## Formulas-1

Credit Aarush Magic

$$ec{a} \cdot ec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 + \cdots + a_n b_n$$
 $ec{a} imes ec{b} = \begin{vmatrix} ec{i} & ec{j} & ec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ 
 $= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} ec{\mathbf{i}} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} ec{\mathbf{j}} + \begin{vmatrix} a_1 & a_2 \\ b_1 & n_2 \end{vmatrix} \mathbf{k}$ 
 $= (a_2 b_3 - b_2 a_3) ec{i} - (a_1 b_3 + b_1 a_3) ec{j} + (a_1 b_2 + b_1 a_2) ec{k}$ 

# **Properties:**

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos(\theta)$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin(\theta)$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times (\vec{b} + \vec{c})$$

$$\vec{a} \times \vec{b} = \vec{a} \times (\vec{c} \vec{b})$$

$$(c\vec{a}) \cdot \vec{b} = \vec{a} \cdot (c\vec{b})$$

$$(c\vec{a}) \times \vec{b} = \vec{a} \times (c\vec{b})$$

$$\vec{a} \cdot \vec{a} = ||\vec{a}||^2$$

$$\vec{a} \times \vec{a} = \vec{0}$$
If  $\vec{a} \perp \vec{b}$  then  $\vec{a} \times \vec{b} = \vec{0}$ 

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w}) \vec{v} - (\vec{u} \cdot \vec{v}) \vec{w} \qquad nbsp;$$

# **Projectile Motion:**

$$ext{Max Height} = rac{(v_0 \sin( heta))^2}{2g} \ ext{Range} = rac{v_0^2 \sin(2 heta)}{g} \ ext{Flight Time} = rac{2v_0 \sin( heta)}{g}$$

# **Graphs**

Туре	Equaions
Elliptical Paraboloid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$
Elliptical Cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$
Ellipsoid	$rac{x^2}{a^2} + rac{y^2}{b^2} + rac{z^2}{c^2} = 1$
Hyperboloid of 1 sheet	$rac{x^2}{a^2} + rac{y^2}{b^2} - rac{z^2}{c^2} = 1$
Hyperboloid of 2 sheets	$-rac{x^2}{a^2} - rac{y^2}{b^2} + rac{z^2}{c^2} = 1$
Hyperbolic Paraboloid	$-rac{x^2}{a^2}+rac{y^2}{b^2}=rac{z}{c},c>0$

## **Other Formulas**

Line through  $P(p_1,p_2,p_3)$  and parallel to  $ec{v}=ec{ai}+ec{bj}+ec{ck}$  when  $t\in\mathbb{R}$ :

$$x=at+p_1$$
  $y=bt+p_2$   $z=ct+p_3$   $\langle at+p_1,bt+p_2,ct+p_3
angle =\langle a,b,c
angle t+\langle p_1,p_2,p_3
angle$ 

Line through  $P(p_1,p_2,p_3)$  and perpendicular to  $ec{n}=aec{i}+bec{j}+cec{k}$  :

$$a(x-p_1) + b(y-p_2) + c(z-p_3) = 0$$

Distance between line and point:

$$d = rac{||ec{PS} imes v||}{||v||}$$

Distance between Point S and a Plane,

$$d = \left| ec{PS} \cdot rac{n}{||n||} 
ight|$$

Projection,

$$\mathrm{proj}_b a = \left(rac{a \cdot b}{||b||}
ight)rac{b}{||b||}$$

Angle between planes or vectors:

$$heta = \cos^{-1}\left(\left|rac{ec{n_1}\cdotec{n_2}}{||ec{n_1}||\cdot||ec{n_2}||}
ight|
ight)$$

Arc Length (s(t)):

$$L = \int_a^b \sqrt{\left(rac{dx}{dt}
ight)^2 + \left(rac{dy}{dt}
ight)^2 + \left(rac{dz}{dt}
ight)^2} dt \qquad = \int_a^b ||ec{r}'(t)|| dt \ s(t) = \int_{t_0}^t \sqrt{\left(rac{dx}{d au}
ight)^2 + \left(rac{dy}{d au}
ight)^2 + \left(rac{dz}{d au}
ight)^2} d au \qquad = \int_{t_0}^t ||ec{r}'( au)|| d au$$

Speed:

$$rac{ds}{dt} = ||ec{v}(t)||$$

The unit tangent vector (T(t)):

$$ec{T}(t) = rac{ec{r}'(t)}{||ec{r}'(t)||} = rac{ec{v}(t)}{||ec{v}(t)||}$$

The curvature function  $(\kappa(t))$ :

$$egin{aligned} \kappa &= \left\| rac{dec{T}}{ds} 
ight\| \ &= rac{||ec{T}'(t)||}{||ec{v}(t)||} \ &= rac{||ec{r}'(t) imesec{r}''(t)||}{||ec{r}'(t)||^3} \end{aligned}$$

Radius of curvature:

$$p=rac{1}{\kappa}$$

Principal Normal Vector (N(t)):

$$ec{N}(t) = rac{ec{T}'(t)}{||ec{T}'(t)||}$$

Binormal vector (B(t)):

$$ec{B}(t) = ec{T}(t) imes ec{N}(t)$$

### Formulas-3

Fubini's Theorem

$$V = \iint_R f(x,y) dA = \int_c^d \int_a^b f(x,y) dx dy = \int_a^b \int_c^d f(x,y) dy dx$$

if R is defined by  $a \leq x \leq b$  and  $g_1(x) \leq y \leq g_2(x)$  then,

$$V=\int_a^b\int_{g_1(x)}^{g_2(x)}f(x,y)dydx$$

Notice: The functions  $g_n(x)$  are evaluated first.

