1 Theory

1.1 The Earth-Sun system

Before starting coding with object orientation, we will look at the problem by simply using Eulers forward method and something starting with V.. In two dimensions we will have the following for the Earth-Sun system

The gravitational force F_G

$$F = \frac{GM_0 M_E}{r^2} \tag{1}$$

where,

$M_E = 6 \times 10^{24}\text{Kg}, M_0 = 2 \times 10^{30}\text{Kg} \quad \text{and} \quad r = 1.5 \times 10^{11}\text{m}$

The force acting on Earth is given by Newtons 2. law, here given in x- and y- direction

$$\frac{d^2x}{dt^2} = \frac{F_x}{M_E}, \quad \frac{d^2y}{dt^2} = \frac{F_y}{M_E}$$

by using the following equalities

$$x = r\cos(\theta), \quad y = r\sin(\theta), \quad \text{and} \quad r = \sqrt{x^2 + y^2}$$

we obtain

$$F_x = -\frac{GM_0 M_E}{r^2}\cos(\theta) = -\frac{GM_0 M_E}{r^3}x \tag{2}$$

$$F_y = -\frac{GM_0 M_E}{r^2}\sin(\theta) = -\frac{GM_0 M_E}{r^3}y, \tag{3}$$

This gives the following first order coupled differential equations:

$$\frac{dv_x}{dt} = -\frac{GM_0}{r^3}x \tag{4}$$

$$\frac{dx}{dt} = v_x \tag{5}$$

$$\frac{dv_y}{dt} = -\frac{GM_0}{r^3}y \tag{6}$$

$$\frac{dy}{dt} = v_y, \tag{7}$$

In order to simplify we will use Astronomical units AU defined by r, which is the avarage distance between Earth and the sun.

Introducing astronomical units: 1 AU = r = 1.5×10^{11}

$$\frac{M_e v^2}{r} = F = \frac{GM_0 M_E}{r^2} \tag{8}$$

Since $GM_= v^2 r$ and the velocity of Earth, assuming it moves in circular motion:

$$v = 2\pi \text{r}/\text{years} = 2\pi \text{AU}/\text{years}$$

Then we have the following relationship

$$GM_0 = v^2 r = 4\pi^2 \frac{(\text{AU})^2}{\text{years}^2}$$

Bulding code- discretized equations:

$$v_{x,i+1} = v_{x,i} - h\frac{4\pi^2}{r_i^3} x_i$$

$$x_{i+1} = x_i + hv_{x,i}$$

$$v_{y,i+1} = v_{xy,i} - h\frac{4\pi^2}{r_i^3} y_i$$

$$y_{i+1} = y_i + hv_{y,i}$$