Part A: Optimizing Chessmon Go trip

1. Optimal Plan of Trips for Karlsen family

This problem has a lot of similarities with the TSP. However, this problem involves more than one person. One way to solve this is to combine the TSP model and the set partitioning model. We can define the alternatives in the set partitioning model as different travel routes, where each alternative contains between 25 to 50 objects which would represent the locations of the Chessmon GO pieces. Each Chessmon GO piece must be included in at least one of the alternatives to ensure that when the set partitioning model is run, each piece can be included exactly once. The cost of each alternative would be represented as the distance travelled in order to fetch all the objects in the given alternative. To find the minimum distance travelled for an alternative containing a given set of objects we would use the TSP model. Having done this for all alternatives we could run the set partitioning model to find a set of alternatives that minimize the total distance travelled while at the same time include each location/Chessmon Go piece exactly once. However, we know that there are only four people in the family, so only four alternatives can be chosen. We would therefore add the following constraint before running the set partitioning model:

$$\sum_{j \epsilon J} x_j = 4$$

Where x_j is a binary variable that takes the value 1 if alternative j is chosen and 0 otherwise.

2. Family vs Jacob Ingebrigtzen

There will be used a minimum of n + 1 arcs if one person collects all the pieces, where n is locations containing pieces. This is because of the travel from and to the starting location. When adding one person to the team, there will be an additional travel from and to the starting location. Four team members would therefore result in three additional trips from a location to the hotel. These trips will increase the total distance without resulting in extra chess pieces. Since each family member can´t collect more than 50 pieces in an afternoon, there must be more than one member involved in the trip. Since the family also set a target that each member should collect at least 25 pieces each, all the members must participate in the trip. This means that they would travel 132 arcs (128 + 4) while Jacob would only travel 129 arcs (128 + 1). As long as Jacob can travel the same arcs as the family in one trip, Jacob´s claim is true.

Part B: Solving the External Candidates Exam Schedule in Norway

1. Excluding Professor and Student who are Relatives

The binary variable x_{et} is equal to one if exam e ∈ E is carried out in space-time slot t ∈ T, and zero otherwise. Likewise, the binary variable z_{pt} is equal to one if professor p ∈ P is assigned to space-time slot t ∈ T, and zero otherwise. The subset $E_C(c) \subset$ E contains the set of exams of candidate c ∈ C. Thus, $E_C(c1)$ is the set of exams for candidate c1. We can write a new constraint that ensures that candidate c1 and professor p1 can´t be assigned to the same space-time slot:

$$x_{et} + z_{1,t} \leq 1 \quad \forall\, t \in T, e \in E_C(c1)$$

If both candidate c1 and professor p1 is assigned to the same space-time slot, both x_{et} *and* $z_{1,t}$ will be 1. Thus, the constraint would not hold. This constraint therefore ensures that at least one of these binary variables are zero.

2. Bad-luck sequence modelling

$$b_{c,d} = \begin{cases} 1 & jf\ candidate\ c\ is\ scheduled\ for\ an\ exam\ on\ day\ d\ and\ d+2 \\ 0 & otherwise \end{cases}$$

If the candidate is scheduled for an exam on day d and d+1 we know that they have a resting day on day d+1 due to the constraint: $\sum_{e \in E_c(c)} \sum_{t \in T(d) \cup T(d+1)} x_{et} \leq 1$.
Taking this into account we can construct equations that ensures that $b_{c,t}$ = 1 if the number of exams candidate c has in day d and day d + 2 = 2, as well as ensuring that each candidate has a maximum of two bad-luck sequences. We also want to make sure that at least 60% of students experience no bad-luck sequences.

$$(1)\ \sum_{e \in E_c(c)} \sum_{t \in T(d) \cup T(d+2)} x_{et} \leq 1 + b_{c,d} \qquad \forall\, c \in C, d \in D{:}\ d \leq |D| - 2$$
$$(2)\ \sum_{d \in D:\ d \leq |D|-2} b_{c,d} \leq 2 \qquad \forall\, c \in C$$

From equation 1, it follows that if $\sum_{e \in E_c(c)} \sum_{t \in T(d) \cup T(d+2)} x_{et} = 2$ (the candidate has two exams with only one resting day in between), then $b_{c,d}$ must equal 1 for the equation to hold.

In equation 2 we ensure that the sum of bad-luck sequences for each candidate must be less than or equal to two. Although we do ensure that constraint 1 is respected, we do not ensure that $\sum_{c\in C}\sum_{d\in D:\, d\leq|D|-2} b_{c,d}$ (the sum of all bad-luck sequences) is minimized. Note for instance that if $\sum_{e\in E_c(c)}\sum_{t\in T(d)\cup T(d+2)} x_{et} = 1$ then $b_{c,d}$ can take the value 0 and 1. This will be addressed shortly. First we introduce a new binary variable which will be used to ensure that a minimum of 60% of the candidates must have zero bad-luck sequences:

$$(3)\ \gamma_c = \begin{cases} 1\, jf\ candidate\ c\ experienced\ at\ least\ one\ bad\ luck\ sequence \\ 0\ otherwise \end{cases}$$

We add a constraint that ensures that γ_c takes the value 1 if the candidate has had at least one bad-luck sequence:

$$(4)\ \sum\nolimits_{d\in D:\, d\leq|D|-2} b_{c,d} \leq 2\gamma_c\ \forall\ c \in C$$

Finally, we ensure that at least 60% of the students do not experience a bad-luck sequence:

$$(5)\ \sum_{c\epsilon \mathrm{C}} \gamma_c \leq 0.4 * |C|$$

We still haven't addressed the issue that both $b_{c,d}$ and γ_c can take the value one and zero if the left-hand side of their respective equations (1 and 4) are equal to zero. To ensure that the value is 0 when this is the case, we could incorporate a term in the objective function that "punishes" the optimal objective function value if these variables take the value 1. For instance the objective function can be defined as in the paper with the inclusion of two additional terms:

$$\min \sum_{t\epsilon T_{>}(d*)} d(t) \sum_{s\epsilon S} y_{st} - \epsilon \sum_{e\epsilon E} \sum_{t\epsilon T_l(e)} x_{et} + \rho * \sum_{c\epsilon \mathrm{C}} \gamma_c + \xi \sum_{c\in C} \sum_{d\in D:\, d\leq|D|-2} b_{c,d}$$

Where ρ and $\xi > 0$ and punishes the count of unique candidates who experienced at least one bad-luck sequence and the total number of bad-luck sequences among all candidates respectively.

Part C: Supply Chain Model for PetroBAN

1. Supply Chain Optimization Model

The model have the following sets, parameters, decision variables, objective function and constraints:

Sets:

$i \in I: Set\ of\ crude\ oils$

$b \in B: Set\ of\ components$

$p \in P: Set\ of\ final\ products$

$j \in J: Set\ of\ Crude\ Distilling\ Units\ (CDUs)$

$d \in D: Set\ of\ depots$

$k \in K: Set\ of\ markets$

$m \in M: Set\ of\ running\ modes$

$t \in T: Set\ of\ days$

Parameters:

$C_{i,t}^{CRU}: Cost\ of\ purchasing\ one\ unit\ of\ crude\ oil\ i\ on\ day\ t$

$R_{i,b,j,m}: Amount\ of\ component\ b\ obtained\ from\ refining\ one\ unit\ of\ crude\ oil\ i\ in\ CDU\ j\ at\ running\ mode\ m$

$N_{b,p}: Amount\ of\ component\ b\ needed\ to\ produce\ one\ unit\ of\ product\ p$

$S_p: Sales\ price\ for\ one\ unit\ of\ product\ p$

$S^{Lowqc}: Sales\ price\ for\ one\ unit\ of\ lowqc$

$C_{i,j,m}^{REF}: Cost\ of\ refining\ one\ unit\ of\ crude\ oil\ i\ in\ CDU\ j\ in\ mode\ m$

