Formulas-3

Fubini's Theorem

$$V=\iint_R f(x,y)dA=\int_c^d \int_a^b f(x,y)dxdy=\int_a^b \int_c^d f(x,y)dydx$$

if R is defined by $a \leq x \leq b$ and $g_1(x) \leq y \leq g_2(x)$ then,

$$V=\int_a^b\int_{q_1(x)}^{g_2(x)}f(x,y)dydx$$

Notice: The functions $g_n(x)$ are evaluated first.

Properties

$$egin{aligned} &\iint_R cf(x,y)dA = c\iint_R f(x,y)dA \ &\iint_R f(x,y) \pm g(x,y)dA = \iint_R f(x,y)dA \pm \iint_R g(x,y)dA \ &\iint_R f(x,y)dA \geq 0 ext{ if } f(x,y) \geq 0 ext{ on } R \ &\iint_R f(x,y)dA \geq \iint_R g(x,y)dA ext{ if } f(x,y) \geq g(x,y) ext{ on } R \end{aligned}$$

If R_1 and R_2 are non-overlapping regions,

$$egin{aligned} &\iint_R f(x,y) dA = \iint_{R_1} f(x,y) dA + \iint_{R_2} f(x,y) dA \ &A = \iint_R dA = \int_{x_1}^{x_2} \int_{y_1}^{y_2} dy dx = \int_{ heta_1}^{ heta_2} \int_{r_1}^{r_2} r \ dr d heta \ &V = \iint_R f(x,y) dA \end{aligned}$$
 Average Value $= \frac{1}{ ext{Area of }R} \iint_R f dA \quad ext{OR} \quad \frac{1}{ ext{Volume of }D} \iiint_D F dV \ &V = \iiint_D dV \ &= \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} dz dy dx \ &= \int_{ heta_1}^{ heta_2} \int_{r_1}^{r_2} \int_{z_1}^{z_2} r \ dz dr d heta \ &= \int_{ heta_1}^{ heta_2} \int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2}
ho^2 \sin(\phi) \ d heta d\phi d \phi d
ho$

Mass and First Moments

3D Solids

 δ is the density function.

$$\text{Mass: } M = \iiint_D \delta \ dV$$
 First Moments: $M_{yz} = \iiint_D x \delta \ dV, M_{xz} = \iiint_D y \delta \ dV, M_{xy} = \iiint_D z \delta \ dV$ Center of mass: $\bar{x} = \frac{M_{yz}}{M}, \bar{y} = \frac{M_{xz}}{M}, \bar{z} = \frac{M_{xy}}{M}$

2D Plates

$$\text{Mass: } M = \iint_R \delta \ dA$$

$$\text{First Moments: } M_y = \iint_R x \delta \ dA, M_x = \iint_R y \delta \ dA$$

$$\text{Center of mass: } \bar{x} = \frac{M_y}{M}, \bar{y} = \frac{M_x}{M}$$

Moments of Inertia

3D Solids

About x-axis:
$$I_x = \iiint_D (y^2 + z^2) \delta dV$$

About y-axis: $I_y = \iiint_D (x^2 + z^2) \delta dV$
About z-axis: $I_z = \iiint_D (x^2 + y^2) \delta dV$
About a line L: $I_L = \iiint_D r^2(x, y, z) \delta dV$

2D Plates

About x-axis:
$$I_x=\iint_R y^2\delta dA$$

About y-axis: $I_y=\iint_R x^2\delta dA$
About a line L: $I_L=\iint_R r^2(x,y)\delta dA$
About the origin: $I_O=\iint_R (x^2+y^2)\delta dA=I_x+I_y$

Joint probability density function

Conditions

$$f(x,y)\geq 0 \ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)dxdy=1 \ P((X,Y)\in R)=\iint_{R}f(x,y)dxdy$$

Mean and expected value

$$egin{aligned} \mu_X &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x f(x,y) dx dy \ \ \mu_Y &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y f(x,y) dx dy \end{aligned}$$

Spherical Coordinates (ρ, ϕ, θ)

$$egin{aligned} r &=
ho \sin(\phi), \ x &= r \cos(heta) =
ho \sin(\phi) \cos(heta) \ z &=
ho \cos(\phi), \ y &= r \sin(heta) =
ho \sin(\phi) \sin(heta) \
ho &= \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2} \end{aligned}$$

Jacobian:

$$J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial(x,y)}{\partial(u,v)}$$

$$\iint_{R} f(x,y) dx dy = \iint_{G} f(g(u,v),h(u,v)) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du dv$$

$$J(u,v,w) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \frac{\partial(x,y,z)}{\partial(u,v,w)}$$

$$\iiint_{D} f(x,y,z) dx dy dz = \iiint_{B} f(g(u,v,w),h(u,v,w),k(u,v,w)) \left| \frac{\partial(x,y,z)}{\partial(u,v,w)} \right| du dv dw$$