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Part A

A.1 Linear programming model

In order to define the linear programming model, we first need to define the sets, parameters and variables we need. The notation for the sets, parameters and decision variables are listed below. To see the relevant data for each set and parameter, please consult the file “PartA.dat” in folder “PartA”.

Sets:
$f \in F\text{: } Set\ of\ fish\ processing\ facilities$
$p \in P\text{: } Set\ of\ pollutants$
Parameters:
$m_p\text{: } Minimum\ pollution\ reduction\ requirement\ of\ pollutant\ p\ set\ by\ the\ government$
$c_f\text{: } Cost\ of\ processing\ one\ ton\ fish\ with\ the\ new\ technology\ at\ facility\ f$
$r_{p,f}\text{: } Reduction\ of\ pollutant\ p\ per\ ton\ fish\ produced\ at\ facility\ f$
Variables:
$x_f\text{: } quantity\ of\ fish\ in\ tons\ produced\ at\ facility\ f$

We have now defined the decision variables, sets and parameters we need to formulate the model mathematically. We start by defining the objective function whose objective is to minimize the total processing costs. The total processing costs can be calculated by multiplying the quantity (in tons) of fish produced at each facility with the processing cost (per ton) at each facility and adding together the products. Mathematically, this can be written as:

$$Min\ TC = \sum_{f \in F} x_f c_f$$

We are also informed that the reduction of pollutant p at facility f is directly proportional to tons of fish produced at facility f. Therefore, by summing the products of the reduction of pollutant p per ton fish produced at facility f and quantity of fish produced at facility f, we can find the total amount reduced of pollutant p. This sum must be greater or equal to the requirement set by the government. Mathematically this can be written as:

$$\sum_{f \in F} x_f r_{p,f} \geq m_p \quad \forall p \in P$$

Finally, we assume that there is no upper limit to how much can be produced as well as stating a non-negative condition. This is done as there is no real interpretational value to producing a negative amount of fish.

The model is summarized below:

Objective function:

$$Min\ TC = \sum_{f \in F} x_f c_f$$

Constraints:

Requirement for pollutant reduction:

$$(1.1)\ \sum_{f \in F} x_f p_{p,f} \geq m_p \quad \forall p \in P$$

Non-negativity condition:

$$(1.2)\ x_f \geq 0 \quad \forall f \in F$$

The implementation of the model in AMPL can be found by opening the file "PartA.mod" in folder "PartA". In the same folder the output relevant for problems 1-4 can be found in file "PartA_solution". This output is shown in Figure 1:

```
total_cost = 2730.77

x [*] :=
F1   11.5385
F2  119.231
F3    0
;

:  restriction.body restriction.ub restriction.slack    :=
P1        25          Infinity           0
P2        35          Infinity           0
;

:  restriction restriction.down restriction.current restriction.up    :=
P1     23.0769        7.77778             25                28
P2     61.5385       31.25                35               112.5
;

:    x.down  x.current      x.up       :=
F1    10        30         36
F2    16.6667   20         25.3333
F3    27.6923   40       1e+20
;

x.rc [*] :=
F1    0
F2    0
F3  12.3077
;
```

Figure 1 - Optimization output from AMPL for Part A

The optimal solution is to process approximately 11.54 tons of fish at facility 1 and 119.23 tons at facility 2. Facility 3 should not process any fish. The total cost (objective value) at these production levels is $2,730.77.

A.2 Discussion on constraints

There is no slack for the constraints regarding minimum pollution reduction. In other words, the reduction in pollution is equal to the minimum targets set by the government of 25 tons for P1 and 35 tons for P2. According to this model, the only way to reduce pollution is to increase costs. Therefore, it is logical that there will be no more processing than necessary if cost minimization is the objective. We can also note that the non-negative condition is satisfied as there are no negative amounts of fish being produced at any facility.