$Cap_{j,m}: Maximum\ processing\ capacity\ of\ CDU\ j\ in\ running\ mode\ m$

$C_p^{PRO}: Cost\ of\ producing\ one\ unit\ of\ product\ p$

$C^{TRA}: Cost\ of\ transporting\ one\ unit\ of\ any\ component\ from\ refining\ to\ blending\ department$

$C_d^{TRA}: Cost\ of\ transporting\ for\ one\ unit\ of\ any\ product\ from\ blending\ department\ to\ depot\ d$

$C_{d,k}^{TRA}: Shipping\ cost\ for\ one\ unit\ of\ any\ product\ from\ depot\ to\ market\ k$

$C^{INVI}: Cost\ of\ storing\ one\ unit\ of\ any\ type\ of\ crude\ oil\ at\ the\ refining\ department\ per\ day.$

$C^{INVB}: Daily\ cost\ of\ storing\ one\ unit\ of\ any\ component\ at\ the\ refining\ department\ (excl. lowqc)$

$C_d^{INVP}: cost\ of\ storing\ one\ unit\ of\ any\ type\ of\ product\ at\ depot\ d$

$\delta_{p,k,t}: Maximum\ demand\ limit\ for\ product\ p\ from\ market\ k\ in\ day\ t$

$C_{j,m}^{Mode}: Fixed\ cost\ incurred\ per\ day\ from\ operation\ of\ CDU\ j\ in\ mode\ m$

$C^{Change}: Cost\ of\ changing\ running\ mode\ at\ CDU$

$IO_i^{Zero}: Initial\ inventory\ of\ crude\ i$

$IC_b^{Zero}: Initial\ inventory\ of\ component\ b$

$IP_{p,d}^{Zero}: Initial\ inventory\ of\ product\ p\ at\ depot\ d$

$IC_b^{Final}: Minimum\ final\ inventory\ requirement\ for\ component\ b$

$IP^{Final}_{p,d}: Minimum\ final\ inventory\ requirement\ for\ product\ p\ at\ depot\ d$

$y^{Zero}_{b}: Initial\ component\ b\ sent\ to\ blending$

$x^{Zero}_{p,d}: Initial\ product\ p\ sent\ to\ depot\ d$

$v^{Zero}_{p,d,k}: Initial\ product\ p\ sent\ to\ market\ k$

$m^{Zero}_{j,m}: Initial\ mode\ m\ of\ CDU\ j$

Decision variables:

$u_{i,t}: Units\ of\ crude\ oil\ i\ purchased\ at\ day\ t$

$z_{i,j,m,t}: Units\ of\ crude\ oil\ i\ destilled\ in\ CDU\ j\ in\ mode\ m\ in\ period\ t$

$y_{b,t}: Units\ of\ component\ b\ sent\ to\ blending\ in\ period\ t\ (for\ blending\ in\ t+1)$

$w_{p,t}$: Units of product p produced at the blending department in period t

$x_{p,d,t}: Units\ of\ product\ p\ sent\ to\ depot\ d\ in\ period\ t\ (available\ at\ depot\ in\ t+1)$

$v_{p,d,k,t}: Units\ of\ product\ p\ sent\ from\ depot\ d\ to\ market\ k\ in\ period\ t\ (satisfies\ demand\ in\ t+1)$

$s_{b,t}: Amount\ of\ component\ b\ sold\ immediately\ after\ refining\ (only\ lowqc)$

$IO_{i,t}: Inventory\ of\ crude\ oil\ i\ at\ the\ refinery\ in\ the\ end\ of\ period\ t$

$IC_{b,t}: Inventory\ of\ component\ b\ at\ the\ refinery\ in\ the\ end\ of\ period\ t$

$IP_{p,d,t}: Inventory\ of\ product\ p\ at\ depot\ d\ at\ the\ end\ of\ period\ t$

$$L_{j,m,t}: \begin{cases} 1\ if\ mode\ m\ is\ used\ on\ CDU\ j\ in\ period\ t \\ 0\ otherwise \end{cases}$$

$$E_{j,m,t}: \begin{cases} 1\ if\ mode\ m\ used\ on\ CDU\ j\ in\ period\ t\ is\ different\ than\ in\ period\ t-1 \\ 0\ otherwise \end{cases}$$

f: $parameter\ equal\ to$ 0.5 $to\ account\ for\ the\ double\ count\ for\ each\ change\ in\ CDU\ mode$ m

Objective function:

$Max\ contribution$

$$= \sum_{p\in P}\sum_{d\in D}\sum_{k\in K}\sum_{t\in T:t>0\ and\ t\leq|T|-2} S_p\, v_{p,d,k,t} + \sum_{t\in T:t>0} s_{lowqc,t}S^{Lowqc}$$
$$-\sum_{i\in I}\sum_{t\in T:t>0} C^{CRU}_{i,t}u_{i,t} - \sum_{i\in I}\sum_{j\in J}\sum_{m\in M}\sum_{t\in T:t>0} C^{REF}_{i,j,m}z_{i,j,m,t} - \sum_{p\in P}\sum_{t\in T:t>0} C^{PROD}_{p}w_{p,t}$$
$$-\sum_{i\in I}\sum_{t\in T:t>0} C^{INVI}IO_{i,t} - \sum_{b\in B}\sum_{t\in T:t>0} C^{INVB}IC_{b,t} - \sum_{p\in P}\sum_{d\in D}\sum_{t\in T:t>0} C^{INVP}_{d}IP_{p,d,t}$$
$$-\sum_{b\in B}\sum_{t\in T:t>0} C^{TRA}y_{b,t} - \sum_{p\in P}\sum_{d\in D}\sum_{t\in T:t>0} C^{TRA}_{d}x_{p,d,t} - \sum_{p\in P}\sum_{d\in D}\sum_{k\in K}\sum_{t\in T:t>0} C^{TRA}_{d,k}v_{p,d,k,t}$$
$$-\sum_{j\in J}\sum_{m\in M}\sum_{t\in T:t>0} L_{j,m,t}C^{Mode}_{j,m} - \sum_{j\in J}\sum_{m\in M}\sum_{t\in T:t>0} E_{j,m,t}C^{Change}\, f$$

Constraints:

Balance of crude oils at the refinery:

Constraints:

Balance of crude oils at the refinery:

(1) $IO_{i,t} = IO_{i,t-1} + u_{i,t} - \sum_{j \in J} \sum_{m \in M} z_{i,j,m,t} \quad \forall\, i \in I, t \in T: t > 0$

Maximum units of crudes i processed by CDU j in running mode m per day:

(2) $\sum_{i \in I} z_{i,j,m,t} \leq Cap_{j,m} \quad \forall\, j \in J, m \in M, t \in T: t > 0$

Balance of components at distillation:

(3) $IC_{b,t} = IC_{b,t-1} - y_{b,t} + \sum_{i \in I} \sum_{j \in J} \sum_{m \in M} R_{i,b,j,m} z_{i,j,m,t} \quad \forall\, b \in B, t \in T: t > 0, b \neq lowqc$

All production of component lowqc is sold in time period t:

(4) $s_{lowqc,t} = \sum_{i \in I} \sum_{j \in J} \sum_{m \in M} R_{i,lowqc,j,m} z_{i,j,m,t} \quad \forall\, t \in T: t > 0$

Component b sent to blending according to product recipes:

(5) $y_{b,t-1} = \sum_{p \in P} N_{b,p} w_{p,t} \quad \forall\, b \in B, t \in T: t > 0$

All production at blending sent to depots (no storage at blending facility):

(6) $w_{p,t} = \sum_{d \in D} x_{p,d,t} \quad \forall\, p \in P, t \in T: t > 0$

Balance of products at depots inflow from blender, stored, to markets:

(7) $IP_{p,d,t} = IP_{p,d,t-1} + x_{p,d,t-1} - \sum_{k \in K} v_{p,d,k,t} \quad \forall\, p \in P, d \in D, t \in T: t > 0$

Products shipped to market cannot be more than demand:

(8) $\sum_{d \in D} v_{p,d,k,t-1} \leq \delta_{p,k,t} \quad \forall\, p \in P, k \in K, t \in T: t > 0$

Initial inventory of crude oil i:

(9) $IO_{i,0} = IO_i^{Zero} \quad \forall\, i \in I$

Initial inventory of component b:

(10) $IC_{b,0} = IC_b^{Zero} \quad \forall\, b \in B$

Initial inventory of product p at depot d:

(11) $IP_{p,d,0} = IP_{p,d}^{Zero} \quad \forall\, p \in P, d \in D$

Initial component b sent to blending (available in t=1):

(12) $y_{b,0} = y_b^{Zero} \quad \forall\, b \in B$

Initial product p sent to depot d (available in t=1):

(13) $x_{p,d,0} = x_{p,d}^{Zero} \quad \forall\, p \in P, d \in D$

Initial product p sent from depot d to market k (available in t=1):

(14) $v_{p,d,k,0} = v_{p,d,k}^{Zero} \quad \forall\, p \in P, d \in D, k \in K$

Minimum final inventory of component b:

(15) $IC_{b,12} \geq IC_{b}^{Final} \quad \forall\, b \in B$

Minimum final inventory of product p at depot d:

(16) $IP_{p,d,12} \geq IP_{p,d}^{Final} \quad \forall\, p \in P, d \in D$

Initial starting mode m for CDU j:

(17) $L_{j,m,0} = m_{j,m}^{Zero} \quad \forall\, j \in J, m \in M$

If CDU j is used in mode m at a given day t, $L_{j,m,t}$ take the value 1:

(18) $\sum_{i \in I} z_{i,j,m,t} \leq L_{j,m,t} * 1050 \quad \forall\, j \in J, m \in M, t \in T: t > 0$

Exactly one mode of CDU j is chosen for a given day:

(19) $\sum_{m \in M} L_{j,m,t} = 1 \quad \forall\, j \in J, t \in T: t > 0$

Constraints (20) to (22) ensures that the binary variable E takes the value 1 if mode m used on CDU j in period t is different from previous period (t-1):

(20) $L_{j,m,t} + L_{j,m,t-1} \leq 2 - E_{j,m,t} \quad \forall\, j \in J, m \in M, t \in T: t > 0$

(21) $L_{j,m,t} - L_{j,m,t-1} \leq E_{j,m,t} \quad \forall\, j \in J, m \in M, t \in T: t > 0$

(22)$L_{j,m,t-1} - L_{j,m,t} \leq E_{j,m,t} \quad \forall\, j \in J, m \in M, t \in T: t > 0$

We run the model in AMPL using the solver cplex. The files can be found in the attached folder, in the subfolder “AMPL” in folder “PartC” with the name “Proj2C” followed by “.mod”, “.dat”, and “.run”. The results can be found in the file “results.txt”. Running the model provides us an optimal profit of **$5,201,258.72.**

Table 1 shows the different modes selected for the CDUs throughout the different time periods as well as the available capacity for each CDU in the selected mode. The table also shows how much of this available capacity is used. This is shown by the utilization rate which is defined as the capacity used divided by the available capacity.

Mode Chosen and Capacity Used for the CDUs at Each Period

CDU/Time Period	0	1	2	3	4	5	6	7	8	9	10	11	12
CDU1	Shutdown	High mode	High mode	High mode	High mode	High mode	High mode	High mode	High mode	Shutdown	Shutdown	Shutdown	Shutdown
CDU2	Shutdown	High mode	Low mode	High mode	High mode	High mode	High mode	High mode	Low mode	Low mode	Low mode	Low mode	Low mode
Available Capacity CDU1	0	950	950	950	950	950	950	950	950	0	0	0	0
Available Capacity CDU2	0	900	1000	900	900	900	900	900	1000	1000	1000	1000	1000
CDU1 Capacity Used	0	929.1	950	950	918.31	950	592.6	950	924.5	0	0	0	0
CDU2 Capacity Used	0	900	1000	900	900	900	900	900	1000	1000	286.67	0	266.67
Utilization Rate CDU1	N/A	**97.8%**	**100%**	**100%**	**96.66%**	**100%**	**62.38%**	**100%**	**97.32%**	N/A	N/A	N/A	N/A
Utilization Rate CDU2	N/A	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**28.67%**	**0**	**26.67%**

Table 1; Mode Chosen and Capacity Used for the CDUs at each period

From Table 2 we see that no crude oils are stored. This is reasonable as crude oils which are not sold in the planning horizon will only represent a cost which the maximization model will remove if possible. This is also true for products. However, there were given instructions requiring minimum inventory quantities for the different products. Unsurprisingly, the ending inventory equals the lower bound of this constraint as the model is forced to produce products, which incur costs. Products in the final inventory are not sold and thus do not contribute to maximizing the objective function value. The same argument could be made for the components. Interestingly we find that unlike their counterparts who assume values equal to their lower bound, distilA and distilB have significantly larger ending inventories. Looking at the data for parameter $N_{b,p}$ and the sales prices we obtain some insight as to why this might be the case.

For three of the products (premium, regular and super), the amount requirements for naphta1 and naphta2 are in general greater than the requirements for distilA and distilB. These products also have a higher price than distilF, which is the only product where the requirement for distilA and distilB is greater than the naphtha-components. The excess production of distilA and distilB can therefore be seen as a bi-product caused by maximizing the sale of the more expensive products where the requirement for naphta-components is greater than the distil-components. Note that distilF only requires distilA and distilB. Therefore, this excess production could have been sold to the markets. However, the markets are fully saturated (1537 products of distilF) and the excess production therefore must remain stored.

Ending Inventory for Crudes, Components and Products

Inventory Item	Refinery	Depot 1	Depot 2
Crude oils			
CrA	0		
CrB	0		
Total	**0**		
Components			
distilA	1594.61		
distilB	1088.16		
lowqc	0		
naphtha1	80		
naphtha2	80		
Total	**2842.77**		
Products			
distilF		20	30
premium		25	25
regular		50	50
super		15	25
Total	**240**		

Table 2: Ending Inventory for Crudes, Components and Products

2. Scenario Analysis with Purchasing Cost Increase in Crude Oil

We use the “let” command in AMPL to change the price of crude oil by 10%, 30% and 50%. The results are stored in the files “results10percent.txt”, “results30percent.txt”, “results50percent.txt”. The mode selection for the two CDUs at the different time periods are shown in Table 3, where the selection of mode "shutdown" is shown in bold. The original profit was $5,201,258.72 and the optimal profit in the three scenarios 10%, 30% and 50% are $5,126,640.47 (-1.46%), $4,978,846.99 (-4.28%), $4,833,432.3 (-7.1%) respectively. As for the selection of modes for the CDUs, the general trend is that as prices increase, the high mode selection also increase. The intuition behind this is that although it is more expensive to produce in high mode, the output for the scarce factors (naphtha 1 and 2) is greater than in the low mode.

Mode Chosen the CDUs at Each Period for Different Inflation Levels

CDU/Time Period	0	1	2	3	4	5	6	7	8	9	10	11	12
CDU1	**Shutdown**	High	High	High	High	High	High	High	High	**Shutdown**	**Shutdown**	**Shutdown**	**Shutdown**
CDU1 (10%)	**Shutdown**	High	High	High	High	High	High	High	High	**Shutdown**	**Shutdown**	**Shutdown**	**Shutdown**
CDU1 (30%)	**Shutdown**	High	High	High	High	High	High	High	High	High	**Shutdown**	**Shutdown**	**Shutdown**
CDU1 (50%)	**Shutdown**	High	High	High	High	High	High	High	High	High	**Shutdown**	**Shutdown**	**Shutdown**
CDU2	**Shutdown**	High	Low	High	High	High	High	High	Low	Low	Low	Low	Low
CDU2 (10%)	**Shutdown**	High	Low	High	High	High	High	High	Low	Low	Low	Low	Low
CDU2 (30%)	**Shutdown**	High	High	High	High	High	High	High	High	Low	Low	Low	Low
CDU2 (50%)	**Shutdown**	High	High	High	High	High	High	High	High	High	High	**Shutdown**	Low

Table 3: Mode Chosen at the CDUs at Each Period for Different Inflation Levels

This intuition can further be explained through a simple example: If the price of crude oil is close to zero, you would buy many barrels of oil and distil them through the cheapest alternative. This is because the benefits from producing cheaply outweighs the downside of having to buy more barrels to obtain a given quantity of output. When prices increase, ceteris paribus, this benefit decreases. The opposite extreme case is where a barrel of oil has a very high relative price. In that case the downside of having to buy one more barrel of oil far outweighs the benefit from producing cheaply. One would therefore favor extracting as many scarce factor components as possible from each barrel of oil, which requires using the high mode. We can also see this looking at the number of barrels bought, which decreases for each of the three price scenarios while the share of high mode selection increases.

