Bytecode Interpreters

Concepts of Programming Languages

CAS CS 320

Outline

- Course evaluations
- Discuss **stack-based languages** and **stack machines**
- Look briefly at how to compile **variables** and **functions**
- Finish up the dang course

Course evaluations

Overview

Abstract/Virtual Machines

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A **virtual machine** is a computational abstraction, like a Turing machine (but usually *easier to implement*)

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Virtual machines are typically implemented as **bytecode interpreters**, where "programs" are streams of bytes and a command is represented as a byte (plus possibly some extra data)

One common abstraction is **stack machines**

Benefits of Virtual (Stack) Machines

state program

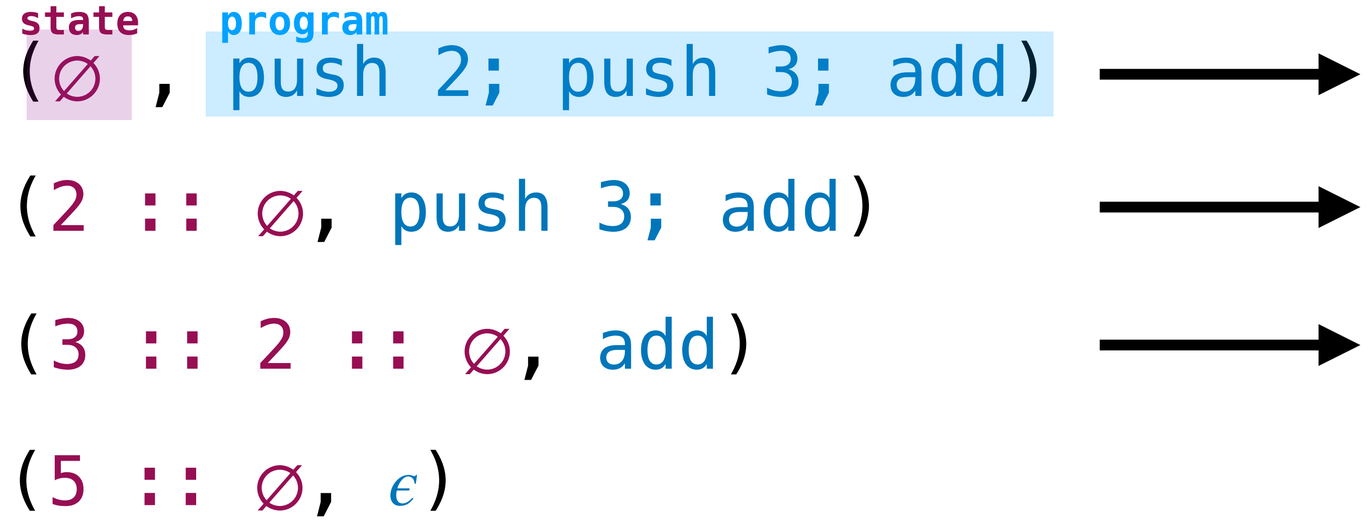
(∅ , push 2; push 3; add) ⟶

(2 :: ∅, push 3; add) ⟶

(3 :: 2 :: ∅, add) ⟶

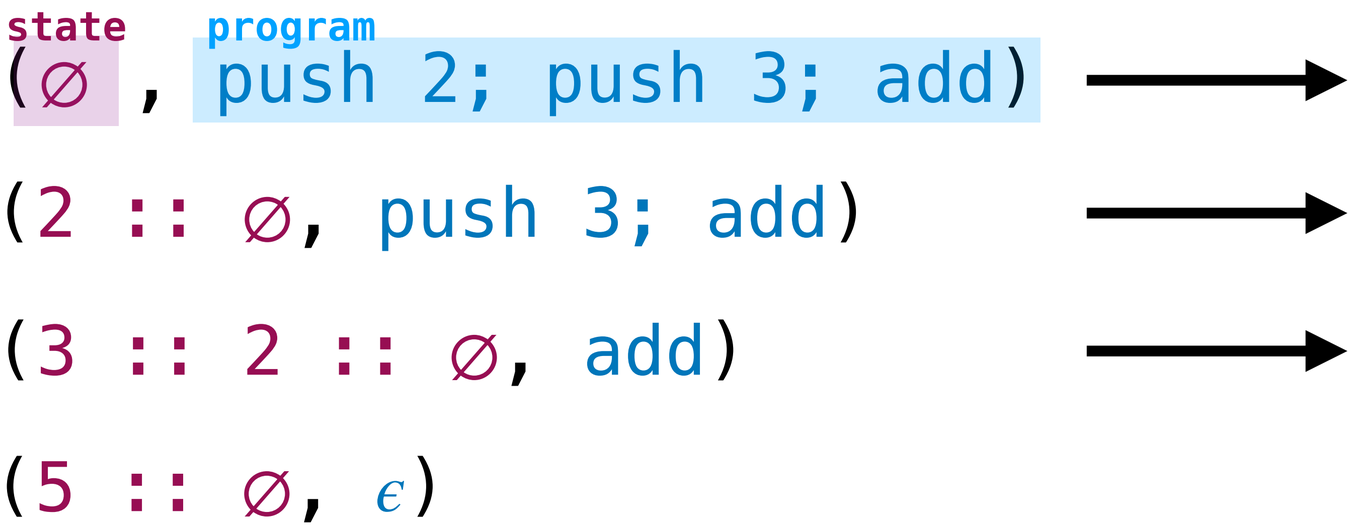
(5 :: ∅, ϵ)

Benefits of Virtual (Stack) Machines



Simplicity: Stacks aren't too complicated

Benefits of Virtual (Stack) Machines



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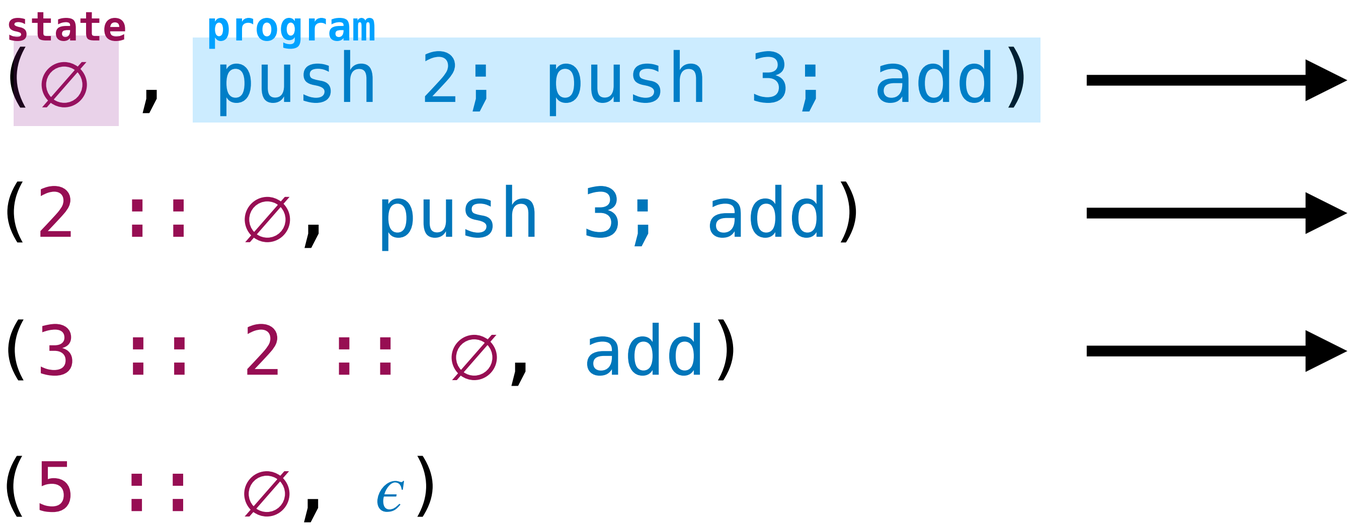
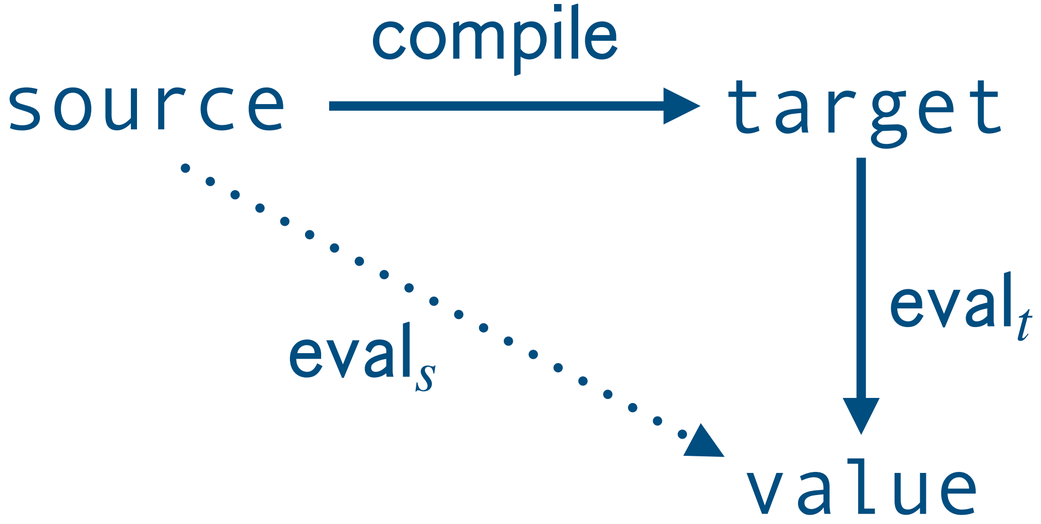


Simplicity: Stacks aren't too complicated

Portability: Any OS should be able to handle a stream of bytes, so the machine dependent part of our programming language can be simplified

Efficiency (sort of): They can be implemented in low-level languages, and so will generally be faster than the interpreters we build in this course (though not as fast as natively compiled code)

Compilation (High Level)