The shadow price shows the change in cost if the targets change by 1 ton, ceteris paribus. The shadow prices are approximately 23.08 for P1 and 61.54 for P2 (as can be seen in line 15 and 16 of Figure 1). This applies for pollution levels within the allowable range, which is between 7.78 and 28 tons for P1 and between 31.25 and 112.5 tons for P2.

A.3 Sensitivity analysis with changes in pollution reduction targets

Since a P2 value of 25 is outside the allowable range for P2, we cannot conclude what the effect on the optimal cost is without running the model again. A P2 value of 70 is however within the allowable range, so we can calculate the effect in the optimal cost in the following way:

$$\Delta TC = (70 - 35) * 61.5385 = 2153.85$$

If the target of pollutant 2 increases to 70, the optimal total cost will increase by $2,153.85. Note that this does not guarantee the same optimal solution for the decision variables.

A.4 Sensitivity analysis with changes in cost coefficients

The allowable range for the cost coefficients are [10, 36] for F1, [16.67, 25.33] for F2 and [27.69, ∞] for F2. If the cost of processing at facility 1 decrease to $20, there will be no changes to the optimal solution since 20 is within the allowable range for F1. The total cost would however change by:

$$\Delta TC = 11.5385 * (20 - 30) = -115.39$$

The optimal cost would decrease by $115.39 if the cost of processing at Facility 1 decreased from $30 to $20.

Part B

B.1 Linear programming model

For the model we define the following sets, parameters and decision variables:

Sets:

$c \in C: Set\ of\ crude\ oil\ types$

$g \in G: Set\ of\ gasoline\ types$

$m \in M: Set\ of\ markets$

Parameters:

$p_{g,m}^{Gasoline} = Price\ per\ barrel\ of\ gasoline\ type\ g\ sold\ to\ market\ m$

$p_c^{Crude} = Purchase\ price\ per\ barrel\ of\ crude\ oil\ type\ c$

$PC_g = Production\ cost\ of\ one\ barrel\ of\ gasoline\ type\ g$

$b_{g,m} = Minimum\ demand\ for\ gasoline\ type\ g\ that\ must\ be\ satisfied\ in\ market\ m\ per\ week$

$a_c = Maximum\ weekly\ purchase\ amount\ of\ crude\ oil\ type\ c$

$t_g = Required\ hours\ of\ supervision\ for\ producing\ one\ barrel\ of\ gasoline\ type\ g\ per\ week$

$Cap = Maximum\ available\ hours\ for\ supervision\ per\ week$

$r_c^{Crude} = Octane\ rating\ for\ crude\ oil\ type\ c$

$s_c^{Crude} = Sulfur\ content\ in\ crude\ oil\ type\ c$

$r_g^{Gasoline} = Minimum\ octane\ rating\ required\ for\ gasoline\ type\ g$

$s_g^{Gasoline} = Maximum\ sulfur\ content\ allowed\ for\ gasoline\ type\ g$

Decision variables:

$q_{c,g}^{Crude} = Barrels\ of\ crude\ oil\ type\ c\ used\ to\ produce\ gasoline\ type\ g\ per\ week$

$q_{g,m}^{Gasoline} = Barrels\ of\ gasoline\ type\ g\ sold\ to\ market\ m\ per\ week$

Since there is unlimited demand for the gasoline, it is assumed that all produced gasoline is sold, thus the variable $q_{g,m}^{Gasoline}$ shows the quantity produced and sold. It is also assumed that there can be sold fractional barrels. Based on the linearity of yield assumption, it is further assumed that the sum of barrels of crude bought must equal the sum of barrels of gasoline produced and sold. Finally, we assume that practical capacity equals theoretical capacity meaning that all available hours for supervision per week can be fully exploited. This entails that total hours available for supervision is equal to 12*40+ 20*20 = 520.