A final comment can be made on the optimal solution suggestion for the 50% price increase, where CDU2 is shut down in period 11 before turned back on in period 12. It distils crude oil in period 12 in order to satisfy the minimum requirement of the naphtha components. This can be seen by looking at the purchase schedule and the inventory of the naphtha components which consists of buying and distilling 266.67 of CrA in period 12 to satisfy the minimum requirement of 80 for these components as they are not satisfied in period 11. The choice of shutting down in period 11 for CDU2 is because shutting down is less expensive than keeping the CDU running even if this results in two changes (on-off, off-on).

Part D: Smart Charging Model for Optidriver

1. General Model

The linear programming model for this problem contains the following sets, subsets, parameters, decision variables, objective function and constraints:

Sets:

$d \epsilon D$: *Set of days*

$h \epsilon H$: *Set of hours*

Subsets:

$D_{Weekdays} \subset D$: Set of weekdays (Monday-Friday)

$D_{Weekends} \subset D$: Set of weekends (Saturday-Sunday)

Parameters:

$C_{d,h}$: Charging cost (per kWh) on day d at hour h

y^{Zero}: Initial state-of-charge

y^{Final}: Final state-of-charge

x^{MAX}: Maximum possible charge rate per hour

k^{WORK}: kWh used by travelling one way to/from work

Decision variables:

$x_{d,h}$: kWh charged on day d at hour h

$y_{d,h}$: Battery capacity (in kWh) on day d at end of hour h[1]

$$g_{d,h}: \begin{cases} 1 \text{ } if \text{ } car \text{ } is \text{ } driven \text{ } at \text{ } day \text{ } d \text{ } at \text{ } hour \text{ } h \\ 0 \text{ } otherwise \end{cases}$$

Objective function:

$$Min\ Cost = \sum_{d \in D: d>0} \sum_{h \in H} C_{d,h} x_{d,h}$$

Constraints:

kWh charged on day d at hour h can´t exceed the maximum possible hourly charge rate:

(1) $x_{d,h} \leq x^{MAX} \quad \forall d \in D, h \in H: d > 0$

The car can´t be charged between 08:00 and 17:00 on weekdays:

(2) $x_{d,h} = 0 \quad \forall d \in D_{Weekdays}, h \in \{8, \dots, 17\}$

The car is used from 07:00-08:00 (h = 8) and 16:00-17:00 (h = 17) every weekday:

[1] Our model is defined by days and hours, where hour 1 indicates hour 00:00-01:00. The state of charge at hour h is therefore the ending state-of-charge for this hour. This is particularly important to keep in mind in problem D4, because whether or not charging can happen depends on the state-of-charge value for the previous hour.

(3) $G_{d,h} = 1 \quad \forall d \in D_{Weekdays}, h \in \{8,17\}$

The car cannot be used other times in weekdays:

(4) $G_{d,h} = 0 \quad \forall d \in D, h \in H: h \neq 8, h \neq 17, d > 0$

The car cannot be used in weekends:

(5) $G_{d,h} = 0 \quad \forall d \in D_{Weekends}, h \in H: d > 0$

State-of-charge must equal capacity at previous period + charge – consumption:

(6) $y_{d,h} = y_{d,h-1} + x_{d,h} - k^{WORK} G_{d,h} \quad \forall\, d \in D, h \in H: d > 0, h > 1$

State-of-charge must be transferred from one day to another ($G_{d,h}$ is not taken into account here since we know that the car can´t be used at this time of the day):

(7) $y_{d,1} = y_{d-1,24} + x_{d,1} \quad \forall\, d \in D: d > 0$

Initial state-of-charge (right before 00:00 on 1st of July):

(8) $y_{0,24} = \mathrm{y}^{\mathrm{Zero}}$

Final state-of-charge (right before 00:00 on 1st of October)

(9) $y_{92,24} = \mathrm{y}^{\mathrm{Final}}$

State of charge cannot exceed the maximum battery capacity (64 kWh)

(10) $y_{d,h} \leq 64 \quad \forall\, d \in D, h \in H: d > 0$

State of charge cannot exceed the minimum battery requirement (12.8 kWh)

(11) $y_{d,h} \geq 12.8 \quad \forall\, d \in D, h \in H: d > 0$

We run the model in AMPL using the solver gurobi. The files can be found in the attached folder, in the subfolder "AMPL" in folder "PartD" with the name "Proj2.1D" followed by ".mod", ".dat", and ".run". The price-data has been converted to EUR/kWh using R. The results of the optimization are stored in the "results1.txt"-file in the same folder. The optimal cost is 116.743 Euro, with a total consumption of 726 kWh (0.726 mWh). We can use these numbers to calculate the average price in EUR/mWh:

$$EUR/mWh_{AVG} = \frac{116.743}{0.726} = 160.8$$

Optidriver has an average price per mWh of €160.8 during the three-month period July-September 2022. This is significantly lower than the average price of all hours in the period, which is €288.21.

To compute the optimal solution when maximizing the cost of charging during the three month period, we simply copy the mod-file and swap “minimize” for “maximize” in the objective function. The file is called “Proj2.1D_maximize.mod” and the results are stored in “results1_maximize.txt”. The optimal solution when maximizing the cost of charging during the period is €272.579, with the same consumption as it was when minimizing the total cost of charging (726 kWh). The reason that the charging does not increase is because the driver must drive exactly 10 trips per week (2 times a day, five days a week). Thus, the total amount charged must also be the same because of the final state of charge requirement (constraint (9)). The differences in the maximization and minimization of the total cost of charging in the period is summarized in Table 4:

Optimization/Minimization Differences

Measure	Minimization	Maximization	Difference	Difference (in %)
Total cost	€116.74	€272.58	€155.84	133.49
Total amount charged (in kWh)	726	726	0	0
EUR/mWh	€160.8	€375.45	€214.65	133.49

Table 4: Maximization/Minimization Differences

To further analyze the differences between the optimal solution to the minimization and maximization problem, we have plotted the aggregated kWh that is charged, as well as the average price for each weekday throughout the period. The plots are shown in figure 1 and 2. The code for all plots can be found in the “Plot.R” file in the subfolder “R” in folder “PartD”. In the optimal solution to the minimization problem, the majority of charging occurs in weekends, particularly on Sundays. This seems to be reasonable for two main reasons:

1. The car is available for charging in all hours during the weekends
2. The price is on average lowest in weekends, especially on Sundays

In the optimal solution to the maximization problem, the majority of charging occurs on weekdays, with very little charging in weekends. This seems reasonable, as the price is highest in weekdays.