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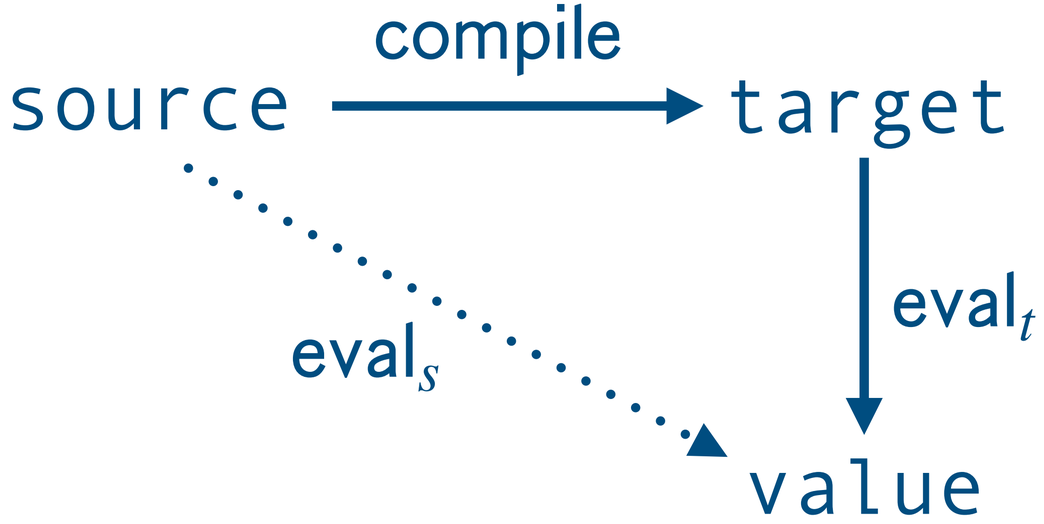
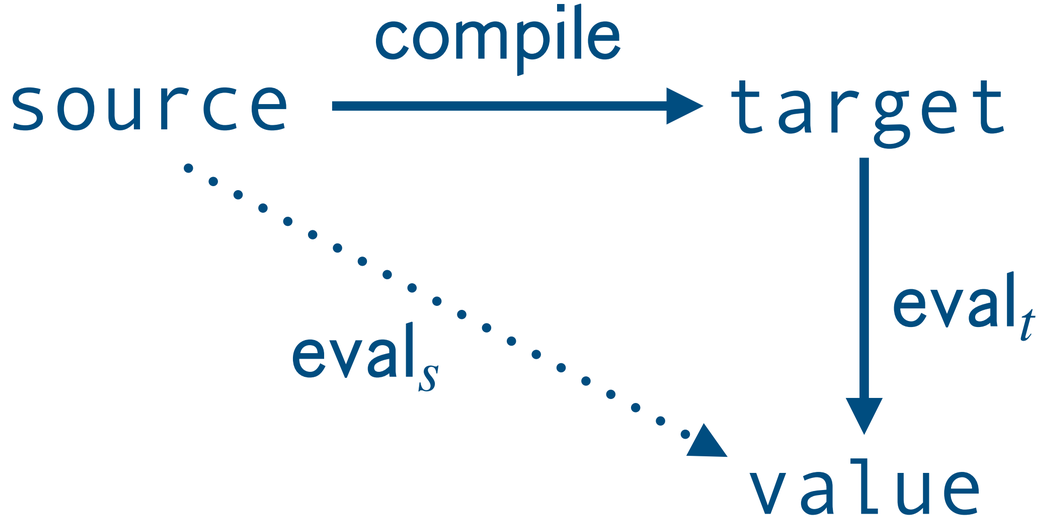


Compilation is the process of translating a program in one language to another, maintaining semantic behavior

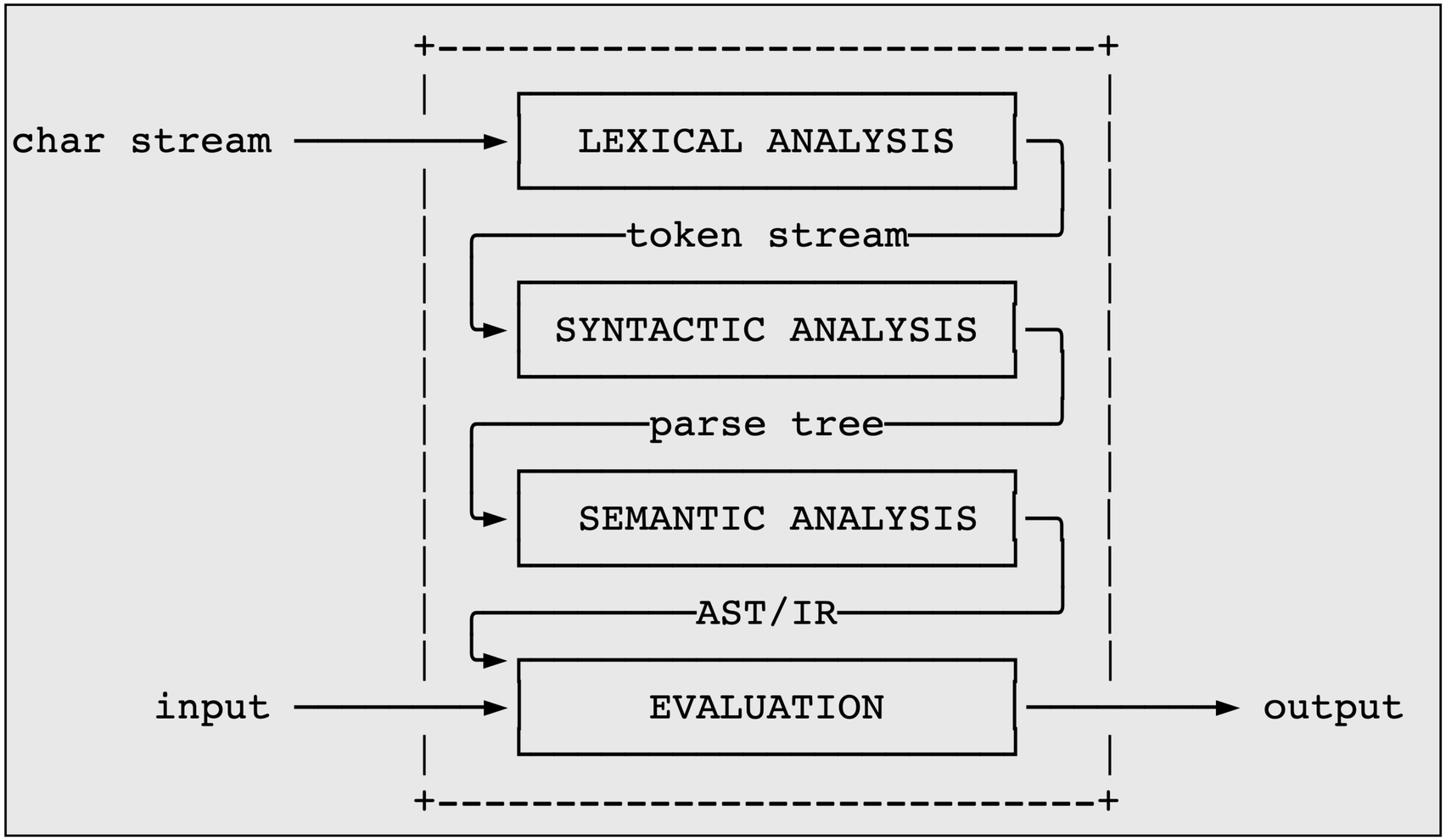
Compilation (High Level)



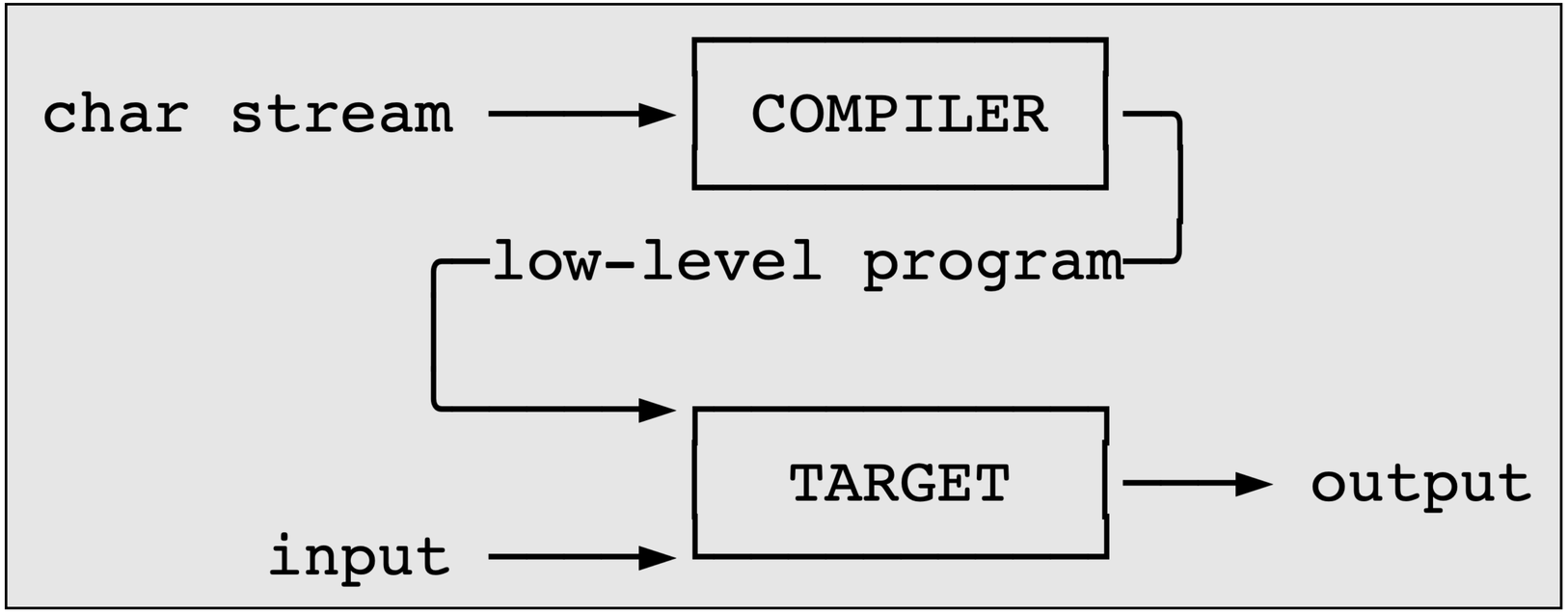
Compilation is the process of translating a program in one language to another, maintaining semantic behavior

