n_2}}{||\vec{n_1}||\cdot||\vec{n_2}||}\right|\right)$$

Distance between Point S and a plane:

$$d=\left|\vec{PS}\cdot\frac{\vec{n}}{||\vec{n}||}\right|$$

The triangle property of integrals:

$$\left\|\int_a^b \vec{f}(t)dt\right\|\leq\int_a^b \|\vec{f}(t)\|dt$$

Arc Length $(s(t))$:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt \qquad = \int_a^b ||\vec{r}'(t)|| dt$$

$$s(t) = \int_{t_0}^t \sqrt{\left(\frac{dx}{d\tau}\right)^2 + \left(\frac{dy}{d\tau}\right)^2 + \left(\frac{dz}{d\tau}\right)^2} d\tau \qquad = \int_{t_0}^t ||\vec{r}'(\tau)|| d\tau$$

Speed:

$$\frac{ds}{dt} = ||\vec{v}(t)||$$

The unit tangent vector ($T(t)$):

$$\vec{T}(t) = \frac{\vec{r}'(t)}{||\vec{r}'(t)||} = \frac{\vec{v}(t)}{||\vec{v}(t)||}$$

The curvature function ($\kappa(t)$):

$$\begin{aligned}
\kappa &= \left\| \frac{d\vec{T}}{ds} \right\| T \text{ is the unit tangent vector, } s \text{ is the arc length} \\
&= \frac{||\vec{T}'(t)||}{||\vec{v}(t)||} \quad \text{note: } \frac{ds}{dt} = ||v|| \\
&= \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3}
\end{aligned}$$

Radius of curvature:

$$p = \frac{1}{\kappa}$$

Principal Normal Vector ($N(t)$):

$$\vec{N}(t) = \frac{\vec{T}'(t)}{||\vec{T}'(t)||}$$

Binormal vector ($B(t)$):

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

Projectile Motion:

$\text{Max Height} = \frac{(v_0 \sin(\theta))^2}{2g}$

$\text{Range} = \frac{v_0^2 \sin(2\theta)}{g}$

$\text{Flight Time} = \frac{2v_0 \sin(\theta)}{g}$

Formulas-2

$\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y) = L$ if for every ε>0, there exists a corresponding δ>0, such that for all (x,y) in the domain of f, $|f(x,y)-L|<\epsilon$ whenever $0<\sqrt{(x-x_0)^2+(y-y_0)^2}<\delta$

$$\frac{\partial f}{\partial x}|_{(x_0,y_0)} = \lim_{h\to 0}\frac{f(x_0+h,y_0)-f(x_0,y_0)}{h} = f_x(x_0,y_0)$$

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$$
$$f_{yy} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)$$
$$f_{xy} = \frac{\partial^2 f}{\partial yx} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$
$$f_{yx} = \frac{\partial^2 f}{\partial xy} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$
$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial w}{\partial z}\cdot\frac{dz}{dt}$$
$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

$$$$ $f'_u(x,y) = D_u f(x,y) = \lim\limits_{s\to 0}\frac{f(x+su_1,y+su_2)-f(x,y)}{s} = \nabla$