# Binary Image Classifier Report

Abdullah Mujawar (20006991)

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*Note: only the content included from the abstract to the conclusion counts towards the 10-page limit, excluding the title page, the table of contents and the bibliography.*

# 1. Abstract

Pattern analysis is a branch of machine learning which focuses on recognizing patterns in data. This project focuses on an application of pattern analysis called classifiers, undertaking the objective of building one to classify binary-images. The classifier is built in MATLAB, a programming language specialised in numerical computing, being the main reason why it was chosen to build the classifier. Using supervised GMM training and classification, a set of approximately 220 binary images is used to train the classifier, which is then able to successfully classify over 80% of the images fed to it during testing. The classifier itself never been 100% certain about the class of the input, it returns the class which is most likely to match the input image. The quality of 83% achieved after optimising the classification within the code itself is consequently a solid result, meaning over four fifths of the input is correctly classified after optimising it. This project includes a wide variety of mathematical theories and applications, which are covered throughout the theory and methodology used to build this classifier.

# 2. Theory (Introduction)

A precisely-followed succession of steps is required to create the binary-shape classifier. The first step consisting in training the classifier using a set of training images, is followed by testing that trained classifier using another set of images and calculating the likelihood of each image being close to one of the classes the classifier can determine. The last step consists in assessing the overall quality of the classifier. All these steps are further analysed in the methodology section, but beforehand many aspects of the theory and the mathematics behind making the classifier function properly are discussed in this part, such as understanding how the classifier recognizes different binary shapes and how it can classify new shapes after having been trained.

## 2.1. Introduction to classifiers

A classifier is a computer program which takes some data *x*, 'learns' it and outputs a discrete class label *c* such as: $c = f(x)$. The learning role consists in analysing the input data *x* to create an inferred function *f*, corresponding to the training step, which can then be used in the testing step to output correct class labels for new input data. To go further with the mathematical definition of a classifier, it can be defined as a function with the following parameters: $i = classify(F(x), D)$ where $x$ is a data point, $F(x)$ is the feature, $D$ is the parameter and $i(\in \mathbb{Z})$ is the result label, or output.

The input data *x* which is fed to the classifier can either be labelled or not. On one hand, labelled data such as: $D = \{(x_1, i_1), (x_2, i_2), \ldots\}$ is used for supervised learning, while on the other hand, data with no labels such as: $D = \{x_1, x_2, \ldots\}$ is used for unsupervised learning. A quick real-life example can be used to illustrate the difference between the two types of classifier learnings, assuming the classifier is a human being, the data to be labelled is a basket with different types of fruits, and the task is to arrange them per type. In the case of supervised learning, the human already knows the type of each fruit since the given data is labelled. This labelled data is usually discrete and comes in the form of a list of pairs, with each pair $(x, i)$ consisting of an input object $x$ and a desired output value $i$. In the case of unsupervised learning, the basket of fruits has four new types of fruits unknown to the classifier. He will therefore try to label them after having analysed features he could use to tell the fruits apart from each other such as size and colour (E. McNulty, 2015). Unsupervised learning being harder to implement, supervised learning is the chosen learning method to build this classifier.

## 2.2. Feature vectors, chain codes, Fourier Transform and filters

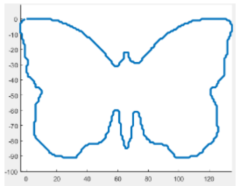
Feature vectors are used to both train and test the classifier. They are numerical representations of objects, which can be used in classification to recognize similar objects, with each of the input images having their own.

In this classifier, the features used for the input binary images are found by performing a succession of transformations and filtering, the first being a chain code transformation. Since the input is in binary images, meaning each pixel of the image can only take two different values (black and white), a chain code transformation can be applied to those images to create a numerical representation of the boundary of the shape in each binary image. The chain code itself is composed of a set of eight symbols (*right, left, up down, up-right, up-left, down-right, down-left*), each representing the direction the boundary of the shape is taking. These symbols are recorded as numbers (from 0 to 7), thus creating a signal. This means that the extremity of the shape in the binary image can be reconstructed from the chain code only, as shown in the images below:

Raw binary image

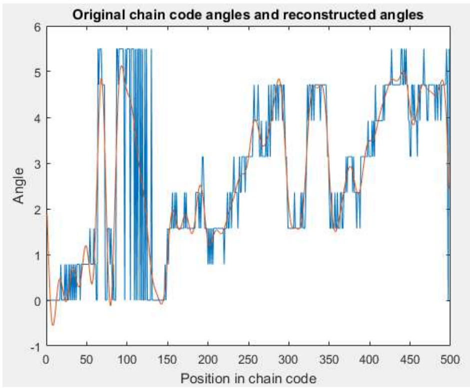
Image with boundary overlay

Reconstructed image

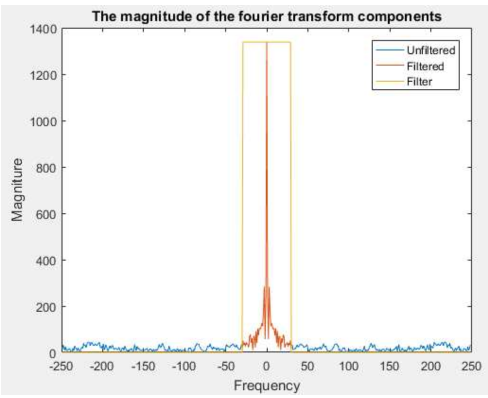


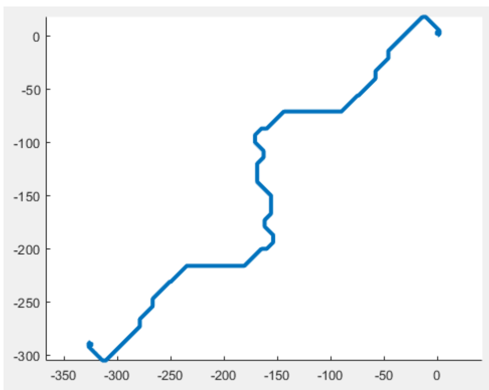
Once the chain code of an image is created, it can be converted to the frequency domain using the Fourier Transform (FT). The FT converts data from the time (signal) domain to frequency domain without any loss of information using the following formula: $F(\omega) = \mathcal{F}[f](\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x)e^{-i\omega x}dx$. The formula being in the complex domain and complicated to implement, built-in functions in MATLAB are used to perform the FT and its inverse of signals.

Converting signals such as the chain code allows for new operations to be carried out which could not have been in the previously. In this case, a low-pass filter, which attenuates frequencies higher than a certain threshhold and keeps the lower ones, is applied to the chain code. This kind of operation, which is more efficient to carry out in the frequency domain, smoothens the signal by removing the short-term fluctuations and leaving the long-term trend (CM20219, lecture 4). It can be seen in the graph below that the chain code (blue) is not smooth before the low-pass filter is applied since it is based on pixels, but becomes smooth (orange) once the filter is applied:




The last step towards retrieving the feature vectors from the filtered chain code in the frequency domain consists in filtering it again using a top-hat filter. A top-hat filter has different uses in the real space and the frequency space, and in the latter, it is used to select a band of desired frequencies from the signal after specifying the lower and upper bounds. Applying the top-hat filter (yellow) to the filtered chain code creates the left graph below (red). At this point, attempting to reconstruct the original binary shape is impossible due to the filtering applied to the chain code. Because the top-hat filter gets rid of the data not "under" it, the chain code data which closes the shape is lost, causing the ends not to join up as depicted in the right graph below. The feature vector itselft corresponds to the part highlighted in red in the left graph, with only the absolute value and the real values retrieved since the signal is complex after the FT has been applied to it.



## 2.3. Supervised GMM Training

The classifier can be trained using a multivariate Gaussian, which corresponds to a generalisation of a normal distribution to $d$ dimensions: $N(x\,|\mu, C) = \frac{1}{(2\pi)^{d/2}\,|C|^{1/2}} \exp(-\frac{1}{2}\,(x-\mu)^T\,C^{-1}(x-\mu))$, where $\mu$ is the mean, $C$ the covariance and $d$ the number of dimensions ($d = 2$ in this case since the data corresponds to images). A Gaussian is fitted for each class, meaning the mean µ and the covariance $C$ are determined for each class.

A Gaussian Mixture Model (GMM) $p(x)$ being the linear sum of $N$ Gaussians, the Gaussians previously determined can be added together using $p(x) = \sum_{i=1}^{N} \alpha_i \times N(x|\mu_i, C_i)$. This is achieved by adding all the Gaussians together and calculating the prior $\alpha_i$ of each Gaussian, which corresponds to the weight of each class in the GMM. The prior is found by dividing the number of images for a class $c$ by the total number of images $M$ in the training set using formula (1):

$$(1)\ \ \alpha^C = \frac{1}{M}\sum i\ [l_i = c]$$

For this binary-image classifier, a Maximum Likelihood Estimation (MLE) technique is used, meaning the Gaussian parameters mean $\mu_i$ and covariance $C_i$ are found with the goal of maximising the sum of the likelihood for every image $n$ over the entire training set $N$. They can be determined by using their usual formulae, and adapting it for each class of the GMM using the MLE, yielding formula (2) for the mean and formula (3) for the covariance:

$$(2)\ \ \mu_i = \frac{1}{N}\sum n\ d_n \qquad\qquad (3)\ C_i = \frac{1}{N}\sum n\ (d - \mu_i)^T(d-\mu_i).$$