Compilation can be a part of interpretation as well, like with **bytecode interpretation**

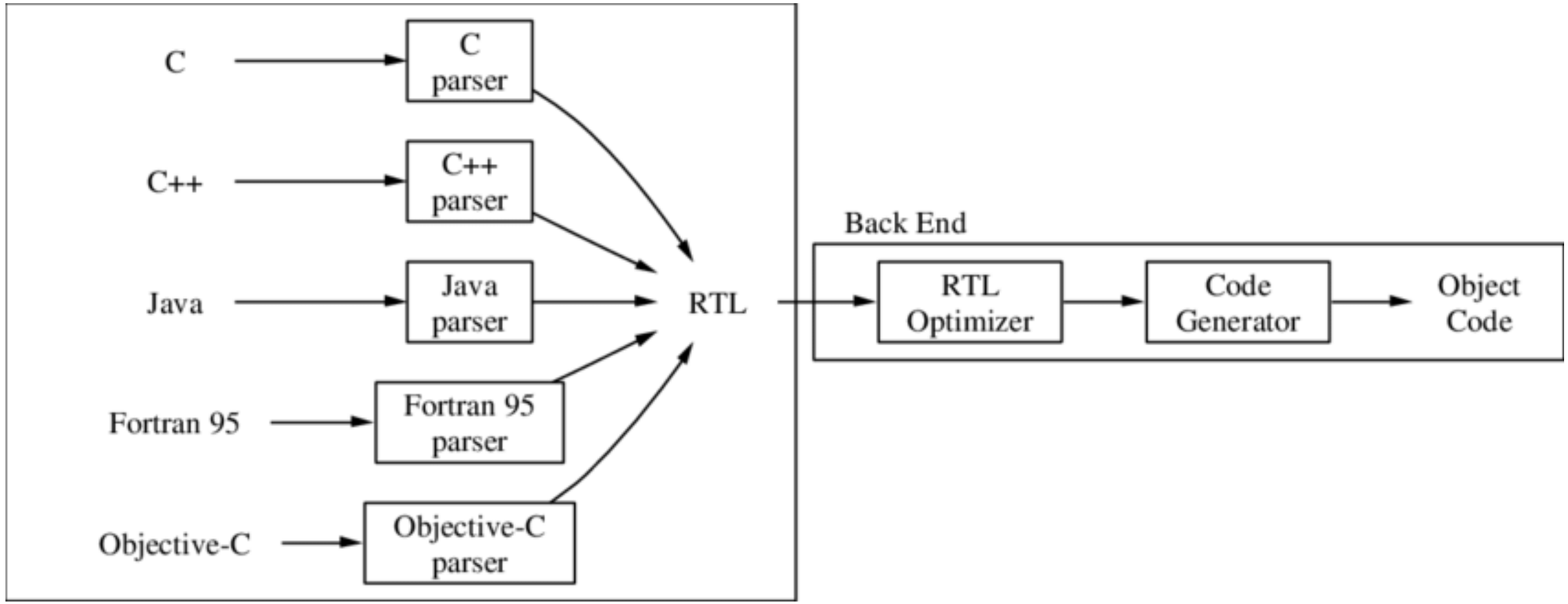
Recall: The Interpretation Pipeline



The Compiler Pipeline



Example Pipelines: gcc



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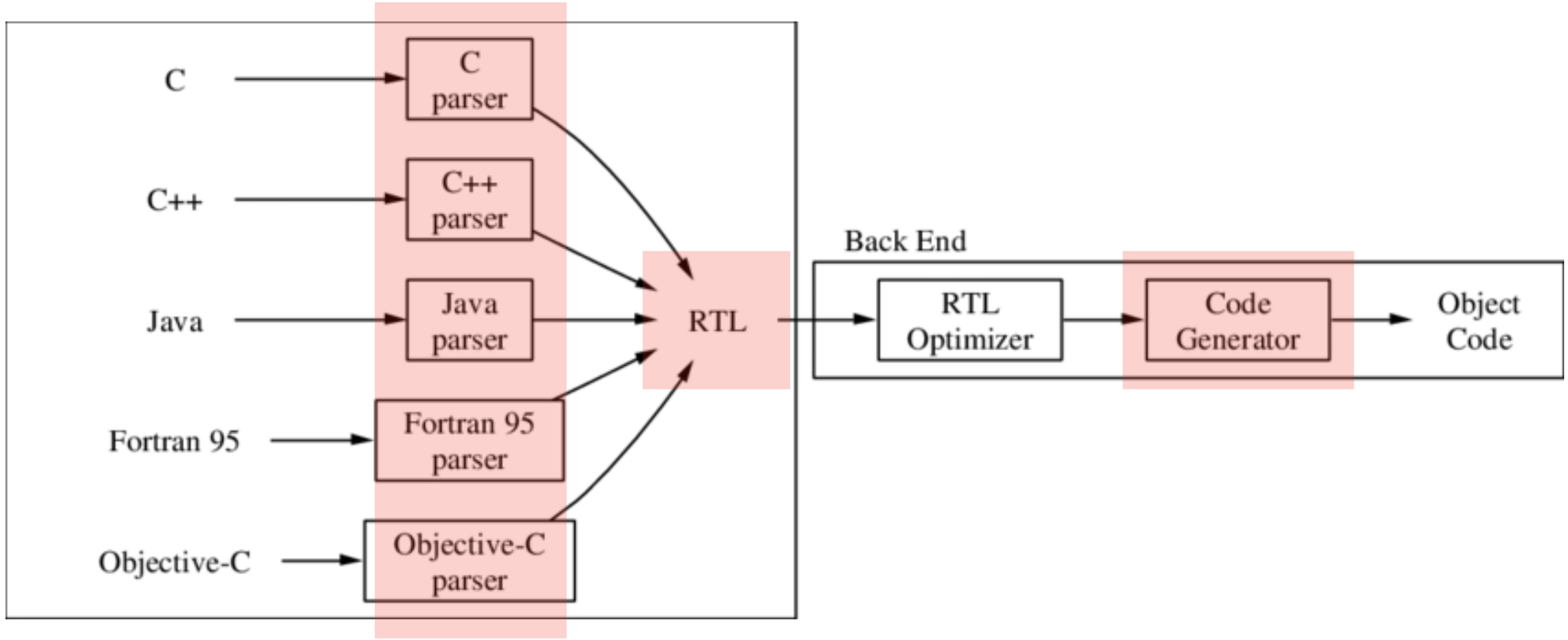
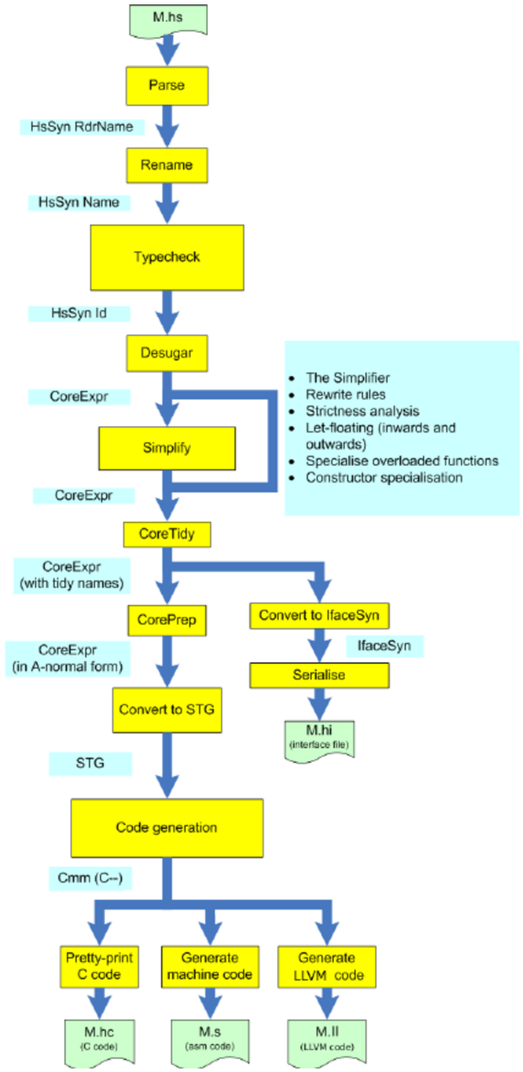


Example Pipelines: GHC (Haskell)



