

Formulas-1

Credit Aarush Magic

$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3 + \cdots + a_nb_n$$

$$\begin{aligned}\vec{a} \times \vec{b} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \\ &= \begin{bmatrix} u_2 & u_3 \\ v_2 & v_3 \end{bmatrix} \mathbf{i} - \begin{bmatrix} u_1 & u_3 \\ v_1 & v_3 \end{bmatrix} \mathbf{j} + \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix} \mathbf{k} \\ &= (a_2b_3 - b_2a_3)\vec{i} - (a_1b_3 + b_1a_3)\vec{j} + (a_1b_2 + b_1a_2)\vec{k}\end{aligned}$$

Properties:

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos(\theta)$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin(\theta)$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\vec{a} \cdot \vec{0} = 0$$

$$\vec{a} \times \vec{0} = \vec{0}$$

$$(c\vec{a}) \cdot \vec{b} = \vec{a} \cdot (c\vec{b})$$

$$(c\vec{a}) \times \vec{b} = \vec{a} \times (c\vec{b})$$

$$\vec{a} \cdot \vec{a} = ||\vec{a}||^2$$

$$\vec{a} \times \vec{a} = \vec{0}$$

$$\text{If } \vec{a} \perp \vec{b} \text{ then } \vec{a} \cdot \vec{b} = 0$$

$$\text{If } \vec{a} \parallel \vec{b} \text{ then } \vec{a} \times \vec{b} = \vec{0}$$

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w} \quad \text{nbsp;}$$

Graphs

$$\text{Cylinder: } ax^n + by^m = c, ax^n + bz^m = c, ay^n + bz^m = c$$

$$\text{Elliptical Paraboloid: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$

$$\text{Elliptical Cone: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$

$$\text{Ellipsoid: } \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Hyperboloid of 1 sheet: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

Hyperboloid of 2 sheets: $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Hyperbolic Paraboloid: $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}, c > 0$

Other Formulas

Line through $P(p_1, p_2, p_3)$ and parallel to $\vec{v} = a\vec{i} + b\vec{j} + c\vec{k}$:

$$x = at + p_1 \quad y = bt + p_2 \quad z = ct + p_3$$

$$\langle at + p_1, bt + p_2, ct + p_3 \rangle = \langle a, b, c \rangle t + \langle p_1, p_2, p_3 \rangle \quad \forall t \in \mathbb{R}$$

Distance between line and point Q : $d = \frac{\|\vec{PQ} \times \vec{v}\|}{\|\vec{v}\|}$

Line through $P(p_1, p_2, p_3)$ and perpendicular to $\vec{n} = a\vec{i} + b\vec{j} + c\vec{k}$:

$$a(x - p_1) + b(y - p_2) + c(z - p_3) = 0$$

Angle between planes: $\theta = \cos^{-1} \left(\left| \frac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \cdot \|\vec{n}_2\|} \right| \right)$

Distance between Point S and a plane: $d = \left| \vec{PS} \cdot \frac{\vec{n}}{\|\vec{n}\|} \right|$

The triangle property of integrals: $\left\| \int_a^b \vec{f}(t) dt \right\| \leq \int_a^b \|\vec{f}(t)\| dt$

Arc Length, $s(t)$ is the function for arc length:

$$\begin{aligned} L &= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt &= \int_a^b \|\vec{r}'(t)\| dt \\ s(t) &= \int_{t_0}^t \sqrt{\left(\frac{dx}{d\tau}\right)^2 + \left(\frac{dy}{d\tau}\right)^2 + \left(\frac{dz}{d\tau}\right)^2} d\tau &= \int_{t_0}^t \|\vec{r}'(\tau)\| d\tau \end{aligned}$$

Speed:

$$\frac{ds}{dt} = \|\vec{v}(t)\|$$

The unit tangent vector:

$$\vec{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \frac{\vec{v}(t)}{\|\vec{v}(t)\|}$$

The curvature function:

$$\begin{aligned}
\kappa &= \left\| \frac{d\vec{T}}{ds} \right\| \quad T \text{ is the unit tangent vector, } s \text{ is the arc length} \\
&= \frac{\|\vec{T}'(t)\|}{\|\vec{v}(t)\|} \quad \text{note: } \frac{ds}{dt} = v \\
&= \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|^3} \\
\kappa &= \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|^3} \\
\kappa &= \left\| \frac{d\vec{T}}{ds} \right\| = \frac{\|\vec{T}'(t)\|}{\|\vec{r}'(t)\|} = \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|^3}
\end{aligned}$$

Radius of curvature:

$$p = \frac{1}{\kappa}$$

Principal Normal Vector:

$$\vec{N}(t) = \frac{\vec{T}'(t)}{\|\vec{T}'(t)\|}$$

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

$$\vec{a} = a_T \vec{T} + a_N \vec{N}$$

$$a_t = \frac{d^2 s}{dt^2} = \frac{d}{dt} \|\vec{r}'(t)\|$$

$$a_N = \|\vec{T}'(t)\| \cdot \frac{ds}{dt} = \kappa \left(\frac{ds}{dt} \right)^2 = \kappa \|\vec{r}'(t)\|^2 = \sqrt{\|\vec{a}\|^2 - a_T^2}$$

Projectile Motion:

$$\text{Max Height} = \frac{(v_0 \sin(\theta))^2}{2g}$$

$$\text{Range} = \frac{v_0^2 \sin(2\theta)}{g}$$

$$\text{Flight Time} = \frac{2v_0 \sin(\theta)}{g}$$

Formulas-2