The formula to calculate the covariance can be broken down into two steps (Stat Trek, 2017). The first one consists in calculating the deviation matrix, defined in formula (4), and then calculating the covariance matrix, defined in formula (5), which can be done by dividing each element of the deviation matrix by the sum of squares matrix by the number of rows in the size of the feature vector. $x$ corresponds to all the feature vectors of a given class.

$$(4)\ dev = x - \frac{ones \times x}{n} \qquad\qquad (5)\ C = \frac{dev^T \times dev}{n-1}$$

Once the prior, mean and covariance have been determined for each class, the likelihood $p(x|\theta_i) = \frac{1}{(2\pi)^{K/2}\,|C_i|^{1/2}} \exp(-\frac{1}{2}\,(x-\mu_i)^T\,{C_i}^{-1}(x-\mu_i))$ can be found, which corresponds to the probability of a an image corresponding to a class. Note that the first difference $(d - \mu_i)^T$ is transposed instead of the second one since the vectors are stored in a row. The combination of the parameters for each Gaussian constitute the GMM, which is the data used to classify new images from the testing set.

## 2.4. GMM Classification

Given a new test point/image $x$, the classifier is at this point able to find the class with the highest posterior likelihood. This is done using a Maximum a Posteriori (MAP) classification. For each new image from the testing set, a class which maximises the posterior $p(x|\theta_i)$ is selected, corresponding to the class that the classifier outputted based on the image it was fed. This is done using: $\arg\max[p(\theta|x)] \propto \arg\max[\alpha^\theta \times p(x|\theta)]$, where $p(x|\theta)$ is equal to the multivariate Gaussian for image $x$ and class $\theta$, which can be found using the formulas defined in section 2.3. The likelihood can be calculated using formula (6), where $\alpha_\theta$, $\mu_\theta$ and $C_\theta$ are the prior, mean and covariance (respectively) for a class θ:

$$(6)\ \ \arg\max p(\theta\,|x) = \arg\max[(\log\alpha_\theta) - (\tfrac{1}{2}\log|C_\theta|) - (\tfrac{1}{2}(x-\mu_\theta)^T)\ {C_\theta}^{-1}\ (x-\mu_\theta)].$$

"*arg max*" is used to only keep into account terms depending on the class θ, while "log" is used to prevent the likelihoods of being rounded to zero when stored on a computer.

## 2.5. Confusion Matrices

Confusion matrices are used to assess the quality of the classifier during the testing step. A confusion matrix is a table of numbers which is filled each time an image from the testing set has been classified. The columns correspond to the different classes which can be outputted by the classifier (the classes it has been trained to identify), while the rows correspond to the classification returned after an input image has been analysed by the classifier.

This confusion matrix can be achieved by separating the testing set into the same groups that were used during the supervised GMM training. Each testing group represents a row of the matrix, while the classification returned by the classifier represents a column on this specific row. For example, in the case of the binary-image classifier which has been trained to classify images from six different classes, if an image from the group 'Alien' is classified as 'Alien', then the element on row: 'Alien' and column: 'Alien' of the matrix is incremented. If another image from the group 'Alien' is misclassified as 'Butterfly', the element on row: 'Alien' and column: 'Butterfly' is incremented.

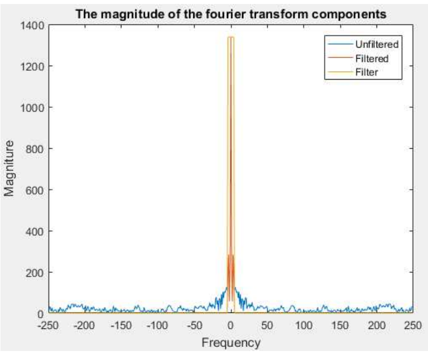
After the entire testing set has been classified, the completed matrix can be analysed to assess the classifier's quality. The number of images correctly classified can be found in the leading diagonal of the matrix, whereas misclassified images are scattered outside that diagonal. Formula (7) can be used to assess the quality of the classifier in the form of a percentage, where *"sum(c)"* is the sum of all correctly classified images in the diagonal of the matrix:

$$(7)\quad quality = \frac{sum(c)}{N} \times 100$$

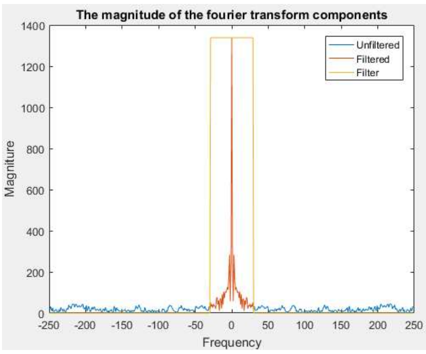
## 2.6. Classifier optimisation

The size of the feature vector is key to achieving a high-quality classifier with over 80% of the images from the testing set correctly classified. Indeed, the feature vector is the element used to train different classes of the GMM and then classify new images by comparing the features with the ones of each class. Therefore, the feature can neither be too short, nor too long, it must be the perfect length. If it is too short, then not enough data will be used to train the GMM, whereas if it is too long, the features for different classes will start looking alike with a peak at the centre and decreasing magnitudes away from the centre, causing more misclassifications than correct ones. This is demonstrated in the three features in red in the graph below. The first one corresponds to the feature vector of the image *Butterfly001* with a feature of size $N = 5$, the second one to the feature vector of the same image with a size $N = 30$, and the last one to the feature vector of the image *Face008* with a feature of size $N = 30$. For $N = 5$ there is a lack of information, whereas for $N = 30$ the two different images' features look similar, especially towards the zero frequency where they both have a powerful magnitude peak. When the feature becomes too long, an issue referred to as the "curse of dimensionality" occurs, causing the classifier's performance to decrease after a certain length due to the accumulation of data making harder for the algorithm to process efficiently.

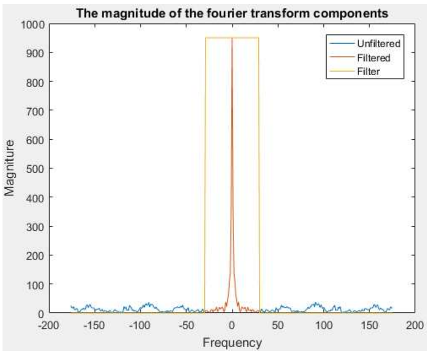
Butterfly, $N = 5$



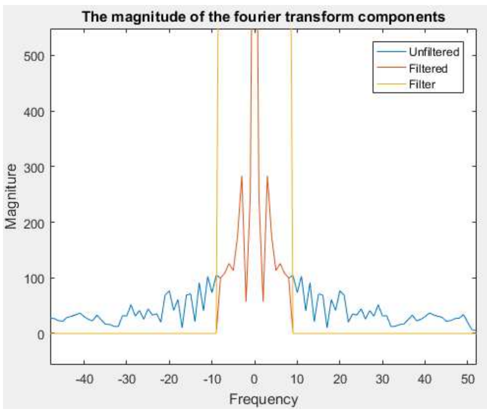

Butterfly, $N = 30$




Face, $N = 30$




Nonetheless, smaller feature vectors are more efficient than bigger feature vectors since the key elements characterizing an image's feature vector can be found at around the center of the peak (not the peak itself). Therefore to further optimise the classifier, the features can be retrieved starting three frequencies to the right of the zero frequency (peak), up to the $N + 3$ frequency, for feature vector of size $N$.




Additionally, it can be seen that the magnitudes are symmetric about the zero frequency, therefore the absolute value of the signal can be used to get more data while keeping the same feature vector length. In the case of the example above, the top-hat filter would be applied starting at the third frequency and go up to N-frequency.

## 2.7. Other Models

Gaussian distributions are used in this project, but many other models exist and are worth mentioning. One of them is the Bernoulli distribution, which is the probability distribution of a variable taking the value 1 for a probability $p$, else 0 for a probability $q = 1 - p$. Being the simplest probability distribution compare to Gaussians, it is unfortunately impossible to use for this project since the Bernoulli distribution is only suitable for binary events, which is not the case with this classifier.

# 3. Methodology

Now that all the theory and mathematics aspects needed to understand the functioning of the binary-image classifier have been covered, the different steps undertaken which go towards building one can be covered.

*Note: the code snippets in this section represent only trimmed parts of the submitted code. Only the most important sections of the code are shown in the snippets.*

## 3.1. Environment setup

Before actually training and testing the classifier, the environment must be setup. In the project directory, the code used to train and test the classifier is stored in the "*src*" folder, while the images are stored in the "*images*" folder and separated in three different sub-folders, one corresponding to the training set, another to the testing set, and the last one to a set of extra test cases.

The entire code is run from one main script called "*script*.m" which starts by storing all the different directories used in strings, as shown below, then trains the classifier, tests it, assesses its quality and performs extra test cases.

```
%% Image paths
imagedir = '../images';
verifyImageDir(imagedir);
imagedir_train = [imagedir '/train'];
imagedir_test = [imagedir '/test'];
imagedir_extra = [imagedir '/extra'];
```

## 3.2. Training

The steps in the training part follow the mathematics explained in sections 2.2 and 2.3 of the report. The script calls the "*train*.m" function, passing two arguments to the function. The first is the path to the testing set of images and the second a variable called $N$, which corresponds to the number of lower frequencies to keep for each feature vector. In other words, $N$ represents the length of each feature vector to retrieve.

```
%% Train classifier with training set of binary images
N = 9; % = number of lowest frequencies to keep
train(imagedir_train, N);
```

