Formulas-4

Module 4 Formulas

$$\text{Line Integral:}$$

$$ds = ||\vec{r}'(t)||dt$$

$$\int_C f(x,y,z)ds = \int_a^b f(g(t),h(t),k(t))|\vec{v}(t)|dt$$

$$\text{Mass (Thin wire)} = \int_C \delta ds$$

$$\text{First Moments (Thin wire):}$$

$$M_{yz} = \int_C x\delta ds$$

$$M_{xz} = \int_C y\delta ds$$

$$M_{xy} = \int_C z\delta ds$$

$$\text{Moment of inertia (Thin wire):}$$

$$I_x = \int_C (y^2+z^2)\delta ds$$

$$I_y = \int_C (x^2+z^2)\delta ds$$

$$I_z = \int_C (x^2+y^2)\delta ds$$

$$I_L = \int_C r^2\delta ds \text{ where r(x,y,z) = distance from point (x,y,z) to line L}$$

$$\text{Line Integral of vector field } \vec{F} \text{ along } C:$$

$$\int_C \vec{F}\cdot\vec{T}ds = \int_C \vec{F}\cdot\frac{d\vec{r}}{ds}ds = \int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\frac{d\vec{r}}{dt}dt$$

$$\int_C Mdx + Ndy + Pdz = \int_C M(x,y,z)dx + \int_C N(x,y,z)dy + \int_C P(x,y,z)dz$$

$$\text{Where } \int_C M(x,y,z)dx = \int_C M(g(t),h(t),k(t))g'(t)dt$$

$$\text{Work} = \int_C \vec{F}\cdot\vec{T}ds$$

$$\text{Flow} = \int_C \vec{F} \cdot \vec{T} ds = \int_C M dx + N dy$$

$$\text{Flux} = \int_C \vec{F} \cdot \vec{T} ds = \oint_C M dy - N dx$$

Conservative Fields:

Fields are conservative if $\int_C \vec{F} \cdot d\vec{r}$ is path independent

Potential Function: If $\vec{F} = \nabla f$ then f is the potential function for $\vec{F}$

$\vec{F}$ is conservative if and only if $\vec{F}$ is a gradient field ∇f for differentiable function f

$\vec{F}$ is conservative if and only if $\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}, \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$ and $\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$

For a conservative vector field, $\vec{F}$, $\int_C \vec{F} \cdot d\vec{r} = f(B) - f(A)$

$\oint \vec{F} \cdot d\vec{r} = 0 \Leftrightarrow$ The field $\vec{F}$ is conservative on D

Differential form: $M(x,y,z)dx + N(x,y,z)dy + P(x,y,z)dz$

Exact form: $M dx + N dy + P dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz = df$ ($\vec{F}$ is conservative)

Circulation density (k-component of curl): (curl $\vec{F}$) $\cdot \vec{k} = \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}$

Flux density (divergence): div $\vec{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}$

Green's Theorem:

Circulation-Curl (counterclockwise circulation): $\oint_C \vec{F} \cdot \vec{T} ds = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy$

Flux Divergence (outward flux of F): $\oint_C \vec{F} \cdot \vec{n} ds = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dx dy$

$$\text{Area of R} = \frac{1}{2} \oint x dy - y dx$$

Surface Area:

$$SA = \iint_S d\sigma$$

$$d\sigma = ||\vec{r_u} \times \vec{r_v}|| du dv = \frac{||\nabla F||}{\nabla F \cdot \vec{p}} dA = \sqrt{{f_x}^2 + {f_y}^2 + 1} dx dy$$

Surface Integral:

$$\iint_S G(x,y,z) d\sigma$$

Surface integral of F over S (Flux): $\iint_S \vec{F}\cdot\vec{n}d\sigma = \iint_S \vec{F}\cdot(\vec{r_u}\times\vec{r_v})dudv$

$$\text{Mass (Thin shell)} = \iint_S \delta d\sigma$$

First Moments (Thin shell):

$$M_{yz} = \iint_S x\delta d\sigma$$

$$M_{xz} = \iint_S y\delta d\sigma$$

$$M_{xy} = \iint_S z\delta d\sigma$$

Moment of inertia (Thin shell):

$$I_x = \iint_S (y^2+z^2)\delta d\sigma$$

$$I_y = \iint_S (x^2+z^2)\delta d\sigma$$

$$I_z = \iint_S (x^2+y^2)\delta d\sigma$$

$$I_L = \iint_S r^2\delta d\sigma \text{ where r(x,y,z) = distance from point (x,y,z) to line L}$$

$$\text{The del operator: } \nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

$$\text{curl } \vec{F} = \nabla\times\vec{F}$$

$$\text{div } \vec{F} = \nabla\cdot\vec{F}$$

$$\text{Important properties: curl(grad } f) = \nabla\times\nabla f = \vec{0}$$

,

$$\text{div(curl } \vec{F}) = \nabla\cdot(\nabla\times\vec{F}) = 0$$

Stoke's Theorem and Divergence Theorem:

$$\text{Circulation of } \vec{F} = \text{Flux of curl } \vec{F}\text{: } \oint_C \vec{F}\cdot d\vec{r} = \iint_S (\nabla\times\vec{F})\cdot\vec{n}d\sigma$$

$$\nabla\times\vec{F} = \vec{0} \text{ at every point } \Leftrightarrow \oint_C \vec{F}\cdot d\vec{r} = 0 \Leftrightarrow \text{ The field } f \text{ is conservative}$$

Outward flux of $\vec{F}$ along surface $S = \iint_S \vec{F}\cdot\vec{n}d\sigma = \iiint_D(\nabla\cdot\vec{F})dV$