The profit is equal to the revenue generated from selling the gasoline minus the production costs and purchase cost of crude oil. Mathematically, the objective function can be written in the following manner:

$$Max\ Profit = \sum_{g\in G}\sum_{m\in M} q_{g,m}^{Gasoline} p_{g,m}^{Gasoline} - \sum_{c\in C} q_c^{Crude} p_c^{Crude} - \sum_{g\in G} q_g^{Gasoline} PC_g$$

Constraints:

Maximum weekly crude oil purchase:

(2.1) $\quad \sum_{g\in G} q_{c,g}^{Crude} \leq a_c \qquad \forall c \in C$

Maximum hours of supervision available:

(2.2) $\sum_{c \in C} \sum_{g \in G} q_{c,g}^{Crude} t_g \leq Cap$

Barrels of crude bought must equal barrels of gasoline produced and sold:

(2.3) $\sum_{c \in C} q_{c,g}^{Crude} = \sum_{m \in M} q_{g,m}^{Gasoline} \quad \forall g \in G$

Demand for each gasoline g must be satisfied in all markets m:

(2.4) $q_{g,m}^{Gasoline} \geq b_{g,m} \; \forall g \in G,\; m \in M$

Octane rating must be above the minimum rating for all gasoline types:

(2.5) $\sum_{c \in C} q_{c,g}^{Crude} r_c^{Crude} \geq \sum_{m \in M} q_{g,m}^{Gasoline} r_g^{Gasoline} \quad \forall g \in G$

Sulfur levels must be below the maximum allowance for all gasoline types:

(2.6) $\sum_{c \in C} q_{c,g}^{Crude} s_c^{Crude} \leq \sum_{m \in M} q_{g,m}^{Gasoline} s_g^{Gasoline} \quad \forall g \in G$

Non-negativity condition

(2.7) $q_{g,m}^{Gasoline}, q_{c,g}^{Crude} \geq 0 \quad \forall g \in G,\; m \in M,\; c \in C$

The implementation of the model in AMPL can be found by opening the file "PartB.mod" in folder "PartB". The solution is found in file "PartB_solution", but is also presented in a transposed format in the report to make it more readable. The model yields the following optimal weekly blending plan:

Optimal Weekly Blending Plan

Crude/Gasoline type	G1	G2	G3	G4	G5	Sum
C1	125	4,625	8,250	5,714.29	1,542.86	**20,257.15**
C2	10,375	4,625	0	0.00	0.00	**15,000.00**
C3	0	0	0	0.00	1,542.86	**1,542.86**
C4	0	0	0	2,285.71	914.28	**3,199.99**
Sum	**10,500**	**9,250**	**8,250**	**8,000.00**	**4,000.00**	**40,000.00**

Table 1 - Weekly blending plan of crude oil c to produce gasoline g, values in number of barrels

The corresponding distribution of the produced gasoline types is summarized below:

Optimal Gasoline Distribution to Market

Market/Gasoline type	G1	G2	G3	G4	G5	Sum
M1	3,000	3,000	1,500	2,000	1,000	**10,500**
M2	2,500	2,000	1,000	2,000	1,000	**8,500**
M3	5,000	4,250	5,750	4,000	2,000	**21,000**
Sum	**10,500**	**9,250**	**8,250**	**8,000**	**4,000**	**40,000**

Table 2 - Weekly distribution plan of gasoline g to market m, values in number of barrels

The optimal solution leads to a profit of $1,371,570 (rounded).

B.2 Scenario analysis

B.2.1 Scenario 1

For Scenario 1 we only make modifications to the datafile where we increase the price of G1 in M1 from 75 to 78 and decrease the demand requirement for G1 in M1 from 3000 to 2700. The changes are illustrated in Figure 2:

```
param rev :
     G1     G2     G3     G4     G5:=
M1   75     75     75     85     90
M2   75     80     85     90     92
M3   80     85     90     95     95
;
```

```
param demand :
     G1     G2     G3     G4     G5:=
M1   3000   3000   1500   2000   1000
M2   2500   2000   1000   2000   1000
M3   5000   4250   3000   4000   2000
;
```

```
param rev :
     G1     G2     G3     G4     G5:=
M1   78     75     75     85     90
M2   75     80     85     90     92
M3   80     85     90     95     95
```

```
param demand :
     G1     G2     G3     G4     G5:=
M1   2700   3000   1500   2000   1000
M2   2500   2000   1000   2000   1000
M3   5000   4250   3000   4000   2000
;
```

