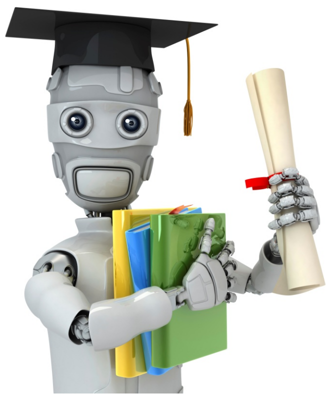


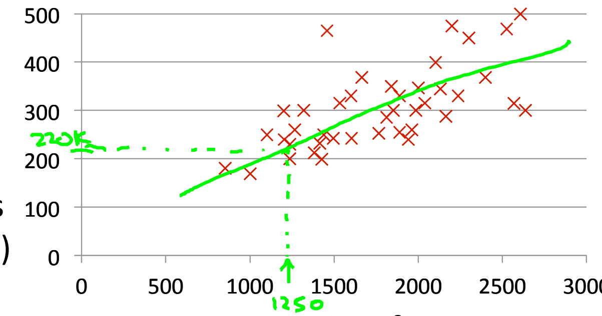
Machine Learning

Linear regression with one variable

Model representation

Housing Prices (Portland, OR)

Price (in 1000s of dollars)



Size (feet^2)

Supervised Learning

Given the "right answer" for each example in the data.

Regression Problem

Predict real-valued output

Classification: Discrete-valued output

Andrew Ng

Training set of housing prices (Portland, OR)

Size in feet^2 (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

m = 47

Notation:

- m = Number of training examples
- x's = "input" variable / features
- y's = "output" variable / "target" variable

(x,y) - one training example

$(x^{(i)}, y^{(i)})$ - i^{th} training example

$x^{(1)} = 2104$

$x^{(2)} = 1416$

$y^{(1)} = 460$

Andrew Ng

Training Set

↓

Learning Algorithm

↓

Size of house → h → Estimated price

x

hypothesis

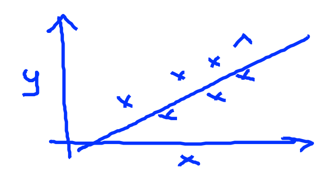
(estimated value of y)

h maps from x's to y's.

How do we represent h ?

$h_\theta(x) = \theta_0 + \theta_1 x$

Shorthand: $h(x)$



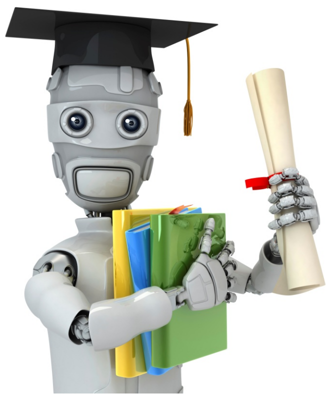
$h(x) = \theta_0 + \theta_1 x$

Linear regression with one variable. (x)

Univariate linear regression.

one variable

Andrew Ng



Machine Learning

Linear regression with one variable

Cost function

Training Set

Size in feet² (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

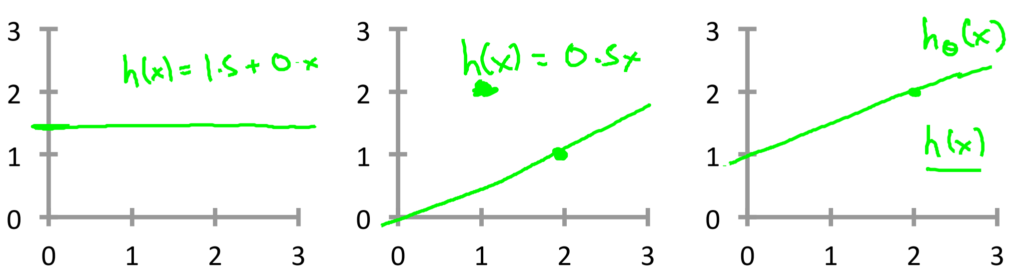
m = 47

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

θ_i's: Parameters

How to choose θ_i's ?

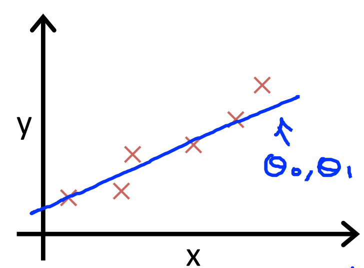
$h_\theta(x) = \theta_0 + \theta_1 x$



$\theta_0 = 1.5$, $\theta_1 = 0$

$\theta_0 = 0$, $\theta_1 = 0.5$

$\theta_0 = 1$, $\theta_1 = 0.5$



$(x^{(i)}, y^{(i)})$

Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ is close to y for our training examples (x, y)

x, y

training examples

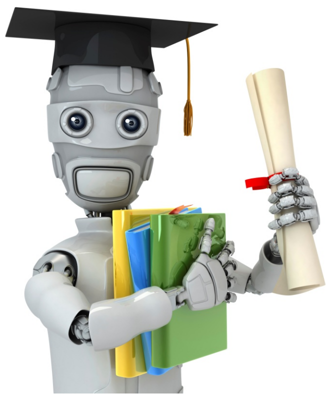
$$\boxed{\underset{\theta_0\theta_1}{\text{minimize}}} \ \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$h_\theta(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}$$

$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$\underset{\theta_0,\theta_1}{\text{minimize}} \ \underbrace{J(\theta_0,\theta_1)}_{\text{Cost function}}$$

