

# Formulas-1

Credit Aarush Magic

$$\begin{aligned}\vec{a} \cdot \vec{b} &= a_1b_1 + a_2b_2 + a_3b_3 + \cdots + a_nb_n \\ \vec{a} \times \vec{b} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \\ &= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k} \\ &= (a_2b_3 - b_2a_3)\vec{i} - (a_1b_3 + b_1a_3)\vec{j} + (a_1b_2 + b_1a_2)\vec{k}\end{aligned}$$

## Properties:

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos(\theta)$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin(\theta)$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\vec{a} \cdot \vec{0} = 0$$

$$\vec{a} \times \vec{0} = \vec{0}$$

$$(c\vec{a}) \cdot \vec{b} = \vec{a} \cdot (c\vec{b})$$

$$(c\vec{a}) \times \vec{b} = \vec{a} \times (c\vec{b})$$

$$\vec{a} \cdot \vec{a} = ||\vec{a}||^2$$

$$\vec{a} \times \vec{a} = \vec{0}$$

$$\text{If } \vec{a} \perp \vec{b} \text{ then } \vec{a} \cdot \vec{b} = 0$$

$$\text{If } \vec{a} \parallel \vec{b} \text{ then } \vec{a} \times \vec{b} = \vec{0}$$

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w} \quad \text{nbsp;}$$

## Projectile Motion:

$$\text{Max Height} = \frac{(v_0 \sin(\theta))^2}{2g}$$

$$\text{Range} = \frac{v_0^2 \sin(2\theta)}{g}$$

$$\text{Flight Time} = \frac{2v_0 \sin(\theta)}{g}$$

# Graphs

Type	Equations
Cylinder	$ax^n + by^m = c, ax^n + bz^m = c, ay^n + bz^m = c$
Elliptical Paraboloid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$
Elliptical Cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Hyperboloid of 1 sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
Hyperboloid of 2 sheets	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Hyperbolic Paraboloid	$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}, c > 0$

## Other Formulas

Line through  $P(p_1, p_2, p_3)$  and parallel to  $\vec{v} = a\vec{i} + b\vec{j} + c\vec{k}$  when  $t \in \mathbb{R}$ :

$$x = at + p_1 \quad y = bt + p_2 \quad z = ct + p_3$$

$$\langle at + p_1, bt + p_2, ct + p_3 \rangle = \langle a, b, c \rangle t + \langle p_1, p_2, p_3 \rangle$$

Line through  $P(p_1, p_2, p_3)$  and perpendicular to  $\vec{n} = a\vec{i} + b\vec{j} + c\vec{k}$ :

$$a(x - p_1) + b(y - p_2) + c(z - p_3) = 0$$

Distance between line and point:

$$d = \frac{||\vec{PS} \times v||}{||v||}$$

Distance from a Point to a Plane,

$$d = \left| \vec{PS} \cdot \frac{n}{||n||} \right|$$

Projection,

$$\text{proj}_b a = \left( \frac{a \cdot b}{||b||} \right) \frac{b}{||b||}$$

Angle between planes or vectors:

$$\theta = \cos^{-1} \left( \left| \frac{\vec{n}_1 \cdot \vec{n}_2}{||\vec{n}_1|| \cdot ||\vec{n}_2||} \right| \right)$$

Distance between Point  $S$  and a plane:

$$d = \left| \vec{PS} \cdot \frac{\vec{n}}{||\vec{n}||} \right|$$

The triangle property of integrals:

$$\left\| \int_a^b \vec{f}(t) dt \right\| \leq \int_a^b \|\vec{f}(t)\| dt$$

Arc Length ( $s(t)$ ):

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt = \int_a^b \|\vec{r}'(t)\| dt$$

$$s(t) = \int_{t_0}^t \sqrt{\left(\frac{dx}{d\tau}\right)^2 + \left(\frac{dy}{d\tau}\right)^2 + \left(\frac{dz}{d\tau}\right)^2} d\tau = \int_{t_0}^t \|\vec{r}'(\tau)\| d\tau$$

Speed:

$$\frac{ds}{dt} = \|\vec{v}(t)\|$$

The unit tangent vector ( $T(t)$ ):

$$\vec{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \frac{\vec{v}(t)}{\|\vec{v}(t)\|}$$

The curvature function ( $\kappa(t)$ ):

$$\kappa = \left\| \frac{d\vec{T}}{ds} \right\| \quad T \text{ is the unit tangent vector, } s \text{ is the arc length}$$

$$= \frac{\|\vec{T}'(t)\|}{\|\vec{v}(t)\|} \quad \text{note: } \frac{ds}{dt} = \|\vec{v}\|$$

$$= \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|^3}$$

Radius of curvature:

$$\rho = \frac{1}{\kappa}$$

Principal Normal Vector ( $N(t)$ ):

$$\vec{N}(t) = \frac{\vec{T}'(t)}{\|\vec{T}'(t)\|}$$

Binormal vector ( $B(t)$ ):

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$