Formulas-1

Credit Aarush Magic

$$\begin{aligned}
\vec{a}\cdot\vec{b} &= a_1b_1 + a_2b_2 + a_3b_3 + \cdots + a_nb_n \\
\vec{a}\times\vec{b} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \\
&= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & n_2 \end{vmatrix}\mathbf{k} \\
&= (a_2b_3 - b_2a_3)\vec{i} - (a_1b_3 + b_1a_3)\vec{j} + (a_1b_2 + b_1a_2)\vec{k}
\end{aligned}$$

Properties:

$$\begin{aligned}
\vec{u}\cdot(\vec{v}\times\vec{w}) &= \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \\
\vec{a}\cdot\vec{b} &= \vec{b}\cdot\vec{a} \\
\vec{a}\times\vec{b} &= -\vec{b}\times\vec{a} \\
\vec{a}\cdot\vec{b} &= |\vec{a}|\cdot|\vec{b}|\cos(\theta) \\
|\vec{a}\times\vec{b}| &= |\vec{a}|\cdot|\vec{b}|\sin(\theta) \\
\vec{a}\cdot(\vec{b}+\vec{c}) &= \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} \\
\vec{a}\times(\vec{b}+\vec{c}) &= \vec{a}\times\vec{b} + \vec{a}\times\vec{c} \\
\vec{a}\cdot\vec{0} &= 0 \\
\vec{a}\times\vec{0} &= \vec{0} \\
(c\vec{a})\cdot\vec{b} &= \vec{a}\cdot(c\vec{b}) \\
(c\vec{a})\times\vec{b} &= \vec{a}\times(c\vec{b}) \\
\vec{a}\cdot\vec{a} &= ||\vec{a}||^2 \\
\vec{a}\times\vec{a} &= \vec{0} \\
\text{If } \vec{a}\perp\vec{b} \text{ then } \vec{a}\cdot\vec{b} &= 0 \\
\text{If } \vec{a}\parallel\vec{b} \text{ then } \vec{a}\times\vec{b} &= \vec{0} \\
\vec{u}\times(\vec{v}\times\vec{w}) &= (\vec{u}\cdot\vec{w})\vec{v} - (\vec{u}\cdot\vec{v})\vec{w} \qquad nbsp;
\end{aligned}$$

Projectile Motion:

$\text{Max Height} = \frac{(v_0\sin(\theta))^2}{2g}$
$\text{Range} = \frac{v_0^2\sin(2\theta)}{g}$
$\text{Flight Time} = \frac{2v_0\sin(\theta)}{g}$

Graphs

Type	Equaions
Elliptical Paraboloid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}$
Elliptical Cone	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$
Ellipsoid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
Hyperboloid of 1 sheet	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$
Hyperboloid of 2 sheets	$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
Hyperbolic Paraboloid	$-\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}, c>0$

Other Formulas

Line through $P(p_1, p_2, p_3)$ and parallel to $\vec{v}=a\vec{i}+b\vec{j}+c\vec{k}$ when $t\in\mathbb{R}$:

$$x=at+p_1 \quad y=bt+p_2 \quad z=ct+p_3$$

$$\langle at+p_1, bt+p_2, ct+p_3\rangle=\langle a,b,c\rangle t+\langle p_1,p_2,p_3\rangle$$

Line through $P(p_1, p_2, p_3)$ and perpendicular to $\vec{n}=a\vec{i}+b\vec{j}+c\vec{k}$:

$$a(x-p_1)+b(y-p_2)+c(z-p_3)=0$$

Distance between line and point:

$$d=\frac{||\vec{PS}\times v||}{||v||}$$

Distance between Point S and a Plane,

$$d=\left|\vec{PS}\cdot\frac{n}{||n||}\right|$$

Projection,

$$\text{proj}_b a=\left(\frac{a\cdot b}{||b||}\right)\frac{b}{||b||}$$

Angle between planes or vectors:

$$\theta=\cos^{-1}\left(\left|\frac{\vec{n_1}\cdot\vec{n_2}}{||\vec{n_1}||\cdot||\vec{n_2}||}\right|\right)$$

Arc Length $(s(t))$:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt \qquad = \int_a^b ||\vec{r}'(t)|| dt$$

$$s(t) = \int_{t_0}^t \sqrt{\left(\frac{dx}{d\tau}\right)^2 + \left(\frac{dy}{d\tau}\right)^2 + \left(\frac{dz}{d\tau}\right)^2} d\tau \qquad = \int_{t_0}^t ||\vec{r}'(\tau)|| d\tau$$

