Formulas-1

Credit Aarush Magic

$$ec{a} \cdot ec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 + \cdots + a_n b_n \ ec{a} imes ec{b} = egin{bmatrix} ec{i} & ec{j} & ec{k} \ a_1 & a_2 & a_3 \ b_1 & b_2 & b_3 \ \end{bmatrix} \ = egin{bmatrix} u_2 & u_3 \ v_2 & v_3 \end{bmatrix} \mathbf{i} - egin{bmatrix} u_1 & u_3 \ v_1 & v_3 \end{bmatrix} \mathbf{j} + egin{bmatrix} u_1 & u_2 \ v_1 & v_2 \end{bmatrix} \mathbf{k} \ = (a_2 b_3 - b_2 a_3) ec{i} - (a_1 b_3 + b_1 a_3) ec{j} + (a_1 b_2 + b_1 a_2) ec{k} \ \end{bmatrix}$$

Properties:

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos(\theta)$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin(\theta)$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times (\vec{b} + \vec{a} \times \vec{c})$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times (\vec{b} + \vec{c})$$

$$\vec{a} \times \vec{b} = \vec{a} \times (\vec{c} \vec{b})$$

$$(c\vec{a}) \cdot \vec{b} = \vec{a} \cdot (c\vec{b})$$

$$(c\vec{a}) \times \vec{b} = \vec{a} \times (c\vec{b})$$

$$\vec{a} \cdot \vec{a} = ||\vec{a}||^2$$

$$\vec{a} \times \vec{a} = \vec{0}$$
If $\vec{a} \perp \vec{b}$ then $\vec{a} \times \vec{b} = \vec{0}$

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w}) \vec{v} - (\vec{u} \cdot \vec{v}) \vec{w} \qquad nbsp;$$

Graphs

Cylinder: $ax^n + by^m = c$, $ax^n + bz^m = c$, $ay^n + bz^m = c$

Elliptical Paraboloid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$

Elliptical Cone: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$ Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Hyperboloid of 1 sheet: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ Hyperboloid of 2 sheets: $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Hyperbolic Paraboloid: $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c},c>0$

Other Formulas

Line through $P(p_1,p_2,p_3)$ and parallel to $ec{v}=aec{i}+bec{j}+cec{k}$ when $t\in\mathbb{R}$:

$$egin{aligned} x = at + p_1 \quad y = bt + p_2 \quad z = ct + p_3 \ & \langle at + p_1, bt + p_2, ct + p_3
angle = \langle a, b, c
angle t + \langle p_1, p_2, p_3
angle \end{aligned}$$

Line through $P(p_1,p_2,p_3)$ and perpendicular to $ec{n}=aec{i}+bec{j}+cec{k}$:

$$a(x-p_1) + b(y-p_2) + c(z-p_3) = 0$$

Distance between line and point:

$$d = rac{||ec{PS} imes v||}{||v||}$$

Distance from a Point to a Plane,

$$d = \left| ec{PS} \cdot rac{n}{||n||}
ight|$$

Projection,

$$\mathrm{proj}_b a = \left(\frac{a \cdot b}{||b||}\right) \frac{b}{||b||}$$

Angle between planes:

$$heta = \cos^{-1}\left(\left|rac{ec{n_1}\cdotec{n_2}}{||ec{n_1}||\cdot||ec{n_2}||}
ight|
ight)$$

Distance between Point S and a plane:

$$d = \left| ec{PS} \cdot rac{ec{n}}{||ec{n}||}
ight|$$

The triangle property of integrals:

$$\left\|\int_a^b ec{f}(t)dt
ight\| \leq \int_a^b \|ec{f}(t)\|dt$$

Arc Length (s(t)):

$$L = \int_a^b \sqrt{\left(rac{dx}{dt}
ight)^2 + \left(rac{dy}{dt}
ight)^2 + \left(rac{dz}{dt}
ight)^2} dt \qquad = \int_a^b ||ec{r}'(t)|| dt \ s(t) = \int_{t_0}^t \sqrt{\left(rac{dx}{d au}
ight)^2 + \left(rac{dy}{d au}
ight)^2 + \left(rac{dz}{d au}
ight)^2} d au \qquad = \int_{t_0}^t ||ec{r}'(au)|| d au$$

