Formulas-4

Line Integral

$$ds = ||\vec{r}'(t)||dt$$
$$\int_C f(x,y,z)ds = \int_a^b f(g(t),h(t),k(t))|\vec{v}(t)|dt$$

Thin wire

Mass

$$\int_C \delta ds$$

First Moments

$$M_{yz} = \int_C x\delta ds$$
$$M_{xz} = \int_C y\delta ds$$
$$M_{xy} = \int_C z\delta ds$$

Moment of inertia

$$I_x = \int_C (y^2+z^2)\delta ds$$
$$I_y = \int_C (x^2+z^2)\delta ds$$
$$I_z = \int_C (x^2+y^2)\delta ds$$
$$r(x,y,z) = \text{distance from point } (x,y,z) \text{ to line } L$$
$$I_L = \int_C r^2\delta ds$$

Line Integral of vector field

$\vec{F}$ along C :

$$\begin{aligned}\int_C \vec{F}\cdot\vec{T}ds &= \int_C \vec{F}\cdot\frac{d\vec{r}}{ds}ds \\ &= \int_C \vec{F}\cdot d\vec{r} \\ &= \int_a^b \vec{F}(\vec{r}(t))\cdot\frac{d\vec{r}}{dt}dt\end{aligned}$$

$$\int_C Mdx+Ndy+Pdz = \int_C M(x,y,z)dx + \int_C N(x,y,z)dy + \int_C P(x,y,z)dz$$

$$\text{Where, } \int_C M(x,y,z)dx = \int_C M(g(t),h(t),k(t))g'(t)dt$$

Applications

$$\begin{aligned}\text{Work} &= \int_C \vec{F}\cdot\vec{T}ds \\ \text{Flow} &= \int_C \vec{F}\cdot\vec{T}ds = \int_C Mdx+Ndy \\ \text{Flux} &= \int_C \vec{F}\cdot\vec{T}ds = \oint_C Mdy-Ndx\end{aligned}$$