This is Calculus

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Intro

1.1 Some words of encouragement

When people think of "Calculus", they think of an incredibly complex field of mathematics, only incredibly gifted minds. But, this could not be further from the truth. In reality, calculus is something anyone who has done basic algebra can understand, to an extent.

1.2 Why?

A lot of people ask questions like "Why do I have to learn this?" or "Where will I use this?", and a big part of learning "Calculus" is learning how to actually use it, not just how to do it.

1.3 What is it?

The question "What is Calculus" is very very broad. There are 4 different "sections" of Calculus: Differential [Calculus], Integral [Calculus], Multi-variable [Calculus], and Differential Equations. Differential Calculus follows topics like Rates of Change and Slope.

Understanding the Derivative

2.1 Slope and Derivatives

When we think of slope, we think of how far on the vertical axis we have to move, divided by and how far on the horizontal axis we have to move to get from one point, A, to another point, B.

But does it really matter how close these points are on the line? No. The formula for slope is:

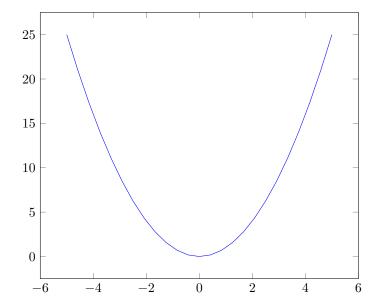
$$\frac{y_1 - y}{x_1 - x}$$

The two ordered pairs $((x_1, y_1))$ and (x, y) can be any distance apart, and no matter what the slope will be the same as long as the two points lie on the line, and the line is *linear*.

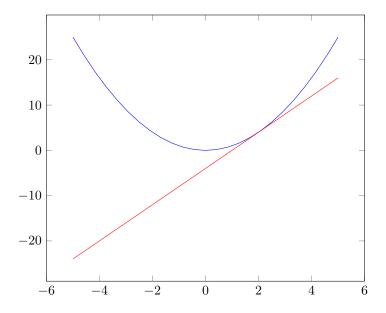
The *derivative* really is just the slope of a line when the points are REALLY close together. If the points are close enough, it's effectively the slope at a *single point*.

You can imagine how this would be helpful when finding the slope of *non-linear* lines because the slope on a non-linear line is always changing. That's why linear lines have a *constant* rate of change and non-linear lines don't.

2.2 Further into instantaneous slope



As you can see, the slope is negative near x=4 and positive past x=2. This means the slope is different at every point. So instead of the slope/RoC being a constant number (scalar), it's a function. We won't talk about why, yet, but the "slope" of the graph x^2 is 2x. For example, at -4 on the x-axis, the slope of the parabola is negative and very steep. Plugging -4 into 2x returns -8. Looks about right. The slope starts to even out at about -0.5, and plugging it into our slope/derivative function returns 1. The slope at 0 on the x-axis is 0, and the slope at 2 on the x-axis is 4. That definitely looks right.



We can draw something called a tangent line at the point, and the slope of the tangent line is the same as the slope at that point. We will dig deeper into these later.

Limits and continuity

Calculating derivatives

Derivative rules

L'Hôpital's rule

Rates of change for things other than graphs

Implicit Differentiation

Related Rates

Physics

Understanding Integrals

Areas under graphs

Approximating Integrals

Solving Integrals

Integral Rules

The Fundamental Theorem of Calculus

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