"*train*.m" fits a multivariate Gaussian for each class found in the supervised data (in the "*images/training*" folder) using the provided "*getClasses.m*" function. The Gaussians are fitted by calculating the mean, the covariance and the prior of the features of each class in a *for* loop, which goes through each class. The features are retrieved by calling the "*getDataMatrix.m*" function, which creates a matrix storing all the features of length $N$ for a given class. The function returns a matrix, with an example which can be found in section 4.1, with features stored in the columns and each row representing a shape:

```
function [D] = getDataMatrix(imagedir,class,N)
    imagelist = dir(sprintf('%s/%s*.gif', imagedir, class));
    for idx = 1:length(imagelist)
        imagepath = sprintf('%s/%s', imagedir, imagelist(idx).name);
        D(idx, :) = getFeatures(imagepath, N)'; %data matrix returned
    end
end
```

The features retrieved from the output of the "*getDataMatrix.m*" function are created using the "*getFeatures*" function. This function uses the same steps described in section 2.2 of the report. It starts by converting the input binary image from the training set into logical 0's and 1's, then calculates the chain code signal. The. A fast Fourier Transform is applied to the signal which is then filtered using a top-hat filter. The values chosen for the filter are explained in sections 2.6 and 3.4, since they correspond to the quality optimisation section. Once the filter has been applied, the real values of the absolute value of the filtered chain code constitute the feature vectors, as mentioned in section 2.2.

```
function [ features ] = getFeatures( image_path , N)
    im = imread(image_path);
    im = logical(im); % Convert intensity values to 1s and 0s

    %   calculate chain code
    c = chainCode(im);

    % Filter using the FT of the angles of the chaincode
    angles = c(3,:)*(2*pi/8);
    anglesFFT = fft(angles); %fft

    % top-hat filter
    filter = zeros(size(angles));
    filter(3:N+2) = 1;

    % A pply the filter by scalar multipliacation
    filteredFFT = anglesFFT .* filter;

    % absolute valye
    absFiltered = real(abs(filteredFFT));

     % transpose features
    features = (absFiltered(3:N+2))';
end
```

Once each feature of the images for a given class have been stored in the data matrix, the "*train.*m" function calculates the mean, covariance and prior of each classes' data matrix.

The mean, which corresponds to the average of all elements in a column, is calculated using formula (2) from section 2.3 with the "*calcMean.m*" function below. It returns a row vector the length of the number of feature vectors, or $N$, with each value equal to the average of all elements in a column of the data matrix.

```
function [mean] = calcMean(data)
    columns = size(data, 1); % # of feature vectors in input
    rows = size(data, 2);    % # of data points in input
    mean = zeros(1, rows);   % initialize mean vector
    for i = 1:rows
        total = 0; % sum of all elements in a column
        for j = 1:columns
            total = total + data(j,i);
        end
        mean(1,i) = total/columns;
    end
end
```

The covariance is calculated using formulas (4) and (5) from in section 2.3 with the "*calcCov.m*" function. It returns a square matrix the size of the number of features. The leading diagonal shows the covariance matrix corresponds to the variance between the features of the images of a same class from the training.

```
function [covariance] = calcCov(data)
    % size of input matrix column
    columns = size(data,1);
    % square matrix of 1's with size of column of input matrix
    one = ones(columns,columns);

    % deviation matrix
    deviation = data - (one*data/(columns));
    % covariance matrix
    covariance = (transpose(deviation) * deviation)/(columns-1);
end
```

The prior is the last step towards having a complete GMM. It is calculated using formula (1) (section 2.3) within the "*train.m*" function. Corresponding to the weight of a class in the GMM, it can be found by dividing the number of training images for a given class by the number of images in the training set:

```
for idx = 1:length(classes)
    class = classes{idx};
    models(idx).name = class;
    dataMatrix = getDataMatrix(imagedir, class, N);
    models(idx).mean = transpose(calcMean(dataMatrix));
    models(idx).cov = ensurePSD(calcCov(dataMatrix));
    models(idx).prior = ((getNumImagesForClass(imagedir,class)) / totalImages);
end
save('models');
```

When "*train.m*" finishes running for each class, it stores each Gaussian in a MATLAB structure named "*models.mat*" corresponding to the GMM, which can be found in section 4.2. This is the GMM used to classify the testing set of images.

## 3.3. Testing and Quality assessment

At this point, the classifier can be considered as functional but it is yet to be tested with a new set of images to verify that it works correctly and to assess its quality. This can be done by using the GMM which has just been fitted previously. After having called the training function, the main script next calls "*getConfusionMatrix.m*", which is a function that classifies all the images in the testing set and assesses the overall quality of the classifier using confusion matrices.

```
%% Test classifier with new testing set of binary images
confusion_matrix = getConfusionMatrix(imagedir_test);
disp(confusion_matrix);
```