Squared error function



Machine Learning

Linear regression with one variable

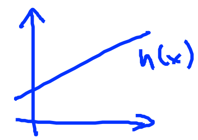
Cost function intuition I

Hypothesis:

$h_\theta(x) = \theta_0 + \theta_1 x$

Parameters:

θ_0, θ_1



Cost Function:

$\rightarrow J(\theta_0, \theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

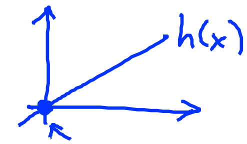
Goal: $\underset{\theta_0,\theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

Simplified

$h_\theta(x) = \theta_1 x$

$\theta_0 = 0$

θ_1



$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

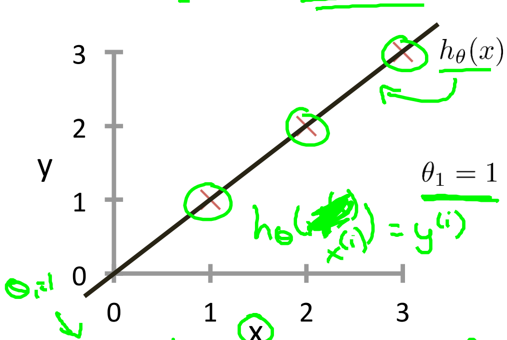
$\theta_1 x^{(i)}$

$\underset{\theta_1}{\text{minimize}}\ J(\theta_1)$

Andrew Ng

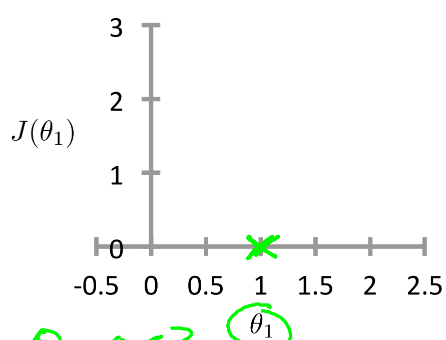
$\rightarrow h_\theta(x)$

(for fixed θ_1, this is a function of x)



$\rightarrow J(\theta_1)$

(function of the parameter θ_1)



$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

$= \frac{1}{2m}\sum_{i=1}^{m}\left(\theta_1 x^{(i)} - y^{(i)}\right)^2 = \frac{1}{2m}(0^2+0^2+0^2) = 0$

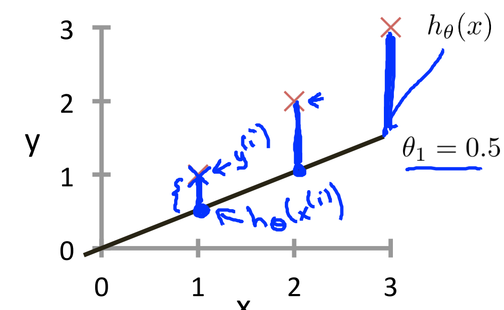
$\theta_1 = 0.5?$

$J(1) = 0$

Andrew Ng

$h_\theta(x)$

(for fixed θ_1, this is a function of x)

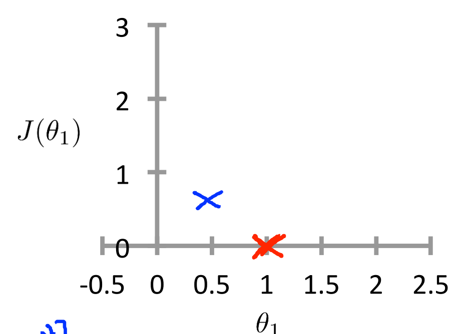


$J(0.5) = \frac{1}{2m}\left[(0.5-1)^2 + (1-2)^2 + (1.5-3)^2\right]$

$= \frac{1}{2 \times 3}(3.5) = \frac{3.5}{6} \approx 0.58$

$J(\theta_1)$

(function of the parameter θ_1)



$\theta_1 = 0?$

$J(0) = ?$

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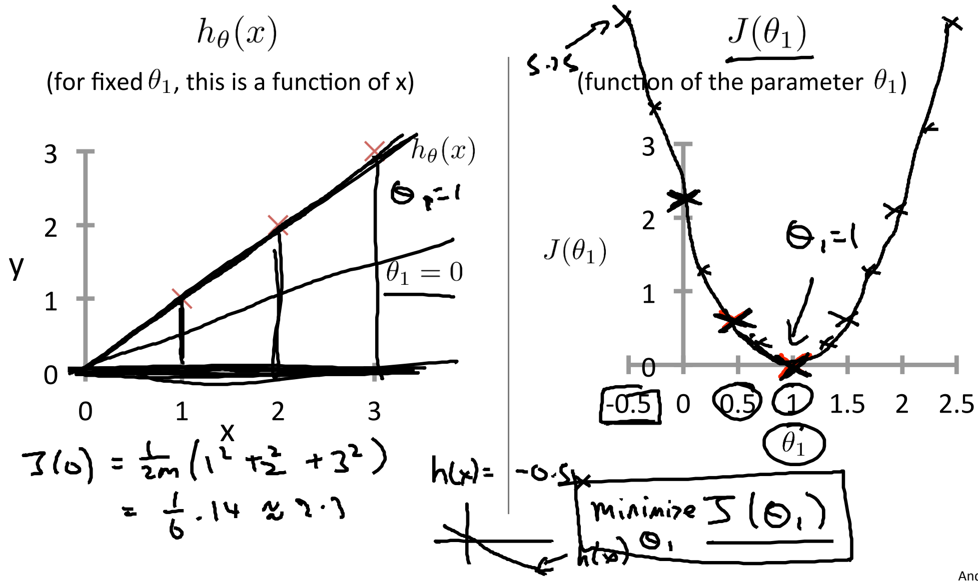
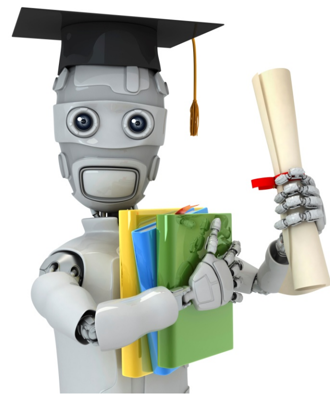
$h_\theta(x)$

