Formulas

Credit Aarush Magic

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 + \cdots + a_n b_n$$

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \\ &= \begin{bmatrix} u_2 & u_3 \\ v_2 & v_3 \end{bmatrix} \mathbf{i} - \begin{bmatrix} u_1 & u_3 \\ v_1 & v_3 \end{bmatrix} \mathbf{j} + \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix} \mathbf{k} \\ &= (a_2 b_3 - b_2 a_3) \vec{i} - (a_1 b_3 + b_1 a_3) \vec{j} + (a_1 b_2 + b_1 a_2) \vec{k} \end{aligned}$$

Properties:

$$\begin{gathered} \vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \\ \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} \\ \vec{a} \times \vec{b} = -\vec{b} \times \vec{a} \\ \vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos(\theta) \\ |\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin(\theta) \\ \vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} \\ \vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c} \\ \vec{a} \cdot \vec{0} = 0 \\ \vec{a} \times \vec{0} = \vec{0} \\ (c\vec{a}) \cdot \vec{b} = \vec{a} \cdot (c\vec{b}) \\ (c\vec{a}) \times \vec{b} = \vec{a} \times (c\vec{b}) \\ \vec{a} \cdot \vec{a} = ||\vec{a}||^2 \\ \vec{a} \times \vec{a} = \vec{0} \\ \text{If } \vec{a} \perp \vec{b} \text{ then } \vec{a} \cdot \vec{b} = 0 \\ \text{If } \vec{a} \parallel \vec{b} \text{ then } \vec{a} \times \vec{b} = \vec{0} \\ \vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w}) \vec{v} - (\vec{u} \cdot \vec{v}) \vec{w} \qquad nbsp; \end{gathered}$$

Graphs

Cylinder: $ax^n + by^m = c, ax^n + bz^m = c, ay^n + bz^m = c$

Elliptical Paraboloid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$

Elliptical Cone: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$
Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Hyperboloid of 1 sheet: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
Hyperboloid of 2 sheets: $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Hyperbolic Paraboloid: $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}, c > 0$

Line through $P(p_0, p_1, p_2)$ and parallel to $\vec{v} = a\vec{i} + b\vec{j} + c\vec{k}$:

$$x = at + p_0, y = bt + p_1, z = ct + p_2$$

$$\langle at + p_0, bt + p_1, ct + p_3 \rangle = \langle a, b, c \rangle t + \langle p_0, p_1, p_2 \rangle \forall -\infty < t < \infty$$

Distance between line and point $Q : d = \frac{||\vec{PQ} \times \vec{v}||}{||\vec{v}||}$
Line through $P(p_0, p_1, p_2)$ and perpendicular to $\vec{n} = a\vec{i} + b\vec{j} + c\vec{k}$:

$$a(x - p_0) + b(y - p_1) + c(z - p_3) = 0$$

Angle between planes: $\theta = \cos^{-1}\left(\left|\frac{\vec{n}_1 \cdot \vec{n}_2}{||\vec{n}_1|| \cdot ||\vec{n}_2||}\right|\right)$
Distance between Point S and plane: $d = \left|\vec{PS} \cdot \frac{\vec{n}}{||\vec{n}||}\right|$

$$\left\| \int_a^b \vec{f}(t) dt \right\| \leq \int_a^b \|\vec{f}(t)\| dt$$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt = \int_a^b ||\vec{r}'(t)|| dt$$

$$s(t) = \int_{t_0}^t \sqrt{\left(\frac{dx}{d\tau}\right)^2 + \left(\frac{dy}{d\tau}\right)^2 + \left(\frac{dz}{d\tau}\right)^2} d\tau = \int_{t_0}^t ||\vec{r}'(\tau)|| d\tau$$

$$\frac{ds}{dt} = ||\vec{v}(t)||$$

$$\vec{T}(t) = \frac{\vec{r}'(t)}{||\vec{r}'(t)||} = \frac{\vec{v}(t)}{||\vec{v}(t)||}$$

$$\kappa = \left\| \frac{d\vec{T}}{ds} \right\| = \frac{||\vec{T}'(t)||}{||\vec{r}'(t)||} = \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3}$$

$$p = \frac{1}{\kappa}$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{||\vec{T}'(t)||}$$

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

$$\vec{a} = a_T\vec{T} + a_N\vec{N}$$

$$a_t = \frac{d^2s}{dt^2} = \frac{d}{dt}||\vec{r}'(t)||$$

$$a_N = ||\vec{T}'(t)|| \cdot \frac{ds}{dt} = \kappa\left(\frac{ds}{dt}\right)^2 = \kappa||\vec{r}'(t)||^2 = \sqrt{||\vec{a}||^2 - a_T^2}$$

$$||\vec{a}||^2 = a_T^2 + a_N^2$$

$$\tau = \frac{-d\vec{B}}{ds} \cdot \vec{N}'(t) = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \dddot{x} & \dddot{y} & \dddot{z} \end{vmatrix}}{||\vec{r}'(t) \times \vec{r}''(t)||^2} = \frac{\vec{r}'(t) \cdot (\vec{r}''(t) \times \vec{r}'''(t))}{||\vec{r}'(t) \times \vec{r}''(t)||^2}$$

Projectile Motion:

Max Height= $\frac{(v_0\sin(\theta))^2}{2g}$
Range= $\frac{v_0^2\sin(2\theta)}{g}$
Flight time= $\frac{2v_0\sin(\theta)}{g}$

Polar and cylindrical equations:

$\vec{u_r} = \cos\theta\vec{i} + \sin\theta\vec{j}$

$\vec{u_\theta} = -\sin\theta\vec{i} + \cos\theta\vec{j}$

$\vec{r}(t) = r\vec{u_r}$

$\vec{r}'(t) = \dot{r}\vec{u_r} + r\dot{\theta}\vec{u_\theta}$

$\vec{r}''(t) = (\ddot{r} - r\dot{\theta}^2)\vec{u_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u_\theta}$

$\vec{r}(t) = r\vec{u_r} + z\vec{k}$

$\vec{r}'(t) = \dot{r}\vec{u_r} + r\dot{\theta}\vec{u_\theta} + \dot{z}\vec{k}$

$\vec{r}''(t) = (\ddot{r} - r\dot{\theta}^2)\vec{u_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u_\theta} + \ddot{z}\vec{k}$