PROJECT 2
Integer Linear Programming

Submit your project through *WISEflow*. The submission deadline is Wednesday October 19th, at 14:00 hrs. The project can be done individually or in a group of at most 2 students. No cooperation between people who are not submitting this project as a group is allowed. It is possible to change groups throughout the semester and it is also possible to do some project(s) alone and other project(s) in a group. Provide **all your AMPL files** (model code, data, running commands, solution file, etc.) compressed in a single file (.zip). Include all files needed to run all parts of the project, even if from one to another task the changes are just marginal (we need all files to be able to run without modifying what you submitted). Include in the zip file also the source files of the plots required in Part D. In addition, provide a written report with your model formulations and the answers to the questions required in each part. The formulation of your models can be typed in a text editor (e.g. Word, LaTeX), written by hand and scanned, or copied directly as text or screenshot from the AMPL code files when it applies (please just be careful the presentation must be clear enough for a reader). In the written report, it is fine that when there is just a marginal change from one task to another, in the latter you include just the modified part of the formulation (e.g., in task 2 you just defined a new variable or modified one constraint of the model you formulated in task 1, then it is fine that you included the full model formulation in task 1 and only the new variable definition and new constraint that you modified in task 2). Provide a short description (no more than two sentences, e.g. "#demand fulfillment") for every objective function and constraint in your formulations. All model formulations in this project, either involving continuous and/or integer variables, must be **linear**. Only as a reference (not as a requirement), the expected length of your report is: Part A half of a page, Part B half of a page, Part C three pages, Part D four pages.

Part A (10%)

The Karlsen Family consists of four persons (mother, father, daughter, son). All of them enjoy playing an augmented reality mobile game called *Chessmon GO*, where virtual chess pieces with fancy designs are hidden around locations on a digital map which coincides with the map of the real-world location of the players. To collect a chess piece, players must stop with their mobile phones at the corresponding physical location of the piece and pronounce the words "chess speaks for itself". While on vacations in a large and well-known city, the Karlsen Family would like to spend an afternoon collecting by walking all the *Chessmon GO* pieces hidden in an area called *Manjatan*. In total, there are 128 pieces to collect. The family has retrieved data from a website, with the location and distance it takes to travel between each of the chess pieces. Due to the time limitation and the large distances to walk, they estimate that each of them can collect at most 50 pieces during the afternoon. They also set as a target that each of them should collect at least 25 pieces. All of them are to depart from the celebre *Timez Skuare* hotel and perform a *trip* independent from each other. A trip is a tour which starts at the hotel, then it departs to collect a group of chess pieces travelling from one to another piece's location, and at the end it comes back to the hotel. The location of the chess pieces do not coincide with each other, neither with the hotel location. The wish of the Karlsen Family is to collect the 128 pieces while minimizing the walking distance traversed in total by the family. The son, whose name is Magnuz, has carefully studied the model formulations to eight problems of lectures 7 and 8 of the BAN402 course at NHH, namely: fixed cost, knapsack, facility location, assignment, set partitioning, set covering, set packing, and TSP. He claims that by using two of these models, perhaps with an additional constraint in one of them, and solving the models in AMPL for perhaps many data instances, would help to devise an optimal plan of trips to each of the family members.

1. Using the model formulations of lectures 7 and 8, can you also identify a procedure to find the optimal plan of trips for the Karlsen Family? Similarly as Magnuz's plan, your procedure should use at most two model formulations of those lectures, perhaps with an additional constraint in one of them, and the models could be run more than once.
2. When The Karlsen Family members are about to depart to perform the optimal trips found by Magnuz, they meet with a national fellow called Jacob Ingebrigtzen at the reception of the hotel. When they tell

him about their plans, he tells them he is eventually also about to start a trip to collect the *Chessmon GO* pieces in *Manjatan*. He is a top runner, so he is confident that during the afternoon he will manage to collect all the 128 pieces on his own. Suppose that Jacob has computed the trip that minimizes the total distance he needs to travel to achieve such a mammoth task. He claims that he will for sure travel less distance than the total distance travelled by the Karlsen Family. Briefly discuss wether Jacob's claim is true or not.

