## Formulas-2

 $\lim_{(x,y)\to(x_0,y_0)}f(x,y)=L \text{ if for every } \epsilon>0, \text{ there exists a corresponding } \delta>0, \text{ such that for all } (x,y) \text{ in the domain of f, } |f(x,y)-L|<\epsilon \text{ whenever } 0<\sqrt{(x-x_0)^2+(y-y_0)^2}<\delta$ 

$$egin{aligned} rac{\partial f}{\partial x}|_{(x_0,y_0)} &= \lim_{h o 0} rac{f\left(x_0+h,y_0
ight)-f\left(x_0,y_0
ight)}{h} = f_x\left(x_0,y_0
ight) \ f_{xx} &= rac{\partial^2 f}{\partial x^2} = rac{\partial}{\partial x}\left(rac{\partial f}{\partial x}
ight) \ f_{yy} &= rac{\partial^2 f}{\partial y^2} = rac{\partial}{\partial y}\left(rac{\partial f}{\partial y}
ight) \ f_{xy} &= rac{\partial^2 f}{\partial yx} = rac{\partial}{\partial y}\left(rac{\partial f}{\partial x}
ight) \ f_{yx} &= rac{\partial^2 f}{\partial xy} = rac{\partial}{\partial x}\left(rac{\partial f}{\partial y}
ight) \ rac{dw}{dt} &= rac{\partial w}{\partial x} \cdot rac{dx}{dt} + rac{\partial w}{\partial y} \cdot rac{dy}{dt} + rac{\partial w}{\partial z} \cdot rac{dz}{dt} \ rac{dy}{dx} &= -rac{F_x}{F_x} \end{aligned}$$

Directional Derivative of the unit vector  $u = u_1 \mathbf{i} + u_2 \mathbf{j}$ 

$$egin{aligned} f_u'\left(x,y
ight) &= D_u f\left(x,y
ight) = \lim_{s o 0} rac{f\left(x+s u_1,y+s u_2
ight) - f\left(x,y
ight)}{s} \ &= 
abla f\left(x,y
ight) \cdot u \ &= ||
abla f|| \cos heta \end{aligned}$$

The Gradient

$$abla \, f \left( x,y,z 
ight) = rac{\partial f}{\partial x} i + rac{\partial f}{\partial y} j + rac{\partial f}{\partial z} k$$

Tangent line to a level curve of the form f(x,y)=0 at a point  $(x_0,y_0)$ 

$$egin{split} f_x(x_0,y_0)\left(x-x_0
ight) + f_y(x_0,y_0)\left(y-y_0
ight) = 0 \ & rac{d}{dt}(f(r(t)) = 
abla f(r(t)) \cdot r'(t) \end{split}$$

Tangent plane to f(x, y, z) = c at the point  $P(x_0, y_0, z_0)$ ,

$$f_x(P)(x-x_0) + f_y(P)(y-y_0) + f_z(P)(z-z_0) = 0$$

Or when f(x,y) = z at the point  $P(x_0, y_0)$ ,

$$f_x(P)(x-x_0) + f_y(P)(y-y_0) - (z-z_0) = 0$$

Normal line to  $f\left(x,y,z
ight)=c$  at the point  $P(x_{0},y_{0},z_{0})$ ,

$$egin{aligned} x &= x_0 + f_x(P)t \ y &= y_0 + f_y(P)t \ z &= z_0 + f_z(P)t \end{aligned}$$

Linear Approximation (f(x,y) pprox L(x,y)) of f(x,y) at  $(x_0,y_0)$ ,

$$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

Total Differential where M is the upper bound of the second partials on a rectangle centered at point P,

$$df=f_{x}\left( x_{0},y_{0}
ight) dx+f_{y}\left( x_{0},y_{0}
ight) dy$$

Standard Linear Approximation Error,

$$|E| \leq \frac{1}{2} M (|x-x_0| + |y-y_0|)^2$$