Formulas-1

Credit Aarush Magic

$$ec{a} \cdot ec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 + \cdots + a_n b_n$$
 $ec{a} imes ec{b} = \begin{vmatrix} ec{i} & ec{j} & ec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
 $= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} ec{\mathbf{i}} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} ec{\mathbf{j}} + \begin{vmatrix} a_1 & a_2 \\ b_1 & n_2 \end{vmatrix} \mathbf{k}$
 $= (a_2 b_3 - b_2 a_3) ec{i} - (a_1 b_3 + b_1 a_3) ec{j} + (a_1 b_2 + b_1 a_2) ec{k}$

Properties:

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos(\theta)$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin(\theta)$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times (\vec{b} + \vec{c})$$

$$\vec{a} \times \vec{b} = \vec{a} \times (\vec{c} \vec{b})$$

$$(c\vec{a}) \cdot \vec{b} = \vec{a} \cdot (c\vec{b})$$

$$(c\vec{a}) \times \vec{b} = \vec{a} \times (c\vec{b})$$

$$\vec{a} \cdot \vec{a} = ||\vec{a}||^2$$

$$\vec{a} \times \vec{a} = \vec{0}$$
If $\vec{a} \perp \vec{b}$ then $\vec{a} \times \vec{b} = \vec{0}$

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w}) \vec{v} - (\vec{u} \cdot \vec{v}) \vec{w} \qquad nbsp;$$

Projectile Motion:

$$ext{Max Height} = rac{(v_0 \sin(heta))^2}{2g} \ ext{Range} = rac{v_0^2 \sin(2 heta)}{g} \ ext{Flight Time} = rac{2v_0 \sin(heta)}{g}$$

Graphs

Туре	Equaions
Elliptical Paraboloid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$
Elliptical Cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$
Ellipsoid	$rac{x^2}{a^2} + rac{y^2}{b^2} + rac{z^2}{c^2} = 1$
Hyperboloid of 1 sheet	$rac{x^2}{a^2} + rac{y^2}{b^2} - rac{z^2}{c^2} = 1$
Hyperboloid of 2 sheets	$-rac{x^2}{a^2} - rac{y^2}{b^2} + rac{z^2}{c^2} = 1$
Hyperbolic Paraboloid	$-rac{x^2}{a^2}+rac{y^2}{b^2}=rac{z}{c},c>0$

Other Formulas

Line through $P(p_1,p_2,p_3)$ and parallel to $ec{v}=ec{ai}+ec{bj}+ec{ck}$ when $t\in\mathbb{R}$:

$$egin{aligned} x &= at + p_1 \quad y = bt + p_2 \quad z = ct + p_3 \ &\langle at + p_1, bt + p_2, ct + p_3
angle = \langle a, b, c
angle t + \langle p_1, p_2, p_3
angle \end{aligned}$$

Line through $P(p_1,p_2,p_3)$ and perpendicular to $ec{n}=aec{i}+bec{j}+cec{k}$:

$$a(x-p_1) + b(y-p_2) + c(z-p_3) = 0$$

Distance between line and point:

$$d = rac{||ec{PS} imes v||}{||v||}$$

Distance between Point S and a Plane,

$$d = \left| ec{PS} \cdot rac{n}{||n||}
ight|$$

Projection,

$$\mathrm{proj}_b a = \left(rac{a \cdot b}{||b||}
ight)rac{b}{||b||}$$

Angle between planes or vectors:

$$heta = \cos^{-1}\left(\left|rac{ec{n_1}\cdotec{n_2}}{||ec{n_1}||\cdot||ec{n_2}||}
ight|
ight)$$

Arc Length (s(t)):

$$L = \int_a^b \sqrt{\left(rac{dx}{dt}
ight)^2 + \left(rac{dy}{dt}
ight)^2 + \left(rac{dz}{dt}
ight)^2} dt \qquad = \int_a^b ||ec{r}'(t)|| dt \ s(t) = \int_{t_0}^t \sqrt{\left(rac{dx}{d au}
ight)^2 + \left(rac{dy}{d au}
ight)^2 + \left(rac{dz}{d au}
ight)^2} d au \qquad = \int_{t_0}^t ||ec{r}'(au)|| d au$$

Speed:

$$rac{ds}{dt} = ||ec{v}(t)||$$

The unit tangent vector (T(t)):

$$ec{T}(t) = rac{ec{r}'(t)}{||ec{r}'(t)||} = rac{ec{v}(t)}{||ec{v}(t)||}$$

The curvature function $(\kappa(t))$:

$$egin{aligned} \kappa &= \left\| rac{dec{T}}{ds}
ight\| \ &= rac{||ec{T}'(t)||}{||ec{v}(t)||} \ &= rac{||ec{r}'(t) imesec{r}''(t)||}{||ec{r}'(t)||^3} \end{aligned}$$