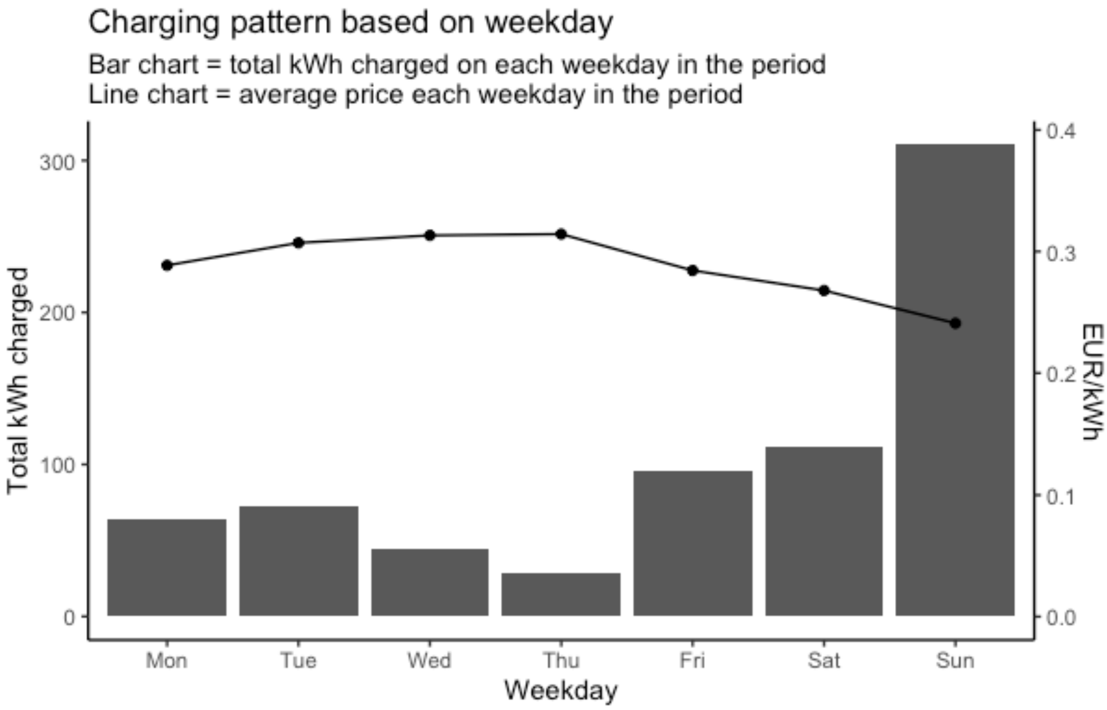

Figure 1: Charging pattern based on Weekday for Minimization Model

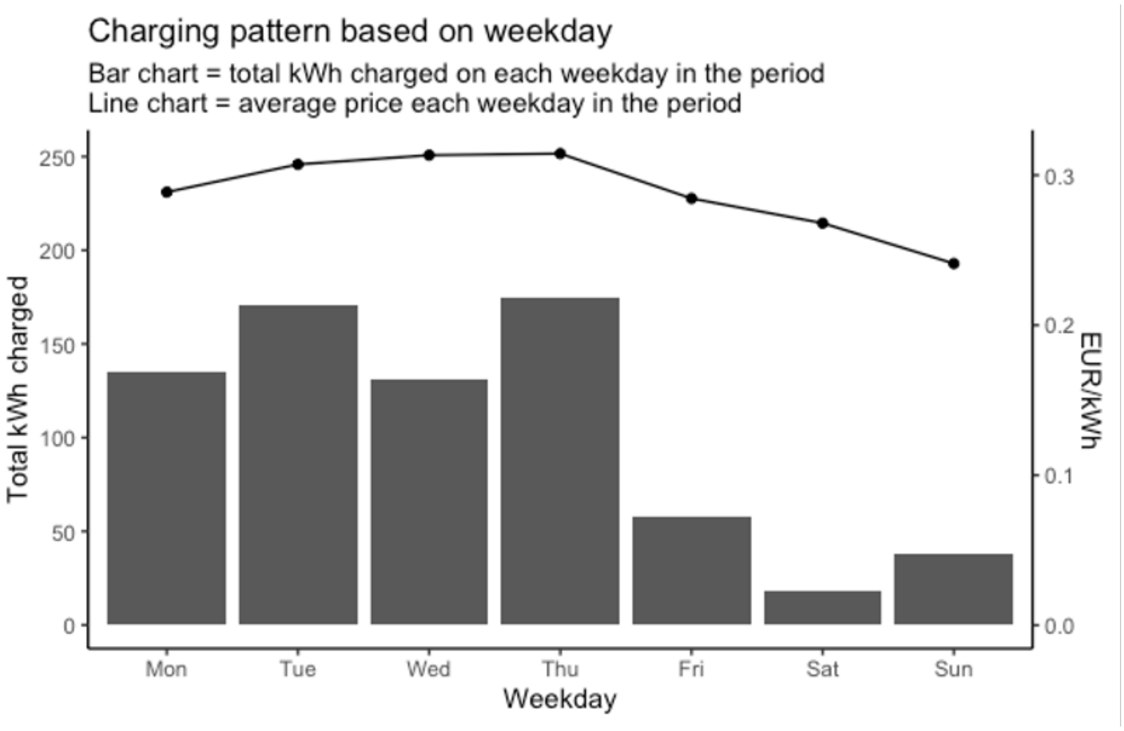

Figure 2 : Charging pattern based on Weekday for Minimization Model

2. Model with recreation activities every Sunday

For this problem, we build upon the general model from the previous problem. Since the rest of the models are minimization problems only, we will not use constraints (4) and (5) from the general model. The reason for this is that it will conflict with new constraints that will be implemented in these problems. We add a new set I with values {1, 2, 3, 4}, which is the four different time intervals Optidriver can choose to do recreation activities in. We also create a new subset $D_{Sundays}$ containing all d values in set D that are Sundays. A parameter k^{REC} is also added, which is the kWh used by travelling one way to/from recreation activities. We also add two new binary variables. The first binary variable $z_{d,i}$ is 1 if the car is used for recreation activites at day d in interval i, and 0 otherwise. The second binary variable $u_{d,h}$ is 1 if the car is used for recreation activites at day d in hour h.

The biggest difference from the previous model are the constraints. Constraint (10) ensures that exactly one interval is chosen every Sunday:

$$(10) \sum_{i \in I} z_{d,i} = 1 \quad \forall\, d \in D_{Sundays}$$

Logical constraints (11) – (14) ensures that the car is driven in the first and last hour of each interval chosen for recreation activities:

$$(11)\ u_{d,11} + u_{d,14} = 2z_{d,1} \quad \forall\, d \in D_{Sundays}$$

$$(12)\ u_{d,12} + u_{d,15} = 2z_{d,2} \quad \forall\, d \in D_{Sundays}$$

$$(13)\ u_{d,13} + u_{d,16} = 2z_{d,3} \quad \forall\, d \in D_{Sundays}$$

$$(14)\ u_{d,14} + u_{d,17} = 2z_{d,4} \quad \forall\, d \in D_{Sundays}$$

Logical constraints (15) – (18) ensures that the car cannot be charged in the 4-hour interval that the car is being used for recreation activities:

$$(15)\ x^{MAX}(1 - z_{d,1}) \geq x_{d,h} \quad \forall\, d \in D_{Sundays}, h \in \{11, \dots, 14\}$$

$$(16)\ x^{MAX}(1 - z_{d,2}) \geq x_{d,h} \quad \forall\, d \in D_{Sundays}, h \in \{12, \dots, 15\}$$

$$(17)\ x^{MAX}(1 - z_{d,3}) \geq x_{d,h} \quad \forall\, d \in D_{Sundays}, h \in \{13, \dots, 16\}$$

$$(18)\ x^{MAX}(1 - z_{d,4}) \geq x_{d,h} \quad \forall\, d \in D_{Sundays}, h \in \{14, \dots, 17\}$$

The full model implemented in AMPL can be found in the attached folder, in the subfolder "PartD" with the name "Proj2.2D" followed by ".mod", ".dat", and ".run". The results are stored in the file called "results2.txt". The problem is solved using gurobi. The optimal cost is €137.564, with 797.5 kWh of charging in total. The average price incurred by the optimal solution (in EUR/mWh) is then:

$EUR/mWh_{AVG} = \frac{137.564}{0.7975} = 172.49$

The car is used one extra day a week which requires additional charging, thus increasing the total cost. Optidriver also experience a €11.69 increase on the average price per kWh compared to the optimal solution in D1. This is likely a consequence of the car being unavailable for charging four hours every Sunday, which is the cheapest day to charge on average.