Part B (20%)

In the article "Practice Summary: Solving the External Candidates Exam Schedule in Norway" (available on a link in the *Complementary readings* folder in *Canvas*), the authors describe the use of a decision model to plan exam sessions in the Vestfold region of Norway. The problem setting and decision variables are explained in Section 2 of the article, and the mathematical formulation of the model is outlined in the Appendix. As an expert in decision modelling, you have been asked to modify the model to address some new situations. Your formulations must be linear and may involve new definitions (e.g. of variables), new expressions (e.g. in the objective function and/or constraints), the modification of some expressions in the original formulation, etc. These must be formulated in mathematical terms (not in AMPL code). The new situations are described below (each situation is independent from each other).

1. Suppose that there is a candidate and a professor who are relatives. We refer to them as candidate $c1$ and professor $p1$, respectively ($c1 \in C, p1 \in P$). To avoid any skepticism, they have asked the county authority to not schedule any of their activities at the same space-time slot. This means that the exams taken by candidate $c1$ cannot be carried out in a space-time slot to which professor $p1$ is assigned (and equivalently, professor $p1$ cannot be assigned to a space-time slot where candidate $c1$ takes an exam). Which modification(s) would you introduce in the model to address this new scenario?

2. The model in the Appendix secures that every candidate does not undergo two exams in the same day or in two adjacent days. This is nice, but considering that exams take a lot of preparation and effort from the candidates, we may try to conceive schedules that secure candidates to have not only one but two resting days in between exams. Imposing this condition, however, might be too restrictive risking the model to become infeasible. Thus, we try to find a mild approach here, requiring a couple of more relaxed conditions. Let us say that a *bad-luck sequence* for candidate c occurs whenever two exams of candidate c are scheduled with only one resting day between them (that is, there exist a day d in D such that candidate c undergoes exams on day d and $d + 2$). Note that with the current model many candidates might have many bad-luck sequences in the schedule. Thus, we would like impose two new conditions:

   - each candidate should have at most two bad-luck sequences;
   - at least 60% of the candidates have no bad-luck sequences.

   Which modification(s) would you introduce in the model to capture these conditions?

Part C (30%)

The company PetroBAN refines two types of crude oil. The crude oil is converted to components in the Crude Distilling Units (CDUs). Different setup or characteristics in the process define different *running modes* at which the CDUs can work. The crude oil which goes through a CDU at a given running mode will provide proportions of different components. This is illustrated with a simple numerical example in Figure 1. Here one unit of crude oil 1 will separate into 60% of component 1 and 40% of component 2 if it is processed in CDU 1 at running mode *LowMode*. The corresponding proportions will be 70% and 30% if it is processed in CDU 2 at running mode *HighMode*. Each crude oil and running mode will give different proportions of components.

In general, we will refer by $R_{i,b,j,m}$ to the amount of component b obtained from refining one unit of crude oil i in CDU j at running mode m.

The company has two CDUs, both located at the same refining department. Each CDU can be set in any of the three running modes: *LowMode*, *HighMode*, *Shutdown*. One running mode per day must be chosen for each CDU. If the mode used on day t is different than the mode used on day $t-1$, a cost *Cchange* is incurred due to the change of running mode. We will assume that right before the first

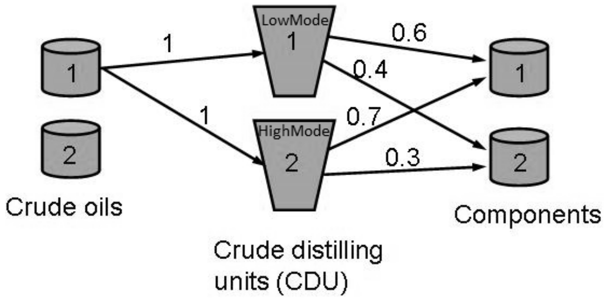

Figure 1: A numerical example of the CDU process.

day of our planning horizon, the mode of both CDUs is *Shutdown*. There is a fixed cost $Cmode_{j,m}$ per day incurred because of the operation of CDU j in mode m (which is positive for modes *LowMode* and *HighMode*, and zero for mode *Shutdown*).