Radius of curvature:

$$p=rac{1}{\kappa}$$

Principal Normal Vector (N(t)):

$$ec{N}(t) = rac{ec{T}'(t)}{||ec{T}'(t)||}$$

Binormal vector (B(t)):

$$ec{B}(t) = ec{T}(t) imes ec{N}(t)$$

Formulas-2

 $\lim_{(x,y)\to(x_0,y_0)}f(x,y)=L \text{ if for every } \epsilon>0, \text{ there exists a corresponding } \delta>0, \text{ such that for all } (x,y) \text{ in the domain of f, } |f(x,y)-L|<\epsilon \text{ whenever } 0<\sqrt{(x-x_0)^2+(y-y_0)^2}<\delta$

$$egin{aligned} rac{\partial f}{\partial x}|_{(x_0,y_0)} &= \lim_{h o 0} rac{f\left(x_0+h,y_0
ight)-f\left(x_0,y_0
ight)}{h} = f_x\left(x_0,y_0
ight) \ f_{xx} &= rac{\partial^2 f}{\partial x^2} = rac{\partial}{\partial x}\left(rac{\partial f}{\partial x}
ight) \ f_{yy} &= rac{\partial^2 f}{\partial y^2} = rac{\partial}{\partial y}\left(rac{\partial f}{\partial y}
ight) \ f_{xy} &= rac{\partial^2 f}{\partial yx} = rac{\partial}{\partial y}\left(rac{\partial f}{\partial x}
ight) \ f_{yx} &= rac{\partial^2 f}{\partial xy} = rac{\partial}{\partial x}\left(rac{\partial f}{\partial y}
ight) \ rac{dw}{dt} &= rac{\partial w}{\partial x} \cdot rac{dx}{dt} + rac{\partial w}{\partial y} \cdot rac{dy}{dt} + rac{\partial w}{\partial z} \cdot rac{dz}{dt} \ rac{dy}{dx} &= -rac{F_x}{F_x} \end{aligned}$$

Directional Derivative of the unit vector $u = u_1 \mathbf{i} + u_2 \mathbf{j}$

$$egin{aligned} f_u'\left(x,y
ight) &= D_u f\left(x,y
ight) = \lim_{s o 0} rac{f\left(x+s u_1,y+s u_2
ight) - f\left(x,y
ight)}{s} \ &=
abla f\left(x,y
ight) \cdot u \ &= ||
abla f|| \cos heta \end{aligned}$$

The Gradient

$$abla \, f \left(x,y,z
ight) = rac{\partial f}{\partial x} i + rac{\partial f}{\partial y} j + rac{\partial f}{\partial z} k$$

Tangent line to a level curve of the form f(x,y)=0 at a point (x_0,y_0)

$$egin{split} f_x(x_0,y_0)\left(x-x_0
ight) + f_y(x_0,y_0)\left(y-y_0
ight) = 0 \ & rac{d}{dt}(f(r(t)) =
abla f(r(t)) \cdot r'(t) \end{split}$$

Tangent plane to f(x, y, z) = c at the point $P(x_0, y_0, z_0)$,

$$f_x(P)(x-x_0) + f_y(P)(y-y_0) + f_z(P)(z-z_0) = 0$$

Or when f(x,y) = z at the point $P(x_0, y_0)$,

$$f_x(P)\left(x-x_0
ight)+f_y(P)\left(y-y_0
ight)-\left(z-z_0
ight)=0$$

Normal line to f(x, y, z) = c at the point $P(x_0, y_0, z_0)$,

$$egin{aligned} x &= x_0 + f_x(P)t \ y &= y_0 + f_y(P)t \ z &= z_0 + f_z(P)t \end{aligned}$$

Linear Approximation $(f(x,y) \approx L(x,y))$ of f(x,y) at (x_0,y_0) ,

$$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

Total Differential,

$$egin{aligned} df &= f_x\left(x_0, y_0
ight) dx + f_y\left(x_0, y_0
ight) dy \ df &= \left(
abla f(P_0) \cdot u
ight) ds \end{aligned}$$

Standard Linear Approximation Error where M is the upper bound of the second partials on a rectangle centered at point P,

$$|E| \leq rac{1}{2} M (|x-x_0| + |y-y_0|)^2$$

Second Partials Test at (x_0, y_0) assuming $\nabla f = 0$,

$$A=f_{xx}(x_0,y_0),\quad B=f_{xy}(x_0,y_0)\quad, C=f_{yy}(x_0,y_0)$$
 $D=AC-B^2$
 $D<0 ext{ Saddle point }$
 $D>0 ext{ Relative Extrema }$
 $D=0 ext{ Indecisive }$

For the relative extrema, if D>0 and A>0 then you have a local min, if A<0 you have a local max.

Lagrange Multipliers when g(x, y) = 0,

$$abla f = \lambda
abla g$$