Formulas-1

Credit Aarush Magic

$$ec{a} \cdot ec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 + \cdots + a_n b_n \ ec{a} imes ec{b} = egin{bmatrix} ec{i} & ec{j} & ec{k} \ a_1 & a_2 & a_3 \ b_1 & b_2 & b_3 \ \end{bmatrix} \ = egin{bmatrix} u_2 & u_3 \ v_2 & v_3 \end{bmatrix} \mathbf{i} - egin{bmatrix} u_1 & u_3 \ v_1 & v_3 \end{bmatrix} \mathbf{j} + egin{bmatrix} u_1 & u_2 \ v_1 & v_2 \end{bmatrix} \mathbf{k} \ = (a_2 b_3 - b_2 a_3) ec{i} - (a_1 b_3 + b_1 a_3) ec{j} + (a_1 b_2 + b_1 a_2) ec{k} \ \end{bmatrix}$$

Properties:

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos(\theta)$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin(\theta)$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times (\vec{b} + \vec{a} \times \vec{c})$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times (\vec{b} + \vec{c})$$

$$\vec{a} \times \vec{b} = \vec{a} \times (\vec{c} \vec{b})$$

$$(c\vec{a}) \cdot \vec{b} = \vec{a} \cdot (c\vec{b})$$

$$(c\vec{a}) \times \vec{b} = \vec{a} \times (c\vec{b})$$

$$\vec{a} \cdot \vec{a} = ||\vec{a}||^2$$

$$\vec{a} \times \vec{a} = \vec{0}$$
If $\vec{a} \perp \vec{b}$ then $\vec{a} \times \vec{b} = \vec{0}$

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w}) \vec{v} - (\vec{u} \cdot \vec{v}) \vec{w} \qquad nbsp;$$

Graphs

Cylinder: $ax^n + by^m = c$, $ax^n + bz^m = c$, $ay^n + bz^m = c$

Elliptical Paraboloid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$

Elliptical Cone: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$ Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Hyperboloid of 1 sheet: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ Hyperboloid of 2 sheets: $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Hyperbolic Paraboloid: $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c},c>0$

Other Formulas

Line through $P(p_1,p_2,p_3)$ and parallel to $ec{v}=aec{i}+bec{j}+cec{k}$:

$$egin{aligned} x = at + p_1 \quad y = bt + p_2 \quad z = ct + p_3 \ & \langle at + p_1, bt + p_2, ct + p_3
angle = \langle a, b, c
angle t + \langle p_1, p_2, p_3
angle \ orall \ t \in \mathbb{R} \end{aligned}$$

Distance between line and point $Q:d=rac{||ec{PQ} imesec{v}||}{||ec{v}||}$

Line through $P(p_1,p_2,p_3)$ and perpendicular to $ec{n}=aec{i}+bec{j}+cec{k}$:

$$a(x-p_1) + b(y-p_2) + c(z-p_3) = 0$$

Angle between planes: $heta = \cos^{-1}\left(\left| \frac{ec{n_1} \cdot ec{n_2}}{||ec{n_1}|| \cdot ||ec{n_2}||} \right|
ight)$

Distance between Point S and a plane: $d = \left| \vec{PS} \cdot \frac{\vec{n}}{||\vec{n}||} \right|$

The triangle property of integrals: $\left\|\int_a^b \vec{f}(t)dt\right\| \leq \int_a^b \|\vec{f}(t)\|dt$

Arc Length, s(t) is the function for arc length:

$$L = \int_a^b \sqrt{\left(rac{dx}{dt}
ight)^2 + \left(rac{dy}{dt}
ight)^2 + \left(rac{dz}{dt}
ight)^2} dt \qquad = \int_a^b ||ec{r}'(t)|| dt \ s(t) = \int_{t_0}^t \sqrt{\left(rac{dx}{d au}
ight)^2 + \left(rac{dy}{d au}
ight)^2 + \left(rac{dz}{d au}
ight)^2} d au \qquad = \int_{t_0}^t ||ec{r}'(au)|| d au$$

Speed:

$$rac{ds}{dt} = ||ec{v}(t)||$$

The unit tangent vector:

$$ec{T}(t) = rac{ec{r}'(t)}{||ec{r}'(t)||} = rac{ec{v}(t)}{||ec{v}(t)||}$$

The curvature function:

$$\kappa = \left\| \frac{d\vec{T}}{ds} \right\| T \text{ is the unit tangent vector, } s \text{ is the arc length}$$

$$= \frac{||\vec{T}'(t)||}{||\vec{v}(t)||} \quad \text{note: } \frac{ds}{dt} = v$$

$$= \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3}$$

$$\kappa = \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3}$$

$$\kappa = \left\| \frac{d\vec{T}}{ds} \right\| = \frac{||\vec{T}'(t)||}{||\vec{r}'(t)||} = \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3}$$

Radius of curvature:

$$p=rac{1}{\kappa}$$

Principal Normal Vector:

$$ec{N}(t) = rac{ec{T}'(t)}{||ec{T}'(t)||} \ ec{B}(t) = ec{T}(t) imes ec{N}(t) \ ec{a} = a_T ec{T} + a_N ec{N} \ a_t = rac{d^2 s}{dt^2} = rac{d}{dt} ||ec{r}'(t)|| \ a_N = ||ec{T}'(t)|| \cdot rac{ds}{dt} = \kappa igg(rac{ds}{dt}igg)^2 = \kappa ||ec{r}'(t)||^2 = \sqrt{||ec{a}||^2 - a_T^2} \ ||ec{a}||^2 = a_T^2 + a_N^2$$

Projectile Motion:

$$egin{aligned} ext{Max Height} &= rac{(v_0\sin(heta))^2}{2g} \ ext{Range} &= rac{v_0^2\sin(2 heta)}{g} \ ext{Flight Time} &= rac{2v_0\sin(heta)}{g} \end{aligned}$$