(for fixed θ_1, this is a function of x)

$J(\theta_1)$

(function of the parameter θ_1)





Machine Learning

Linear regression with one variable

Cost function intuition II

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

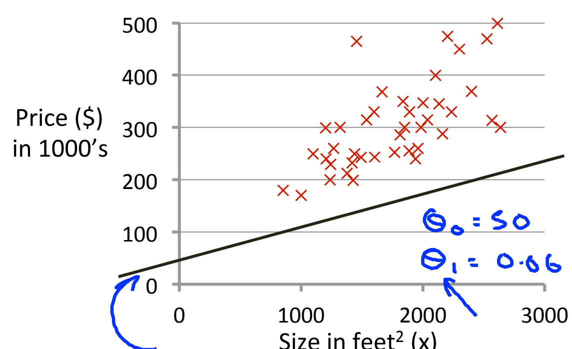
Parameters: θ_0, θ_1

Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

$h_\theta(x)$

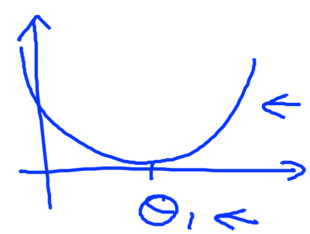
(for fixed θ_0, θ_1, this is a function of x)



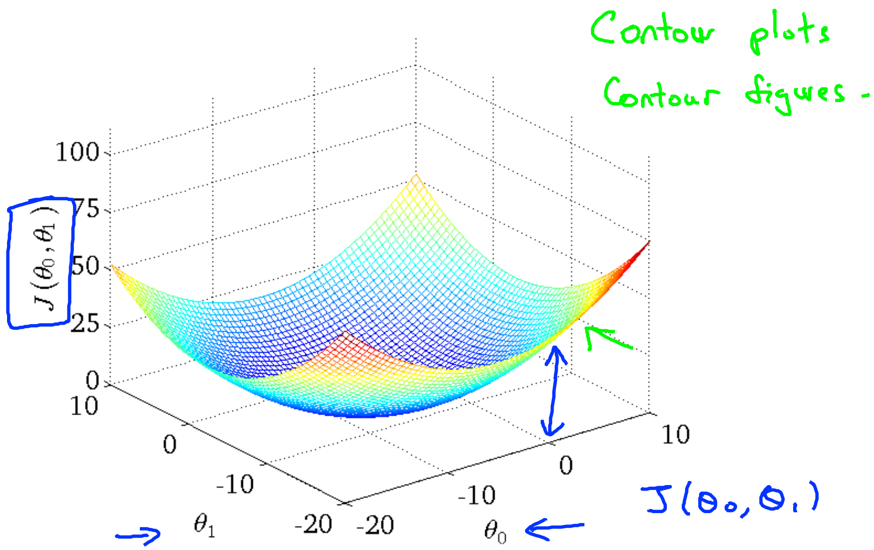
$h_\theta(x) = 50 + 0.06x$

$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



θ_0, θ_1



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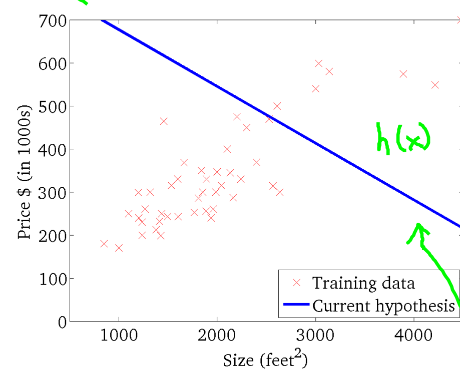
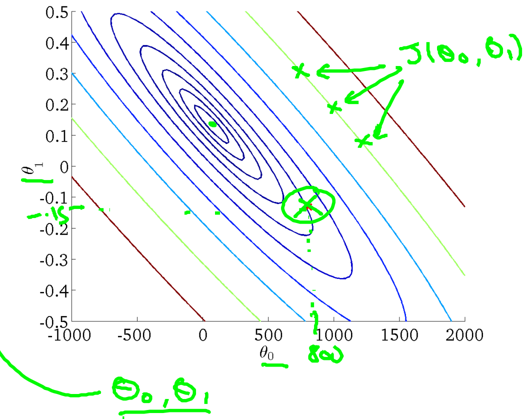
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



