Formulas-1

Credit Aarush Magic

$$\begin{aligned}
\vec{a} \cdot \vec{b} &= a_1b_1 + a_2b_2 + a_3b_3 + \cdots + a_nb_n \\
\vec{a} \times \vec{b} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \\
&= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & n_2 \end{vmatrix} \mathbf{k} \\
&= (a_2b_3 - b_2a_3)\vec{i} - (a_1b_3 + b_1a_3)\vec{j} + (a_1b_2 + b_1a_2)\vec{k}
\end{aligned}$$

Properties:

$$\begin{aligned}
\vec{u} \cdot (\vec{v} \times \vec{w}) &= \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \\
\vec{a} \cdot \vec{b} &= \vec{b} \cdot \vec{a} \\
\vec{a} \times \vec{b} &= -\vec{b} \times \vec{a} \\
\vec{a} \cdot \vec{b} &= |\vec{a}| \cdot |\vec{b}| \cos(\theta) \\
|\vec{a} \times \vec{b}| &= |\vec{a}| \cdot |\vec{b}| \sin(\theta) \\
\vec{a} \cdot (\vec{b} + \vec{c}) &= \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} \\
\vec{a} \times (\vec{b} + \vec{c}) &= \vec{a} \times \vec{b} + \vec{a} \times \vec{c} \\
\vec{a} \cdot \vec{0} &= 0 \\
\vec{a} \times \vec{0} &= \vec{0} \\
(c\vec{a}) \cdot \vec{b} &= \vec{a} \cdot (c\vec{b}) \\
(c\vec{a}) \times \vec{b} &= \vec{a} \times (c\vec{b}) \\
\vec{a} \cdot \vec{a} &= ||\vec{a}||^2 \\
\vec{a} \times \vec{a} &= \vec{0} \\
\text{If } \vec{a} \perp \vec{b} \text{ then } \vec{a} \cdot \vec{b} &= 0 \\
\text{If } \vec{a} \parallel \vec{b} \text{ then } \vec{a} \times \vec{b} &= \vec{0} \\
\vec{u} \times (\vec{v} \times \vec{w}) &= (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w} \qquad nbsp;
\end{aligned}$$

Projectile Motion:

$\text{Max Height} = \frac{(v_0 \sin(\theta))^2}{2g}$
$\text{Range} = \frac{v_0^2 \sin(2\theta)}{g}$
$\text{Flight Time} = \frac{2v_0 \sin(\theta)}{g}$

Graphs

Type	Equaions
Elliptical Paraboloid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}$
Elliptical Cone	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$
Ellipsoid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
Hyperboloid of 1 sheet	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$
Hyperboloid of 2 sheets	$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
Hyperbolic Paraboloid	$-\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}, c>0$

Other Formulas

Line through $P(p_1,p_2,p_3)$ and parallel to $\vec{v}=a\vec{i}+b\vec{j}+c\vec{k}$ when $t\in\mathbb{R}$:

$$x=at+p_1 \quad y=bt+p_2 \quad z=ct+p_3$$

$$\langle at+p_1, bt+p_2, ct+p_3\rangle=\langle a,b,c\rangle t+\langle p_1,p_2,p_3\rangle$$

Line through $P(p_1,p_2,p_3)$ and perpendicular to $\vec{n}=a\vec{i}+b\vec{j}+c\vec{k}$:

$$a(x-p_1)+b(y-p_2)+c(z-p_3)=0$$

Distance between line and point:

$$d=\frac{||\vec{PS}\times v||}{||v||}$$

Distance between Point S and a Plane,

$$d=\left|\vec{PS}\cdot\frac{n}{||n||}\right|$$

Projection,

$$\text{proj}_b a=\left(\frac{a\cdot b}{||b||}\right)\frac{b}{||b||}$$

Angle between planes or vectors:

$$\theta=\cos^{-1}\left(\left|\frac{\vec{n_1}\cdot\vec{n_2}}{||\vec{n_1}||\cdot||\vec{n_2}||}\right|\right)$$

Arc Length $(s(t))$:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt \qquad = \int_a^b ||\vec{r}'(t)|| dt$$

$$s(t) = \int_{t_0}^t \sqrt{\left(\frac{dx}{d\tau}\right)^2 + \left(\frac{dy}{d\tau}\right)^2 + \left(\frac{dz}{d\tau}\right)^2} d\tau \qquad = \int_{t_0}^t ||\vec{r}'(\tau)|| d\tau$$

Speed:

$$\frac{ds}{dt} = ||\vec{v}(t)||$$

The unit tangent vector $(T(t))$:

$$\vec{T}(t) = \frac{\vec{r}'(t)}{||\vec{r}'(t)||} = \frac{\vec{v}(t)}{||\vec{v}(t)||}$$

The curvature function $(\kappa(t))$:

$$\begin{aligned} \kappa &= \left\| \frac{d\vec{T}}{ds} \right\| \\ &= \frac{||\vec{T}'(t)||}{||\vec{v}(t)||} \\ &= \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3} \end{aligned}$$

Radius of curvature:

$$p = \frac{1}{\kappa}$$

Principal Normal Vector $(N(t))$:

$$\vec{N}(t) = \frac{\vec{T}'(t)}{||\vec{T}'(t)||}$$

Binormal vector $(B(t))$:

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$