

# **Principle Types**

## **Concepts of Programming Languages**

# Outline

- » Demo an implementation of **unification**
- » Discuss **principle types**

# Practice Problem

$$\cdot \vdash \lambda x. xx : \tau \dashv \mathcal{C}$$

Determine the type  $\tau$  and constraints  $\mathcal{C}$  such that the above judgment is derivable

$$\frac{(x : \forall_{\alpha_1} \forall_{\alpha_2} \forall_{\alpha} \tau) \in \Gamma \quad \alpha_1, \alpha_2, \alpha \text{ are fresh}}{\Gamma \vdash x : \forall_{\alpha_1} \forall_{\alpha_2} \forall_{\alpha} \tau \dashv \emptyset} \text{ (var)}$$

$$\frac{\alpha \text{ is fresh} \quad \Gamma, x : \alpha \vdash e : \tau \dashv \mathcal{C}}{\Gamma \vdash \lambda x. e : \alpha \rightarrow \tau \dashv \mathcal{C}} \text{ (fun)}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \dashv \mathcal{C}_1 \quad \Gamma \vdash e_2 : \tau_2 \dashv \mathcal{C}_2 \quad \alpha \text{ is fresh}}{\Gamma \vdash e_1 e_2 : \alpha \dashv \tau_1 \doteq \tau_2 \rightarrow \alpha, \mathcal{C}_1, \mathcal{C}_2} \text{ (app)}$$

# Answer

$\cdot \vdash \lambda x. xx : \tau \vdash C$

$\emptyset \vdash \lambda x. xx : \alpha \rightarrow \beta \vdash \alpha \doteq \alpha \rightarrow \beta$

$\vdash \{x : \alpha\} \vdash x x : \beta \vdash \alpha \doteq \alpha \rightarrow \beta$

$\vdash \{x : \alpha\} \vdash x : \alpha \vdash \emptyset$

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# **Recap**

# Recall: Unification

$$a \doteq d \rightarrow e$$

$$c \doteq \text{int} \rightarrow d$$

$$\text{int} \rightarrow \text{int} \rightarrow \text{int} \doteq b \rightarrow c$$

**Unification** is the process of solving a system of equations over *symbolic* expressions

# Recall: Type Unification Problem

A **unification problem** is a collection of equations of the form

$$\begin{aligned}s_1 &\doteq t_1 \\ s_2 &\doteq t_2 \\ &\vdots \\ s_k &\doteq t_k\end{aligned}$$

where  $s_1, \dots, s_k$  and  $t_1, \dots, t_k$  are **types**

# Recall: Unifiers

A **unifier** is a sequence of substitutions to variables, typically written

$$\mathcal{S} = \{x_1 \mapsto t_1, x_2 \mapsto t_2, \dots, x_n \mapsto t_n\}$$



ordered

s.t.

$$\mathcal{S}t_1 = \mathcal{S}s_1$$

$$\mathcal{S}s_2 = \mathcal{S}t_2$$

⋮

$$\mathcal{S}s_k = \mathcal{S}t_k$$

# Recall: Most General Unifiers

A **most general unifier** of a unification problem is a solution  $\mathcal{S}$  such that, for any solution  $\mathcal{S}'$ , there is another solution  $\mathcal{S}''$  such that  $\mathcal{S}' = \mathcal{S}\mathcal{S}''$

In other words,  $\mathcal{S}'$  is  $\mathcal{S}$  *with more substitutions*

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perform the substitution  $\alpha \mapsto t$  to every equation in  $\mathcal{U}$

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**OTHERWISE**  $\implies$  **FAIL**

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**OTHERWISE**  $\implies$  **FAIL**

**RETURN**  $\mathcal{S}$

# Another Practice Problem

$$\beta \doteq \eta$$

$$\alpha \rightarrow \beta \doteq \alpha \rightarrow \gamma$$

$$\alpha \rightarrow \beta \doteq \gamma \rightarrow \eta$$

$$\alpha \rightarrow \beta \doteq \text{int} \rightarrow \eta$$

*Determine a most general unifier to the above type unification problem using the algorithm we just gave*

# Answer

$$S = \{$$

$$\beta \mapsto \gamma$$

$$\gamma \mapsto \gamma$$

$$\alpha \mapsto \gamma$$

$$\gamma \mapsto \text{int}$$

$$\beta \dot{=} \eta \quad (\text{asgn})$$

$$\alpha \xrightarrow{\gamma} \beta \dot{=} \alpha \Rightarrow \gamma \quad (\text{fun})$$

$$\alpha \xrightarrow{\gamma} \beta \dot{=} \gamma \rightarrow \eta \quad (\text{fun})$$

$$\alpha \xrightarrow{\gamma} \beta \dot{=} \text{int} \rightarrow \eta \quad (\text{fun})$$

$$\alpha \dot{=} \alpha$$

$$(eq)$$

$$\alpha \dot{=} \gamma$$

$$(\text{asgn})$$

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$$(\text{asgn})$$

$$\gamma \dot{=} \gamma \quad (\text{eq})$$

$$\gamma \dot{=} \text{int} \quad (\text{asgn})$$

demo  
(unification)