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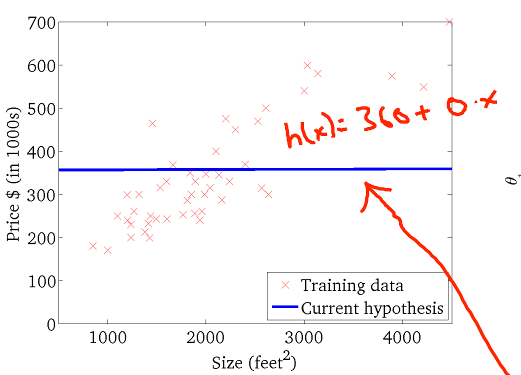
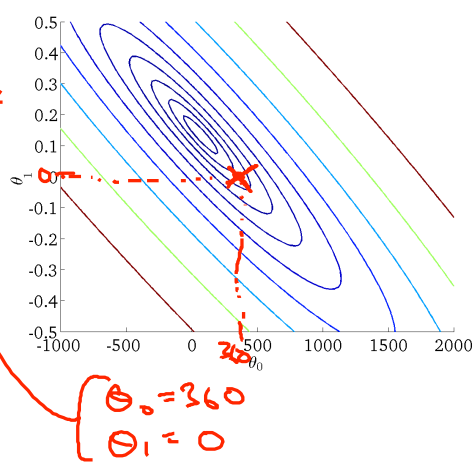
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



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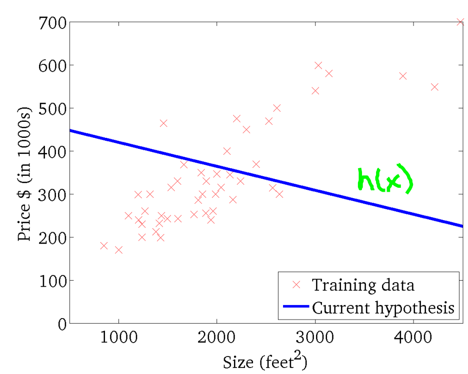
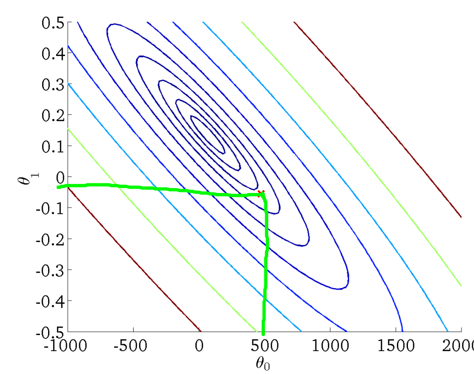
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



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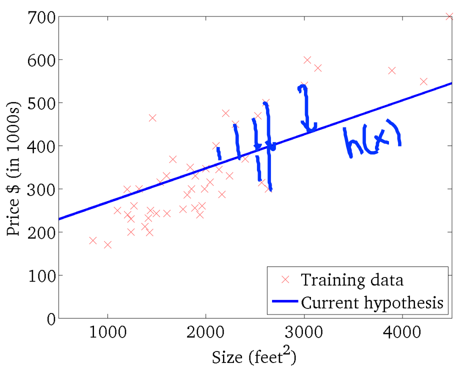
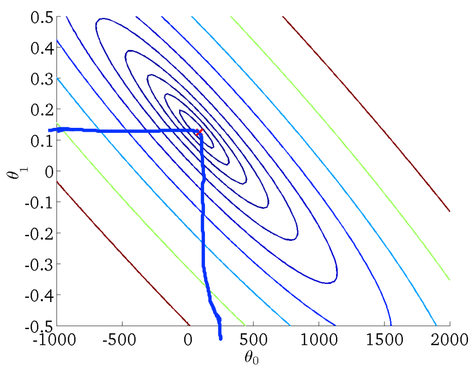
$h_\theta(x)$

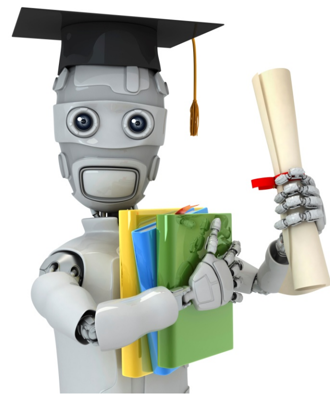
(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)





Machine Learning

Linear regression with one variable

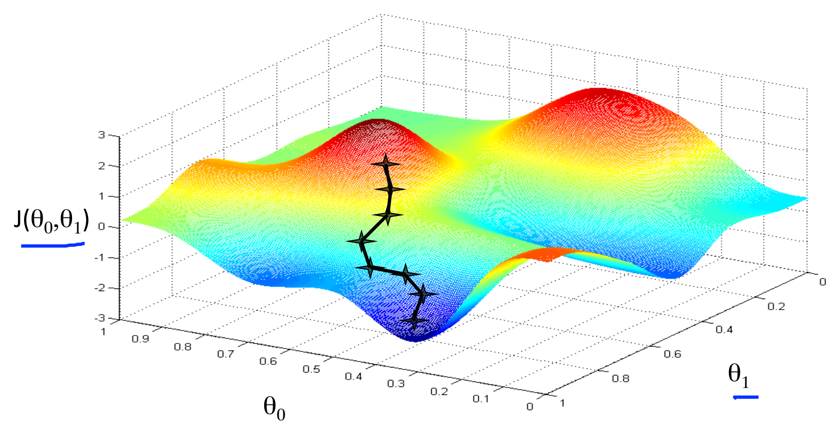
Gradient descent

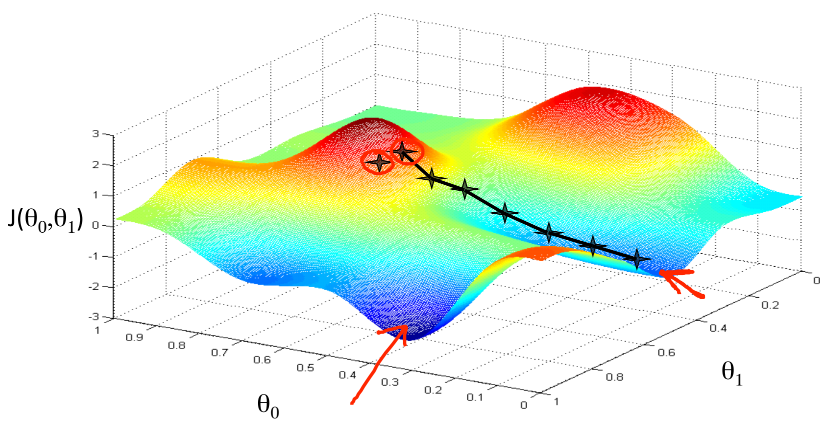
Have some function $J(\theta_0, \theta_1)$ $J(\theta_0, \theta_1, \theta_2, \dots, \theta_n)$