Figure 2 - Changes made to data file in Part B for Scenario 1

The resulting changes from running Scenario 1 can be briefly summarized as:

- Profit increases by $12,900 to $1,384,470
- C1 is no longer used to produce G1 (-125 barrels)
    - There is a corresponding increase in C1 used to produce G3 (+125 barrels)
- Decrease in C2 used to produce G1 (-175 barrels)
    - There is a corresponding increase in C2 used to produce G3 (+175 barrels)

- The decrease in production of G1 of (175+125 = 300) is reflected in a decrease in the amount of G1 going to market M1
- The increase in production of G3 of (175+125 = 300) is reflected in an increase in the amount of G3 going to market M3

Although the price of G1 in market M1 increases to $78, the margin for producing and selling G3 in market M3 is still greater than the margin of selling G1 in market M1. The relaxation of the minimum demand quantity of G1 to M1 is therefore used to increase sales of G3 in M3. This can also be seen by looking at the shadow price. The shadow price for the demand requirement for G1 in M1 is -16. That means that if we enforce an increase from 3000 to 3001, we will decrease the profit by 16 and if we relax the restriction to 2999 the profit would increase by 16. The increase/decrease of 16 would be a result of producing one more/less barrel of G3 and selling it in M3. We can state this because only G3 to M3 has a non-negative shadow price and a non-zero slack value. Note that this is only true within the interval [2875, 5750] in the original model. The argument still holds for a relaxation to values below 2875 as we can see from the sensitivity analysis of the demand requirement restriction in file "PartB_S1_solution". A full decomposition of the changes in profit can be found in Table 3.

		Cost of crude	Cost of production	Sales price	Margin	Change in barrels sold	Effect on total profit
C1, G1	G1, M1	45	6	75	24	-125	-3000
C2, G1		40	6	75	29	-175	-5075
C2, G3	G3, M3	40	5	90	45	175	7875
C1, G3		45	5	90	40	125	5000
Sum						0	4800
Effect of price increase							8100*
Total							**12900**

*2700x(78-75)

Table 3 - Decomposition of the changes made to total profit in scenario 1

From Table 3 we see that about 63% of the change in total profit is due to the price increase of G1 in M1, whereas about 37% of the change is due to more cost-efficient production and increased sales of the most profitable combination (G3 to M3).

B.2.2 Scenario 2

We implement the changes in the minimum demand requirement parameter (see Figure 3) and run our model again. The solution can be found in file "PartB_S2_solution".

```
param demand :
     G1      G2      G3      G4      G5:=
M1   3000    3000    1500    2000    1000
M2   2500    2000    1000    2000    1000
M3   5000    4250    3000    4000    2000
;
param demand :
     G1      G2      G3      G4      G5:=
M1   3100    3000    1500    2000    1000
M2   2400    2000    1000    2000    1000
M3   5000    4250    4000    4000    2000
;
```

Figure 3 - Changes made to data file in Part B for Scenario 2

Since the production of gasoline G3 to market M3 already were higher than 4000 barrels, this constraint restriction won´t change the output, ceteris paribus.

Due to the minimum demand requirements, the capacity released by the relaxation of the decrease in demand for G1 going to M2 by 100 barrels is used to satisfy the 100-barrel increase in the demand requirement for G1 going to M1. There are no differences in which crudes that are used, as the cheapest way of producing these 100 barrels of G1 does not change by this change in demand. Since the price of G1 is $75 in both these markets, the change in output does not affect the total profit which is still $1,371,570.

