PROJECT 3

Submit your project through *WISEflow*. The submission deadline is Wednesday November 9th, at 14:00 hrs. The project can be done individually or in a group of at most 2 students. No cooperation between people who are not submitting this project as a group is allowed. It is possible to change groups throughout the semester and it is also possible to do some project(s) alone and other project(s) in a group. Provide **all your AMPL files** (model code, data, running commands, solution file, etc.) and other files you have used in your computations (e.g. Excel file) compressed in a single file (.zip or .rar). Include all files needed to calculate and run all parts of the project, even if from one to another task the changes are just marginal (we need all files to be able to run without modifying what you submit). In addition, provide a written report in **pdf** with your model formulations and the answers to the questions required in each part. The formulation of your models in the report can be typed using a text editor (e.g. Word, LaTeX), written by hand and scanned, or copied directly as text or screenshot from the AMPL code files when it applies (please just be careful the presentation must be clear enough for a reader). In the written report, it is fine that when there is just a marginal change from one task to another, in the latter you include just the modified part of the formulation (e.g., in task 2 you just defined a new variable or modified one constraint of the model you formulated in task 1, then it is fine that you included the full model formulation in task 1 and only the new variable definition and new constraint that you modified in task 2). Provide a short description (no more than two sentences, e.g. "#demand fulfillment") for every objective function and constraint in your formulations. Expected (not required) length of your report: Part A two pages, Part B one page, Part C two pages, Part D one page.

Part A

The worldwide known band *Golden Mayden* has recently announced their next European tour, which luckily for their fans in Norway includes a concert date in Bergen. The organizers of the concert are considering different alternatives to price the tickets for the event, and being aware of your modelling skills they ask you to help them evaluating these alternatives.

We will assume that the capacity of the venue allows to sell up to 55,000 tickets. Using historical data, the organizers have been able to segment the market into two groups: *general* and *students*. The estimated demand for tickets from each of these segments has been modeled by a linear function of the price. The demand function for the general segment is $Q_g = 120000 - 3000p_g$, and the demand function for the student segment is $Q_s = 20000 - 1250p_s$, where Q_i is the demand quantity from segment i and p_i is the price of the ticket charged to segment i (the price is expressed in hundreds of Norwegian kroner).

The goal is to maximize the total revenue from selling tickets. Assume the decision variables in this problem are continuous.

1. Suppose that the first alternative under assessment consists of charging the same price to all attendees to the concert. In this alternative, even if the price is the same to both segments, the organizers would like to secure that at least 20% of the capacity of the venue is sold to students and that at least 20% of the capacity of the venue is sold to general public. Formulate a non-linear model to help the organizers maximizing total revenues in this case. Implement the model in AMPL and solve it using the solver **minos**. How much is the optimal revenue? How many tickets are sold to students and how many to general public?

2. In a second alternative, the organizers may charge different prices to different segments, but still require that each segment gets allocated at least 20% of the capacity of the venue. How would you modify your model and your code to address this new alternative? How much is the optimal revenue? How many tickets are sold to students and how many to general public?

3. Given the successful and long career of *Golden Mayden*, the organizers have identified a third segment of *senior* people, who have followed the band from the early years back in 1975. The demand from this segment can be modelled by the function $Q_r = 15000 - 1400p_r$. To include senior attendance, the organizers would like to assess the three alternatives below.

- Alternative 3a: The price for seniors and students must be the same, but the price for general public can differ from the price of the other two segments. The tickets allocated to general public must be at least 20% of the venue capacity, and the sum of the tickets allocated in total to seniors and students must be at least 20% of the venue capacity.
- Alternative 3b: The price can vary for all segments, the tickets allocated to general public must be at least 20% of the venue capacity, the tickets allocated to seniors must be at least 10% of the venue capacity, and the tickets allocated to students must be at least 10% of the venue capacity.
- Alternative 3c: The price can vary for all segments, but for every segment it has to be at least six hundred Norwegian kroner and it cannot happen that the price for one segment is more than two times the price for another segment. Each segment must be allocated at least 5% of the venue capacity.

How would you modify your model and your code to address these three alternatives (3a, 3b, 3c) and which one of them would you suggest to the organizers?