Figure 3 illustrates when Optidriver tends to charge his car. The kWh charged each hour in the period is illustrated by the black points, where a darker color indicates that there are many points in this area. The hours that have 0 kWh charged are ignored in the chart, to make it easier to interpret. The price (in EUR/kWh) is illustrated by the blue line. By studying this chart, we observe that there are many full hours (7.5 kWh) charged when the price has large downward movements. The opposite is true when the price has large upward movements, where we observe very few hours of charging. Hours where the price is below 0.05€ is indicated by a vertical red dashed line. We observe many data points where charging is 7.5 kWh at these hours.

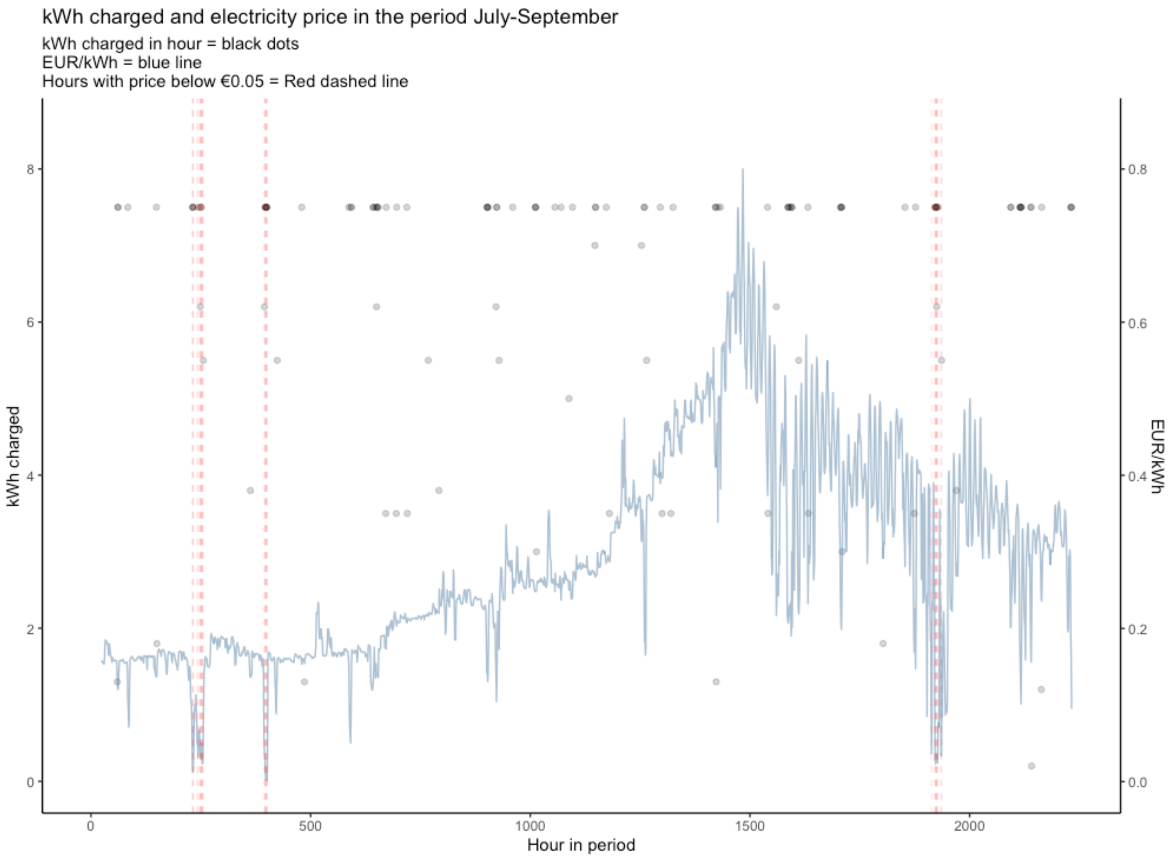

Figure 3: kWh charged and Electricity Price in period

3. Model with home office once a week

For this model, we build on the model from problem D2. We add new subsets for every week $D_1, \ldots, D_{13} \subset D$ with all weekdays of every week 1 to 13. The first week starts on day 4, and the last week ends on day 92. There are no additional parameters, but we add a new binary variable:

$$l_d = \begin{cases} 1\ if\ car\ is\ driven\ to\ work\ on\ weekday\ d \\ 0\ otherwise \end{cases}$$

There are multiple new constraints in this model, and some modifications of constraints from the general model. These will be explained briefly below.

Modified constraints from general model:

Constraint (2) is modified to ensure that the car cannot be charged between 07:00-17:00 if driven on weekday:

$$(2^{MOD})\ x_{d,h} \leq x^{MAX}(1 - l_d) \quad \forall d \in D_{Weekdays}, h \in \{8, \ldots, 17\}$$

Constraint (3) is modified to ensure that the car is driving at 07:00-08:00 and 16:00-17:00 the first day:

$(3^{MOD})\ G_{1,h} = 1 \quad \forall\, h \in \{8,17\}$

New constraints

l_d must take the value 1 if the car is driven to work on a weekday:

$(19)\ G_{d,8} + G_{d,17} \leq 2l_d \quad \forall d \in D_{Weekdays}$

Optidriver must go to work at least 4 times a week:

$$(20) \sum_{d \in D_1} G_{d,h} \geq 4 \quad \forall h \in \{8,17\}$$

$$(21) \sum_{d \in D_2} G_{d,h} \geq 4 \quad \forall h \in \{8,17\}$$

$$\dots$$

$$(31) \sum_{d \in D_{12}} G_{d,h} \geq 4 \quad \forall h \in \{8,17\}$$

$$(32) \sum_{d \in D_{13}} G_{d,h} \geq 4 \quad \forall h \in \{8,17\}$$

If Optidriver travels one way, he has to travel the same way back for a given day:

$(33)\ G_{d,8} = G_{d,17} \quad \forall d \in D_1$

$(34)\ G_{d,8} = G_{d,17} \quad \forall d \in D_2$

$$\dots$$

$(44)\ G_{d,8} = G_{d,17} \quad \forall d \in D_{12}$

$(45)\ G_{d,8} = G_{d,17} \quad \forall d \in D_{13}$

The full model implemented in AMPL can be found in the attached folder, in the subfolder "AMPL" in folder "PartD" with the name "Proj2.3D" followed by ".mod", ".dat", and ".run". The results are stored in the file called "results3.txt". The problem is solved using gurobi. The optimal cost is €102.072, with 654.5 kWh of charging in total. The average price incurred by the optimal solution (in EUR/mWh) is then:

$$EUR/mWh_{AVG} = \frac{102.072}{0.6545} = 155.95$$

This model has a lower total cost and average cost (in EUR/mWh) than both the previous models. Optidriver can choose to have home office once a week, which he will do every week when cost minimization is the objective. The weekly electricity consumption will therefore be 5.5 kWh lower than the first model and 11 kWh lower than the second model. There is also a relaxation in the constraint for when he can charge his car, since he can do so when he has home office. To check whether or not he has used this opportunity in the optimal plan, we use R to filter out weekends and hours that are not in the time range 07:00-17:00. We find that Optidriver has charged his car three weekdays between 07:00-17:00, as show in Table 5.

Days for Which the Car is Charged During Home Office

Date	Time	kWh charged	Weekday
26.07.2022	12:00-17:00	37.5 kWh	Tuesday
03.08.2022	11:00-12:00	7.5 kWh	Wednesday
03.08.2022	13:00-14:00	7.5 kWh	Wednesday
30.09.2022	15:00-16:00	5.7 kWh	Friday

Table 5: Days for Which the Car is Charged During Home Office

4. Model with limited number of ON-OFF charging sequences

To solve this problem we use the model from D2 and add new binary variables as well as new constraints. The binary variables added are:

$$m_{d,h} = \begin{cases} 1\ if\ SOC\ is\ less\ than\ 32\ kWh\ at\ day\ d\ in\ h-1\ (ingoing\ SOC\ in\ h) \\ 0\ otherwise^2 \end{cases}$$

$$j_{d,h} = \begin{cases} 1\ if\ SOC\ is\ greater\ than\ 51.2\ kWh\ at\ day\ d\ in\ h-1\ (ingoing\ SOC\ in\ h) \\ 0\ otherwise^2 \end{cases}$$

$$t_{d,h} = \begin{cases} 1\ if\ SOC\ is\ less\ than\ 51.2\ kWh\ at\ day\ d\ in\ h-1\ (ingoing\ SOC\ in\ h) \\ 0\ otherwise^2 \end{cases}$$

$$r_{d,h} = \begin{cases} 1\ if\ SOC\ is\ greater\ than\ 32\ kWh\ at\ day\ d\ in\ h-1\ (ingoing\ SOC\ in\ h) \\ 0\ otherwise^2 \end{cases}$$

[2] Note that in the case that state-of-charge is equal to 32, both variables $m_{d,h}$ and $r_{d,h}$can take the value 0 and 1. This entails that m can be equal to 0 in the case where the state of charge is equal to 32. However, the model will select the value 1 for $m_{d,h}$ if this contributes to minimizing the objective function value, meaning that charging can start when the state-of-charge is equal to 32. This logic also applies for $j_{d,h}$ and $t_{d,h}$.