B.2.3 Scenario 3

The restriction on the maximum barrel constraint for C3 and C4 leads to an infeasible model. This occurs because C3 and C4 have the highest octane ratings and lowest sulfur content, which make them crucial for blending gasolines that require high octane ratings and/or low sulfur content, such as G4 and G5. Restricting these constraints leads to an infeasible model, since there can´t be produced enough gasoline to fulfil the minimum demand requirements in all markets.

```
param cap :=
C1 25000
C2 15000
C3 1800
C4 3200
;
param cap :=
C1 25000
C2 15000
C3 1350
C4 2050
;
```

Figure 4 - Changes made to data file in Part B for Scenario 3

B.2.4 Scenario 4

We implement this extra crude oil source by adding a new variable of crude oil C2_new that possess the same properties as C2, but for which the price and available quantity is different (see data file "PartB_S4.dat"). Running the model again shows us that the profit is $1,378,670 which is an increase of $7,100 from the original profit of $1,371,570. The full solution can be found in file "PartB_S4_solution". This increase is not due an increase in number of barrels sold, as the total production does not change. It is rather due to a change in the composition that makes up the different gasolines. More specifically there is a change in which crude oils are used to produce gasoline 1 and 3. We see that we are shifting production from the more expensive crude oil C1 to the less expensive crude oil C2, see Table 4.

	Cost of crude	Production cost	Total cost	Change in barrels sold	Change in cost
C1, G1	45	6	51	-125	-6375
C1, G3	45	5	50	-1650	-82500
C2, G1	40	6	46	-1650	-75900
C2, G3	40	5	45	1650	74250
C2_new, G1	41	6	47	1775	83425
Total				**0**	**-7100**

Table 4 - Decomposition of the changes made to total profit in scenario 4

Part C:

C.1 Linear programming model

As in part A and B we start by defining sets, parameters and decision variables which are needed to define the linear programming model:

Sets:

$r \in R$: *Set of regions*
$p \in P$: *Set of ports*
$k \in K$: *Set of markets*

Parameters:

$\mathrm{d_k}$ = Demand of apples in market K (in 1000kgs)
$\mathrm{c}_{\mathrm{r,p}}^{ROUTE1}$ = Cost of shipping per 1000kg of apples from region r to port p
$\mathrm{c}_{\mathrm{p,k}}^{ROUTE2}$ = Cost of shipping per 1000kg of apples from port p to market k
$\mathrm{c}_{\mathrm{r,k}}^{ROUTE3}$ = Cost of shipping per 1000kg of apples directly from region r to market k
$\mathrm{s_r}$ = Supply limit 1000kg for region r (in 1000kgs)

Variables:

$\mathrm{x_{r,p}}$ = Apples shipped from region r to port p (in 1000kgs)
$\mathrm{y_{p,k}}$ = Apples shipped from port p to market k (in 1000kgs)
$\mathrm{z_{r,k}}$ = Apples shipped from region r to market k (in 1000kgs)

To find the optimal solution we want to minimize the total transportation cost. Mathematically we can define this cost as:

Objective function:

$$Min\ Total\ Transport\ Cost = \sum_{r\in R}\sum_{p\in P} c_{r,p}^{ROUTE1} x_{r,p} + \sum_{p\in P}\sum_{k\in K} c_{p,k}^{ROUTE2} y_{p,k} + \sum_{r\in R}\sum_{k\in K} c_{r,k}^{ROUTE3} z_{r,k}$$

Constraints:

Everything that exits port p must equal everything that enters port p (no storing capacity):

(3.1) $\sum_{r\in R} x_{r,p} = \sum_{k\in K} \mathrm{y_{p,k}} \quad \forall p \in P$

The demand must be met in all the markets:

(3.2) $\sum_{p\in P} \mathrm{y_{p,k}} + \sum_{r\in R} \mathrm{z_{r,k}} \geq \mathrm{d_k} \ \ \forall k \in K$

The total amount of apples leaving region r cannot be greater than the supply limit:

(3.3) $\sum_{p\in P} x_{r,p} + \sum_{k\in K} \mathrm{z_{r,k}} \leq \mathrm{s_r} \ \ \forall r \in R$

Non-negativity condition:

(3.4) $x_{r,p}, \mathrm{y_{p,k}}, \mathrm{z_{r,k}} \geq 0 \ \forall r \in R,\ p \in P,\ k \in K$

For all tasks in Part C, please see the corresponding data-, mod-, run- and solution files in folder "PartC" for more detailed information regarding implementation. We enter the data and solve the model in AMPL, which returns the following optimal shipping plan:

Optimal Shipping Plan

From/To	P1	P2	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10
P1	-	-	24	30	0	0	0	52	0	0	20	0
P2	-	-	0	0	40	0	15	0	42	12	0	0
R1	110	0	0	0	0	0	0	0	0	0	0	40
R2	16	109	0	0	0	35	0	0	0	0	0	0

Table 5 - Optimal Shipping Plan of apples expressed in 1000 kg

The value of the objective function corresponding to this shipment is $10,760.

C.2 Evaluation of renovation decision for P1

As we have a minimization problem and the optimal solution contains shipments to P1, we should expect the cost to increase if we put further restrictions on the model by removing the possibility of shipping to P1.

We remove P1 and run the model again to determine the optimal shipping plan which can be seen in Table 6:

Optimal Shipping Plan without Port P1

From/To	P1	P2	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10
P1	-	-	-	-	-	-	-	-	-	-	-	-
P2	-	-	0	30	40	0	15	52	42	12	0	0
R1	-	26	24	0	0	0	0	0	0	0	20	40
R2	-	165	0	0	0	35	0	0	0	0	0	0

Table 6 - Optimal Shipping Plan of apples without Port P1 expressed in 1000 kg

As expected, we find that the cost increases by $18,438-$10,760 = $7,678.

C.3 Decrease in weekly apple supply

By changing the weekly supply of apples to 125 in region R1 and 175 in region R2 we cannot run the model because there is no feasible solution. This can also be seen easily by comparing the total available supply to the total minimum demand requirement for the markets. When the weekly supply drops to 125 and 175 for region 1 and 2 respectively while the demand remains unaffected, the available supply is not sufficient to cover the demand. Given the available supply and demand this equates to:

$$\sum_{r \in R} s_r < \sum_{k \in K} d_k$$

Where the sum of supply is 300 and the sum of demand is 310. Given that all demand must be met in the model. There is no feasible solution.

In order to implement the proposal from the manager to define a maximum level of unsatisfied demand, we include a new parameter b_k. We define this parameter as 1 minus the maximum percentage of unsatisfied demand for market k. Subsequently we multiply it with the demand parameter in the requirement restriction. This reduces the minimum requirement for each market by the maximum percentage of unsatisfied demand. For instance, the sum of shipping to market K9 must now at least accumulate to 20*(1-0.2) = 16. For the AMPL implementation we have to make some modifications to our model. Mathematically we change our model by modifying equation 3.2 (see also file “PartC_task3.mod” in folder “PartC”):

$$(3.2)^* \quad \sum_{p \in P} \mathrm{y}_{\mathrm{p,k}} + \sum_{r \in R} \mathrm{z}_{\mathrm{r,k}} \geq \mathrm{d}_{\mathrm{k}} * b_k \ \ \forall k \in K$$

The optimal shipping plan under these modifications are shown in Table 7:

Optimal Shipping Plan with Decreased Supply

From/To	P1	P2	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10
P1	-	-	21.6	27	0	0.0	0	49.4	0.0	0.0	16	0
P2	-	-	0.0	0	38	0.0	12	0.0	39.9	9.6	0	0
R1	87	0	0.0	0	0	0.0	0	0.0	0.0	0.0	0	38
R2	27	99.5	0.0	0	0	31.5	0	0.0	0.0	0.0	0	0

Table 7 - Optimal Shipping Plan of apples with reduced supply and the manager's proposal expressed in 1000 kg

The resulting optimal value for the objective function is $9,894.6 which is expectedly lower as this implementation in practice is a relaxation of restrictions that have slack values of 0 (see line 56-65 in file “PartC_solution).