Part B

In the article "Menu Engineering for Continuing Care Senior Living Facilities with Captive Dining Patrons" (available on a link in the *Complementary readings* folder in *Canvas*), the authors describe the use of a decision model to optimize the combination of menu items to be included in the meals provided to senior population at continuing care facilities in USA. The Appendix A of the article outlines a model formulation. As an expert in decision modelling, you have been approached by Romeo and Juliet, two individuals eating at these facilities. They have asked you to modify the model to address some new situations. Your formulations must be linear and may involve new definitions (e.g. of variables), new expressions (e.g. in the objective function and/or constraints), the modification of some expressions in the original formulation, etc. These must be formulated in mathematical terms (not in AMPL code). The new situations are described below (each situation is independent from each other). Romeo and Juliet have exactly the same preferences and nutritional needs, so for all purposes we treat them below as a single individual or diet type, which we denote as p_1.

1. Romeo and Juliet have got a new medical requirement about a special nutrient, which we denote by k_1. The requirement states that in every set of three consecutive days during the time horizon, Romeo and Juliet must consume an amount of at least 250 units of nutrient k_1. Which modification(s) would you introduce in the model to capture this condition?

2. Romeo and Juliet love the item "pasta bolognese" as a main dish in their menu. They also love the item "vanilla ice cream" and the item "strawberry ice cream". They get particularly excited when any of these two ice cream items are served in the same meal as "pasta bolognese" and, moreover, if this happens on a Saturday the story is just superb and they watch a movie together. Thus, we define a *Happy Saturday* as a Saturday in which there exists a meal serving "pasta bolognese" together with either "vanilla ice cream" or "strawberry ice cream". Romeo and Juliet ask you to please make sure there are at least two *Happy Saturdays* in the planning horizon. Which modification(s) would you introduce in the model to capture this condition?

Part C

In this part we will study some of the key aspects of the Nord Pool day-ahead market. These are the basis behind system prices and two bid types: hourly bids and block bids. As a basis for the questions, the file model.mod is provided. This file is ready to run (so you don't have to create your own .run file). Since some Linux and Mac operative systems use the syntax *./minos* to call the solver, you might need to edit line 36 in file model.mod by adding the characters "./". The models implemented in this file attempt to maximize the social surplus on the electricity market. It uses two different descriptions of the supply and demand curves. The first description is based on the step functions obtained from the total volume demand (supply) given a price below (above) each distinct price of any bid. The other resembles the approach at Nord Pool by using linear approximations of the step functions to guarantee a unique intersection between the demand and supply, which defines the equilibrium price. The code also implements a procedure for block bids, which works as follows. First, the problem without block bids is solved. Then, the block bids are studied iteratively, where the price of each block bid is compared against the average system prices over the time periods included in each block bid. If there is more than one block bid that should be accepted (better block price than the average price) the one with lowest price is selected. Then, the volume of the selected block bid is assumed to be fixed and the model (without block bids) is run again. This procedure is repeated until no more block bids are accepted.

Day-ahead market without block bids

1. In the data file hour.dat there is data for five time periods or hours. This instance includes 20 supply bids and 20 demand bids for each period. Study the file and draw a figure of the supply and demand curves for one of these five periods[1]. Draw also the linearized curves. Based on your figure, which price (approximately) would you propose as the system price for this period and which bids should be accepted? Briefly (no more than 200 words) discuss the advantages and disadvantages with the step function curves and with the linearized curves in this example.
   Note: You can run model.mod and use the report file plot.txt to draw the figures in Excel (follow instructions in lines 15-30 of model.mod).

2. Run the model using the file hour.dat and study the results. Suppose you are a new supplier with a power plant with a capacity of 1200 MW per time period. Your own production cost is 11 euro per MWh. Decide the best bid you can come up with in order to maximize your profit in each of these two scenarios:

   a) You must place exactly one bid per period, the volume must be the same for all periods and can be any non-negative integer number, and the price may vary from one to another period but it always has to be an integer number greater than 1 euro.
   b) You can place at most one bid per period, the price may vary from one to another period but it always has to be an integer number greater than 1 euro, and the volume may also vary from one to another period but it always must be a multiple of 10.
   Calculate the profit you obtain in each scenario and explain the reasoning behind your strategy.