$c_{d,h} = \begin{cases} 1\ if\ car\ is\ charging\ at\ d\ in\ h \\ 0\ otherwise \end{cases}$

$l_{d,h} = \begin{cases} 1\ if\ car\ goes\ from\ not\ charging\ in\ h-1\ to\ charging\ in\ h\ at\ day\ d \\ 0\ otherwise \end{cases}$

$o_{d,h} = \begin{cases} 1\ if\ car\ goes\ from\ charging\ in\ h-1\ to\ not\ charging\ in\ h\ at\ day\ d \\ 0\ otherwise \end{cases}$

$s_{d,h} = \begin{cases} 1\ if\ charging\ event\ start\ at\ day\ d\ in\ h \\ 0\ otherwise \end{cases}$

$e_{d,h} = \begin{cases} 1\ if\ charging\ event\ stops\ at\ day\ d\ in\ h \\ 0\ otherwise \end{cases}$

$f_{d,h} = \begin{cases} 1\ if\ there\ is\ a\ charging\ event\ at\ day\ d\ in\ h \\ 0\ otherwise \end{cases}$

$p_{d,h} = \begin{cases} 1\ if\ h\ in\ day\ d\ is\ the\ last\ hour\ of\ the\ charging\ event \\ 0\ otherwise \end{cases}$

Constraints (19)-(23) ensures that $m_{d,h}$ must take the value 1 if $y_{d,h-1} < 32$ and that $r_{d,h}$ must take the value 1 if the $y_{d,h-1} > 32$. If $y_{d,h-1} = 32$, one of these will be equal to 1, based on what leads to the minimum total cost (see footnote 1 for further explanation). Also note that equation (20) and (22) ensures that the logical constraint also applies from one day to another. The same approach will be used in all logical constraints where the next (h+1) or previous hour (h-1) is involved.

(19) $y_{d,h-1} \geq 32(1 - m_{d,h}) \quad \forall d \in D, h \in H: h > 1$

(20) $y_{d-1,24} \geq 32(1 - m_{d,1}) \quad \forall d \in D: d > 32$

(21) $y_{d,h-1} - 32 \leq 64r_{d,h} \quad \forall d \in D, h \in H: h > 1$

(22) $y_{d-1,24} - 32 \leq 64r_{d,1} \quad \forall d \in D: d > 32$

(23) $m_{d,h} + r_{d,h} = 1 \quad \forall d \in D, h \in H$

Constraints (24)-(28) ensures that $t_{d,h}$ must take the value 1 if $y_{d,h-1} < 51.2$ and that $j_{d,h}$ must take the value 1 if the $y_{d,h-1} > 51.2$. In the same manner as in constraints (19)-(23), if $y_{d,h-1} = 51.2$, one of these will be equal to 1, based on what leads to the minimum total cost.

(24) $y_{d,h-1} \geq 51.2(1 - t_{d,h}) \quad \forall d \in D, h \in H: h > 1$

(25) $y_{d-1,24} \geq 51.2(1 - m_{d,1}) \quad \forall d \in D: d > 32$

(26) $y_{d,h-1} - 51.2 \leq 64r_{d,h} \quad \forall d \in D, h \in H: h > 1$

(27) $y_{d-1,24} - 51.2 \leq 64r_{d,1} \quad \forall d \in D: d > 32$

(28) $t_{d,h} + j_{d,h} = 1 \quad \forall d \in D, h \in H$

Constraints (29) and (30) ensures that $c_{d,h}$ takes the value 1 if $x_{d,h}$ > 0 and zero if $x_{d,h}$ = 0:

(29) $x_{d,h} \leq x^{MAX} c_{d,h} \quad \forall d \in D, h \in H$

(30) $x_{d,h} \geq c_{d,h} \quad \forall d \in D, h \in H$

Constraints (31)-(32) ensures that $l_{d,h}$ takes the value 1 only if the car goes from not charging to charging:

(31) $2c_{d,h} - c_{d,h-1} \leq 1 + l_{d,h} \quad \forall d \in D, h \in H: h > 1$

(32) $2c_{d,1} - c_{d-1,24} \leq 1 + l_{d,1} \quad \forall d \in D: d > 32$

(33) $l_{d,h} \leq 1 - c_{d,h-1} \quad \forall d \in D, h \in H: h > 1$

(34) $l_{d,1} \leq 1 - c_{d-1,24} \quad \forall d \in D: d > 32$

(35) $l_{d,h} \leq c_{d,h} \quad \forall d \in D, h \in H$

Constraints(36)-(38) ensures that $s_{d,h}$ takes the value 1 if a charging event starts and 0 otherwise:

(36) $l_{d,h} + m_{d,h} \leq 1 + s_{d,h} \quad \forall d \in D, h \in H$

(37) $s_{d,h} \leq l_{d,h} \quad \forall d \in D, h \in H$

(38) $s_{d,h} \leq m_{d,h} \quad \forall d \in D, h \in H$

Constraints (39)-(43) ensures that $o_{d,h}$ takes the value 1 only if the car goes from charging to not charging:

(39) $2c_{d,h-1} - c_{d,h} \leq 1 + o_{d,h} \quad \forall d \in D, h \in H: h > 1$

(40) $2c_{d-1,24} - c_{d,1} \leq 1 + o_{d,1} \quad \forall d \in D: d > 32$

(41) $o_{d,h} \leq 1 - c_{d,h} \quad \forall d \in D, h \in H: h > 1$

(42) $o_{d,h} \leq c_{d,h-1} \quad \forall d \in D, h \in H: h > 1$

(43) $o_{d,1} \leq c_{d-1,24} \quad \forall d \in D: d > 32$

Constraints (44)-(46) ensures that $e_{d,h}$ takes the value 1 if a charging event stops and 0 otherwise:

(44) $o_{d,h} + j_{d,h} \leq 1 + e_{d,h} \quad \forall d \in D, h \in H$

(45) $e_{d,h} \leq o_{d,h} \quad \forall d \in D, h \in H$

(46) $e_{d,h} \leq j_{d,h} \quad \forall d \in D, h \in H$

Constraints (47) and (48) ensures that $f_{d,h}$ takes the value 1 if there is a charging event and 0 otherwise:

(47) $f_{d,h} = f_{d,h-1} + s_{d,h} - e_{d,h} \quad \forall d \in D, h \in H: h > 1$

(48) $f_{d,1} = f_{d-1,24} + s_{d,1} - e_{d,1} \quad \forall d \in D: d > 32$

Constraints (49)-(54) ensures that the car must charge 7.5 all hours of the charging event, except the last one:

(49) $2f_{d,h} - e_{d,h+1} \leq 1 + p_{d,h} \quad \forall d \in D, h \in H: h < 24$