# **Properties**

$$egin{aligned} &\iint_R cf(x,y)dA = c\iint_R f(x,y)dA \ &\iint_R f(x,y) \pm g(x,y)dA = \iint_R f(x,y)dA \pm \iint_R g(x,y)dA \ &\iint_R f(x,y)dA \geq 0 ext{ if } f(x,y) \geq 0 ext{ on } R \ &\iint_R f(x,y)dA \geq \iint_R g(x,y)dA ext{ if } f(x,y) \geq g(x,y) ext{ on } R \end{aligned}$$

If  $R_1$  and  $R_2$  are non-overlapping regions,

$$egin{aligned} &\iint_R f(x,y) dA = \iint_{R_1} f(x,y) dA + \iint_{R_2} f(x,y) dA \ &A = \iint_R dA = \int_{x_1}^{x_2} \int_{y_1}^{y_2} dy dx = \int_{ heta_1}^{ heta_2} \int_{r_1}^{r_2} r \ dr d heta \ &V = \iint_R f(x,y) dA \end{aligned}$$
 Average Value  $= \frac{1}{ ext{Area of } R} \iint_R f dA \quad ext{OR} \quad \frac{1}{ ext{Volume of } D} \iiint_D F dV \ &V = \iiint_D dV \ &= \int_{x_2}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} dz dy dx \ &= \int_{ heta_1}^{ heta_2} \int_{r_1}^{r_2} \int_{z_1}^{z_2} r \ dz dr d heta \ &= \int_{ heta_1}^{ heta_2} \int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} 
ho^2 \sin(\phi) \ d\theta d\phi d
ho \end{aligned}$ 

### **Mass and First Moments**

#### 3D Solids

 $\delta$  is the density function.

$$\text{Mass: } M = \iiint_D \delta \ dV$$
 First Moments:  $M_{yz} = \iiint_D x \delta \ dV, M_{xz} = \iiint_D y \delta \ dV, M_{xy} = \iiint_D z \delta \ dV$  Center of mass:  $\bar{x} = \frac{M_{yz}}{M}, \bar{y} = \frac{M_{xz}}{M}, \bar{z} = \frac{M_{xy}}{M}$ 

#### 2D Plates

$$\text{Mass: } M = \iint_R \delta \ dA$$
 
$$\text{First Moments: } M_y = \iint_R x \delta \ dA, M_x = \iint_R y \delta \ dA$$
 
$$\text{Center of mass: } \bar{x} = \frac{M_y}{M}, \bar{y} = \frac{M_x}{M}$$

### **Moments of Inertia**

#### 3D Solids

About x-axis: 
$$I_x = \iiint_D (y^2 + z^2) \delta dV$$
  
About y-axis:  $I_y = \iiint_D (x^2 + z^2) \delta dV$   
About z-axis:  $I_z = \iiint_D (x^2 + y^2) \delta dV$   
About a line L:  $I_L = \iiint_D r^2(x, y, z) \delta dV$ 

#### **2D Plates**

About x-axis: 
$$I_x=\iint_R y^2\delta dA$$
  
About y-axis:  $I_y=\iint_R x^2\delta dA$   
About a line L:  $I_L=\iint_R r^2(x,y)\delta dA$   
About the origin:  $I_O=\iint_R (x^2+y^2)\delta dA=I_x+I_y$ 

# Joint probability density function

### **Conditions**

$$f(x,y)\geq 0 \ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)dxdy=1 \ P((X,Y)\in R)=\iint_{R}f(x,y)dxdy$$

## Mean and expected value

$$\mu_X = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x f(x,y) dx dy \ \mu_Y = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y f(x,y) dx dy$$

# Spherical Coordinates $(\rho, \phi, \theta)$

$$egin{aligned} r &= 
ho \sin(\phi), \ x &= r \cos( heta) = 
ho \sin(\phi) \cos( heta) \ z &= 
ho \cos(\phi), \ y &= r \sin( heta) = 
ho \sin(\phi) \sin( heta) \ 
ho &= \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2} \end{aligned}$$