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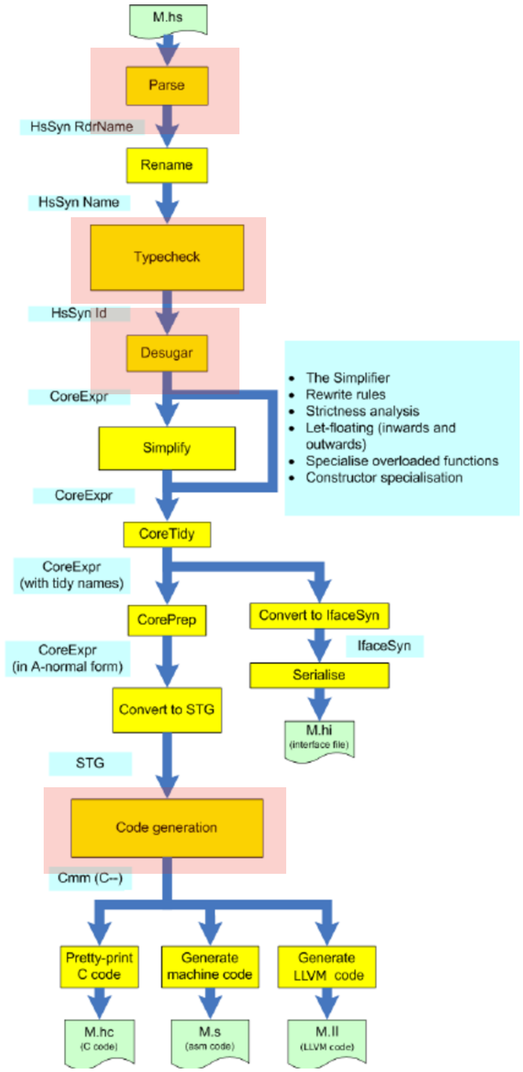
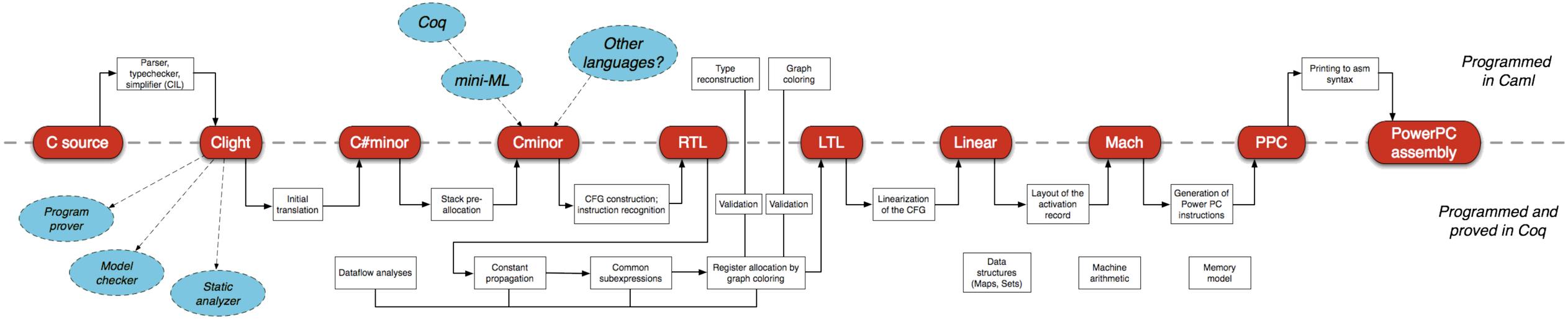
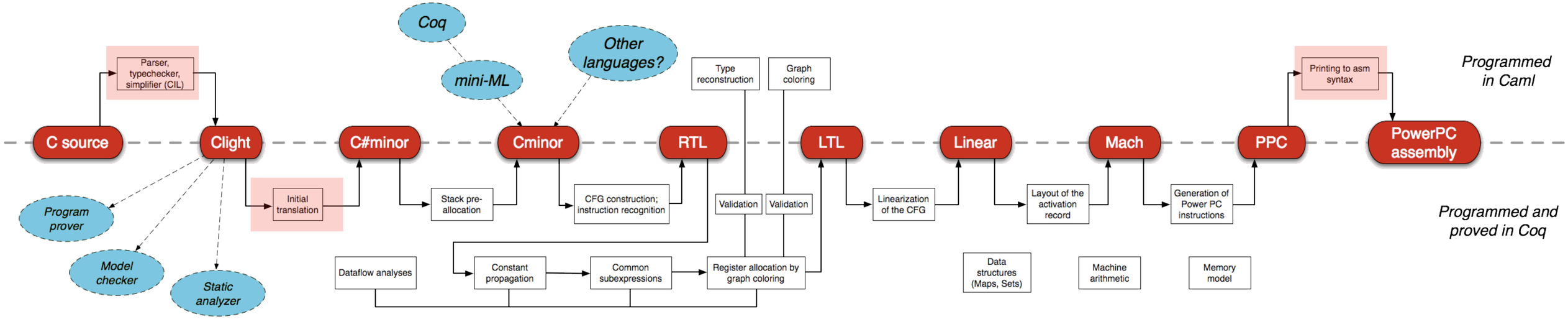


Example Pipelines: CompCert (C)



Example Pipelines: CompCert (C)



Stack-Based Arithmetic

Stack-Based Arithmetic (Syntax)

$$
\begin{aligned}
\texttt{<prog>} &::= \{\texttt{<com>}\} \\
\texttt{<com>} &::= \texttt{ADD} \mid \texttt{SUB} \mid \texttt{MUL} \mid \texttt{DIV} \mid \texttt{PUSH}\ \texttt{<num>} \\
\texttt{<num>} &::= \mathbb{Z}
\end{aligned}
$$

Stack-Based Arithmetic (Semantics)

$$\langle \mathcal{S}, P \rangle$$

A **value** is an integer ($\mathbb{Z}$)

A **configuration** is made up of a stack ($\mathcal{S}$) of values and a program (P) given by **<prog>**

Stack-Based Arithmetic (Semantics)

$$\frac{}{\langle m :: n :: \mathcal{S}, \texttt{ADD}\ P\rangle \longrightarrow \langle (m+n) :: \mathcal{S}, P\rangle}\,(\texttt{add})$$

$$\frac{}{\langle m :: n :: \mathcal{S}, \texttt{SUB}\ P\rangle \longrightarrow \langle (m-n) :: \mathcal{S}, P\rangle}\,(\texttt{sub})$$

$$\frac{}{\langle m :: n :: \mathcal{S}, \texttt{MUL}\ P\rangle \longrightarrow \langle (m\times n) :: \mathcal{S}, P\rangle}\,(\texttt{mul})$$

$$\frac{n \neq 0}{\langle m :: n :: \mathcal{S}, \texttt{DIV}\ P\rangle \longrightarrow \langle (m/n) :: \mathcal{S}, P\rangle}\,(\texttt{div})$$

$$\frac{}{\langle \mathcal{S}, \texttt{PUSH}\ n\ P\rangle \longrightarrow \langle n :: \mathcal{S}, P\rangle}\,(\texttt{push})$$

Example (Evaluation)

$\langle\ \varnothing\ ,$ `PUSH 2 PUSH 3 SUB PUSH 4 MUL` $\rangle \longrightarrow$

Example (Evaluation)

$\langle\ \varnothing\ ,$ `PUSH 2 PUSH 3 SUB PUSH 4 MUL` $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,$ `PUSH 3 SUB PUSH 4 MUL` $\rangle \longrightarrow$

Example (Evaluation)

$\langle\ \varnothing\ ,$ **PUSH 2 PUSH 3 SUB PUSH 4 MUL** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,$ **PUSH 3 SUB PUSH 4 MUL** $\rangle \longrightarrow$

$\langle\ 3 :: 2 :: \varnothing\ ,$ **SUB PUSH 4 MUL** $\rangle \longrightarrow$

Example (Evaluation)

$\langle\ \varnothing\ ,$ `PUSH 2 PUSH 3 SUB PUSH 4 MUL` $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,$ `PUSH 3 SUB PUSH 4 MUL` $\rangle \longrightarrow$

$\langle\ 3 :: 2 :: \varnothing\ ,$ `SUB PUSH 4 MUL` $\rangle \longrightarrow$

$\langle\ -1 :: \varnothing\ ,$ `PUSH 4 MUL` $\rangle \longrightarrow$

Example (Evaluation)

$\langle\ \varnothing\ ,\ \texttt{PUSH 2 PUSH 3 SUB PUSH 4 MUL}\ \rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \texttt{PUSH 3 SUB PUSH 4 MUL}\ \rangle \longrightarrow$

$\langle\ 3 :: 2 :: \varnothing\ ,\ \texttt{SUB PUSH 4 MUL}\ \rangle \longrightarrow$

$\langle\ -1 :: \varnothing\ ,\ \texttt{PUSH 4 MUL}\ \rangle \longrightarrow$

$\langle\ 4 :: -1 :: \varnothing\ ,\ \texttt{MUL}\ \rangle \longrightarrow$

Example (Evaluation)

$\langle\ \varnothing\ ,\ \texttt{PUSH 2 PUSH 3 SUB PUSH 4 MUL}\ \rangle \longrightarrow$

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$\langle\ 3 :: 2 :: \varnothing\ ,\ \texttt{SUB PUSH 4 MUL}\ \rangle \longrightarrow$

$\langle\ -1 :: \varnothing\ ,\ \texttt{PUSH 4 MUL}\ \rangle \longrightarrow$

$\langle\ 4 :: -1 :: \varnothing\ ,\ \texttt{MUL}\ \rangle \longrightarrow$

$\langle\ -4 :: \varnothing\ ,\ \epsilon\ \rangle \longrightarrow$

Example (Evaluation)

$\langle\ \varnothing\ ,\ \texttt{PUSH 2 PUSH 3 SUB PUSH 4 MUL}\ \rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \texttt{PUSH 3 SUB PUSH 4 MUL}\ \rangle \longrightarrow$

$\langle\ 3 :: 2 :: \varnothing\ ,\ \texttt{SUB PUSH 4 MUL}\ \rangle \longrightarrow$

$\langle\ -1 :: \varnothing\ ,\ \texttt{PUSH 4 MUL}\ \rangle \longrightarrow$

$\langle\ 4 :: -1 :: \varnothing\ ,\ \texttt{MUL}\ \rangle \longrightarrow$

$\langle\ -4 :: \varnothing\ ,\ \epsilon\ \rangle \longrightarrow\ -4$

Compiling Arithmetic Expressions

$$\texttt{n} \Longrightarrow \texttt{PUSH n}$$

$$e_1 \texttt{ + } e_2 \Longrightarrow \mathscr{C}(e_2)\ \mathscr{C}(e_1)\ \texttt{ADD}$$

$$e_1 \texttt{ - } e_2 \Longrightarrow \mathscr{C}(e_2)\ \mathscr{C}(e_1)\ \texttt{SUB}$$

$$e_1 \texttt{ * } e_2 \Longrightarrow \mathscr{C}(e_2)\ \mathscr{C}(e_1)\ \texttt{MUL}$$

$$e_1 \texttt{ / } e_2 \Longrightarrow \mathscr{C}(e_2)\ \mathscr{C}(e_1)\ \texttt{DIV}$$

We need a procedure $\mathscr{C}$ for converting an arithmetic expression into a stack program. *Note the order!*

Example (Compilation)

$\mathscr{C}$(`4 * (2 - 3)`)

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$\Longrightarrow$ $\mathscr{C}$(`4`) $\mathscr{C}$(`2 - 3`) `MUL`

