Formulas-2

$\lim\limits_{(x,y)\to(x_0,y_0)} f\left(x,y\right)=L$ if for every ε>0, there exists a corresponding δ>0, such that for all (x,y) in the domain of f, $\left|f\left(x,y\right)-L\right|<\epsilon$ whenever $0<\sqrt{(x-x_0)^2+(y-y_0)^2}<\delta$

$$\frac{\partial f}{\partial x}|_{(x_0,y_0)}=\lim_{h\to 0}\frac{f\left(x_0+h,y_0\right)-f\left(x_0,y_0\right)}{h}=f_x\left(x_0,y_0\right)$$

$$\begin{aligned}
f_{xx}&=\frac{\partial^2 f}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)\\
f_{yy}&=\frac{\partial^2 f}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)\\
f_{xy}&=\frac{\partial^2 f}{\partial yx}=\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)\\
f_{yx}&=\frac{\partial^2 f}{\partial xy}=\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)\\
\frac{dw}{dt}&=\frac{\partial w}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial w}{\partial y}\cdot\frac{dy}{dt}+\frac{\partial w}{\partial z}\cdot\frac{dz}{dt}\\
\frac{dy}{dx}&=-\frac{F_x}{F_y}
\end{aligned}$$

Directional Derivative of the unit vector $u=u_1\mathbf{i}+u_2\mathbf{j}$

$$\begin{aligned}
f'_u\left(x,y\right)=D_u f\left(x,y\right)&=\lim_{s\to 0}\frac{f\left(x+su_1,y+su_2\right)-f\left(x,y\right)}{s}\\
&=\nabla\mathrm{f}\left(x,y\right)\cdot u\\
&=||\nabla f||\cos\theta
\end{aligned}$$

The Gradient

$$\nabla\ f\left(x,y,z\right)=\frac{\partial f}{\partial x}i+\frac{\partial f}{\partial y}j+\frac{\partial f}{\partial z}k$$

Tangent line to a level curve of the form $f(x,y)=0$ at a point (x_0,y_0),

$$f_x(x_0,y_0)\left(x-x_0\right)+f_y(x_0,y_0)\left(y-y_0\right)=0$$

$$\frac{d}{dt}(f(r(t))=\nabla f(r(t))\cdot r'(t)$$

Tangent plane to $f\left(x,y,z\right)=c$ at the point $P(x_0,y_0,z_0)$,

$$f_x(P)\left(x-x_0\right)+f_y(P)\left(y-y_0\right)+f_z(P)\left(z-z_0\right)=0$$

Or when $f(x,y)=z$ at the point $P(x_0,y_0)$,

$$f_x(P)\,(x-x_0)+f_y(P)\,(y-y_0)-(z-z_0)=0$$

Normal line to $f\,(x,y,z)=c$ at the point $P(x_0,y_0,z_0)$,

$$\begin{aligned} x &= x_0+f_x(P)t \\ y &= y_0+f_y(P)t \\ z &= z_0+f_z(P)t \end{aligned}$$

Linear Approximation ($f(x,y)\approx L(x,y)$) of $f(x,y)$ at (x_0,y_0),

$$L(x,y)=f(x_0,y_0)+f_x(x_0,y_0)(x-x_0)+f_y(x_0,y_0)(y-y_0)$$

Total Differential,

$$\begin{aligned} df &= f_x\,(x_0,y_0)dx+f_y\,(x_0,y_0)dy \\ df &= (\nabla f(P_0)\cdot u)ds \end{aligned}$$

Standard Linear Approximation Error where M is the upper bound of the second partials on a rectangle centered at point P,

$$|E|\leq\frac{1}{2}M(|x-x_0|+|y-y_0|)^2$$

Second Partials Test at (x_0,y_0) assuming $\nabla f=0$,

$$\begin{gathered} A=f_{xx}(x_0,y_0),\quad B=f_{xy}(x_0,y_0)\quad ,C=f_{yy}(x_0,y_0) \\ D=AC-B^2 \\ D<0 \text{ Saddle point} \\ D>0 \text{ Relative Extrema} \\ D=0 \text{ Indecisive} \end{gathered}$$

For the relative extrema, if $D>0$ and $A>0$ then you have a local min, if $A<0$ you have a local max.

Lagrange Multipliers when $g(x,y)=0$,

$$\nabla f=\lambda\nabla g$$