"*getConfusionMatrix.m*" loops through each image in the testing set and attempts to classify them, then increments the appropriate element in the confusion matrix. The function makes use of the provided function "*classify.m*", which follows the Maximum a Posteriori (MAP) classification technique described in section 2.4. Using the formula (6), the feature of each new image from the testing set is compared with each class the classifier can classify, and the class which maximises the posterior for the feature of the input image is returned by the function:

```
function [classname] = classify(imagepath)
    load('models'); % trained GMM
    N = length(models(1).mean);  %Assume models use the same number of features
    features = getFeatures(imagepath, N); % feature of input image
    maxscore = -inf;

    for idx = 1:length(models) % which class has the highest score
        model = models(idx);

        score = log(model.prior) - 0.5*(log(det(model.cov))) -
                0.5 * (transpose(features-model.mean)) *
                (inv(model.cov)) * (features-model.mean);

        % keep class which maximises posterior
        if score > maxscore
            maxscore = score;
            bestidx = idx;
        end

    end
    classname = classes(bestidx); % classification returned
end
```

Each returned classification by the "*classify.m*" function is next used to increment the confusion matrix. This is done by creating three nested for loops. The first loop goes through the six classes and separates the images from the testing set into groups corresponding to those classes. The second loop classifies each image from the groups in the testing set and compares the classification returned by "*classify.m*" with the classes in the third loop. This is where the confusion matrix slot with the row corresponding to the testing group and the column corresponding the classification returned, is incremented.

```
classes = getClasses(imagedir);
numClasses = size(classes,2);
totalImages = getNumImages(imagedir);
accumulator = 0;
confusion_matrix = zeros(numClasses,numClasses); %initialize
for i = 1:numClasses % loop through each class
    imagelist = dir(sprintf('%s/%s*.gif', imagedir, classes{i}));
    length_list = size(imagelist,1); % # of images in created list

    for j = 1:length_list % loop through each image in class i
        imagepath = sprintf('%s/%s', imagedir, imagelist(j).name);
        classification = classify(imagepath); %classify images individually

        for k = 1:numClasses % increment classification matrix
            if(strcmp(char(classification),classes{k}))
                confusion_matrix(k,i) = confusion_matrix(k,i) + 1;
                break;
            end
        end

    end
end
```

Once the entire testing set has been classified, the quality of the classifier can be determined. As mentioned in section 2.5, the images correctly classified are found in the leading diagonal of the confusion matrix, while misclassified images are scattered over the rest of the matrix. To assess the overall quality of the classifier, formula (7) can be used to calculate the sum of all the values in the diagonal, divided by the total number of images that have been classified:

```
% total images which were correctly classified
for l = 1:numClasses
    accumulator = accumulator + confusion_matrix(l,l);
end

% calculate final classifier quality with current testing set
score = (accumulator / totalImages)*100;
disp(['Confusion matrix score = ' int2str(score) '%']); disp(' ');
```

The output of "*getConfusionMatrices.m*" can be found in section 4.3, with the confusion matrix and the overall quality of the classifier.

### 3.4. Optimisation

Once all the previous steps have been carried out, the classifier is fully functional and the confusion matrix is used to output the overall quality. After the first run, the quality of the classifier is at approximately 66%, which can be improved. This is due to the argument $N$ which is passed at the beginning of the script in the "*train.*m" function. As explained in section 2.6, $N$ corresponds to the vector length. A quality of 62% is achieved when $N = 30$, which is too long. Therefore, to optimise the classifier, the size of the feature vectors should not be too long nor too short. After plotting a function in Excel to see for which feature vector the classifier has the best quality, the optimal size found is $N = 8$, with the results shown in section 4.4. The optimal feature vector length, combined with retrieving the features three frequencies away from the zero frequency, leads to an overall quality of 83%.