Jacobian:

$$J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial(x,y)}{\partial(u,v)}$$

$$\iint_{R} f(x,y) dx dy = \iint_{G} f(g(u,v),h(u,v)) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du dv$$

$$J(u,v,w) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \frac{\partial(x,y,z)}{\partial(u,v,w)}$$

$$\iiint_{D} f(x,y,z) dx dy dz = \iiint_{B} f(g(u,v,w),h(u,v,w),k(u,v,w)) \left| \frac{\partial(x,y,z)}{\partial(u,v,w)} \right| du dv dw$$

### Formulas-2

 $\lim_{(x,y)\to(x_0,y_0)}f(x,y)=L \text{ if for every } \epsilon>0, \text{ there exists a corresponding } \delta>0, \text{ such that for all } (x,y) \text{ in the domain of f, } |f(x,y)-L|<\epsilon \text{ whenever } 0<\sqrt{(x-x_0)^2+(y-y_0)^2}<\delta$ 

$$egin{aligned} rac{\partial f}{\partial x}|_{(x_0,y_0)} &= \lim_{h o 0} rac{f\left(x_0+h,y_0
ight)-f\left(x_0,y_0
ight)}{h} = f_x\left(x_0,y_0
ight) \ f_{xx} &= rac{\partial^2 f}{\partial x^2} = rac{\partial}{\partial x}\left(rac{\partial f}{\partial x}
ight) \ f_{yy} &= rac{\partial^2 f}{\partial y^2} = rac{\partial}{\partial y}\left(rac{\partial f}{\partial y}
ight) \ f_{xy} &= rac{\partial^2 f}{\partial yx} = rac{\partial}{\partial y}\left(rac{\partial f}{\partial x}
ight) \ f_{yx} &= rac{\partial^2 f}{\partial xy} = rac{\partial}{\partial x}\left(rac{\partial f}{\partial y}
ight) \ rac{dw}{dt} &= rac{\partial w}{\partial x} \cdot rac{dx}{dt} + rac{\partial w}{\partial y} \cdot rac{dy}{dt} + rac{\partial w}{\partial z} \cdot rac{dz}{dt} \ rac{dy}{dx} &= -rac{F_x}{F_x} \end{aligned}$$

Directional Derivative of the unit vector  $u = u_1 \mathbf{i} + u_2 \mathbf{j}$ 

$$egin{aligned} f_u'\left(x,y
ight) &= D_u f\left(x,y
ight) = \lim_{s o 0} rac{f\left(x+s u_1,y+s u_2
ight) - f\left(x,y
ight)}{s} \ &= 
abla f\left(x,y
ight) \cdot u \ &= ||
abla f|| \cos heta \end{aligned}$$

The Gradient

$$abla \, f \left( x,y,z 
ight) = rac{\partial f}{\partial x} i + rac{\partial f}{\partial y} j + rac{\partial f}{\partial z} k$$

Tangent line to a level curve of the form f(x,y)=0 at a point  $(x_0,y_0)$ 

$$egin{aligned} f_x(x_0,y_0)\left(x-x_0
ight) + f_y(x_0,y_0)\left(y-y_0
ight) = 0 \ & rac{d}{dt}(f(r(t)) = 
abla f(r(t)) \cdot r'(t) \end{aligned}$$

Tangent plane to f(x, y, z) = c at the point  $P(x_0, y_0, z_0)$ ,

$$f_x(P)(x-x_0) + f_y(P)(y-y_0) + f_z(P)(z-z_0) = 0$$

Or when f(x,y) = z at the point  $P(x_0, y_0)$ ,

$$f_x(P)\left(x-x_0
ight)+f_y(P)\left(y-y_0
ight)-\left(z-z_0
ight)=0$$

Normal line to f(x, y, z) = c at the point  $P(x_0, y_0, z_0)$ ,

$$egin{aligned} x &= x_0 + f_x(P)t \ y &= y_0 + f_y(P)t \ z &= z_0 + f_z(P)t \end{aligned}$$

Linear Approximation  $(f(x,y) \approx L(x,y))$  of f(x,y) at  $(x_0,y_0)$ ,

$$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

Total Differential,

$$egin{aligned} df &= f_x\left(x_0, y_0
ight) dx + f_y\left(x_0, y_0
ight) dy \ df &= \left(
abla f(P_0) \cdot u
ight) ds \end{aligned}$$

Standard Linear Approximation Error where M is the upper bound of the second partials on a rectangle centered at point P,

$$|E| \leq rac{1}{2} M (|x-x_0| + |y-y_0|)^2$$

Second Partials Test at  $(x_0, y_0)$  assuming  $\nabla f = 0$ ,

$$A=f_{xx}(x_0,y_0),\quad B=f_{xy}(x_0,y_0)\quad, C=f_{yy}(x_0,y_0)$$
 $D=AC-B^2$ 
 $D<0 ext{ Saddle point }$ 
 $D>0 ext{ Relative Extrema }$ 
 $D=0 ext{ Indecisive }$ 

For the relative extrema, if D>0 and A>0 then you have a local min, if A<0 you have a local max.

Lagrange Multipliers when g(x, y) = 0,

$$abla f = \lambda 
abla g$$