Example (Compilation)

$\mathscr{C}$(`4 * (2 - 3)`)

$\Longrightarrow$ $\mathscr{C}$(`4`) $\mathscr{C}$(`2 - 3`) `MUL`

$\Longrightarrow$ `PUSH 4` $\mathscr{C}$(`2 - 3`) `MUL`

Example (Compilation)

$\mathscr{C}$(4 * (2 - 3))

$\Longrightarrow$ $\mathscr{C}$(4) $\mathscr{C}$(2 - 3) MUL

$\Longrightarrow$ PUSH 4 $\mathscr{C}$(2 - 3) MUL

$\Longrightarrow$ PUSH 4 $\mathscr{C}$(2) $\mathscr{C}$(3) SUB MUL

Example (Compilation)

$\mathscr{C}$(4 * (2 - 3))

$\Longrightarrow$ $\mathscr{C}$(4) $\mathscr{C}$(2 - 3) MUL

$\Longrightarrow$ PUSH 4 $\mathscr{C}$(2 - 3) MUL

$\Longrightarrow$ PUSH 4 $\mathscr{C}$(2) $\mathscr{C}$(3) SUB MUL

$\Longrightarrow$ PUSH 4 PUSH 2 $\mathscr{C}$(3) SUB MUL

Example (Compilation)

$\mathscr{C}$(4 * (2 - 3))

$\Longrightarrow$ $\mathscr{C}$(4) $\mathscr{C}$(2 - 3) MUL

$\Longrightarrow$ PUSH 4 $\mathscr{C}$(2 - 3) MUL

$\Longrightarrow$ PUSH 4 $\mathscr{C}$(2) $\mathscr{C}$(3) SUB MUL

$\Longrightarrow$ PUSH 4 PUSH 2 $\mathscr{C}$(3) SUB MUL

$\Longrightarrow$ PUSH 4 PUSH 2 PUSH 2 SUB MUL

Variables

Variables (Syntax)

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\begin{aligned}
\texttt{<prog>} &::= \{\texttt{<com>}\} \\
\texttt{<com>} &::= \texttt{ADD} \mid \texttt{SUB} \mid \texttt{MUL} \mid \texttt{DIV} \mid \texttt{PUSH}\ \texttt{<num>} \\
&\quad \mid \texttt{ASSIGN}\ \texttt{<var>} \mid \texttt{LOOKUP}\ \texttt{<var>} \\
\texttt{<num>} &::= \mathbb{Z} \\
\texttt{<var>} &::= \mathbb{I}
\end{aligned}
$$

Variables (Semantics)

$$\langle \mathcal{S}, \mathcal{E}, P \rangle$$

A **value** is an integer ($\mathbb{Z}$)

A **configuration** is made up of a stack $\mathcal{S}$ of values, an environment $\mathcal{E}$ (mapping of identifiers to values), and a program P given by **<prog>**

Variables (Semantics)

$$\frac{}{\langle m :: n :: \mathcal{S}, \mathcal{E}, \texttt{ADD}\ P\rangle \longrightarrow \langle (m+n) :: \mathcal{S}, \mathcal{E}, P\rangle}\,(\texttt{add}) \qquad \frac{}{\langle m :: n :: \mathcal{S}, \mathcal{E}, \texttt{SUB}\ P\rangle \longrightarrow \langle (m-n) :: \mathcal{S}, \mathcal{E}, P\rangle}\,(\texttt{sub})$$

$$\frac{}{\langle m :: n :: \mathcal{S}, \mathcal{E}, \texttt{MUL}\ P\rangle \longrightarrow \langle (m \times n) :: \mathcal{S}, \mathcal{E}, P\rangle}\,(\texttt{mul}) \qquad \frac{n \neq 0}{\langle m :: n :: \mathcal{S}, \mathcal{E}, \texttt{DIV}\ P\rangle \longrightarrow \langle (m/n) :: \mathcal{S}, \mathcal{E}, P\rangle}\,(\texttt{div})$$

$$\frac{}{\langle \mathcal{S}, \mathcal{E}, \texttt{PUSH}\ n\ P\rangle \longrightarrow \langle n :: \mathcal{S}, \mathcal{E}, P\rangle}\,(\texttt{push})$$

$$\frac{}{\langle n :: \mathcal{S}, \mathcal{E}, \texttt{ASSIGN}\ x\ P\rangle \longrightarrow \langle \mathcal{S}, \mathcal{E}[x \mapsto n], P\rangle}\,(\texttt{asn}) \qquad \frac{}{\langle n :: \mathcal{S}, \mathcal{E}, \texttt{LOOKUP}\ x\ P\rangle \longrightarrow \langle \mathcal{E}(x) :: \mathcal{S}, \mathcal{E}, P\rangle}\,(\texttt{lkp})$$

Variables (Semantics)

basically the same

$$\frac{}{\langle m :: n :: \mathcal{S}, \mathcal{E}, \texttt{ADD}\ P\rangle \longrightarrow \langle (m+n) :: \mathcal{S}, \mathcal{E}, P\rangle}\,(\texttt{add}) \qquad \frac{}{\langle m :: n :: \mathcal{S}, \mathcal{E}, \texttt{SUB}\ P\rangle \longrightarrow \langle (m-n) :: \mathcal{S}, \mathcal{E}, P\rangle}\,(\texttt{sub})$$

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Variables (Semantics)

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$$\frac{}{\langle \mathcal{S}, \mathcal{E}, \texttt{PUSH}\ n\ P\rangle \longrightarrow \langle n :: \mathcal{S}, \mathcal{E}, P\rangle}\ (\texttt{push})$$

new rules

$$\frac{}{\langle n :: \mathcal{S}, \mathcal{E}, \texttt{ASSIGN}\ x\ P\rangle \longrightarrow \langle \mathcal{S}, \mathcal{E}[x \mapsto n], P\rangle}\ (\texttt{asn}) \qquad \frac{}{\langle n :: \mathcal{S}, \mathcal{E}, \texttt{LOOKUP}\ x\ P\rangle \longrightarrow \langle \mathcal{E}(x) :: \mathcal{S}, \mathcal{E}, P\rangle}\ (\texttt{lkp})$$

Example (Evaluation)

$\langle \varnothing , \varnothing ,$ `PUSH 2 ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD` $\rangle \longrightarrow$

Example (Evaluation)

$\langle\ \varnothing\ ,\ \varnothing\ ,$ **PUSH 2 ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \varnothing\ ,$ **ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

Example (Evaluation)

$\langle\ \varnothing\ ,\ \varnothing\ ,$ **PUSH 2 ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \varnothing\ ,$ **ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{x \mapsto 2\}\ ,$ **PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

Example (Evaluation)

$\langle\ \varnothing\ ,\ \varnothing\ ,$ **PUSH 2 ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \varnothing\ ,$ **ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{x \mapsto 2\}\ ,$ **PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 3 :: \varnothing\ ,\ \{x \mapsto 2\}\ ,$ **ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

Example (Evaluation)

$\langle\ \varnothing\ ,\ \varnothing\ ,$ **PUSH 2 ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \varnothing\ ,$ **ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{x \mapsto 2\}\ ,$ **PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 3 :: \varnothing\ ,\ \{x \mapsto 2\}\ ,$ **ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{x \mapsto 2\ ,\ y \mapsto 3\}\ ,$ **LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

Example (Evaluation)

$\langle\ \varnothing\ ,\ \varnothing\ ,$ **PUSH 2 ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \varnothing\ ,$ **ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{x \mapsto 2\}\ ,$ **PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 3 :: \varnothing\ ,\ \{x \mapsto 2\}\ ,$ **ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{x \mapsto 2\ ,\ y \mapsto 3\}\ ,$ **LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{x \mapsto 2\ ,\ y \mapsto 3\}\ ,$ **LOOKUP y ADD** $\rangle \longrightarrow$

Example (Evaluation)

$\langle\ \varnothing\ ,\ \varnothing\ ,$ **PUSH 2 ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \varnothing\ ,$ **ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{x \mapsto 2\}\ ,$ **PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 3 :: \varnothing\ ,\ \{x \mapsto 2\}\ ,$ **ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{x \mapsto 2\ ,\ y \mapsto 3\}\ ,$ **LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{x \mapsto 2\ ,\ y \mapsto 3\}\ ,$ **LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 3 :: 2 :: \varnothing\ ,\ \{x \mapsto 2\ ,\ y \mapsto 3\}\ ,$ **ADD** $\rangle \longrightarrow$

Example (Evaluation)

$\langle\ \varnothing\ ,\ \varnothing\ ,$ **PUSH 2 ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \varnothing\ ,$ **ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{x \mapsto 2\}\ ,$ **PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 3 :: \varnothing\ ,\ \{x \mapsto 2\}\ ,$ **ASSIGN y LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{x \mapsto 2\ ,\ y \mapsto 3\}\ ,$ **LOOKUP x LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{x \mapsto 2\ ,\ y \mapsto 3\}\ ,$ **LOOKUP y ADD** $\rangle \longrightarrow$

$\langle\ 3 :: 2 :: \varnothing\ ,\ \{x \mapsto 2\ ,\ y \mapsto 3\}\ ,$ **ADD** $\rangle \longrightarrow$

$\langle\ 5 :: \varnothing\ ,\ \{x \mapsto 2\ ,\ y \mapsto 3\}\ ,\ \epsilon\ \rangle \longrightarrow$

Example (Evaluation)

$\langle\ \varnothing\ ,\ \varnothing\ ,\ \texttt{PUSH 2 ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD}\ \rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \varnothing\ ,\ \texttt{ASSIGN x PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD}\ \rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{x \mapsto 2\}\ ,\ \texttt{PUSH 3 ASSIGN y LOOKUP x LOOKUP y ADD}\ \rangle \longrightarrow$

$\langle\ 3 :: \varnothing\ ,\ \{x \mapsto 2\}\ ,\ \texttt{ASSIGN y LOOKUP x LOOKUP y ADD}\ \rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{x \mapsto 2\ ,\ y \mapsto 3\}\ ,\ \texttt{LOOKUP x LOOKUP y ADD}\ \rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{x \mapsto 2\ ,\ y \mapsto 3\}\ ,\ \texttt{LOOKUP y ADD}\ \rangle \longrightarrow$

