Formulas

Credit Aarush Magic

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 + \cdots + a_n b_n$$

$$\begin{aligned}
\vec{a} \times \vec{b} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \\
&= \begin{bmatrix} u_2 & u_3 \\ v_2 & v_3 \end{bmatrix} \mathbf{i} - \begin{bmatrix} u_1 & u_3 \\ v_1 & v_3 \end{bmatrix} \mathbf{j} + \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix} \mathbf{k} \\
&= (a_2 b_3 - b_2 a_3)\vec{i} - (a_1 b_3 + b_1 a_3)\vec{j} + (a_1 b_2 + b_1 a_2)\vec{k}
\end{aligned}$$

Properties:

$$\begin{aligned}
\vec{u} \cdot (\vec{v} \times \vec{w}) &= \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \\
\vec{a} \cdot \vec{b} &= \vec{b} \cdot \vec{a} \\
\vec{a} \times \vec{b} &= -\vec{b} \times \vec{a} \\
\vec{a} \cdot \vec{b} &= |\vec{a}| \cdot |\vec{b}| \cos(\theta) \\
|\vec{a} \times \vec{b}| &= |\vec{a}| \cdot |\vec{b}| \sin(\theta) \\
\vec{a} \cdot (\vec{b} + \vec{c}) &= \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} \\
\vec{a} \times (\vec{b} + \vec{c}) &= \vec{a} \times \vec{b} + \vec{a} \times \vec{c} \\
\vec{a} \cdot \vec{0} &= 0 \\
\vec{a} \times \vec{0} &= \vec{0} \\
(c\vec{a}) \cdot \vec{b} &= \vec{a} \cdot (c\vec{b}) \\
(c\vec{a}) \times \vec{b} &= \vec{a} \times (c\vec{b}) \\
\vec{a} \cdot \vec{a} &= ||\vec{a}||^2 \\
\vec{a} \times \vec{a} &= \vec{0} \\
\text{If } \vec{a} \perp \vec{b} \text{ then } \vec{a} \cdot \vec{b} &= 0 \\
\text{If } \vec{a} \parallel \vec{b} \text{ then } \vec{a} \times \vec{b} &= \vec{0} \\
\vec{u} \times (\vec{v} \times \vec{w}) &= (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w} \qquad nbsp;
\end{aligned}$$

Graphs

Cylinder: $ax^n + by^m = c, ax^n + bz^m = c, ay^n + bz^m = c$

Elliptical Paraboloid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$

Elliptical Cone: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$
Ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
Hyperboloid of 1 sheet: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$
Hyperboloid of 2 sheets: $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
Hyperbolic Paraboloid: $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}, c>0$

Line through $P(p_0,p_1,p_2)$ and parallel to $\vec{v}=a\vec{i}+b\vec{j}+c\vec{k}$:

$$x=at+p_0, y=bt+p_1, z=ct+p_2$$

$$\langle at+p_0, bt+p_1, ct+p_3\rangle=\langle a,b,c\rangle t+\langle p_0,p_1,p_2\rangle \forall -\infty<t<\infty$$

Distance between line and point $Q: d=\frac{||\vec{PQ}\times\vec{v}||}{||\vec{v}||}$
Line through $P(p_0,p_1,p_2)$ and perpendicular to $\vec{n}=a\vec{i}+b\vec{j}+c\vec{k}:$

$$a(x-p_0)+b(y-p_1)+c(z-p_3)=0$$

Angle between planes: $\theta=\cos^{-1}\left(\left|\frac{\vec{n}_1\cdot\vec{n}_2}{||\vec{n}_1||\cdot||\vec{n}_2||}\right|\right)$
Distance between Point S and plane: $d=\left|\vec{PS}\cdot\frac{\vec{n}}{||\vec{n}||}\right|$

$$\left\|\int_a^b \vec{f}(t)dt\right\|\le\int_a^b\|\vec{f}(t)\|dt$$

$$L=\int_a^b\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}dt=\int_a^b||\vec{r}'(t)||dt$$

$$s(t)=\int_{t_0}^t\sqrt{\left(\frac{dx}{d\tau}\right)^2+\left(\frac{dy}{d\tau}\right)^2+\left(\frac{dz}{d\tau}\right)^2}d\tau=\int_{t_0}^t||\vec{r}'(\tau)||d\tau$$

$$\frac{ds}{dt}=||\vec{v}(t)||$$

$$\vec{T}(t)=\frac{\vec{r}'(t)}{||\vec{r}'(t)||}=\frac{\vec{v}(t)}{||\vec{v}(t)||}$$

$$\kappa=\left\|\frac{d\vec{T}}{ds}\right\|=\frac{||\vec{T}'(t)||}{||\vec{r}'(t)||}=\frac{||\vec{r}'(t)\times\vec{r}''(t)||}{||\vec{r}'(t)||^3}$$

$$p=\frac{1}{\kappa}$$

$$\vec{N}(t)=\frac{\vec{T}'(t)}{||\vec{T}'(t)||}$$

$$\vec{B}(t)=\vec{T}(t)\times\vec{N}(t)$$

$$\vec{a} = a_T\vec{T} + a_N\vec{N}$$

$$a_t = \frac{d^2s}{dt^2} = \frac{d}{dt}||\vec{r}'(t)||$$

$$a_N = ||\vec{T}'(t)|| \cdot \frac{ds}{dt} = \kappa\left(\frac{ds}{dt}\right)^2 = \kappa||\vec{r}'(t)||^2 = \sqrt{||\vec{a}||^2 - a_T^2}$$

$$||\vec{a}||^2 = a_T^2 + a_N^2$$

$$\tau = \frac{-d\vec{B}}{ds} \cdot \vec{N}'(t) = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \dddot{x} & \dddot{y} & \dddot{z} \end{vmatrix}}{||\vec{r}'(t) \times \vec{r}''(t)||^2} = \frac{\vec{r}'(t) \cdot (\vec{r}''(t) \times \vec{r}'''(t))}{||\vec{r}'(t) \times \vec{r}''(t)||^2}$$

Projectile Motion:

Max Height= $\frac{(v_0\sin(\theta))^2}{2g}$
Range= $\frac{v_0^2\sin(2\theta)}{g}$
Flight time= $\frac{2v_0\sin(\theta)}{g}$

Polar and cylindrical equations:

$\vec{u_r} = \cos\theta\vec{i} + \sin\theta\vec{j}$

$\vec{u_\theta} = -\sin\theta\vec{i} + \cos\theta\vec{j}$

$\vec{r}(t) = r\vec{u_r}$

$\vec{r}'(t) = \dot{r}\vec{u_r} + r\dot{\theta}\vec{u_\theta}$

$\vec{r}''(t) = (\ddot{r} - r\dot{\theta}^2)\vec{u_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u_\theta}$

$\vec{r}(t) = r\vec{u_r} + z\vec{k}$

$\vec{r}'(t) = \dot{r}\vec{u_r} + r\dot{\theta}\vec{u_\theta} + \dot{z}\vec{k}$

$\vec{r}''(t) = (\ddot{r} - r\dot{\theta}^2)\vec{u_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u_\theta} + \ddot{z}\vec{k}$