Day-ahead market with block bids

3. In the files blockbids_$\{1, 2, 3, 4, 5\}$.dat there is data for hourly and block bids. The data file you have to use is defined in analogous way as for task 1. Run the model and study the results (you don't have to write about this in the report; it is just for you to get familiar with this data instance and the equilibrium). Suppose you are a new supplier with a power plant with a capacity of 1200 MW per time period. Your own production cost is 11 euro per MWh. Suppose you only can place one bid. This bid must be a block bid that covers four consecutive time periods (note there are five periods in the data file). The price of your block bid cannot be equal to the price of any other block bid in the file and must be greater than 1.0 euro (you may use up to one decimal in your price). The volume must be a multiple of 10. Decide the best block bid you can come up with in order to maximize your profit. Explain the reasoning behind your strategy. How much is your profit?

   Hints: You are not expected to create a model to properly "optimize" your bids nor to run thousand different combinations. Rather, try to explore the equilibrium and design a strategy which allows you to progressively arrive at a favourable result. In order to explore the performance of your bids, you can incorporate your bids in the data files: for hourly bids, write the volume and price of your bids as values for parameters *InputQS* and *InputPS* at the end of the file hour.dat; for block bids, do it after line 29 in the file blockbids_$\{1, 2, 3, 4, 5\}$.dat. Note you will have to update the value of parameter *no_supply_bids* and *no_block_bids* in tasks 2 and 3, respectively (yours will be the hourly bid 21 for each period in task 2 and the block bid 11 in task 3). The data file name must be given in lines 44 or 45 of the file model.mod

[1] The period you have to study is given as follows. Consider the ordered set of numbers $N = \{1, 2, 3, 4, 5\}$. Divide the last digit of your candidate number by 2 and approximate it to the closest greater or equal integer in N. If you work in teams of two persons, there might be two different last digits in your candidate numbers. If so, choose the lowest one. For example, if you work alone and your number is 83 or 84 you have to study period 2 (because $3/2 = 1.5$ and $4/2 = 2$ are approximated to 2); if your number is 31 and you work in a team with someone whose number is 77 you have to study period 1 (because 1 is lower than 7, and $1/2 = 0.5$ is approximated to 1).

(do not modify other lines in model.mod). If the price of your hourly bid coincides with the highest price of an accepted supply bid in the corresponding period (which can be seen under the column header "PS" in results.txt), assume that the volume accepted of your bid was either all the volume that you bid or the volume of the last supply bid accepted (the value under the column header "s" in results.txt) in case the latter is less than the former. When calculating your revenue during a period, you must consider the equilibrium price for the corresponding period (which can be seen under the column header "Price" in results.txt).

Part D

With the aim of increasing connectivity and fostering sustainable transportation, many cities provide bike sharing services to their inhabitants. The growth in the number of users, however, depends considerably on the geographical location of the bike racks (where the bikes must be parked) and on the size of the fleet. This motivates us to formulate a model to design the network and the fleet size of a bike-sharing service at an intricate city in the north of the world.

The city government has a number of candidate locations to install the bike racks. We will refer by I to the set of these locations. The cost of installing a bike rack at location i consists of a fixed cost f_i plus a variable cost v_i per each bike initially allocated to this rack (in practice, during a day of operation the users may move bikes from one to another rack, but a frequent repositioning service of the city will return bikes as to match this initial setup). As for the bike fleet, the government can acquire *conventional bikes* and *electric bikes*. The cost of acquiring a conventional bike is c and the cost of acquiring an electric bike is e. The policy of the government is to acquire exactly two electric bikes for every rack installed, while all the other bikes will be conventional bikes.

Let N be the set of inhabitants in the city. The distance of the house of inhabitant n to the candidate rack location i is $d_{i,n}$ km. The government has estimated that an inhabitant will become a *potential user* of the service if the distance from the inhabitant's house to its closest bike rack is at most t km. As a target, the government would like to secure that at least 50% of the inhabitants of the city become potential users.

In deciding the fleet size, the government wants to make sure that the number of bikes allocated to each installed rack must be at least 5% of the potential users *linked* to that rack. For this purpose, note that even if there are two or more racks within t km from the house of a potential user, the potential user is linked to only one rack: naturally, the closest one to his/her house (for the sake of simplicity, assume that there exists no pair of candidate locations at the same distance from an inhabitant's house).

The objective of the city government is to minimize the total budget allocated to this bike-sharing service.

Formulate an integer linear programming model for this problem. (**AMPL** code is not required in this part.)