Formulas-1

Credit Aarush Magic

$$\vec{a}\cdot\vec{b}=a_1b_1+a_2b_2+a_3b_3+\cdots+a_nb_n$$

$$\begin{aligned}\vec{a}\times\vec{b}&=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}\\ &=\begin{bmatrix}u_2 & u_3\\ v_2 & v_3\end{bmatrix}\mathbf{i}-\begin{bmatrix}u_1 & u_3\\ v_1 & v_3\end{bmatrix}\mathbf{j}+\begin{bmatrix}u_1 & u_2\\ v_1 & v_2\end{bmatrix}\mathbf{k}\\ &=(a_2b_3-b_2a_3)\vec{i}-(a_1b_3+b_1a_3)\vec{j}+(a_1b_2+b_1a_2)\vec{k}\end{aligned}$$

Properties:

$$\begin{aligned}\vec{u}\cdot(\vec{v}\times\vec{w})&=\begin{vmatrix}u_1 & u_2 & u_3\\ v_1 & v_2 & v_3\\ w_1 & w_2 & w_3\end{vmatrix}\\ \vec{a}\cdot\vec{b}&=\vec{b}\cdot\vec{a}\\ \vec{a}\times\vec{b}&=-\vec{b}\times\vec{a}\\ \vec{a}\cdot\vec{b}&=|\vec{a}|\cdot|\vec{b}|\cos(\theta)\\ |\vec{a}\times\vec{b}|&=|\vec{a}|\cdot|\vec{b}|\sin(\theta)\\ \vec{a}\cdot(\vec{b}+\vec{c})&=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}\\ \vec{a}\times(\vec{b}+\vec{c})&=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}\\ \vec{a}\cdot\vec{0}&=0\\ \vec{a}\times\vec{0}&=\vec{0}\\ (c\vec{a})\cdot\vec{b}&=\vec{a}\cdot(c\vec{b})\\ (c\vec{a})\times\vec{b}&=\vec{a}\times(c\vec{b})\\ \vec{a}\cdot\vec{a}&=||\vec{a}||^2\\ \vec{a}\times\vec{a}&=\vec{0}\\ \text{If }\vec{a}\perp\vec{b}\text{ then }\vec{a}\cdot\vec{b}&=0\\ \text{If }\vec{a}\parallel\vec{b}\text{ then }\vec{a}\times\vec{b}&=\vec{0}\\ \vec{u}\times(\vec{v}\times\vec{w})&=(\vec{u}\cdot\vec{w})\vec{v}-(\vec{u}\cdot\vec{v})\vec{w}\qquad nbsp;\end{aligned}$$

Graphs

Cylinder: $ax^n+by^m=c, ax^n+bz^m=c, ay^n+bz^m=c$
Elliptical Paraboloid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}$
Elliptical Cone: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$
Ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Hyperboloid of 1 sheet: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$
Hyperboloid of 2 sheets: $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
Hyperbolic Paraboloid: $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}, c>0$

Other Formulas

Line through $P(p_1,p_2,p_3)$ and parallel to $\vec{v}=a\vec{i}+b\vec{j}+c\vec{k}$:

$$x=at+p_1 \quad y=bt+p_2 \quad z=ct+p_3$$

$$\langle at+p_1, bt+p_2, ct+p_3\rangle = \langle a,b,c\rangle t + \langle p_1,p_2,p_3\rangle \;\forall\, t\in\mathbb{R}$$

Distance between line and point $Q: d=\dfrac{||\vec{PQ}\times\vec{v}||}{||\vec{v}||}$

Line through $P(p_1,p_2,p_3)$ and perpendicular to $\vec{n}=a\vec{i}+b\vec{j}+c\vec{k}:$

$$a(x-p_1)+b(y-p_2)+c(z-p_3)=0$$

Angle between planes: $\theta=\cos^{-1}\left(\left|\dfrac{\vec{n_1}\cdot\vec{n_2}}{||\vec{n_1}||\cdot||\vec{n_2}||}\right|\right)$

Distance between Point S and a plane: $d=\left|\vec{PS}\cdot\dfrac{\vec{n}}{||\vec{n}||}\right|$

The triangle property of integrals: $\left\|\int_a^b \vec{f}(t)dt\right\|\le\int_a^b\|\vec{f}(t)\|dt$

Arc Length, $s(t)$ is the function for arc length:

$$L=\int_a^b\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}dt \qquad =\int_a^b ||\vec{r}'(t)||dt$$

$$s(t)=\int_{t_0}^t\sqrt{\left(\frac{dx}{d\tau}\right)^2+\left(\frac{dy}{d\tau}\right)^2+\left(\frac{dz}{d\tau}\right)^2}d\tau \qquad =\int_{t_0}^t ||\vec{r}'(\tau)||d\tau$$

Speed:

$$\frac{ds}{dt}=||\vec{v}(t)||$$

The unit tangent vector:

$$\vec{T}(t)=\frac{\vec{r}'(t)}{||\vec{r}'(t)||}=\frac{\vec{v}(t)}{||\vec{v}(t)||}$$

The curvature function:

$$\kappa = \left\| \frac{d\vec{T}}{ds} \right\| \; T \text{ is the unit tangent vector, } s \text{ is the arc length}$$

$$= \frac{||\vec{T}'(t)||}{||\vec{v}(t)||} \quad \text{note: } \frac{ds}{dt} = v$$

$$= \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3}$$

$$\kappa = \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3}$$

$$\kappa = \left\| \frac{d\vec{T}}{ds} \right\| = \frac{||\vec{T}'(t)||}{||\vec{r}'(t)||} = \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3}$$

Radius of curvature:

$$p = \frac{1}{\kappa}$$

Principal Normal Vector:

$$\vec{N}(t) = \frac{\vec{T}'(t)}{||\vec{T}'(t)||}$$

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

$$\vec{a} = a_T\vec{T} + a_N\vec{N}$$

$$a_t = \frac{d^2s}{dt^2} = \frac{d}{dt}||\vec{r}'(t)||$$

$$a_N = ||\vec{T}'(t)|| \cdot \frac{ds}{dt} = \kappa\left(\frac{ds}{dt}\right)^2 = \kappa||\vec{r}'(t)||^2 = \sqrt{||\vec{a}||^2 - a_T^2}$$

Projectile Motion:

$\text{Max Height} = \frac{(v_0 \sin(\theta))^2}{2g}$
$\text{Range} = \frac{v_0^2 \sin(2\theta)}{g}$
$\text{Flight Time} = \frac{2v_0 \sin(\theta)}{g}$