Want $\min_{\theta_0, \theta_1} J(\theta_0, \theta_1)$ $\min_{\theta_0 \dots \theta_n} J(\theta_0, \dots, \theta_n)$

Outline:

- Start with some θ_0, θ_1 (say $\theta_0 = 0, \theta_1 = 0$)
- Keep changing θ_0, θ_1 to reduce $J(\theta_0, \theta_1)$ until we hopefully end up at a minimum





Gradient descent algorithm

Assignment

$a := b$

$a := a+1$

Truth assertion

$a = b$

$a = a+1$ X

θ_0, θ_1

repeat until convergence {

$$\theta_j := \boxed{\theta_j - \alpha \boxed{\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)}} \quad \text{(for } j = 0 \text{ and } j = 1\text{)}$$

}

learning rate

Simultaneously update θ_0 and θ_1

Correct: Simultaneous update

$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$

$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$

$\theta_0 := \text{temp0}$

$\theta_1 := \text{temp1}$

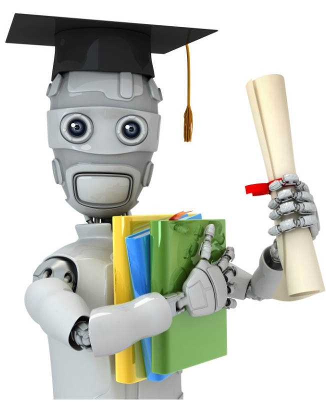
Incorrect:

$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$

$\theta_0 := \text{temp0}$

$\text{temp1} := \theta_1 - \boxed{\alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)}$

$\theta_1 := \text{temp1}$



Linear regression with one variable

Gradient descent intuition

Machine Learning

Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \boxed{\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)} \quad \text{(simultaneously update } j = 0 \text{ and } j = 1\text{)}$$

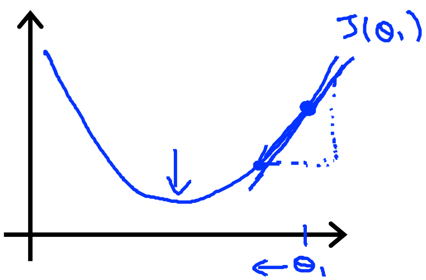
}

learning rate

derivative

$\min_{\theta_1} J(\theta_1)$ $\theta_1 \in \mathbb{R}$.

$(\theta_1 \in \mathbb{R})$

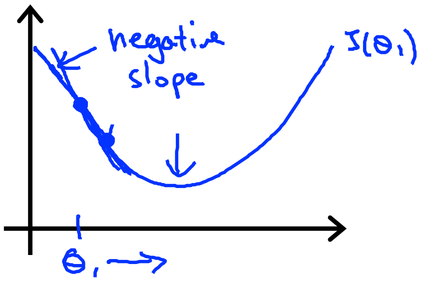


$\frac{\partial}{\partial\theta_1}$ ←

$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$

≥ 0

$\theta_1 := \theta_1 - \alpha \cdot (\text{positive number})$



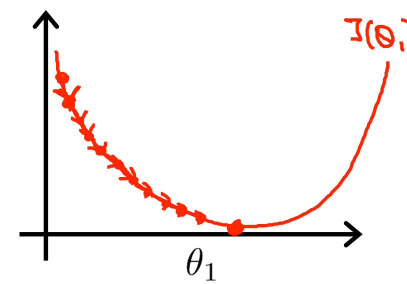
$\frac{d}{d\theta_1} J(\theta_1) \leq 0$

$\theta_1 := \theta_1 - \alpha \, (\text{negative number})$

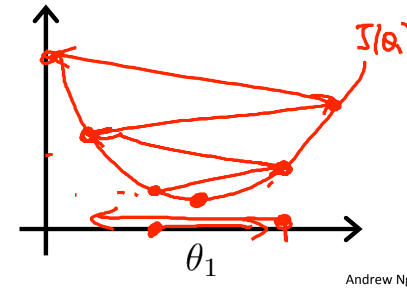
Andrew Ng

$$\boxed{\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)}$$

If α is too small, gradient descent can be slow.



If α is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.



