Formulas-3

Fubini's Theorem

$$V=\iint_R f(x,y)dA=\int_c^d\int_a^b f(x,y)dxdy=\int_a^b\int_c^d f(x,y)dydx$$

if R is defined by $a\leq x\leq b$ and $g_1(x)\leq y\leq g_2(x)$ then,

$$V=\int_a^b\int_{g_1(x)}^{g_2(x)} f(x,y)dydx$$

Properties

$$\iint_R cf(x,y)dA=c\iint_R f(x,y)dA$$

$$\iint_R f(x,y)\pm g(x,y)dA=\iint_R f(x,y)dA\pm\iint_R g(x,y)dA$$

$$\iint_R f(x,y)dA\geq 0 \text{ if } f(x,y)\geq 0 \text{ on } R$$

$$\iint_R f(x,y)dA\geq\iint_R g(x,y)dA \text{ if } f(x,y)\geq g(x,y) \text{ on } R$$

$$\iint_R f(x,y)dA=\iint_{R_1} f(x,y)dA+\iint_{R_2} f(x,y)dA \text{ if } R_1 \text{ and } R_2 \text{ are non-overlapping regions}$$

$$A=\iint_R dA=\int_{x_1}^{x_2}\int_{y_1}^{y_2} dydx=\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} rdrd\theta$$

$$V=\iint_R f(x,y)dA$$

$$\text{Average Value}=\frac{1}{\text{Area of } R}\iint_R fdA \quad \text{OR} \quad \frac{1}{\text{Volume of } D}\iiint_D FdV$$

$$V=\iiint_D dV=\int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{z_1}^{z_2} dzdydx=\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\int_{z_1}^{z_2} rdzdrd\theta=\int_{\rho_1}^{\rho_2}\int_{\phi_1}^{\phi_2}\int_{\theta_1}^{\theta_2}\rho^2\sin(\phi)d\theta d\phi d\rho$$

Mass and First Moments of 3D Solids

$$\text{Mass: } M=\iiint_D \delta dV$$

$$\text{First Moments: } M_{yz}=\iiint_D x\delta dV, M_{xz}=\iiint_D y\delta dV, M_{xy}=\iiint_D z\delta dV$$

$$\text{Center of mass: } \bar{x} = \frac{M_{yz}}{M}, \bar{y} = \frac{M_{xz}}{M}, \bar{z} = \frac{M_{xy}}{M}$$

Mass and First Moments of 2D Plates:

$$\text{Mass: } M = \iint_R \delta dA$$

$$\text{First Moments: } M_y = \iint_R x\delta dA, M_x = \iint_R y\delta dA$$

$$\text{Center of mass: } \bar{x} = \frac{M_y}{M}, \bar{y} = \frac{M_x}{M}$$

Moments of Inertia of 3D Solids:

$$\text{About x-axis: } I_x = \iiint_D (y^2 + z^2)\delta dV$$

$$\text{About y-axis: } I_y = \iiint_D (x^2 + z^2)\delta dV$$

$$\text{About z-axis: } I_z = \iiint_D (x^2 + y^2)\delta dV$$

$$\text{About a line L: } I_L = \iiint_D r^2(x, y, z)\delta dV$$

Moments of Inertia of 2D Plates:

$$\text{About x-axis: } I_x = \iint_R y^2\delta dA$$

$$\text{About y-axis: } I_y = \iint_R x^2\delta dA$$

$$\text{About a line L: } I_L = \iint_R r^2(x, y)\delta dA$$

$$\text{About the origin: } I_O = \iint_R (x^2 + y^2)\delta dA = I_x + I_y$$

Joint probability density function:

Conditions:

$$f(x, y) \geq 0$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)dxdy = 1$$

$$P((X, Y) \in R) = \iint_R f(x, y)dxdy$$

Mean and expected value:

$$\mu_X = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xf(x, y)dxdy$$

$$\mu_Y = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} yf(x,y)dxdy$$

Spherical Coordinates (ρ, ϕ, θ)

$$r = \rho\sin(\phi),$$

$$x = r\cos(\theta) = \rho\sin(\phi)\cos(\theta)$$

$$z = \rho\cos(\phi),$$

$$y = r\sin(\theta) = \rho\sin(\phi)\sin(\theta)$$

$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2}$$

Jacobian:

$$J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial(x,y)}{\partial(u,v)}$$

$$\iint_R f(x,y)dxdy = \iint_G f(g(u,v),h(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| dudv$$

$$J(u,v,w) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \frac{\partial(x,y,z)}{\partial(u,v,w)}$$

$$\iiint_D f(x,y,z)dxdydz = \iiint_B f(g(u,v,w),h(u,v,w),k(u,v,w)) \left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| dudvdw$$