A Couple More Practice Problems

Concepts of Programming Languages

CAS CS 320

S-Combinator

$$\{x : \tau_1, y : \tau_2\} \vdash \lambda z^{\top} . xz(yz) : \top \to \top$$

Determine types τ_1 and τ_2 such that the above judgment is derivable in STLC, then given the derivation. Are there multiple choices for τ_1 and τ_2? If so, give another choice and justify your answer

$$\frac{}{\Gamma \vdash \bullet : \top} \qquad \frac{\Gamma, x : \tau \vdash e : \tau'}{\Gamma \vdash \lambda x^{\tau} . e : \tau \to \tau'}$$

$$\frac{(x : \tau) \in \Gamma}{\Gamma \vdash x : \tau} \qquad \frac{\Gamma \vdash e_1 : \tau \to \tau' \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1 e_2 : \tau'}$$

CBN vs. CBV (Part 1)

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$$\frac{e_1 \longrightarrow e_1'}{e_1 e_2 \longrightarrow e_1' e_2} \qquad \frac{e_2 \longrightarrow e_2'}{(\lambda x\,.\,e_1) e_2 \longrightarrow (\lambda x\,.\,e_1) e_2'}$$

$$\frac{}{(\lambda x\,.\,e)(\lambda y\,.\,e') \longrightarrow [(\lambda y\,.\,e')/x]e}$$

small-step call-by-value

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small-step call-by-value

$$\frac{e_1 \longrightarrow e_1'}{e_1 e_2 \longrightarrow e_1' e_2} \qquad \frac{}{(\lambda x\,.\,e) e' \longrightarrow [e'/x]e}$$

small-step call-by-name

CBN vs. CBV (Part 2)

Let e^k be the untyped λ-calculus expression

$$\underbrace{(\lambda x\,.\,x)\ldots(\lambda x\,.\,x)}_{\text{k times}}$$

How many steps does it take to evaluate e^k using CBN and CBV? What is its value?

Using this expression, determine an expression that take 500 steps to evaluate using CBN and 104 steps to evaluate using CBV