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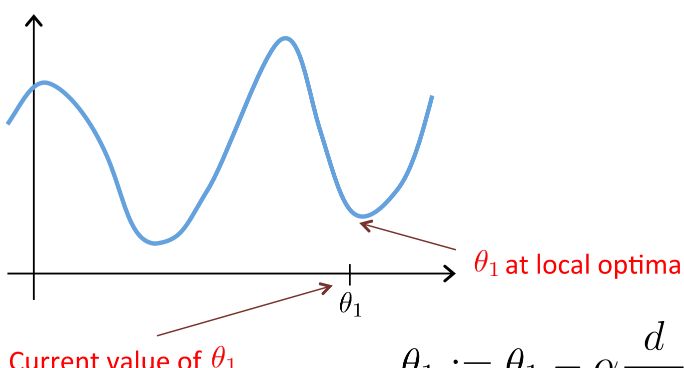

$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

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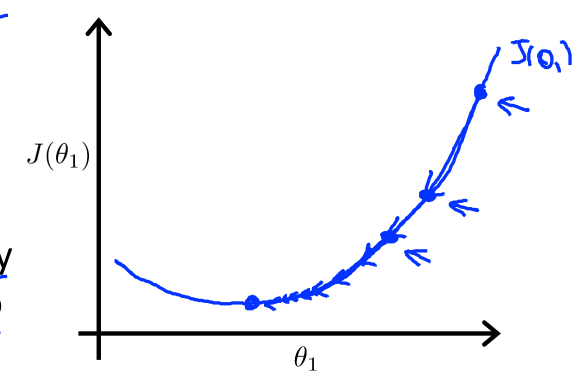
Gradient descent can converge to a local minimum, even with the learning rate α fixed.

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.



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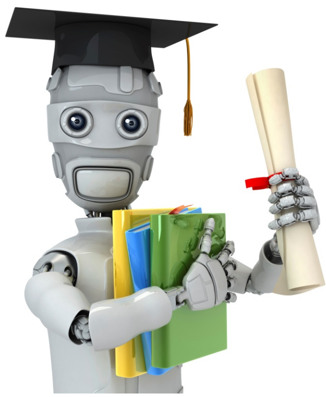


Machine Learning

Linear regression with one variable

Gradient descent for linear regression

Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

$$\text{(for } j = 1 \text{ and } j = 0\text{)}$$

}

Linear Regression Model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Andrew Ng

$$\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) = \frac{\partial}{\partial \theta_j} \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$= \frac{\partial}{\partial \theta_j} \frac{1}{2m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)}\right)^2$$

$$j = 0 : \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)$$

$$j = 1 : \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right) \cdot x^{(i)}$$

Andrew Ng

Gradient descent algorithm

$\frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$

repeat until convergence {

$$\theta_0 := \theta_0 - \alpha \boxed{\frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)}$$

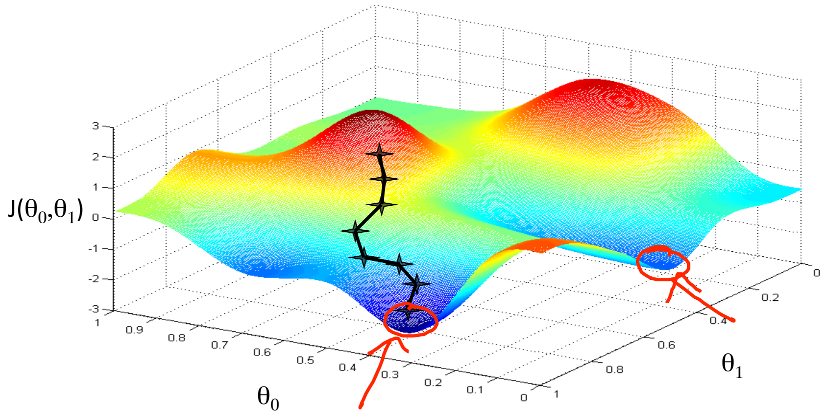
$$\theta_1 := \theta_1 - \alpha \boxed{\frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right) \cdot x^{(i)}}$$

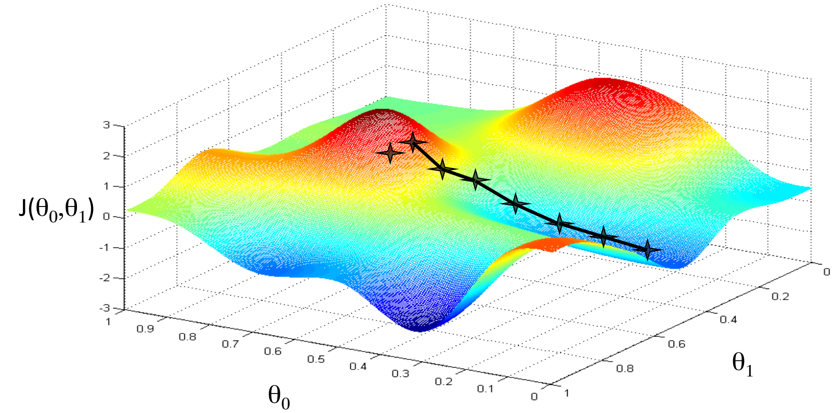
}

update θ_0 and θ_1 simultaneously

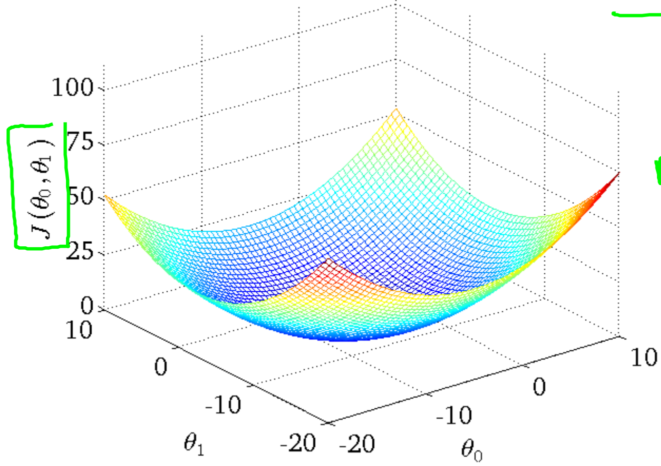
$\frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$