Speed:

$$\frac{ds}{dt} = ||\vec{v}(t)||$$

The unit tangent vector $(T(t))$:

$$\vec{T}(t) = \frac{\vec{r}'(t)}{||\vec{r}'(t)||} = \frac{\vec{v}(t)}{||\vec{v}(t)||}$$

The curvature function $(\kappa(t))$:

$$\begin{aligned}\kappa &= \left\| \frac{d\vec{T}}{ds} \right\| \\ &= \frac{||\vec{T}'(t)||}{||\vec{v}(t)||} \\ &= \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3}\end{aligned}$$

Radius of curvature:

$$p = \frac{1}{\kappa}$$

Principal Normal Vector $(N(t))$:

$$\vec{N}(t) = \frac{\vec{T}'(t)}{||\vec{T}'(t)||}$$

Binormal vector $(B(t))$:

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

Formulas-2

$\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y) = L$ if for every ε>0, there exists a corresponding δ>0, such that for all (x,y) in the domain of f, $|f(x,y) - L| < \epsilon$ whenever $0 < \sqrt{(x-x_0)^2 + (y-y_0)^2} < \delta$

$$\frac{\partial f}{\partial x}\Big|_{(x_0,y_0)} = \lim_{h\to 0} \frac{f(x_0+h, y_0) - f(x_0,y_0)}{h} = f_x(x_0,y_0)$$

$$\begin{aligned}
f_{xx} &= \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) \\
f_{yy} &= \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) \\
f_{xy} &= \frac{\partial^2 f}{\partial yx} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) \\
f_{yx} &= \frac{\partial^2 f}{\partial xy} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) \\
\frac{dw}{dt} &= \frac{\partial w}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial w}{\partial z}\cdot\frac{dz}{dt} \\
\frac{dy}{dx} &= -\frac{F_x}{F_y}
\end{aligned}$$

Directional Derivative of the unit vector $u = u_1\mathbf{i} + u_2\mathbf{j}$

$$\begin{aligned}
f'_u(x,y) = D_u f(x,y) &= \lim_{s\to 0}\frac{f(x+su_1, y+su_2) - f(x,y)}{s} \\
&= \nabla \mathrm{f}(x,y)\cdot u \\
&= ||\nabla f||\cos\theta
\end{aligned}$$

The Gradient

$$\nabla\ f(x,y,z) = \frac{\partial f}{\partial x}i + \frac{\partial f}{\partial y}j + \frac{\partial f}{\partial z}k$$

Tangent line to a level curve of the form $f(x,y) = 0$ at a point (x_0,y_0),

$$f_x(x_0,y_0)\,(x-x_0) + f_y(x_0,y_0)\,(y-y_0) = 0$$

$$\frac{d}{dt}(f(r(t)) = \nabla f(r(t))\cdot r'(t)$$

Tangent plane to $f(x,y,z) = c$ at the point $P(x_0,y_0,z_0)$,

$$f_x(P)\,(x-x_0) + f_y(P)\,(y-y_0) + f_z(P)\,(z-z_0) = 0$$

Or when $f(x,y) = z$ at the point $P(x_0,y_0)$,

$$f_x(P)\,(x-x_0)+f_y(P)\,(y-y_0)-(z-z_0)=0$$

Normal line to $f\,(x,y,z)=c$ at the point $P(x_0,y_0,z_0)$,

$$\begin{gathered}x=x_0+f_x(P)t\\ y=y_0+f_y(P)t\\ z=z_0+f_z(P)t\end{gathered}$$

Linear Approximation ($f(x,y)\approx L(x,y)$) of $f(x,y)$ at (x_0,y_0),

$$L(x,y)=f(x_0,y_0)+f_x(x_0,y_0)(x-x_0)+f_y(x_0,y_0)(y-y_0)$$

Total Differential,

$$\begin{aligned}&df=f_x\,(x_0,y_0)dx+f_y\,(x_0,y_0)dy\\ &df=(\nabla f(P_0)\cdot u)ds\end{aligned}$$

Standard Linear Approximation Error where M is the upper bound of the second partials on a rectangle centered at point P,

$$|E|\leq\frac{1}{2}M(|x-x_0|+|y-y_0|)^2$$

Second Partials Test at (x_0,y_0) assuming $\nabla f=0$,

$$\begin{gathered}A=f_{xx}(x_0,y_0),\quad B=f_{xy}(x_0,y_0)\quad,C=f_{yy}(x_0,y_0)\\ D=AC-B^2\\ D<0\text{ Saddle point}\\ D>0\text{ Relative Extrema}\\ D=0\text{ Indecisive}\end{gathered}$$

For the relative extrema, if $D>0$ and $A>0$ then you have a local min, if $A<0$ you have a local max.

Lagrange Multipliers when $g(x,y)=0$,

$$\nabla f=\lambda\nabla g$$