The processing capacity of CDU j in running mode m is $Cap_{j,m}$ units of crude oil in total per day. The cost of refining one unit of crude oil i in CDU j in mode m is $Cref_{i,j,m}$, while the cost of purchasing one unit of crude oil i on day t is $Ccrude_{i,t}$. The crude oil is ready to be used at the CDUs within the same day of purchase or, alternatively, it can be stored at the refining department.

The components generated can either be stored in tanks located at the same refining department or sent to the blending department of the company. Assume that the components sent from the refining department on day t will be used at the blending department on day $t+1$. The cost of transporting one unit of any component from the refining to the blending department is $Ctra1$. It is not possible to store components at the blending department, so the amount of components sent from the refining department on one day must be the exact amount required for blending the following day.

When the components arrive at the blending department, they are mixed according to recipes for generating final products. The recipe for producing one unit of product p needs $N_{b,p}$ units of component b. The cost of producing one unit of product p is $Cprod_p$.

From the blending department, the products are sent to depots (there is no storage at the blending department). The cost of transporting one unit of any product from the blending department to depot d is $Ctra2_d$. Assume that what is produced during one day arrives to the depots at the beginning of the following day.

Once the products arrive at the depots, they are ready to be shipped to the markets. The cost of shipping one unit of any product from depot d to market k is $Ctra3_{d,k}$. Alternatively, the products may be stored at the depots.

There is a maximum demand limit for product p from market k in day t, which we will refer by $\delta_{p,k,t}$. Assume that what is shipped from the depots one day arrives to the markets the following day. Also, assume that it is possible to partly fulfil demand of a same market by supplying from different depots. The price of one unit of product p in all markets is S_p.

Note there is a component called *lowqc* which is not used in the blending department. Therefore, any unit obtained of this component is sold to a local buyer immediately after the refining process at the relatively low price of $110 per unit.

The cost of storing one unit of any type of crude oil at the refining department is $Cinvi$ per day. The cost of storing one unit of any type of component at the refining department is $Cinvb$ per day (except for *lowqc*, which is not stored). The cost of storing one unit of any type of product at depot d is $Cinvp_d$ per day. (Note all these inventory costs are incurred per any unit stored at the end of each day.)

Suppose there is initial inventory of saleable products at the depots (that is, at the end of period $t=0$), as given in the two columns under the header $Izero_{p,d}$ in Table 1. Assign value zero to any other initial inventory or initial flow variable that you may require in your implementation. In addition, the company would like to have at least some minimum quantity of components and products on inventory at the end of the planning horizon, in order to anticipate future demand. The minimum inventory of products for each depot at the end of the planning horizon is given in Table 1 under the header $Ifinal_{p,d}$. The final inventory for each of the components, except for *lowqc*, must be at least 80 units each. There is no minimum requirement for *lowqc*.

Figure 2 illustrates the different stages of PetroBAN's supply chain.

	$Izero_{p,d}$		$Ifinal_{p,d}$	
	D1	D2	D1	D2
premium	200	200	25	25
regular	480	550	50	50
distilF	128	197	20	30
super	300	235	15	25

Table 1: Initial inventory and minimum final inventory quantities of each product at each depot.

1. Formulate a mixed integer linear programming model for the multi-period planning problem of the company, including decisions on procurement, refining, production, transportation, storage and sales. The objective is to maximize the total profit over the planning horizon. Implement the model in AMPL and solve it using the solver ***cplex*** and the data instance contained in the file "Proj2C.dat" (you may modify it according to your own definitions). Note the set of time periods (expressed in days) is defined as $T = \{0, 1, 2, \ldots, 12\}$, although the relevant decisions are from periods 1 to 12.
(a) What is the optimal profit?
(b) Which running modes are selected for each of the CDUs throughout the different time periods? How much of the capacity of the CDUs is used on each time period?
(c) How much inventory of crude oils, components and final products is left at the end of the planning horizon? If there is any difference among them, briefly discuss why this occur.