$\langle\ 3 :: 2 :: \varnothing\ ,\ \{x \mapsto 2\ ,\ y \mapsto 3\}\ ,\ \texttt{ADD}\ \rangle \longrightarrow$

$\langle\ 5 :: \varnothing\ ,\ \{x \mapsto 2\ ,\ y \mapsto 3\}\ ,\ \epsilon\ \rangle \longrightarrow\ 5$

Compiling Let-Expressions (Attempt)

$$\texttt{x} \quad \Longrightarrow \quad \texttt{LOOKUP}\ x$$

$$\texttt{let}\ x\ \texttt{=}\ e_1\ \texttt{in}\ e_2 \quad \Longrightarrow \quad \mathscr{C}(e_1)\ \texttt{ASSIGN}\ x\ \mathscr{C}(e_2)$$

Compiling Let-Expressions (Attempt)

$$\texttt{x} \quad \Longrightarrow \quad \texttt{LOOKUP}\ x$$

$$\texttt{let}\ x\ \texttt{=}\ e_1\ \texttt{in}\ e_2 \quad \Longrightarrow \quad \mathscr{C}(e_1)\ \texttt{ASSIGN}\ x\ \mathscr{C}(e_2)$$

Except this isn't quite right

Example

$\mathscr{C}$(`let y = 1 in let x = let y = 2 in y in y`) $\Longrightarrow$

Example

$\mathscr{C}$(`let y = 1 in let x = let y = 2 in y in y`) $\Longrightarrow$

$\mathscr{C}$(`1`) `ASSIGN y` $\mathscr{C}$(`let x = let y = 2 in y in y`) $\Longrightarrow$

Example

$\mathscr{C}$(let y = 1 in let x = let y = 2 in y in y) $\Longrightarrow$

$\mathscr{C}$(1) ASSIGN y $\mathscr{C}$(let x = let y = 2 in y in y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(let x = let y = 2 in y in y) $\Longrightarrow$

Example

$\mathscr{C}$(let y = 1 in let x = let y = 2 in y in y) $\Longrightarrow$

$\mathscr{C}$(1) ASSIGN y $\mathscr{C}$(let x = let y = 2 in y in y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(let x = let y = 2 in y in y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(let y = 2 in y) ASSIGN x $\mathscr{C}$(y) $\Longrightarrow$

Example

$\mathscr{C}$(let y = 1 in let x = let y = 2 in y in y) $\Longrightarrow$

$\mathscr{C}$(1) ASSIGN y $\mathscr{C}$(let x = let y = 2 in y in y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(let x = let y = 2 in y in y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(let y = 2 in y) ASSIGN x $\mathscr{C}$(y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(2) ASSIGN y $\mathscr{C}$(y) ASSIGN x $\mathscr{C}$(y) $\Longrightarrow$

Example

$\mathscr{C}$(let y = 1 in let x = let y = 2 in y in y) $\Longrightarrow$

$\mathscr{C}$(1) ASSIGN y $\mathscr{C}$(let x = let y = 2 in y in y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(let x = let y = 2 in y in y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(let y = 2 in y) ASSIGN x $\mathscr{C}$(y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(2) ASSIGN y $\mathscr{C}$(y) ASSIGN x $\mathscr{C}$(y) $\Longrightarrow$

PUSH 1 ASSIGN y PUSH 2 ASSIGN y $\mathscr{C}$(y) ASSIGN x $\mathscr{C}$(y) $\Longrightarrow$

Example

$\mathscr{C}$(let y = 1 in let x = let y = 2 in y in y) $\Longrightarrow$

$\mathscr{C}$(1) ASSIGN y $\mathscr{C}$(let x = let y = 2 in y in y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(let x = let y = 2 in y in y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(let y = 2 in y) ASSIGN x $\mathscr{C}$(y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(2) ASSIGN y $\mathscr{C}$(y) ASSIGN x $\mathscr{C}$(y) $\Longrightarrow$

PUSH 1 ASSIGN y PUSH 2 ASSIGN y $\mathscr{C}$(y) ASSIGN x $\mathscr{C}$(y) $\Longrightarrow$

PUSH 1 ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x $\mathscr{C}$(y) $\Longrightarrow$

Example

$\mathscr{C}$(let y = 1 in let x = let y = 2 in y in y) $\Longrightarrow$

$\mathscr{C}$(1) ASSIGN y $\mathscr{C}$(let x = let y = 2 in y in y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(let x = let y = 2 in y in y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(let y = 2 in y) ASSIGN x $\mathscr{C}$(y) $\Longrightarrow$

PUSH 1 ASSIGN y $\mathscr{C}$(2) ASSIGN y $\mathscr{C}$(y) ASSIGN x $\mathscr{C}$(y) $\Longrightarrow$

PUSH 1 ASSIGN y PUSH 2 ASSIGN y $\mathscr{C}$(y) ASSIGN x $\mathscr{C}$(y) $\Longrightarrow$

PUSH 1 ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x $\mathscr{C}$(y) $\Longrightarrow$

PUSH 1 ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y

Example

$\langle \varnothing , \varnothing ,$ `PUSH 1 ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y` $\rangle \longrightarrow$

Example

$\langle\ \varnothing\ ,\ \varnothing\ ,$ `PUSH 1 ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y` $\rangle \longrightarrow$

$\langle\ 1 :: \varnothing\ ,\ \varnothing\ ,$ `ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y` $\rangle \longrightarrow$

Example

$\langle\ \varnothing\ ,\ \varnothing\ ,$ **PUSH 1 ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 1 :: \varnothing\ ,\ \varnothing\ ,$ **ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 1\}\ ,$ **PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

Example

$\langle\ \varnothing\ ,\ \varnothing\ ,$ `PUSH 1 ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y` $\rangle \longrightarrow$

$\langle\ 1 :: \varnothing\ ,\ \varnothing\ ,$ `ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y` $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 1\}\ ,$ `PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y` $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{y \mapsto 1\}\ ,$ `ASSIGN y LOOKUP y ASSIGN x LOOKUP y` $\rangle \longrightarrow$

Example

$\langle\ \varnothing\ ,\ \varnothing\ ,$ **PUSH 1 ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 1 :: \varnothing\ ,\ \varnothing\ ,$ **ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 1\}\ ,$ **PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{y \mapsto 1\}\ ,$ **ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 2\}\ ,$ **LOOKUP Y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

Example

$\langle\ \varnothing\ ,\ \varnothing\ ,$ **PUSH 1 ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 1 :: \varnothing\ ,\ \varnothing\ ,$ **ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 1\}\ ,$ **PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{y \mapsto 1\}\ ,$ **ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 2\}\ ,$ **LOOKUP Y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{y \mapsto 2\}\ ,$ **ASSIGN x LOOKUP y** $\rangle \longrightarrow$

Example

$\langle\ \varnothing\ ,\ \varnothing\ ,$ **PUSH 1 ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 1 :: \varnothing\ ,\ \varnothing\ ,$ **ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 1\}\ ,$ **PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{y \mapsto 1\}\ ,$ **ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 2\}\ ,$ **LOOKUP Y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{y \mapsto 2\}\ ,$ **ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 2\ ,\ x \mapsto 2\}\ ,$ **LOOKUP y** $\rangle \longrightarrow$

Example

$\langle\ \varnothing\ ,\ \varnothing\ ,$ **PUSH 1 ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 1 :: \varnothing\ ,\ \varnothing\ ,$ **ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 1\}\ ,$ **PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{y \mapsto 1\}\ ,$ **ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 2\}\ ,$ **LOOKUP Y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{y \mapsto 2\}\ ,$ **ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 2\ ,\ x \mapsto 2\}\ ,$ **LOOKUP y** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{y \mapsto 2\ ,\ x \mapsto 2\}\ ,\ \epsilon\ \rangle \longrightarrow$

Example

$\langle\ \varnothing\ ,\ \varnothing\ ,$ **PUSH 1 ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 1 :: \varnothing\ ,\ \varnothing\ ,$ **ASSIGN y PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 1\}\ ,$ **PUSH 2 ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{y \mapsto 1\}\ ,$ **ASSIGN y LOOKUP y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 2\}\ ,$ **LOOKUP Y ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{y \mapsto 2\}\ ,$ **ASSIGN x LOOKUP y** $\rangle \longrightarrow$

$\langle\ \varnothing\ ,\ \{y \mapsto 2\ ,\ x \mapsto 2\}\ ,$ **LOOKUP y** $\rangle \longrightarrow$

$\langle\ 2 :: \varnothing\ ,\ \{y \mapsto 2\ ,\ x \mapsto 2\}\ ,\ \epsilon\ \rangle \longrightarrow\ 2$

Scoping

```
let y = 1 in
let x = let y = 2 in y in
y
```

$\nrightarrow 2$

Scoping

```
let y = 1 in
let x = let y = 2 in y in
y
```

$\nrightarrow 2$

The language we've just described is only good for compiling from languages with **dynamic scoping**

Scoping

```
let y = 1 in
let x = let y = 2 in y in
y
```

$\nrightarrow 2$

The language we've just described is only good for compiling from languages with **dynamic scoping**

We can use closures to deal with lexical scoping

Functions

The Rough Picture

```
let k = fun x -> fun y -> x in
let a = k 2 in
a 3
```

Compilation is just a big sequence of transformations

The Rough Picture

```
(fun k ->
   let a = k 2 in
   a 3)
(fun x -> fun y -> x)
```

We can simulate let expressions with functions
(we did this in lab)

The Rough Picture

```
(fun k ->
   (fun a -> a 3)
   (k 2))
(fun x -> fun y -> x)
```

and again...

The Rough Picture

```
[(fun k ->
   (fun a -> a 3)
   (k 2))
 (fun x -> fun y -> x)]
```

think of [expr] as as compile(expr)

The Rough Picture

```
[(fun x -> fun y -> x)]
[(fun k -> (fun a -> a 3) (k 2))]
CALL
```

We introduce as CALL command to call functions
Note the order, function/argument will go on a stack

The Rough Picture

```
FUN ? X
   [fun y -> x]
   RETURN
[(fun k -> (fun a -> a 3) (k 2))]
CALL
```

We introduce a FUN command to define functions and a RETURN command to return from functions

The Rough Picture

```
FUN ? X
  FUN ? Y
    [x]
    RETURN
  RETURN
[(fun k -> (fun a -> a 3) (k 2))]
CALL
```

and again...