We should note that as we are dealing with a model that minimizes costs and each shipment represents a cost, the model will by definition make as few shipments as possible within the boundaries of the constraints. While the manager’s proposal entails satisfying the maximum levels of unsatisfied demand, it does not ensure that all available supply is shipped. The optimal shipping plan suggests using only 283 000kg out of the 300 000kg available. This is because the lower limit of the manager's constraint results in a minimum total demand of 283 000kg (see Table 8).

Market	Original demand	Maximum % of unsatisfied demand	Minimum Demand
K1	24	10%	21.6
K2	30	10%	27
K3	40	5%	38
K4	35	10%	31.5
K5	15	20%	12
K6	52	5%	49.4
K7	42	5%	39.9
K8	12	20%	9.6
K9	20	20%	16
K10	40	5%	38
Total	310		283

Table 8 - Manager's effect on total minimum demand requirement, values in 1000 kg of apples

Given that they with "normal" supply levels would satisfy the original demand levels, we can assume that it would be beneficial to satisfy as much demand as possible.
The manager should therefore also have stated a constraint that ensured the exhaustion of all available supply or defined the maximum percentage of unsatisfied demand for each market such that the total of the minimum demand requirement equaled total supply available.

Part D

D.1 Discussion on infeasibility and unboundedness of model

a)

This model can be infeasible if the data provided is not able to satisfy all the four constraints simultaneously. The model would for instance be infeasible if demand (d_{ijt}) was negative for any i in I, j in J and t in T, which would make it impossible to satisfy both constraint (3) and (5). Another example could be that the total demand for a group assortment l at the demand points at time t, can't be satisfied by the sum of available supply of assortments k in K_l at time t. This would make it impossible to satisfy constraint (3) and (2) as the flow could not be less than the available supply while at the same time equal to the required demand.

b)

As we have a minimization problem, we need to address the different parameters and variables of the objective function and the conditions that they are subject to, in order to say whether or not the model can be unbounded. The objective function states that we should minimize the total cost of the transportation plan (min C = $\sum_{i\epsilon I}\sum_{j\epsilon J}\sum_{k\epsilon K}\sum_{t\epsilon T} c_{ijkt}x_{ijkt}$).

For the model to be unbounded we would need to be able to improve this objective function without reaching a limit (make it infinitely small while still satisfying all constraints). For the decision variable x_{ijkt} we know from the non-negative condition stated in constraint (5) of the paper that this variable is limited to being strictly positive. We are also informed that c_{ijkt} is positive for all i ∈ I, j ∈ J, k ∈ K, t ∈ T, meaning that the product of c_{ijkt} and $x_{ijkt}$is limited to being non-negative for all i ∈ I, j ∈ J, k ∈ K, t ∈ T. Thus, as we have a minimization problem the model cannot be unbounded as we cannot make the objective function infinitely small.

D.2 Aggregated demand and supply calculation

a)

Under the assumption that all assortments in K also belong to a group assortment in L, we know on an aggregate level that $\sum_{i\epsilon I}\sum_{j\epsilon J}\sum_{k\epsilon K_l}\sum_{t\epsilon T} x_{ijkt} = \sum_{j\epsilon J}\sum_{l\epsilon L}\sum_{t\epsilon T} d_{jlt}$ i.e. the total flow to the demand points, must equal the total aggregated demand.