2. Foreseeing inflation in the market of suppliers, PetroBAN is interested in analyzing three different scenarios on the cost of purchasing crude oil. These scenarios are built by increasing $Ccrude_{i,t}$ by 10%, 30%, and 50% for all crude i and day t. Modify your model formulation and AMPL implementation for these new scenarios and solve it using the solver ***cplex***. What is the optimal profit in each scenario and how does it compare to the one you obtained in task 1? When does the *Shutdown* mode is selected for each of the CDUs in these scenarios?

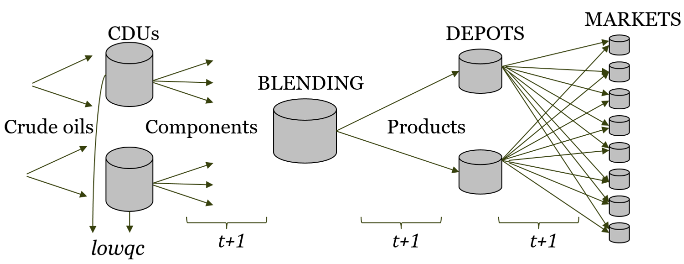

Figure 2: Illustration of the supply chain of PetroBAN.

Note: When solving mathematical programming models, it is interesting to observe how the algorithms of the solvers approach the optimal solution during the optimization process and some statistics on the dimension of the problem and the solution time. For this purpose, when using *cplex*, you can add in your .run file the following lines anywhere before the statement *solve*:
option solver cplex;
option cplex_options 'mipdisplay=4';
option show_stats 1;

Part D (40%)

The market of plug-in electric vehicles (EVs) has grown considerably over the past years. The growth is particularly high in Norway, the first country in the world where the sales of EVs became higher than the sales of other passenger car types. The share of EVs consistently reaches more than 80% of the new car registrations per month in Norway, and nowadays there are about 500,000 EVs on the Norwegian roads. As the market grows, concerns about the potential implications of EVs in the energy grid and high electricity prices have triggered new business models where digital platforms offer *smart charging* services to car owners. A basic principle behind smart charging is to *optimize* the budget of users, by scheduling the charging of EVs at the times where the electricity prices are relatively cheap. These times often coincide with lower congestion in the grid, thus the optimal solution for car owners help shifting load from peak to valley hours, which is also more convenient to the grid operators. This motivates us to address the case of *Optidriver*, an EV owner who lives and works in the Vestland county of Norway.

Optidriver owns an EV whose battery capacity is 64 kWh. The car is charged at home, by the use of a wallbox at rate 7.5 kW per hour. The car is available for charging at any moment of the week (Monday to Sunday), except on weekdays (Monday to Friday) between 7:00 and 17:00 hr, when *Optidriver* uses the car to go to work. The performance of the car on Vestland's roads depend on weather and other factors, but we will assume that the car drives from home to work from 7:00 to 8:00 AM and then from work to home from 16:00 to 17:00, consuming 5.5 kWh on each way. We first focus on a time horizon spanning three months from Friday 1st of July 2022 until Friday 30th September 2022. The data file "Proj2D.xlsx" contains the electricity price for each hour in this time horizon[1]. Consider that at the beginning of the time horizon (that is, right before 00:00 on 1st July), the state-of-charge of the EV battery is 51.2 kWh (equivalent to 80% of its capacity). Foreseeing future trips, by the end of the time horizon (that is, right before 00:00 on 1st October), the state-of-charge must also be 51.2 kWh. Moreover, *Optidriver* is a cautious person and does not want to suffer from *range anxiety*, thus the state-of-charge must never be below 12.8 kWh (equivalent to 20% of the battery capacity). Of course, the state-of-charge can never be above the 64 kWh capacity.