The Rough Picture

```
FUN 4 X
  FUN 2 Y
    LOOKUP X
    RETURN
  RETURN
[(fun k -> (fun a -> a 3) (k 2))]
CALL
```

The familiar LOOKUP command...

And functions let us know how many commands they have

The Rough Picture

```
FUN 4 X
  FUN 2 Y
    LOOKUP X
    RETURN
  RETURN
FUN ? K
  [(fun a -> a 3) (k 2)]
  RETURN
CALL
```

and we can keep going...

The Rough Picture

```
FUN 4 X
  FUN 2 Y
    LOOKUP X
    RETURN
  RETURN
FUN ? K
  [k 2]
  [fun a -> a 3]
  CALL
  RETURN
CALL
```

The Rough Picture

```
FUN 4 X
  FUN 2 Y
    LOOKUP X
    RETURN
  RETURN
FUN ? K
  [2]
  [k]
  CALL
  [fun a -> a 3]
  CALL
  RETURN
CALL
```

The Rough Picture

```
FUN 4 X
  FUN 2 Y
    LOOKUP X
    RETURN
  RETURN
FUN ? K
  PUSH 2
  [k]
  CALL
  [fun a -> a 3]
  CALL
  RETURN
CALL
```

The Rough Picture

```
FUN 4 X
  FUN 2 Y
    LOOKUP X
    RETURN
  RETURN
FUN ? K
  PUSH 2
  LOOKUP K
  CALL
  [fun a -> a 3]
  CALL
  RETURN
CALL
```

The Rough Picture

```
FUN 4 X
  FUN 2 Y
    LOOKUP X
    RETURN
  RETURN
FUN ? K
  PUSH 2
  LOOKUP K
  CALL
  FUN ? A
    [a 3]
    RETURN
  CALL
  RETURN
CALL
```

The Rough Picture

```
FUN 4 X
  FUN 2 Y
    LOOKUP X
    RETURN
  RETURN
FUN ? K
  PUSH 2
  LOOKUP K
  CALL
  FUN ? A
    [3]
    [a]
    CALL
    RETURN
  CALL
  RETURN
CALL
```

The Rough Picture

```
FUN 4 X
  FUN 2 Y
    LOOKUP X
    RETURN
  RETURN
FUN ? K
  PUSH 2
  LOOKUP K
  CALL
  FUN ? A
    PUSH 3
    [a]
    CALL
    RETURN
  CALL
  RETURN
CALL
```

The Rough Picture

```
FUN 4 X
  FUN 2 Y
    LOOKUP X
    RETURN
  RETURN
FUN 10 K
  PUSH 2
  LOOKUP K
  CALL
  FUN 4 A
    PUSH 3
    LOOKUP A
    CALL
    RETURN
  CALL
  RETURN
CALL
```

The Rough Picture

```
let k = fun x -> fun y -> x in
let a = k 2 in
a 3
```

```
FUN 4 X
  FUN 2 Y
    LOOKUP X
    RETURN
  RETURN
FUN 10 K
  PUSH 2
  LOOKUP K
  CALL
  FUN 4 A
    PUSH 3
    LOOKUP A
    CALL
    RETURN
  CALL
  RETURN
CALL
```

Compilation is just a big sequence of transformations

The Rough Picture

```
let k = fun x -> fun y -> x in
let a = k 2 in
a 3
```

```
7 4
7 2
10 1
9
9
7 10
0 2
10 0
8
7 4
0 3
10 0
8
9
8
9
8
```

Byte-code interpretation additionally maps each command to a byte value

Syntax

```
<prog> ::= { <com> }
<com>  ::= PUSH <int>
         | ADD | SUB | MUL | DIV
         | FUN <ident> <int> | CALL | RETURN
         | LOOKUP
```

Semantics (Configurations)

$$\langle \mathcal{S}, \mathcal{E}, P \rangle$$

A **value** is an integer ($\mathbb{Z}$) or a closure ($\mathbb{C}$) of the form $(\mathcal{E}, x, P)$

A **configuration** is made up of a stack $\mathcal{S}$ of values, an environment $\mathcal{E}$ (mapping of identifiers to values) and a program P given by **<prog>**

Semantics (Functions)

$$\frac{}{\langle \mathcal{S}, \mathcal{E}, \mathsf{FUN}\ x\ n\ P \rangle \longrightarrow \langle (\mathcal{E}, x, P[1..n]) :: \mathcal{S}, \mathcal{E}'[x \mapsto v], P[n+1..] \rangle}\ (\texttt{fun})$$

Function definitions carry a **parameter name** and an **offset,** which we use to construct the closure

Semantics (Continuation Passing)

$$\frac{}{\langle (\mathscr{E}', x, P') :: v :: \mathscr{S}, \mathscr{E}, \texttt{CALL}\ P \rangle \longrightarrow \langle (\mathscr{E}, \_, P) :: \mathscr{S}, \mathscr{E}'[x \mapsto v], P' \rangle}\ (\texttt{call})$$

$$\frac{}{\langle v :: (\mathscr{E}', \_, P') :: \mathscr{S}, \mathscr{E}, \texttt{RETURN}\ P \rangle \longrightarrow \langle v :: \mathscr{S}, \mathscr{E}', P' \rangle}\ (\texttt{ret})$$

One challenge: when we call a function, where to we "return" to?

Answer: We put the information on the stack itself in a closure!

Example

Stack:

Env:

```
FUN 4 X
  FUN 2 Y
    LOOKUP X
    RETURN
  RETURN
FUN 10 K
  PUSH 2
  LOOKUP K
  CALL
  FUN 4 A
    PUSH 3
    LOOKUP A
    CALL
    RETURN
  CALL
  RETURN
CALL
```

Example

Stack:

```
∅
x
FUN 2 Y
  LOOKUP X
  RETURN
RETURN
```

Env:

```
FUN 10 K
  PUSH 2
  LOOKUP K
  CALL
  FUN 4 A
    PUSH 3
    LOOKUP A
    CALL
    RETURN
  CALL
  RETURN
CALL
```

Example

Stack:

```
∅
X
FUN 2 Y
  LOOKUP X
  RETURN
RETURN
```

```
∅
K
PUSH 2
LOOKUP K
CALL
FUN 4 A
  PUSH 3
  LOOKUP A
  CALL
  RETURN
CALL
RETURN
```

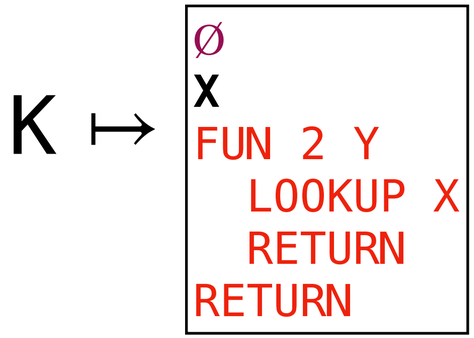
Env:

CALL

Example

Stack:

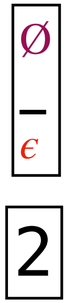
Env:



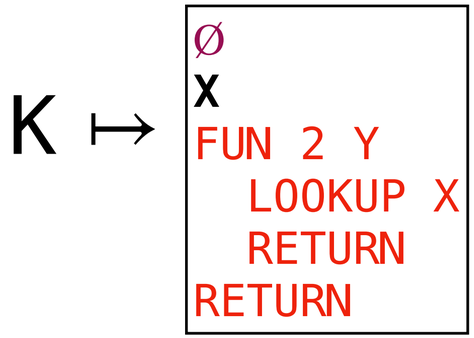

```
PUSH 2
LOOKUP K
CALL
FUN 4 A
  PUSH 3
  LOOKUP A
  CALL
  RETURN
CALL
RETURN
```

Example

Stack:



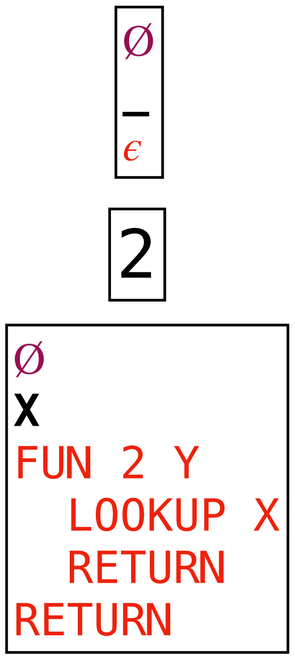
Env:



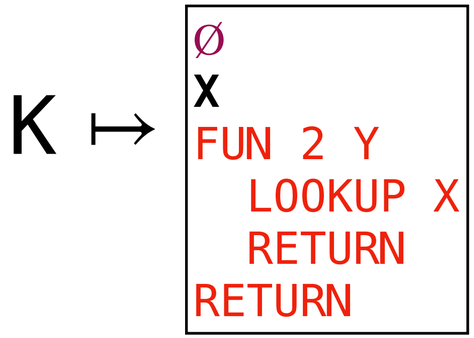

```
LOOKUP K
CALL
FUN 4 A
  PUSH 3
  LOOKUP A
  CALL
  RETURN
CALL
RETURN
```

Example

Stack:



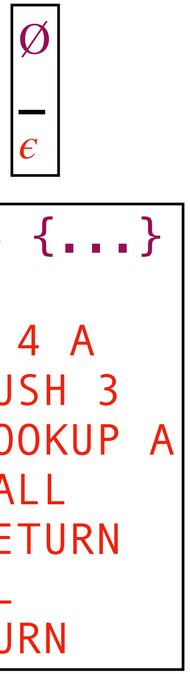
Env:




```
CALL
FUN 4 A
  PUSH 3
  LOOKUP A
  CALL
  RETURN
CALL
RETURN
```

Example

Stack:



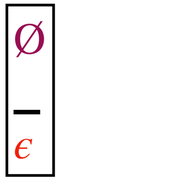
Env:

X ↦ 2

```
FUN 2 Y
  LOOKUP X
  RETURN
RETURN
```

Example

Stack:




```
K ↦ {...}
–
FUN 4 A
  PUSH 3
  LOOKUP A
  CALL
  RETURN
CALL
RETURN
```

```
X ↦ 2
Y
LOOKUP X
RETURN
```

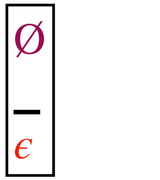
Env:

X ↦ 2

RETURN

Example

Stack:




```
X ↦ 2
Y
LOOKUP X
RETURN
```

Env:

K ↦

```
∅
X
FUN 2 Y
  LOOKUP X
  RETURN
RETURN
```

```
FUN 4 A
  PUSH 3
  LOOKUP A
  CALL
  RETURN
CALL
RETURN
```

Example

Stack:


```
X ↦ 2
Y
LOOKUP X
RETURN
```

```
K ↦ {...}
A
PUSH 3
LOOKUP A
CALL
RETURN
```

Env:

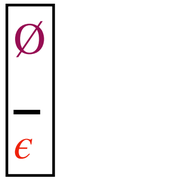
K ↦

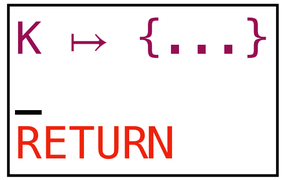
```
Ø
X
FUN 2 Y
  LOOKUP X
  RETURN
RETURN
```

```
CALL
RETURN
```

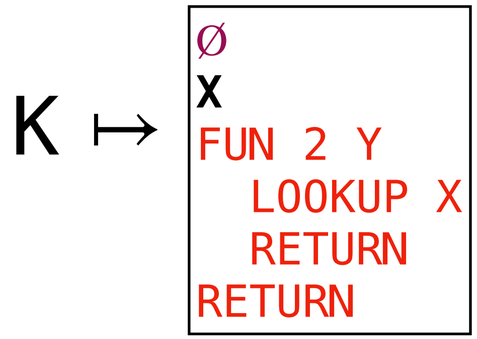
Example

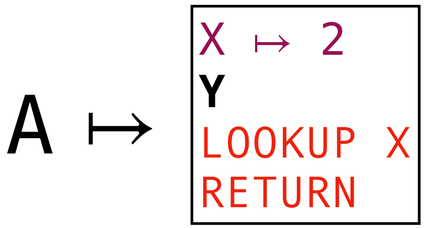
Stack:





Env:

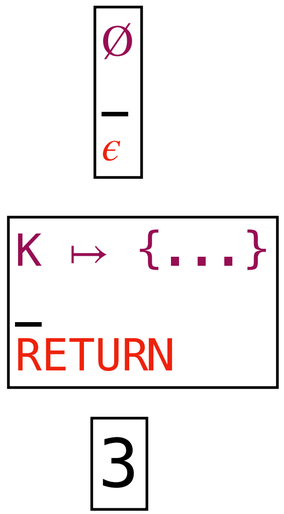




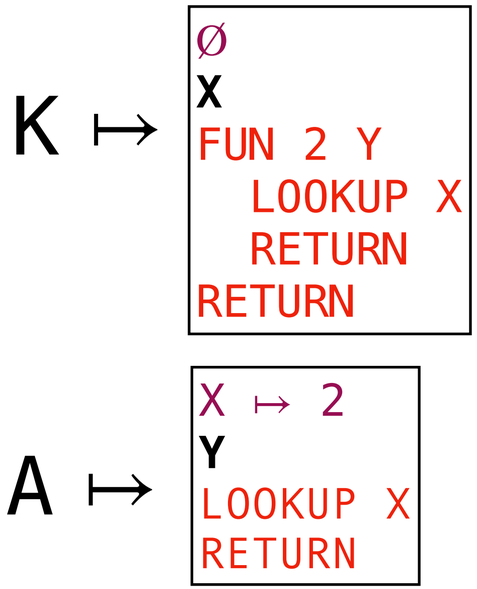

```
PUSH 3
LOOKUP A
CALL
RETURN
```

Example

Stack:



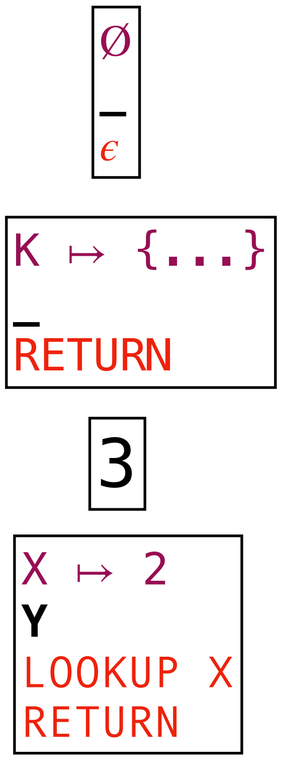
Env:



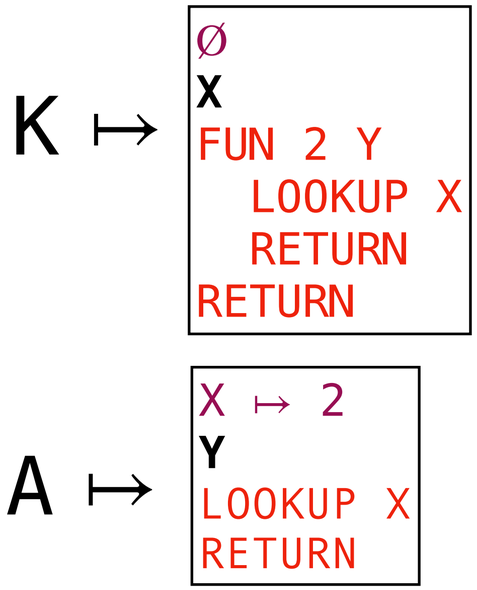

```
LOOKUP A
CALL
RETURN
```

Example

Stack:



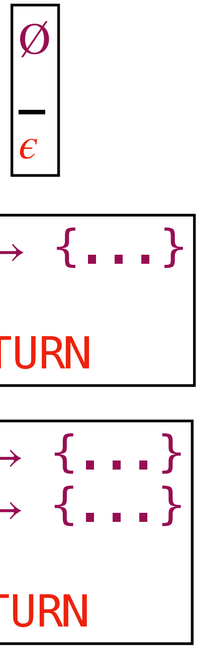
Env:



CALL
RETURN

Example

Stack:



Env:

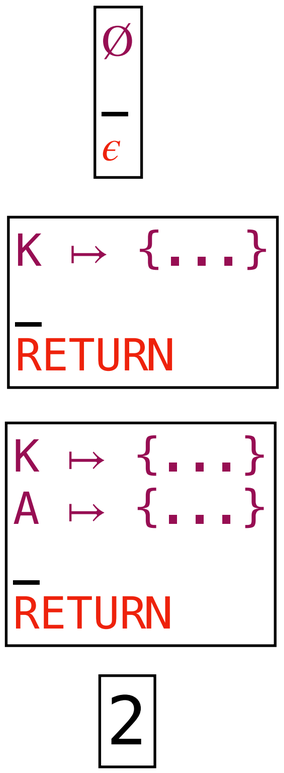
X ↦ 2

Y ↦ 3

```
LOOKUP X
RETURN
```

Example

Stack:



Env:

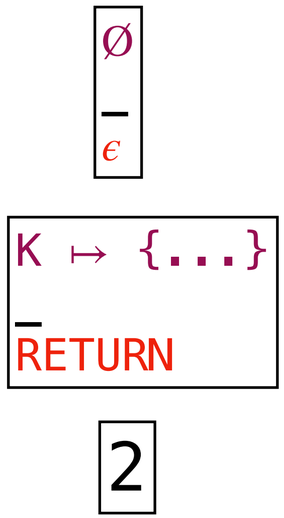
X ↦ 2

Y ↦ 3

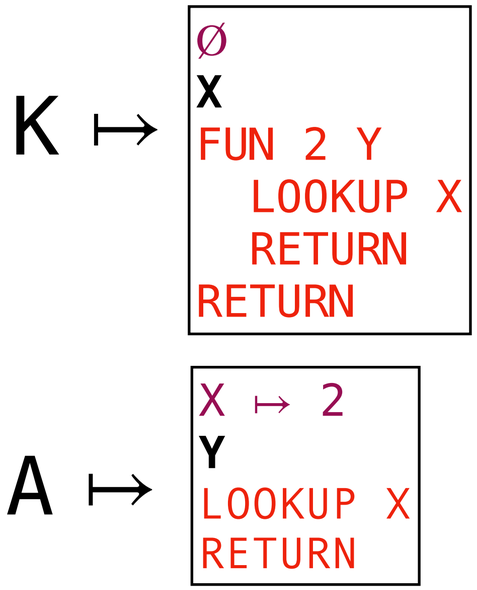
RETURN

Example

Stack:



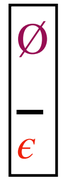
Env:



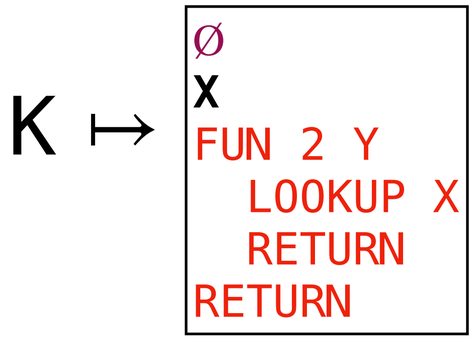
RETURN

Example

Stack:



Env:



RETURN

Example

Stack:

2

Env:

ϵ

demo

What's next?

More OCaml:

» Modules, functional data structures, mutability

» GADTS, effects, parallelism

» applications in ML, linear algebra, scientific computing

More PL:

» Take Professor Xi's compilers course next semester

» Learn Haskell, Elm, Scala, Rust

More Math/Type Theory:

» Go learn about logic with Professor Das next semester!

» Category theory (functors, monads, comonads), Logic, Type theory

» (I do a lot of independent studies)

More Computers:

» Compilers, Linkers, LLVM

» Formal verification

» embedded systems programming, tensor program compilation

fin

(thanks everyone)