## 4. Results

### 4.1. Feature vector & Data Matrix

The "*train.m*" function calls "*getDataMatrix.m*" for each class, which also calls "*getFeatures.m*" for each image in that class. This results in a matrix of size $x \times N$, where $x$ corresponds to the number of input images for a given class, and $N$ to the length of the features to retrieve. In this example, the data matrix retrieved corresponds to the one for the '*Alien*' class, which has 42 images, for features of size 8. The highlighted row corresponds to the feature vector of size 8 for the 13th image of the class '*Alien*'.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | 95.73737 | 58.43914 | 52.25033 | 56.39159 | 78.80954 | 66.46931 | 50.77994 | 35.84649 |
| 2 | 78.0644 | 268.9368 | 67.80256 | 210.7446 | 135.8717 | 125.5557 | 118.2222 | 148.9758 |
| 3 | 44.06166 | 81.2206 | 11.29294 | 158.2132 | 74.71287 | 122.2687 | 44.04089 | 55.27117 |
| 4 | 138.0766 | 176.7387 | 141.2295 | 271.4116 | 101.7774 | 124.6574 | 73.58228 | 76.64733 |
| 5 | 103.9003 | 138.4585 | 16.80655 | 63.74849 | 143.8624 | 59.99934 | 45.41228 | 82.34175 |
| 6 | 94.39859 | 103.9169 | 271.434 | 183.627 | 108.0954 | 179.0352 | 77.55132 | 84.50573 |
| 7 | 231.8699 | 129.876 | 28.50972 | 27.66918 | 141.625 | 37.77739 | 76.66302 | 22.07921 |
| 8 | 156.5857 | 70.81585 | 153.868 | 86.36118 | 160.3936 | 92.22537 | 139.7784 | 23.62581 |
| 9 | 49.01371 | 117.8113 | 19.90042 | 62.4285 | 105.4181 | 54.69346 | 153.1851 | 80.01619 |
| 10 | 90.80663 | 144.5971 | 63.93263 | 148.3484 | 60.26438 | 61.02769 | 53.18465 | 37.65844 |
| 11 | 33.78175 | 90.55072 | 194.786 | 82.37004 | 26.03016 | 21.39026 | 78.75331 | 24.50661 |
| 12 | 132.2353 | 61.19482 | 73.3211 | 6.431513 | 56.10611 | 106.6072 | 3.153754 | 33.79886 |
| 13 | 24.20839 | 75.11311 | 132.157 | 152.9308 | 192.4439 | 98.76548 | 49.34031 | 76.37135 |
| 14 | 82.94862 | 84.57154 | 48.21433 | 68.05846 | 90.95372 | 90.36249 | 64.87328 | 57.67071 |
| 15 | 68.14373 | 60.7009 | 63.98031 | 63.77449 | 62.05649 | 138.9133 | 124.9914 | 48.15893 |
| 16 | 154.1183 | 131.2419 | 421.9038 | 236.8857 | 46.62428 | 51.25669 | 240.8035 | 62.01482 |
| 17 | 109.3789 | 120.2014 | 24.44965 | 89.37524 | 60.43528 | 204.4051 | 55.45601 | 63.66328 |
| 18 | 107.9079 | 94.77834 | 153.1189 | 114.8472 | 0.495391 | 155.7785 | 139.3585 | 19.53816 |
| 19 | 99.62158 | 91.80482 | 121.113 | 91.79557 | 80.769 | 40.04334 | 42.17051 | 13.1904 |
| 20 | 77.72275 | 178.1094 | 74.05351 | 49.14227 | 149.4233 | 73.75769 | 97.94175 | 38.6478 |
| 21 | 73.50963 | 93.5022 | 56.01315 | 93.42591 | 69.50773 | 62.51024 | 19.45267 | 15.3786 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 21 | 73.50963 | 93.5022 | 56.01315 | 93.42591 | 69.50773 | 62.51024 | 19.45267 | 15.3786 |
| 22 | 63.17464 | 64.65367 | 109.8241 | 142.9944 | 139.7539 | 36.78926 | 96.95273 | 48.84178 |
| 23 | 135.8957 | 32.5233 | 153.5136 | 47.68855 | 141.42 | 75.52269 | 53.57674 | 123.4582 |
| 24 | 136.3744 | 73.81764 | 141.9183 | 73.1296 | 124.7294 | 41.30573 | 56.16497 | 31.33775 |
| 25 | 195.5425 | 41.48513 | 109.5332 | 74.69929 | 145.3774 | 73.19919 | 134.0068 | 21.65047 |
| 26 | 176.711 | 126.8807 | 91.62768 | 160.5319 | 121.6368 | 80.70747 | 70.69793 | 20.60706 |
| 27 | 122.6871 | 88.52518 | 47.03868 | 82.99167 | 96.18728 | 56.10816 | 45.86098 | 23.59836 |
| 28 | 61.22393 | 67.02152 | 23.79228 | 62.75952 | 101.9583 | 57.83679 | 111.2634 | 96.86962 |
| 29 | 128.4655 | 89.23225 | 108.6319 | 135.8884 | 104.436 | 132.2858 | 66.09168 | 81.8055 |
| 30 | 72.5659 | 13.55446 | 99.02443 | 39.20181 | 77.10331 | 56.64342 | 13.36645 | 64.62477 |
| 31 | 130.5673 | 74.14481 | 32.35751 | 46.83332 | 69.15429 | 46.71286 | 65.13707 | 91.67118 |
| 32 | 202.991 | 86.14544 | 31.17984 | 158.6493 | 23.16363 | 124.1588 | 209.4575 | 54.96122 |
| 33 | 211.594 | 81.99239 | 131.0529 | 83.22884 | 273.7956 | 135.384 | 25.09465 | 92.62165 |
| 34 | 156.3219 | 70.62089 | 151.1296 | 87.36751 | 136.0262 | 45.28283 | 57.90954 | 16.17195 |
| 35 | 235.656 | 39.00747 | 118.1534 | 88.64763 | 263.5714 | 71.35687 | 41.19308 | 93.91843 |
| 36 | 217.8017 | 88.103 | 60.81636 | 199.2522 | 229.75 | 91.13357 | 49.96948 | 52.69515 |
| 37 | 241.2365 | 100.704 | 36.1108 | 197.2798 | 135.1812 | 102.923 | 98.33618 | 10.12124 |
| 38 | 207.4818 | 18.02004 | 106.4317 | 38.64699 | 300.5114 | 64.2784 | 118.9128 | 107.9689 |
| 39 | 94.9189 | 293.6502 | 256.5881 | 202.3845 | 313.6676 | 116.2877 | 78.39482 | 81.7292 |
| 40 | 107.7442 | 49.24239 | 177.5649 | 60.93478 | 164.9885 | 77.76082 | 110.1465 | 81.85429 |
| 41 | 255.5998 | 219.9035 | 123.5856 | 369.0002 | 233.237 | 147.7104 | 67.38869 | 73.62915 |
| 42 | 130.9497 | 109.7916 | 35.72786 | 30.76031 | 83.31848 | 74.9963 | 29.33674 | 62.74756 |