Andrew Ng





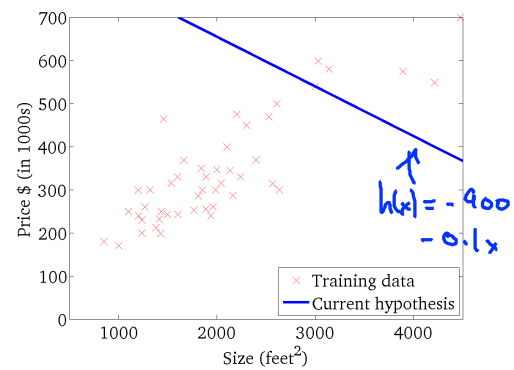
"Convex function"



Bowl-shaped

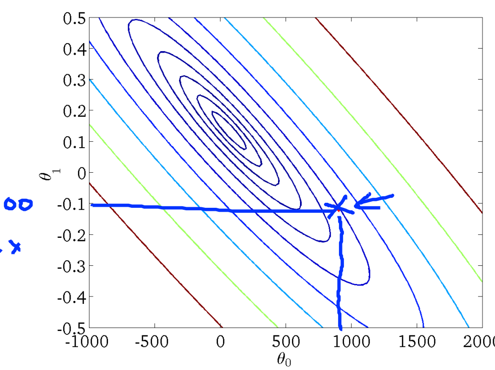
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



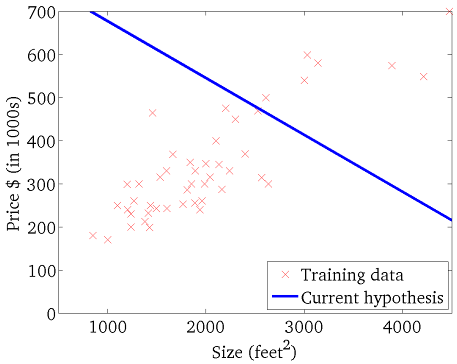
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



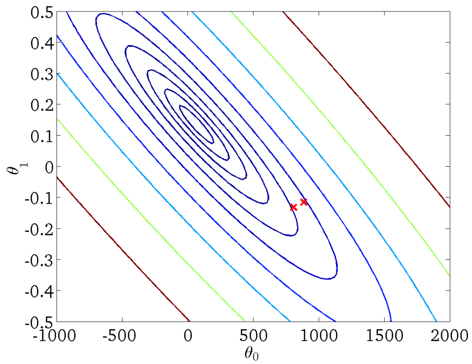
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



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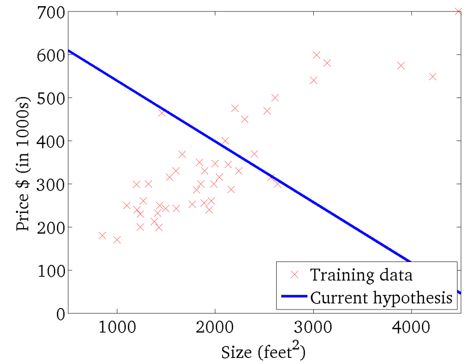
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



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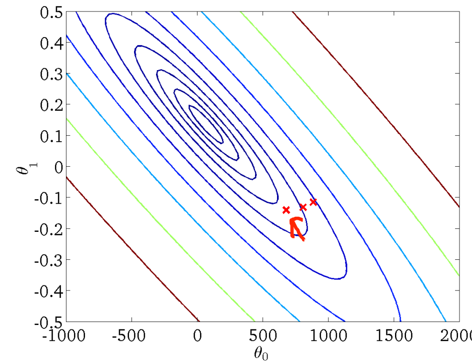
(function of the parameters θ_0, θ_1)



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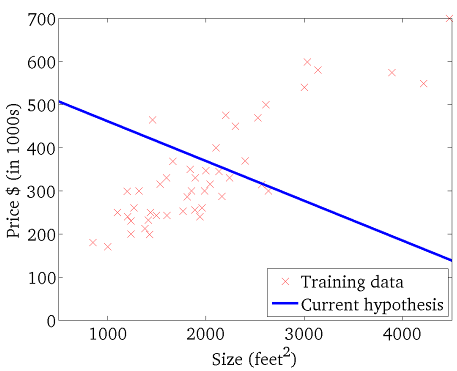
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



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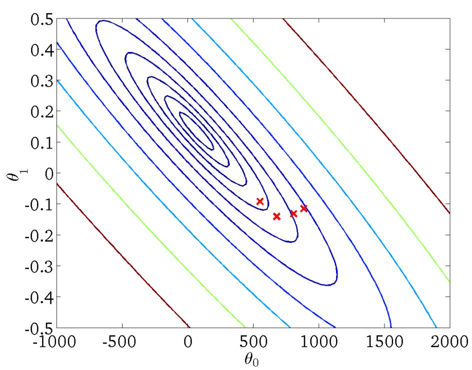
(function of the parameters θ_0, θ_1)



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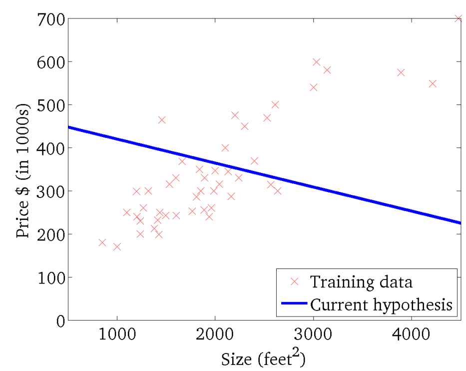
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



