Formulas-1

Credit Aarush Magic

$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3 + \cdots + a_nb_n$$

$$\begin{aligned}\vec{a} \times \vec{b} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \\ &= \begin{bmatrix} u_2 & u_3 \\ v_2 & v_3 \end{bmatrix}\mathbf{i} - \begin{bmatrix} u_1 & u_3 \\ v_1 & v_3 \end{bmatrix}\mathbf{j} + \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix}\mathbf{k} \\ &= (a_2b_3 - b_2a_3)\vec{i} - (a_1b_3 + b_1a_3)\vec{j} + (a_1b_2 + b_1a_2)\vec{k}\end{aligned}$$

Properties:

$$\begin{aligned}\vec{u} \cdot (\vec{v} \times \vec{w}) &= \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \\ \vec{a} \cdot \vec{b} &= \vec{b} \cdot \vec{a} \\ \vec{a} \times \vec{b} &= -\vec{b} \times \vec{a} \\ \vec{a} \cdot \vec{b} &= |\vec{a}| \cdot |\vec{b}| \cos(\theta) \\ |\vec{a} \times \vec{b}| &= |\vec{a}| \cdot |\vec{b}| \sin(\theta) \\ \vec{a} \cdot (\vec{b} + \vec{c}) &= \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} \\ \vec{a} \times (\vec{b} + \vec{c}) &= \vec{a} \times \vec{b} + \vec{a} \times \vec{c} \\ \vec{a} \cdot \vec{0} &= 0 \\ \vec{a} \times \vec{0} &= \vec{0} \\ (c\vec{a}) \cdot \vec{b} &= \vec{a} \cdot (c\vec{b}) \\ (c\vec{a}) \times \vec{b} &= \vec{a} \times (c\vec{b}) \\ \vec{a} \cdot \vec{a} &= ||\vec{a}||^2 \\ \vec{a} \times \vec{a} &= \vec{0} \\ \text{If } \vec{a} \perp \vec{b} \text{ then } \vec{a} \cdot \vec{b} &= 0 \\ \text{If } \vec{a} \parallel \vec{b} \text{ then } \vec{a} \times \vec{b} &= \vec{0} \\ \vec{u} \times (\vec{v} \times \vec{w}) &= (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w} \qquad nbsp;\end{aligned}$$

Projectile Motion:

$\text{Max Height} = \frac{(v_0 \sin(\theta))^2}{2g}$

$\text{Range} = \frac{v_0^2 \sin(2\theta)}{g}$

$\text{Flight Time} = \frac{2v_0 \sin(\theta)}{g}$

Graphs

Type	Equaions
Cylinder	$ax^n + by^m = c, ax^n + bz^m = c, ay^n + bz^m = c$
Elliptical Paraboloid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$
Elliptical Cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Hyperboloid of 1 sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
Hyperboloid of 2 sheets	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Hyperbolic Paraboloid	$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}, c > 0$

Other Formulas

Line through $P(p_1, p_2, p_3)$ and parallel to $\vec{v} = a\vec{i} + b\vec{j} + c\vec{k}$ when $t \in \mathbb{R}$:

$$x = at + p_1 \quad y = bt + p_2 \quad z = ct + p_3$$

$$\langle at + p_1, bt + p_2, ct + p_3 \rangle = \langle a, b, c \rangle t + \langle p_1, p_2, p_3 \rangle$$

Line through $P(p_1, p_2, p_3)$ and perpendicular to $\vec{n} = a\vec{i} + b\vec{j} + c\vec{k}$:

$$a(x - p_1) + b(y - p_2) + c(z - p_3) = 0$$

Distance between line and point:

$$d = \frac{||\vec{PS} \times v||}{||v||}$$

Distance from a Point to a Plane,

$$d = \left| \vec{PS} \cdot \frac{n}{||n||} \right|$$

Projection,

$$\text{proj}_b a = \left(\frac{a \cdot b}{||b||} \right) \frac{b}{||b||}$$

Angle between planes or vectors:

$$\theta = \cos^{-1} \left(\left| \frac{\vec{n_1} \cdot \vec{n_2}}{||\vec{n_1}|| \cdot ||\vec{n_2}||} \right| \right)$$

Distance between Point S and a plane:

$$d = \left| \vec{PS} \cdot \frac{\vec{n}}{||\vec{n}||} \right|$$

The triangle property of integrals:

$$\left\| \int_a^b \vec{f}(t)dt \right\| \leq \int_a^b \|\vec{f}(t)\|dt$$

Arc Length $(s(t))$:

$$\begin{aligned} L &= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt \qquad &= \int_a^b ||\vec{r}'(t)||dt \\ s(t) &= \int_{t_0}^t \sqrt{\left(\frac{dx}{d\tau}\right)^2 + \left(\frac{dy}{d\tau}\right)^2 + \left(\frac{dz}{d\tau}\right)^2} d\tau \qquad &= \int_{t_0}^t ||\vec{r}'(\tau)||d\tau \end{aligned}$$

Speed:

$$\frac{ds}{dt} = ||\vec{v}(t)||$$

The unit tangent vector $(T(t))$:

$$\vec{T}(t) = \frac{\vec{r}'(t)}{||\vec{r}'(t)||} = \frac{\vec{v}(t)}{||\vec{v}(t)||}$$

The curvature function $(\kappa(t))$:

$$\begin{aligned} \kappa &= \left\| \frac{d\vec{T}}{ds} \right\| T \text{ is the unit tangent vector, } s \text{ is the arc length} \\ &= \frac{||\vec{T}'(t)||}{||\vec{v}(t)||} \quad \text{note: } \frac{ds}{dt} = ||v|| \\ &= \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3} \end{aligned}$$

Radius of curvature:

$$p = \frac{1}{\kappa}$$

Principal Normal Vector $(N(t))$:

$$\vec{N}(t) = \frac{\vec{T}'(t)}{||\vec{T}'(t)||}$$

Binormal vector $(B(t))$:

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

Formulas-2

$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$ if for every ε>0, there exists a corresponding δ>0, such that for all (x,y) in the domain of f, $|f(x,y) - L| < \epsilon$ whenever $0 < \sqrt{(x-x_0)^2 + (y-y_0)^2} < \delta$

$$\frac{\partial f}{\partial x}|_{(x_0,y_0)} = \lim_{h\to 0} \frac{f(x_0+h, y_0) - f(x_0,y_0)}{h} = f_x(x_0,y_0)$$

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$$
$$f_{yy} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)$$
$$f_{xy} = \frac{\partial^2 f}{\partial yx} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$
$$f_{yx} = \frac{\partial^2 f}{\partial xy} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$
$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial w}{\partial z}\cdot\frac{dz}{dt}$$
$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

$$f'_u(x,y) = D_u f(x,y) = \lim_{s\to 0}\frac{f(x+su_1, y+su_2) - f(x,y)}{s} = \nabla\ \mathrm{f}(x,y)\cdot u$$

$$\nabla\ f(x,y,z) = \frac{\partial f}{\partial x}i + \frac{\partial f}{\partial y}j + \frac{\partial f}{\partial z}k$$

Tangent line to a level curve: $f_x(x-x_0) + f_y(y-y_0) = 0$

$$\frac{d}{dt}(f(r(t)) = \nabla\ \mathrm{f}(r(t)) \bullet\ r'(t)$$