1. Formulate a linear programming model to find a schedule of charging that minimizes the cost for *Optidriver* during the three month interval, while satisfying the conditions above[2]. Implement and solve the model in AMPL, using the solver ***gurobi***. How much is the optimal cost? What is the average price incurred by the optimal solution (in EUR/MWh)? Just to illustrate the value of optimization in this case, compute what would be the cost if the model would maximize (instead of minimize) the total cost of charging during the three months. Calculate the difference between the two solutions in absolute terms (total cost in EUR), in relative terms (how much in percentage is the most expensive solution with respect to the cheapest solution), and in their average price of energy (EUR/MWh).

2. Now suppose that the car is also used on Sundays, as *Optidriver* likes to do some recreation activities around the city. The use of the car on Sundays is a bit more flexible regarding schedule, and it consumes only half of the energy it consumes on a weekday. Flexibility here means that every Sunday the car is used for recreation activities during an interval of four consecutive hours between 10:00 and 17:00 hr, starting at an o'clock time (*hh*:00) within this interval and arriving home at 17:00 hr at the latest. Half of the energy consumption means 2.25 kWh on the way to recreation and 2.25 kWh on the way back home. For example, it might be that on Sunday 10th July, the car is used for recreation from 12:00 to 16:00 (using 2.25 kWh from 12:00 to 13:00 and 2.25 kWh from 15:00 to 16:00), and on Sunday 17th July the car is used for recreation from 10:00 to 14:00 (using 2.25 kWh from 10:00 to 11:00 and 2.25 kWh from 13:00 to 14:00). In contrast, it may not be that a recreation interval starts at 15:00, because the interval of four consecutive hours would finish after 17:00 hr. During the interval of four hours of recreation, the car cannot be charging. Formulate a mixed integer linear programming model to find a schedule to minimize the cost of charging for this new situation, keeping also all conditions given above. Implement and solve the model in AMPL, using the solver ***gurobi***. How much is the optimal cost? What is the average price incurred by this optimal solution (in EUR/MWh)? In a software of your choice (e.g. Excel, R), illustrate your solution in a plot, where the horizontal axis represents the time and the vertical axis represents the amount charged on every hour of the time horizon. Include the plot in your pdf report (and the source files in your zipped folder). Optional: you may design a combo chart including a second vertical axis with the price per hour and plot the price series in addition to the amount charged per

[1] The prices are expressed in EUR/MWh, and correspond to the day-ahead prices published for the Bergen area in Nord Pool's website: https://www.nordpoolgroup.com/. To convert the prices to EUR/kWh we may divide these data by 1000. In practice, the prices for a given day are not know until 12:42 PM the day before, but we conduct here an analysis on past data. A quick inspection to these data reveal that prices may vary considerably throughout times and days, suggesting that an optimization approach is perhaps worthy.

[2] Note that during each hour where the car is available for charging, the amount charged can be any real number between 0 and 7.5. For example, it might happen that the state-of-charge of the car is 45.4 kWh at 2:59 AM on Tuesday 5th July, and then it charges 7.5 from 3:00 to 4:00, and 2.5 from 4:00 to 5:00, and does not charge more during that morning, so the state-of-charge when the car departs to work at 7:00 is 55.4 kWh. The corresponding cost is 1.6291 EUR and the average price is 162.9100 EUR/MWh. In the same situation, another solution might prescribe to charge 7.5 from 3:00 to 4:00, 7.5 from 4:00 to 5:00, 0 from 5:00 to 6:00, and 3.6 from 6:00 to 7:00, so the car is charged at the full capacity 64kWh when it departs to work on that morning. The corresponding cost in this case is 3.0247 EUR and the average price is 162.6184 EUR/MWh.

hour, which might be insightful to visualize how your optimal solution behave with respect to those prices.

In what follows, we keep all information from the previous tasks (including the condition on Sunday recreations), and we address two new tasks. These two tasks are independent from each other.