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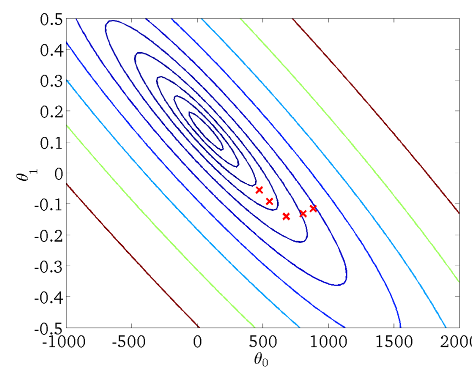
(function of the parameters θ_0, θ_1)



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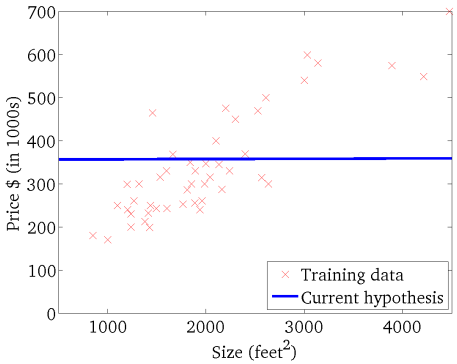
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



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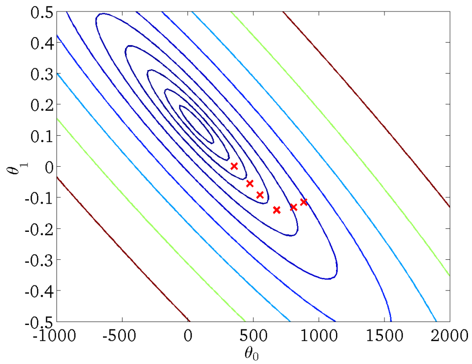
(function of the parameters θ_0, θ_1)



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$h_\theta(x)$

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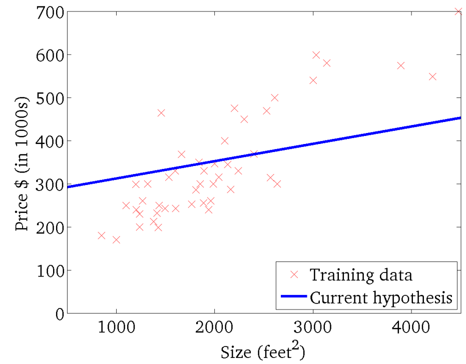
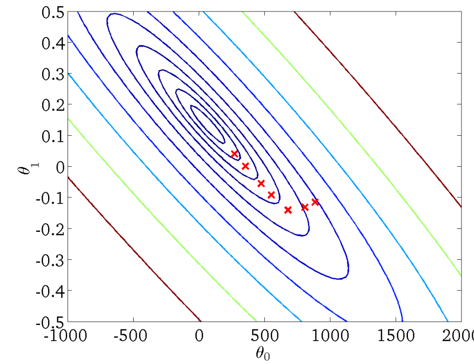


$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



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$h_\theta(x)$

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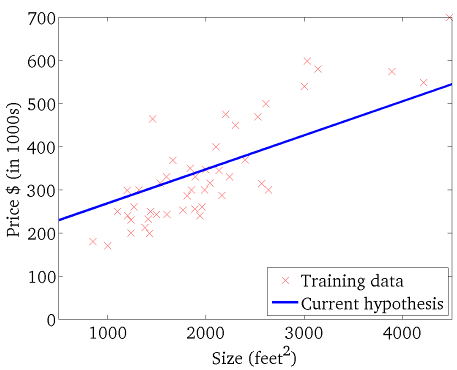

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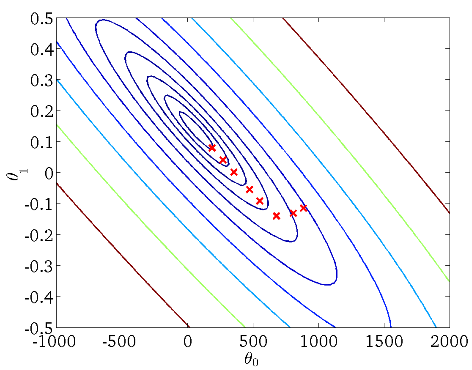
(function of the parameters θ_0, θ_1)



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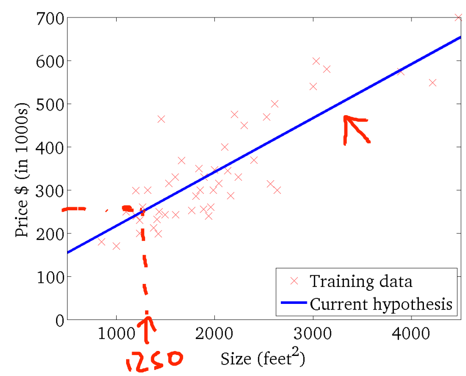

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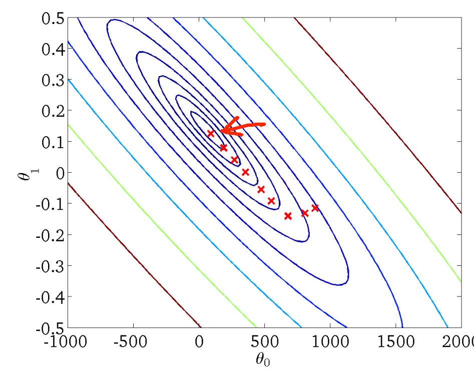
(function of the parameters θ_0, θ_1)



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"Batch" Gradient Descent

"Batch": Each step of gradient descent uses all the training examples.

$$\sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})$$