Knowing that $c_{ijkt} = 1$ for all i ∈ I, j ∈ J, k ∈ K, t ∈ T. We can make the following calculation to find the total aggregated demand:

234,560 = $\sum_{i\epsilon I}\sum_{j\epsilon J}\sum_{k\epsilon K}\sum_{t\epsilon T} c_{ijkt}x_{ijkt} \rightarrow$

234,560 = $\sum_{i\epsilon I}\sum_{j\epsilon J}\sum_{k\epsilon K}\sum_{t\epsilon T} 1 * x_{ijkt} \rightarrow$

$\sum_{i\epsilon I}\sum_{j\epsilon J}\sum_{k\epsilon K}\sum_{t\epsilon T} x_{ijkt} = \sum_{j\epsilon J}\sum_{l\epsilon L}\sum_{t\epsilon T} d_{jlt} = 234{,}560$

The total aggregated demand quantity is equal to 234,560 units.

b)

Unless we assume that the demand quantity is equal for each period, we cannot know the specific total demand quantity for period 6. Since it is likely that the demand for forest fuel will vary depending on conditions such as weather, this assumption is unlikely to hold. Therefore, we cannot conclude what the total demand quantity is for period 6 specifically.

c)
From constraint (2), we know that supply from supply point i must be equal to or greater than the total flow from point i. As a result, the aggregated supply must be equal to or greater than the aggregated total flow.

We also know that the starting inventory, $y_{i,k,0}$, is 100 units for all supply points and all assortments. However, we do not know how many supply points nor assortments there are. We also lack a value for the ending inventory for the supply points and assortments. We are therefore unable to use constraint (4) to calculate the total supply.

We can however create an expression for the total aggregated supply:

$$\sum_{i\epsilon I}\sum_{k\epsilon K}\sum_{t\epsilon T} s_{ikt} = \sum_{i\epsilon I}\sum_{k\epsilon K} y_{ik,12} - \sum_{i\epsilon I}\sum_{k\epsilon K} y_{ik,0} + \sum_{i\epsilon I}\sum_{j\epsilon J}\sum_{k\epsilon K}\sum_{t\epsilon T} x_{ijkt}$$

Because total aggregated supply must be equal to or greater than the aggregated flow, we have that: $\sum_{i\epsilon I}\sum_{k\epsilon K} y_{ik,12} \geq \sum_{i\epsilon I}\sum_{k\epsilon K} y_{ik,0}$. In the case where aggregated starting and ending inventory are equal, the aggregated supply availability should equal the total flow and thereby the total demand quantity (234.560 units). If aggregated ending inventory is greater than aggregated starting inventory, then the aggregated supply availability should be greater than the total flow and thereby the total demand quantity (234.560 units).

We can therefore conclude that total aggregate supply must be equal to or larger than 234,560 units, but more information is needed to calculate the exact number.

d)
We can't calculate the aggregated supply in period 6 because we don't know the exact total aggregated supply. Even if we knew the total aggregated supply, we would be unable to calculate the aggregated supply in period 6 because the supply of forest fuel is likely to vary by month, as mentioned in D.3b).

D.3 New constraints on total flow in time period t

To capture this new condition, there would be added new constraints. The first condition state that the total flow exiting supply point i in time period t must be higher than 90% of the total flow from the same supply point at time period t-1. The second condition state that the total flow exiting supply point i in time period t must be lower than 110% of the total flow from the same supply point at time period t-1. This holds for all time periods where t is larger than zero. We write the constraints as:

$$\sum_{j \epsilon J} \sum_{k \epsilon K} x_{ijkt} > \sum_{j \,\epsilon\, J} \sum_{k \epsilon K} x_{ijkt-1} * 0.9 \quad \forall i \in I, t \in T: t > 0$$

$$\sum_{j \epsilon J} \sum_{k \epsilon K} x_{ijkt} < \sum_{j \,\epsilon\, J} \sum_{k \epsilon K} x_{ijkt-1} * 1.1 \quad \forall i \in I, t \in T: t > 0$$

Because we are imposing new constrictive constraints, the optimal value for the objective function (in a minimization problem) must either be greater than or equal to the optimal value of the objective function before adding the new constraint.