



UNIVERSITY OF THE BASQUE COUNTRY

FINAL YEAR PROJECT

About Tree Depth

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1 Introduction to Graph Theory

1.1 Definition of a graph

A graph is defined as a pair of sets $G = (V, E)$, such that $E \subseteq V^2$. The members of V are called vertices and the ones of E edges. Take into account, that the vertices can be anything, they can even be sets themselves. The usual way to draw a graph is by representing the vertices as individual points and for each edge, draw a link between both elements of that edge. The shape in which a graph is drawn is irrelevant, it will contain the same information.

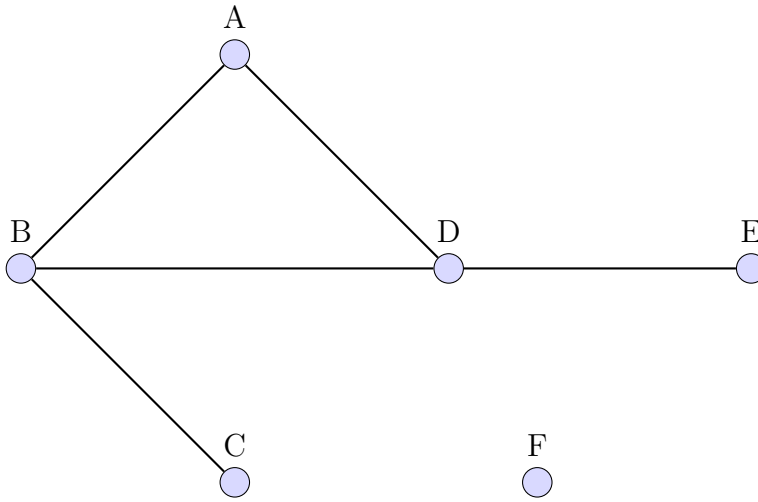


Figure 1: A graph with $V = \{A, B, C, D, E, F\}$ and $E = \{\{A, B\}, \{A, D\}, \{B, D\}, \{B, C\}, \{D, E\}\}$

1.2 Connectivity

An essential concept in graph theory is adjacency. Two vertices $x, y \in V$ are said to be adjacent in G if and only if $\{x, y\} \in E$.

2 Introduction to Tree Depth

2.1 Basic definitions

Vertex x is said to be the ancestor of y in a rooted forest F , if and only if x belongs to the path between y and the root of the component to which x belongs, y included.

The closure of a rooted forest F , expressed as $C = \text{clos}(F)$, is defined as follows:

- $V(C) = V(F)$
- $E(C) = \{ \{x, y\} : x \text{ is an ancestor of } y \text{ in } F, x \neq y \}$

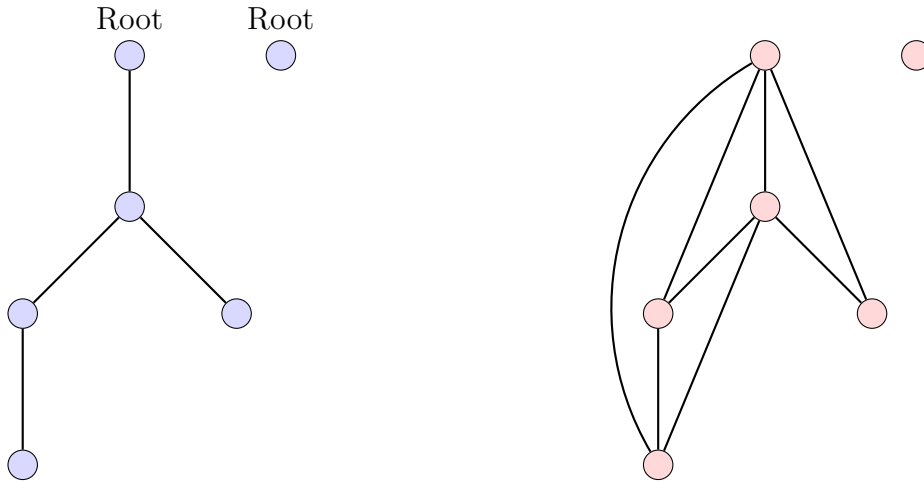


Figure 2: The blue graph at the left is a rooted forest F , the red graph at the right represents $\text{clos}(F)$.

2.2 Tree Depth

Definition: The tree-depth $\text{td}(G)$ of a graph G is the minimum height of a rooted forest F such that $G \subseteq \text{clos}(F)$

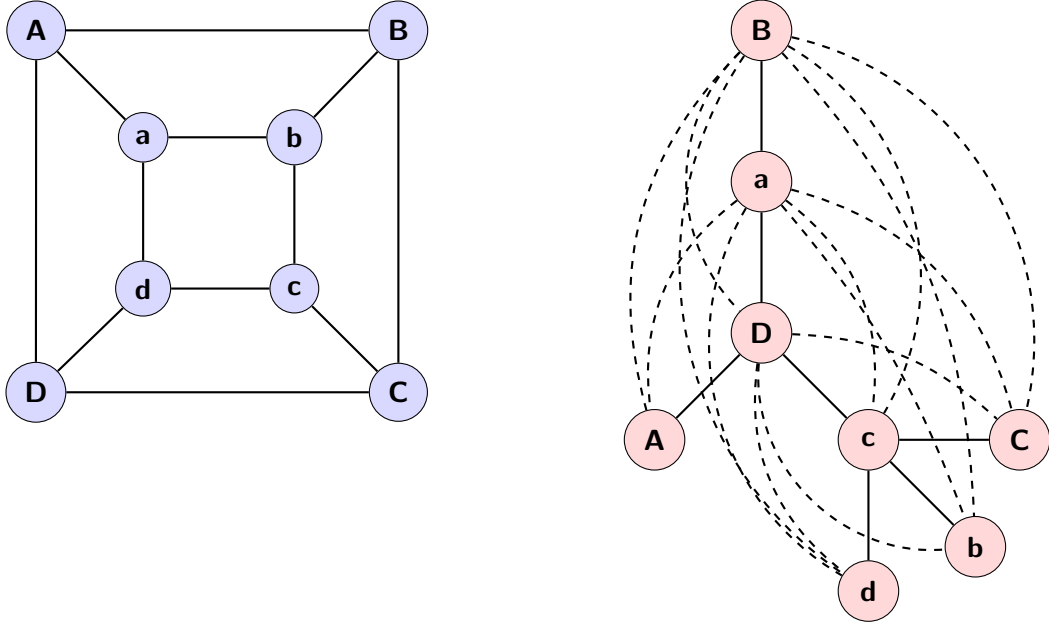


Figure 3: The graph G and tree T are in the left and right respectively. The dotted edges in T , represent the $\text{clos}(T)$. Because $G \subseteq \text{clos}(T)$, we know that $\text{td}(G)$ is at most 4.

The tree depth of a graph G is a numerical invariant of a graph. In other words, the tree depth is a property that depends only on the abstract structure of a graph, not in its representation.

2.3 Elimination tree

An elimination tree Y of a connected graph G is defined recursively as follows:

- If $V(G) = \{x\}$ then Y is just $\{x\}$.
- Otherwise, $r \in V(G)$ is chosen as the root of Y and an elimination tree is created for each component of $G-r$. r will be connected to the root of each of these trees.

The tree T in Figure 3 is an elimination tree for the graph G .