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$$\text{principle}(\tau, \mathcal{C}) = \forall \alpha_1 \dots \forall \alpha_k. \mathcal{S}\tau \text{ where } \text{FV}(\mathcal{S}\tau) = \{\alpha_1, \dots, \alpha_k\}$$

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i.e, the **principle type** of  $e$  (note: it may not exist). Every type we *could* give  $e$  is a *specialization* of  $\forall \alpha_1, \dots, \alpha_k. \mathcal{S}\tau$

# Example

$$S(\alpha \rightarrow \beta \rightarrow \eta) = (\text{int} \rightarrow \boxed{\eta}) \rightarrow \text{int} \rightarrow \boxed{\eta}$$

$$\text{Prin}(\alpha \rightarrow \beta \rightarrow \eta, C) = \forall u. (\text{int} \rightarrow \eta) \rightarrow \text{int} \rightarrow \eta$$

Determine the principle type of  $\lambda f. \lambda x. f(x+1)$

$$\vdash \lambda f. \lambda x. f(x+1) : \alpha \rightarrow \beta \rightarrow \eta + C$$

$$\vdash \{f : \alpha\} \vdash \lambda x. f(x+1) : \beta \rightarrow \eta + C$$

$$\vdash \{f : \alpha, x : \beta\} \vdash f(x+1) : \eta + \boxed{\alpha = \text{int} \rightarrow \eta, \beta = \text{int}, \text{int} = \text{int}}$$

$$\vdash \{f : \alpha, x : \beta\} \vdash f : \alpha + \emptyset$$

$$\vdash \{f : \alpha, x : \beta\} \vdash x+1 : \text{int} + \beta \doteq \text{int}, \text{int} \doteq \text{int}$$

$$\vdash \{ \dots \} \vdash x : \beta + \emptyset$$

$$\vdash \{ \dots \} \vdash 1 : \text{int} + \emptyset$$

$$\alpha \doteq \text{int} \rightarrow \eta$$

$$\beta \doteq \text{int}$$

$$\text{int} \doteq \text{int}$$

$$S = \{$$

$$\alpha \mapsto \text{int} \rightarrow \eta$$

$$\beta \mapsto \text{int}$$

# Example

Show that  $\text{let } f = \lambda x. x \text{ in } f(f\ 2) = 2$  has no principle type

# **Putting everything together (`is_well_typed`)**

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3. *Generalization*: Quantify over the free variables in  $\mathcal{S}\tau$  to get the principle type  $\forall \alpha_1 \dots \forall \alpha_k. \mathcal{S}\tau$  of  $e$
4. Add  $(x : \forall \alpha_1 \dots \forall \alpha_k. \mathcal{S}\tau)$  to  $\Gamma$

# Example

$\text{let id} = \text{fun } x \rightarrow x$   
 $\text{let } _ = \text{id } (\text{id } 2 = 2)$

$$\Gamma = \{ \text{id} : \forall \alpha. \alpha \rightarrow \alpha \}$$



$$\{ \text{id} : \forall \alpha. \alpha \rightarrow \alpha \} \vdash \text{id } (\text{id } 2 = 2) : \tau + C$$

$$\Gamma' = \{ \text{id} : \forall \alpha. \alpha \rightarrow \alpha, \\ - : \text{bool} \}$$



$$\text{prin}(\Gamma, C) = \text{bool}$$

$$\emptyset \vdash \text{fun } x \rightarrow x : \tau \vdash C$$
$$\text{prin}(\tau, C) = \forall \alpha. \alpha \rightarrow \alpha$$

# As a Type System

$$\frac{}{\Gamma \vdash \epsilon} (\text{emptyProg})$$

$$\frac{\Gamma \vdash e : \tau \dashv \mathcal{C} \quad \tau' = \text{principle}(\tau, \mathcal{C}) \quad \Gamma, x : \tau' \vdash P}{\Gamma \vdash \text{let } x = e \ P} (\text{topLet})$$

We can also express this as a type system with judgments of the form  $\Gamma \vdash P$ , where  $P$  is a program (note there is no ":" )

# Example

```
let id = fun x -> x  
let a = id (id 2 = 2)
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```
∅ ⊢ let id = fun x -> x let a = id (id 2 = 2)      (topLet)
  ┌ ──────────────────────────────────────────────────────────
  | ∅ ⊢ fun x -> x : α → α ⊢ ∅                  (...)
  | ...                                         (...)
  └ {id : ∀α.α → α} ⊢ let a = id (id 2 = 2)
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$\vdash \emptyset \vdash \text{fun } x \rightarrow x : \alpha \rightarrow \alpha \dashv \emptyset$	(...)
$\vdash \ldots$	(...)
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$\vdash \ldots$	(...)
$\vdash \{\text{id} : \forall \alpha. \alpha \rightarrow \alpha , \text{ a} : \text{bool}\} \vdash \epsilon$	(emptyProg)

# Summary

The **principle type** of an expression is the most general type we could give it

Our unification algorithm gives us a **most general unifier**, which we use to get the principle type of an expression