

# Formulas-2

$\lim_{(x,y) \rightarrow (x_0,y_0)} f(x,y) = L$  if for every  $\epsilon > 0$ , there exists a corresponding  $\delta > 0$ , such that for all  $(x,y)$  in the domain of  $f$ ,  $|f(x,y) - L| < \epsilon$  whenever  $0 < \sqrt{(x - x_0)^2 + (y - y_0)^2} < \delta$

$$\left. \frac{\partial f}{\partial x} \right|_{(x_0,y_0)} = \lim_{h \rightarrow 0} \frac{f(x_0 + h, y_0) - f(x_0, y_0)}{h} = f_x(x_0, y_0)$$

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial x} \right)$$

$$f_{yy} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial y} \right)$$

$$f_{xy} = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right)$$

$$f_{yx} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right)$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt}$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

Directional Derivative of the unit vector  $u = u_1 \mathbf{i} + u_2 \mathbf{j}$

$$\begin{aligned} f'_u(x, y) &= D_u f(x, y) = \lim_{s \rightarrow 0} \frac{f(x + su_1, y + su_2) - f(x, y)}{s} \\ &= \nabla f(x, y) \cdot u \\ &= \|\nabla f\| \cos \theta \end{aligned}$$

The Gradient

$$\nabla f(x, y, z) = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}$$

Tangent line to a level curve of the form  $f(x, y) = 0$  at a point  $(x_0, y_0)$ ,

$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) = 0$$

$$\frac{d}{dt}(f(r(t))) = \nabla f(r(t)) \cdot r'(t)$$

Tangent plane to  $f(x, y, z) = c$  at the point  $P(x_0, y_0, z_0)$ ,

$$f_x(P)(x - x_0) + f_y(P)(y - y_0) + f_z(P)(z - z_0) = 0$$

Or when  $f(x, y) = z$  at the point  $P(x_0, y_0)$ ,

$$f_x(P)(x - x_0) + f_y(P)(y - y_0) - (z - z_0) = 0$$

Normal line to  $f(x, y, z) = c$  at the point  $P(x_0, y_0, z_0)$ ,

$$x = x_0 + f_x(P)t$$

$$y = y_0 + f_y(P)t$$

$$z = z_0 + f_z(P)t$$

Linear Approximation ( $f(x, y) \approx L(x, y)$ ) of  $f(x, y)$  at  $(x_0, y_0)$ ,

$$L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

Total Differential where  $M$  is the upper bound of the second partials on a rectangle centered at point  $P$ ,

$$df = f_x(x_0, y_0)dx + f_y(x_0, y_0)dy$$

Standard Linear Approximation Error,

$$|E| \leq \frac{1}{2}M(|x - x_0| + |y - y_0|)^2$$