

# Formulas-1

Credit Aarush Magic

$$\begin{aligned}\vec{a} \cdot \vec{b} &= a_1b_1 + a_2b_2 + a_3b_3 + \cdots + a_nb_n \\ \vec{a} \times \vec{b} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \\ &= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k} \\ &= (a_2b_3 - b_2a_3)\vec{i} - (a_1b_3 + b_1a_3)\vec{j} + (a_1b_2 + b_1a_2)\vec{k}\end{aligned}$$

## Properties:

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos(\theta)$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin(\theta)$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\vec{a} \cdot \vec{0} = 0$$

$$\vec{a} \times \vec{0} = \vec{0}$$

$$(c\vec{a}) \cdot \vec{b} = \vec{a} \cdot (c\vec{b})$$

$$(c\vec{a}) \times \vec{b} = \vec{a} \times (c\vec{b})$$

$$\vec{a} \cdot \vec{a} = ||\vec{a}||^2$$

$$\vec{a} \times \vec{a} = \vec{0}$$

$$\text{If } \vec{a} \perp \vec{b} \text{ then } \vec{a} \cdot \vec{b} = 0$$

$$\text{If } \vec{a} \parallel \vec{b} \text{ then } \vec{a} \times \vec{b} = \vec{0}$$

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w} \quad \text{nbsp;}$$

## Projectile Motion:

$$\text{Max Height} = \frac{(v_0 \sin(\theta))^2}{2g}$$

$$\text{Range} = \frac{v_0^2 \sin(2\theta)}{g}$$

$$\text{Flight Time} = \frac{2v_0 \sin(\theta)}{g}$$

# Graphs

Type	Equations
Elliptical Paraboloid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$
Elliptical Cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Hyperboloid of 1 sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
Hyperboloid of 2 sheets	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Hyperbolic Paraboloid	$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}, c > 0$

## Other Formulas

Line through  $P(p_1, p_2, p_3)$  and parallel to  $\vec{v} = a\vec{i} + b\vec{j} + c\vec{k}$  when  $t \in \mathbb{R}$ :

$$x = at + p_1 \quad y = bt + p_2 \quad z = ct + p_3$$

$$\langle at + p_1, bt + p_2, ct + p_3 \rangle = \langle a, b, c \rangle t + \langle p_1, p_2, p_3 \rangle$$

Line through  $P(p_1, p_2, p_3)$  and perpendicular to  $\vec{n} = a\vec{i} + b\vec{j} + c\vec{k}$ :

$$a(x - p_1) + b(y - p_2) + c(z - p_3) = 0$$

Distance between line and point:

$$d = \frac{||\vec{PS} \times v||}{||v||}$$

Distance between Point  $S$  and a Plane,

$$d = \left| \vec{PS} \cdot \frac{n}{||n||} \right|$$

Projection,

$$\text{proj}_b a = \left( \frac{a \cdot b}{||b||} \right) \frac{b}{||b||}$$

Angle between planes or vectors:

$$\theta = \cos^{-1} \left( \left| \frac{\vec{n}_1 \cdot \vec{n}_2}{||\vec{n}_1|| \cdot ||\vec{n}_2||} \right| \right)$$

Arc Length ( $s(t)$ ):

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt = \int_a^b ||\vec{r}'(t)|| dt$$

$$s(t) = \int_{t_0}^t \sqrt{\left(\frac{dx}{d\tau}\right)^2 + \left(\frac{dy}{d\tau}\right)^2 + \left(\frac{dz}{d\tau}\right)^2} d\tau = \int_{t_0}^t ||\vec{r}'(\tau)|| d\tau$$

Speed:

$$\frac{ds}{dt} = ||\vec{v}(t)||$$

The unit tangent vector ( $T(t)$ ):

$$\vec{T}(t) = \frac{\vec{r}'(t)}{||\vec{r}'(t)||} = \frac{\vec{v}(t)}{||\vec{v}(t)||}$$

The curvature function ( $\kappa(t)$ ):

$$\begin{aligned} \kappa &= \left\| \frac{d\vec{T}}{ds} \right\| \\ &= \frac{||\vec{T}'(t)||}{||\vec{v}(t)||} \\ &= \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3} \end{aligned}$$

Radius of curvature:

$$p = \frac{1}{\kappa}$$

Principal Normal Vector ( $N(t)$ ):

$$\vec{N}(t) = \frac{\vec{T}'(t)}{||\vec{T}'(t)||}$$

Binormal vector ( $B(t)$ ):

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

# Formulas-2

$\lim_{(x,y) \rightarrow (x_0,y_0)} f(x,y) = L$  if for every  $\epsilon > 0$ , there exists a corresponding  $\delta > 0$ , such that for all  $(x,y)$  in the domain of  $f$ ,  $|f(x,y) - L| < \epsilon$  whenever  $0 < \sqrt{(x - x_0)^2 + (y - y_0)^2} < \delta$

$$\left. \frac{\partial f}{\partial x} \right|_{(x_0,y_0)} = \lim_{h \rightarrow 0} \frac{f(x_0 + h, y_0) - f(x_0, y_0)}{h} = f_x(x_0, y_0)$$

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial x} \right)$$

$$f_{yy} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial y} \right)$$

$$f_{xy} = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right)$$

$$f_{yx} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right)$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt}$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

Directional Derivative of the unit vector  $u = u_1 \mathbf{i} + u_2 \mathbf{j}$

$$\begin{aligned} f'_u(x, y) &= D_u f(x, y) = \lim_{s \rightarrow 0} \frac{f(x + su_1, y + su_2) - f(x, y)}{s} \\ &= \nabla f(x, y) \cdot u \\ &= \|\nabla f\| \cos \theta \end{aligned}$$

The Gradient

$$\nabla f(x, y, z) = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}$$

Tangent line to a level curve of the form  $f(x, y) = 0$  at a point  $(x_0, y_0)$ ,

$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) = 0$$

$$\frac{d}{dt}(f(r(t))) = \nabla f(r(t)) \cdot r'(t)$$

Tangent plane to  $f(x, y, z) = c$  at the point  $P(x_0, y_0, z_0)$ ,

$$f_x(P)(x - x_0) + f_y(P)(y - y_0) + f_z(P)(z - z_0) = 0$$

Or when  $f(x, y) = z$  at the point  $P(x_0, y_0)$ ,

$$f_x(P)(x - x_0) + f_y(P)(y - y_0) - (z - z_0) = 0$$

Normal line to  $f(x, y, z) = c$  at the point  $P(x_0, y_0, z_0)$ ,

$$x = x_0 + f_x(P)t$$

$$y = y_0 + f_y(P)t$$

$$z = z_0 + f_z(P)t$$

Linear Approximation ( $f(x, y) \approx L(x, y)$ ) of  $f(x, y)$  at  $(x_0, y_0)$ ,

$$L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

Total Differential,

$$df = f_x(x_0, y_0)dx + f_y(x_0, y_0)dy$$

$$df = (\nabla f(P_0) \cdot u)ds$$

Standard Linear Approximation Error where  $M$  is the upper bound of the second partials on a rectangle centered at point  $P$ ,

$$|E| \leq \frac{1}{2}M(|x - x_0| + |y - y_0|)^2$$