

Formulas-3

Fubini's Theorem

$$V = \iint_R f(x, y) dA = \int_c^d \int_a^b f(x, y) dx dy = \int_a^b \int_c^d f(x, y) dy dx$$

if R is defined by $a \leq x \leq b$ and $g_1(x) \leq y \leq g_2(x)$ then,

$$V = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) dy dx$$

Notice: The functions $g_n(x)$ are evaluated first.

Properties

$$\begin{aligned} \iint_R c f(x, y) dA &= c \iint_R f(x, y) dA \\ \iint_R f(x, y) \pm g(x, y) dA &= \iint_R f(x, y) dA \pm \iint_R g(x, y) dA \\ \iint_R f(x, y) dA &\geq 0 \text{ if } f(x, y) \geq 0 \text{ on } R \\ \iint_R f(x, y) dA &\geq \iint_R g(x, y) dA \text{ if } f(x, y) \geq g(x, y) \text{ on } R \end{aligned}$$

If R_1 and R_2 are non-overlapping regions,

$$\iint_R f(x, y) dA = \iint_{R_1} f(x, y) dA + \iint_{R_2} f(x, y) dA$$

$$A = \iint_R dA = \int_{x_1}^{x_2} \int_{y_1}^{y_2} dy dx = \int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} r dr d\theta$$

$$V = \iiint_R f(x, y) dA$$

$$\text{Average Value} = \frac{1}{\text{Area of } R} \iint_R f dA \quad \text{OR} \quad \frac{1}{\text{Volume of } D} \iiint_D F dV$$

$$V = \iiint_D dV = \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} dz dy dx = \int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} \int_{z_1}^{z_2} r dz dr d\theta = \int_{\rho_1}^{\rho_2} \int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} \rho^2 \sin(\phi) d\theta d\phi d\rho$$

Mass and First Moments of 3D Solids

$$\text{Mass: } M = \iiint_D \delta dV$$

First Moments: $M_{yz} = \iiint_D x \delta dV, M_{xz} = \iiint_D y \delta dV, M_{xy} = \iiint_D z \delta dV$

Center of mass: $\bar{x} = \frac{M_{yz}}{M}, \bar{y} = \frac{M_{xz}}{M}, \bar{z} = \frac{M_{xy}}{M}$

Mass and First Moments of 2D Plates:

Mass: $M = \iint_R \delta dA$

First Moments: $M_y = \iint_R x \delta dA, M_x = \iint_R y \delta dA$

Center of mass: $\bar{x} = \frac{M_y}{M}, \bar{y} = \frac{M_x}{M}$

Moments of Inertia of 3D Solids:

About x-axis: $I_x = \iiint_D (y^2 + z^2) \delta dV$

About y-axis: $I_y = \iiint_D (x^2 + z^2) \delta dV$

About z-axis: $I_z = \iiint_D (x^2 + y^2) \delta dV$

About a line L: $I_L = \iiint_D r^2(x, y, z) \delta dV$

Moments of Inertia of 2D Plates:

About x-axis: $I_x = \iint_R y^2 \delta dA$

About y-axis: $I_y = \iint_R x^2 \delta dA$

About a line L: $I_L = \iint_R r^2(x, y) \delta dA$

About the origin: $I_O = \iint_R (x^2 + y^2) \delta dA = I_x + I_y$

Joint probability density function:

Conditions:

$$f(x, y) \geq 0$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$$

$$P((X, Y) \in R) = \iint_R f(x, y) dx dy$$

Mean and expected value:

$$\mu_X = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x f(x, y) dx dy$$

$$\mu_Y = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y f(x, y) dx dy$$

Spherical Coordinates (ρ, ϕ, θ)

$$r = \rho \sin(\phi),$$

$$x = r \cos(\theta) = \rho \sin(\phi) \cos(\theta)$$

$$z = \rho \cos(\phi),$$

$$y = r \sin(\theta) = \rho \sin(\phi) \sin(\theta)$$

$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2}$$

Jacobian:

$$J(u, v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial(x, y)}{\partial(u, v)}$$

$$\iint_R f(x, y) dx dy = \iint_G f(g(u, v), h(u, v)) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| du dv$$

$$J(u, v, w) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \frac{\partial(x, y, z)}{\partial(u, v, w)}$$

$$\iiint_D f(x, y, z) dx dy dz = \iiint_B f(g(u, v, w), h(u, v, w), k(u, v, w)) \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| du dv dw$$