This data matrix is used, along with the five other data matrices for the other classes, to create the GMM.

## 4.2. GMM

The GMM created after training is stored in a MATLAB structure, with fields (columns) for the class name, the classes' mean, covariance and prior. Each row represents the Gaussian fitted for a single class.

| Fields | name | mean | cov | prior |
|---|---|---|---|---|
| 1 | 'Alien' | [126.9427;100.0381;103.2319;111.9250;124.3963;87.5210;79.7132;57.9236] | 8x8 double | 0.1892 |
| 2 | 'Arrow' | [90.8580;129.1431;60.0887;72.7606;23.9065;25.2894;3.6238;12.3178] | 8x8 double | 0.0135 |
| 3 | 'Butterfly' | [91.6441;225.1906;122.8531;128.0920;97.2716;99.3372;71.2372;62.3010] | 8x8 double | 0.2252 |
| 4 | 'Face' | [136.0404;101.9675;69.6703;41.2133;27.2556;30.8056;23.0449;21.8389] | 8x8 double | 0.4505 |
| 5 | 'Star' | [43.4285;70.9617;45.2405;42.6812;61.4408;67.1334;98.1155;121.5870] | 8x8 double | 0.1081 |
| 6 | 'Toonhead' | [112.1748;104.5284;53.0354;52.5145;42.9225;42.2677;19.9245;27.8687] | 8x8 double | 0.0135 |

## 4.3. Confusion Matrix and Quality

The confusion matrix depicts the quality of the classifier. The next screenshot shows the command line output of the main script. The orange line corresponds to images from the training set which have been correctly classified using the "*getConfusionMatrix.m*" function.

```
Confusion matrix score = 83%

    38     1     2    22     4     2
     0     0     0     0     0     0
     4     0    48     1     0     0
     0     1     0    75     0     0
     0     0     0     1    22     0
     0     0     0     0     0     0
```

## 4.4. Optimisation

To determine the feature vector size which provided the highest quality score, the classifier was run with feature vector sizes varying between 2 to 100. The returned quality score was then added to a table in Excel for each of the tested sizes, and then plotted to draw a function in red in the screenshot below. The optimal feature vector size was then determined by taking the peak of the function, as drawn on the right:


## 4.5. Extra testing

Some additional tests were carried out on the classifier. The first one consisted in classifying a binary-image shape which is not part of the six original classes. The second one consisted in classifying a binary-image which was used in the training set. Screenshots of the command line show the classifier's output:

```
Test 1. Image not in sets of classes. Class = Hand. Classified as :  Alien

Test 2. Image from training set. Image = Arrow001. Classified as :  Arrow
```

The hand was classified as an '*Alien*' while the Arrow from the training set was classified as an '*Arrow*'.

## 5. Conclusion

As depicted throughout the report, different steps were carefully undertaken in a specific order in order to build a binary-image classifier. From training the classifier by fitting a Gaussian Mixture Model based on each image's feature vector, to classifying new sets of testing images by comparing their feature vectors with the data from the Gaussian Mixture Model and finally assessing the classifier's quality using confusion matrices, each of these steps required precision and organization during the development process to ensure the highest possible quality classifier within the time constraints.

On top of teaching me more about pattern analysis through classifiers, this project greatly reinforced my mathematical skills with concepts such as vectors, matrices, Gaussians, real & complex domains in general, but especially feature vectors, multivariate Gaussians, Gaussian Mixture Models, Maximum Likelihood Estimation, Maximum a Posteriori and confusion matrices for supervised GMM training and GMM classification. Binary images and Fourier Transforms to convert signals from the time to the frequency domains also contributed greatly from scientific applications point of view. In terms of technical aspects, applying and combining all the mathematics previously stated, into a practical environment with MATLAB to create a functional project greatly contributed towards improving my skills as a software developer.

Achieving a binary-image classifier with a quality of 83% required a lot of time and brainstorming implementing the personal code to work with the provided function. However, if time management was not an issue, improving the classifier to work with unsupervised data instead of supervised data only would have been attempted.