3. Suppose that every week *Optidriver* can choose one weekday to do home office. Since only a fraction of week 26 is involved in the time horizon, we will assume that home office cannot be chosen on the first day of the time horizon (Friday 1st July), but it does apply to all the remaining weeks. The home office day can vary from week to week (for example, home office can be chosen on Thursday 14th July and the following week on Monday 18th July). Note that home office allows to reduce the consumption of the car by one round-trip and it also opens opportunities to charge the car during one weekday from 7:00 to 17:00. Formulate a mixed integer linear programming model to find a schedule to minimize the cost of charging in this new situation. Implement and solve the model in AMPL, using the solver ***gurobi***. How much is the optimal cost? What is the average price incurred by this optimal solution (in EUR/MWh)? Is the car charged on some weekday(s) at some hour(s) between 7:00 and 17:00? If so, in which day(s) it occurs?

4. In this last task, we focus on a time horizon spanning eight weeks from Monday 1st August until Sunday 25th September (inclusive). Assume that the car was not charging between 23:00 and 23:59 on 31st July. Consider that at the beginning of the time horizon (that is, right before 00:00 on 1st August), the state-of-charge of the EV battery is 51.2 kWh and right before 00:00 on 26th September, the state-of-charge must also be 51.2 kWh. Concerned about the battery life, *Optidriver* would like to avoid too many "ON-OFF" charging sequences, that is, intervals of time with positive and zero charging hours alternating with each other (for example, on a same day charging 2.5 from 17:00 to 18:00, 0 from 18:00 to 19:00, 7.5 from 19:00 to 20:00, 0 from 20:00 to 21:00...). With this in mind, a new charging strategy incorporates some new aspects. First, the car cannot start charging if the state-of-charge is more than 50% of the battery capacity. When a charging event occurs, it must start in the beginning of a time period (that is, at a time *hh*:00), and it cannot stop charging before the state-of-charge reaches 80% of the battery capacity. Thus, the car will charge continuously at the maximum charging rate of 7.5 kW per hour during all time periods in a charging event, except possibly in the final time period of the event where the amount charged can be any real number between 0 and 7.5. Of course, the car must be available for charging during all the time periods in a charging event, including the final period.[3] Formulate a mixed integer linear programming model to find a schedule to minimize the cost of charging for this new situation. Implement and solve the model in AMPL, using the solver ***gurobi***. How much is the optimal cost? What is the average price incurred by the optimal solution (in EUR/MWh)? Same as in task 2, illustrate your solution in a plot, where the horizontal axis represents the time and the vertical axis represents the amount charged on every hour of the time horizon (you may again choose to include a second vertical axis with the price series). In addition, build another plot where the horizontal axis represents the time and the vertical axis represents the state-of-charge throughout the time horizon. Include the plots in your pdf report (and the source files in your zipped folder).

Note: Similarly as we noted in Part C for *cplex*, when using *gurobi*, you can add in your .run file the following lines anywhere before the statement *solve*:
option solver gurobi;
option gurobi_options 'outlev=1';
option show_stats 1;

[3] For example, it may happen that the state-of-charge is 25.6 (that is, 40% of the battery capacity) at 17:59 on Wednesday and the car is not charging, and it starts to charge at 18:00 at rate 7.5 kW per hour. Right before 19:00 the state-of-charge would be 33.1, which is only 51.72% of the battery capacity, so the car could not stop charging and it must continue charging at maximum rate for at least two more hours. After then, the charging event might possibly finish between 21:00 and 22:00, when it may charge anything between 3.1 and 7.5 (note 3.1 is because the 80% of battery capacity will be reached at this point); if it charges less than 7.5, then the charging event finished and nothing is charged from 22:00 to 23:00; if it charges 7.5, then from 22:00 to 23:00 anything from 0 to 7.5 can be charged, and so on. Note the 80% is just a minimum required, so possibly the car would keep charging until reaching 100% capacity or anything between 80% and 100% capacity.