Speed:

$$rac{ds}{dt} = ||ec{v}(t)||$$

The unit tangent vector (T(t)):

$$ec{T}(t) = rac{ec{r}'(t)}{||ec{r}'(t)||} = rac{ec{v}(t)}{||ec{v}(t)||}$$

The curvature function $(\kappa(t))$:

$$egin{aligned} \kappa &= \left\| rac{dec{T}}{ds}
ight\| T ext{ is the unit tangent vector, } s ext{ is the arc length} \ &= rac{||ec{T}'(t)||}{||ec{v}(t)||} \quad ext{note: } rac{ds}{dt} = ||v|| \ &= rac{||ec{r}'(t) imes ec{r}''(t)||}{||ec{r}'(t)||^3} \end{aligned}$$

Radius of curvature:

$$p=rac{1}{\kappa}$$

Principal Normal Vector (N(t)):

$$ec{N}(t) = rac{ec{T}'(t)}{||ec{T}'(t)||}$$

Binormal vector (B(t)):

$$ec{B}(t) = ec{T}(t) imes ec{N}(t)$$

Projectile Motion:

$$egin{aligned} ext{Max Height} &= rac{(v_0\sin(heta))^2}{2g} \ ext{Range} &= rac{v_0^2\sin(2 heta)}{g} \ ext{Flight Time} &= rac{2v_0\sin(heta)}{g} \end{aligned}$$

Formulas-2

 $\lim_{(x,y) o(x_0,y_0)}f(x,y)=L$ if for every ϵ >0, there exists a corresponding δ >0, such that for all (x,y) in the domain of f, $|f(x,y)-L|<\epsilon$ whenever $0<\sqrt{(x-x_0)^2+(y-y_0)^2}<\delta$

$$rac{\partial f}{\partial x}|_{(x_0,y_0)} = \lim_{h o 0} rac{f\left(x_0+h,y_0
ight)-f\left(x_0,y_0
ight)}{h} = f_x\left(x_0,y_0
ight)$$

$$egin{align*} f_{xx} &= rac{\partial^2 f}{\partial x^2} = rac{\partial}{\partial x} \left(rac{\partial f}{\partial x}
ight) \ f_{yy} &= rac{\partial^2 f}{\partial y^2} = rac{\partial}{\partial y} \left(rac{\partial f}{\partial y}
ight) \ f_{xy} &= rac{\partial^2 f}{\partial yx} = rac{\partial}{\partial y} \left(rac{\partial f}{\partial x}
ight) \ f_{yx} &= rac{\partial^2 f}{\partial xy} = rac{\partial}{\partial x} \left(rac{\partial f}{\partial y}
ight) \ rac{dw}{dt} &= rac{\partial w}{\partial x} \cdot rac{dx}{dt} + rac{\partial w}{\partial y} \cdot rac{dy}{dt} + rac{\partial w}{\partial z} \cdot rac{dz}{dt} \ rac{dy}{dx} &= -rac{F_x}{F_y} \end{split}$$

$$egin{split} f_u'\left(x,y
ight) &= D_u f\left(x,y
ight) = \lim_{s o 0} rac{f\left(x+su_1,y+su_2
ight) - f\left(x,y
ight)}{s} =
abla \, \mathrm{f}\left(x,y
ight) \cdot u \ & \
abla \, f\left(x,y,z
ight) = rac{\partial f}{\partial x} i + rac{\partial f}{\partial y} j + rac{\partial f}{\partial z} k \end{split}$$

Tangent line to a level curve: $f_{x}\left(x-x_{0}
ight)+f_{y}\left(y-y_{0}
ight)=0$

$$rac{d}{dt}(f(r(t)) =
abla ext{ f}(r(t)) ullet ext{ } r'(t)$$