#### Formulas-1

Credit Aarush Magic

$$ec{a} \cdot ec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 + \cdots + a_n b_n \ ec{a} imes ec{b} = egin{bmatrix} ec{i} & ec{j} & ec{k} \ a_1 & a_2 & a_3 \ b_1 & b_2 & b_3 \ \end{bmatrix} \ &= egin{bmatrix} u_2 & u_3 \ v_2 & v_3 \end{bmatrix} \mathbf{i} - egin{bmatrix} u_1 & u_3 \ v_1 & v_3 \end{bmatrix} \mathbf{j} + egin{bmatrix} u_1 & u_2 \ v_1 & v_2 \end{bmatrix} \mathbf{k} \ &= (a_2 b_3 - b_2 a_3) ec{i} - (a_1 b_3 + b_1 a_3) ec{j} + (a_1 b_2 + b_1 a_2) ec{k} \end{aligned}$$

### **Properties:**

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos(\theta)$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin(\theta)$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times (\vec{c} + \vec{b})$$

$$(c\vec{a}) \cdot \vec{b} = \vec{a} \cdot (c\vec{b})$$

$$(c\vec{a}) \times \vec{b} = \vec{a} \times (c\vec{b})$$

$$\vec{a} \cdot \vec{a} = ||\vec{a}||^2$$

$$\vec{a} \times \vec{a} = \vec{0}$$
If  $\vec{a} \perp \vec{b}$  then  $\vec{a} \times \vec{b} = \vec{0}$ 

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w}) \vec{v} - (\vec{u} \cdot \vec{v}) \vec{w} \qquad nbsp;$$

# **Graphs**

Cylinder:  $ax^n + by^m = c$ ,  $ax^n + bz^m = c$ ,  $ay^n + bz^m = c$ 

Elliptical Paraboloid:  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$ 

Elliptical Cone:  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$  Ellipsoid:  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  Hyperboloid of 1 sheet:  $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$  Hyperboloid of 2 sheets:  $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  Hyperbolic Paraboloid:  $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}, c > 0$ 

### **Other Formulas**

Line through  $P(p_0,p_1,p_2)$  and parallel to  $ec{v}=ec{ai}+ec{bj}+ec{ck}$ :

$$egin{aligned} x = at + p_0 \quad y = bt + p_1 \quad z = ct + p_2 \ & \langle at + p_0, bt + p_1, ct + p_3 
angle = \langle a, b, c 
angle t + \langle p_0, p_1, p_2 
angle orall - \infty < t < \infty \end{aligned}$$

Distance between line and point  $Q:d=rac{||ec{PQ} imesec{v}||}{||ec{v}||}$ 

Line through  $P(p_0,p_1,p_2)$  and perpendicular to  $ec{n}=aec{i}+bec{j}+cec{k}$  :

$$a(x-p_0) + b(y-p_1) + c(z-p_3) = 0$$

Angle between planes: 
$$heta=\cos^{-1}\left(\left|rac{ec{n_1}\cdotec{n_2}}{||ec{n_1}||\cdot||ec{n_2}||}
ight|
ight)$$

Distance between Point S and a plane:  $d = \left| \vec{PS} \cdot \frac{\vec{n}}{||\vec{n}||} \right|$ 

$$\left\| \int_{a}^{b} \vec{f}(t)dt \right\| \leq \int_{a}^{b} \|\vec{f}(t)\|dt$$

$$L = \int_{a}^{b} \sqrt{\left(\frac{dx}{dt}\right)^{2} + \left(\frac{dy}{dt}\right)^{2} + \left(\frac{dz}{dt}\right)^{2}} dt \qquad = \int_{a}^{b} ||\vec{r}'(t)||dt$$

$$s(t) = \int_{t_{0}}^{t} \sqrt{\left(\frac{dx}{d\tau}\right)^{2} + \left(\frac{dy}{d\tau}\right)^{2} + \left(\frac{dz}{d\tau}\right)^{2}} d\tau \qquad = \int_{t_{0}}^{t} ||\vec{r}'(\tau)||d\tau$$

$$\frac{ds}{dt} = ||\vec{v}(t)||$$

$$\vec{T}(t) = \frac{\vec{r}'(t)}{||\vec{r}'(t)||} = \frac{\vec{v}(t)}{||\vec{v}(t)||}$$

$$\kappa = \left\| \frac{d\vec{T}}{ds} \right\| = \frac{||\vec{T}'(t)||}{||\vec{r}'(t)||} = \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^{3}}$$

$$p = \frac{1}{\kappa}$$

$$ec{N}(t) = rac{ec{T}'(t)}{||ec{T}'(t)||} \ ec{B}(t) = ec{T}(t) imes ec{N}(t) \ ec{a} = a_T ec{T} + a_N ec{N} \ a_t = rac{d^2 s}{dt^2} = rac{d}{dt} ||ec{r}'(t)|| \ a_N = ||ec{T}'(t)|| \cdot rac{ds}{dt} = \kappa \left(rac{ds}{dt}
ight)^2 = \kappa ||ec{r}'(t)||^2 = \sqrt{||ec{a}||^2 - a_T^2} \ ||ec{a}||^2 = a_T^2 + a_N^2 \ \dfrac{\left| egin{matrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ ||ec{r}'(t) imes ec{r}''(t) imes ec{r}'''(t)||^2 \end{array} } \ ec{\tau} = rac{-dec{B}}{ds} \cdot ec{N}'(t) = rac{\left| egin{matrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ ||ec{r}''(t) imes ec{r}'''(t)||^2 \end{array} } = rac{ec{r}'(t) \cdot (ec{r}'''(t) imes ec{r}'''(t))}{||ec{r}''(t) imes ec{r}'''(t)||^2}$$

# **Projectile Motion:**

$$ext{Max Height} = rac{(v_0 \sin( heta))^2}{2g} \ ext{Range} = rac{v_0^2 \sin(2 heta)}{g} \ ext{Flight Time} = rac{2v_0 \sin( heta)}{g}$$

# Polar and cylindrical equations:

$$egin{aligned} ec{u_r} &= \cos heta ec{i} + \sin heta ec{j} \ ec{v}_ heta &= -\sin heta ec{i} + \cos heta ec{j} \ ec{r}(t) &= r ec{u_r} \ ec{r}'(t) &= \dot{r} ec{u_r} + r \dot{ heta} ec{u_ heta} \ ec{r}''(t) &= (\ddot{r} - r \dot{ heta}^2) ec{u_r} + (r \ddot{ heta} + 2 \dot{r} \dot{ heta}) ec{u_ heta} \ ec{r}(t) &= r ec{u_r} + z ec{k} \ ec{r}'(t) &= \dot{r} ec{u_r} + r \dot{ heta} ec{u_ heta} + \dot{z} ec{k} \ ec{r}''(t) &= (\ddot{r} - r \dot{ heta}^2) ec{u_r} + (r \ddot{ heta} + 2 \dot{r} \dot{ heta}) ec{u_ heta} + \ddot{z} ec{k} \end{aligned}$$