(50) $2f_{d,24} - e_{d+1,1} \leq 1 + p_{d,24} \quad \forall d \in D : d < 87$

(51) $p_{d,h} \leq 1 - e_{d,h+1} \quad \forall d \in D, h \in H: h < 24$

(52) $p_{d,24} \leq 1 - e_{d+1,1} \quad \forall d \in D : d < 87$

(53) $p_{d,h} \leq f_{d,h} \quad \forall d \in D, h \in H$

(54) $x_{d,h} \leq x^{MAX} p_{d,h} \quad \forall d \in D, h \in H$

The full model implemented in AMPL can be found in the attached folder, in the subfolder "PartD" with the name "Proj2.4D" followed by ".mod", ".dat", and ".run". The results are stored in the file called "results4.txt". The problem is solved using gurobi. Please note that it will require some time to run the model due to the large number of variables and constraints. The optimal cost is €123.497, with 484 kWh of charging in total. The average price incurred by the optimal solution (in EUR/mWh) is then:

$EUR/mWh_{AVG} = \frac{123.497}{0.484} = 255.16$

The total cost is lower than model 2, because the time period is shorter. The average price (in EUR/mWh) is higher than all the previous minimization problems. Figure 5 shows a similar plot as in problem D2, where we observe that the hours that Optidriver charge seems to be quite evenly distributed throughout the period. This makes sense, as he needs to continuously charge from a state-of-charge of 32 kWh or less to a state-of-charge of at least 51.2 kWh. It therefore follows that Optidriver does not have the same flexibility to charge at the lowest prices as in the previous models. This also explains the higher cost and average price. We do observe that the price falls below €0.05 some hours towards the end of the period, indicated by the vertical red dashed line. As expected, there are many points at these lines. However, the negative correlation between price and kWh charged does not seem to be as strong in this model as in previous models.

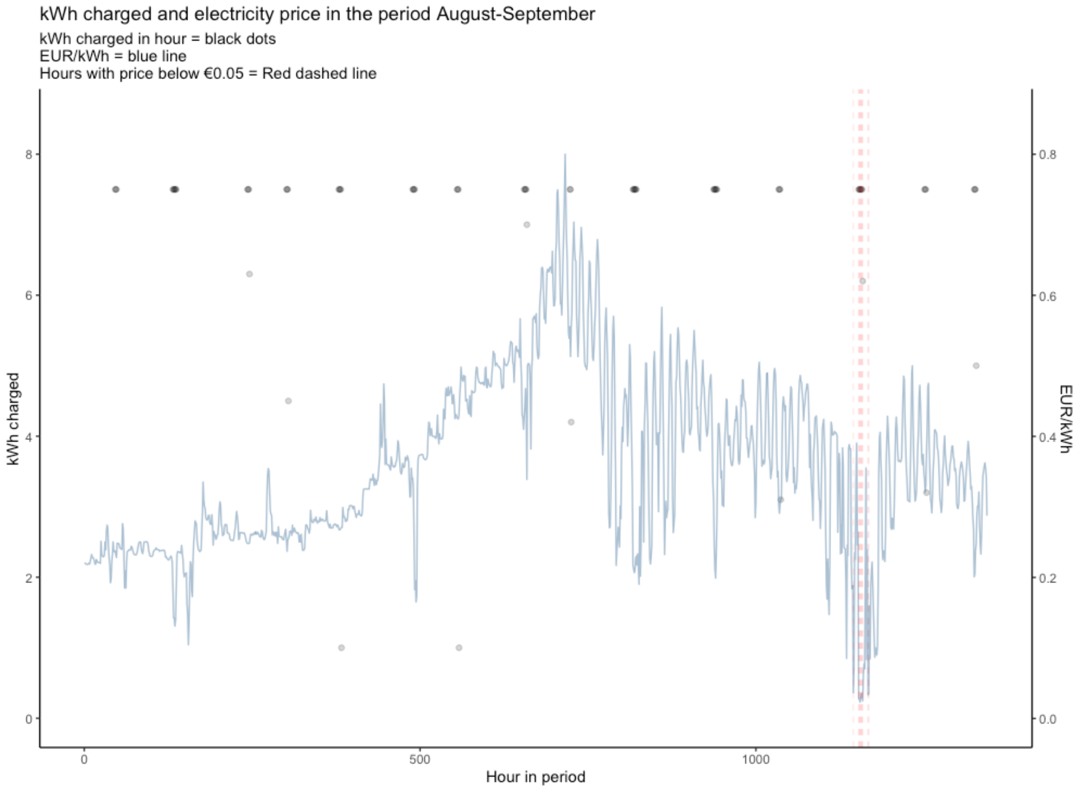

Figure 4: kWh Charged and Electricity Price in the Period August-September

Figure 5 shows the state-of-charge throughout the period, with the same price plot as in Figure 4. We observe that Optidriver never starts charging his car when state-of-charge is above 32 kWh (50% of full capacity), which is indicated by the red horizontal line. Subsequently, once a charging event starts, it does not stop before the state-of-charge is at least 51.2 kWh (80% of full capacity). Note that the negative correlation between price and state-of-charge that we have previously seen is less prominent. However, we still observe this tendency, especially when the price falls significantly around hour 800 and 1200.

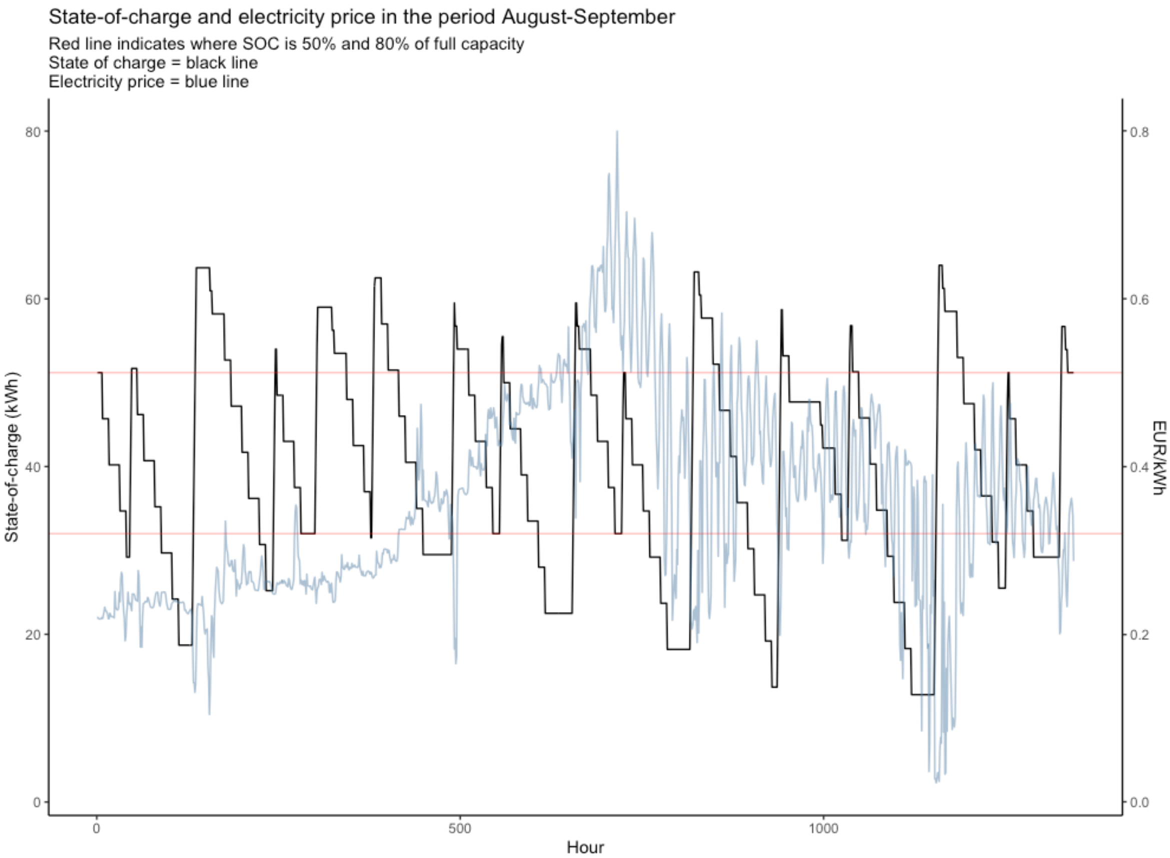

Figure 4: State-of-charge and Electricity Price in the Period August-September