1 Results

1.1 Gauss-Legendre

Solving our integral with Legendre polynomials gives unstable results for $N \in [-5, 5]$ as seen in table 1. Though with a carefull choise of N = 27 and integration limits a = -2.9 and b = 2.9 our results are precice with 4 leading digits after the decimal point.

The results from our Legandre (and Laguerre 1.2) integration program are found at: (main.exe)

Legendre		
N	Approximate integral	Error
11	0.297447	0.104681
15	0.315863	0.123098
21	0.268075	0.075310
25	0.240135	0.047370
27	0.229623	0.036858
27*	0.192725	0.000039

Table 1: Values of the integral for different N's, calculated with Gauss-Legendre. Integration limits are $x \in [-5, 5]$. *: Special case with integration limits $x \in [-2.9, 2.9]$

1.2 Gauss-Laguerre

Improving our algorithm using Legendre polynomials for angles and Laguerre polynomials for radial parts improved accuracy and stability of our results. An increase in $N \in [-5, 5]$ from N = 11 to N = 15 also gives an increase in precision, though for and higer increase the accuracy decrease slightly, which is shown in table 2.

Laguerre		
N	Approximate integral	Error
11	0.183021	0.009743
15	0.193285	0.000520
21	0.194807	0.002050
25	0.194804	0.002030
27	0.194795	0.002029

Table 2: Values of the integral for different N's, calculated with Gauss-Laguerre. Integration limits are $x \in [-5, 5]$.

1.3 Monte Carlo

1.3.1 Naïve approach

The results from our Monte Carlo integration program (`main.exe`, are listed in table 3.

Naïve Monte Carlo			
N	Approximate integral	Standard deviation	Error
10^5	0.21953065	0.154683	0.026764935
10^6	0.14149215	0.0368397	0.051273556
10^7	0.16704012	0.023165	0.025725592
10^8	0.17903453	0.00936631	0.013731177
10^9	0.19105511	0.0041004	0.0017106036

Table 3: Results from running Monte Carlo with cartesian coordinates and integration limits $x \in [-5, 5]$ - our approximation of infinity.

For higher N's, the approximated integral get closer to the actual value and the standard deviation decreases. The error ($|\text{Exact} - \text{Approximated}|$) does however not match up with the standard deviation, and oscillates a bit up and down, despite having a trend of decreasing.

1.3.2 Importance sampling

The results from our Monte Carlo integration program (main.exe), are listed in table 4.

Improved Monte Carlo			
N	Approximate integral	Standard deviation	Error
10^5	0.13773907	0.284624	0.055026645
10^6	0.19068327	0.405372	0.0020824368
10^7	0.2075781	0.381901	0.014812393
10^8	0.19459392	0.092418	0.001828214
10^9	0.20918288	0.0646068	0.016417166

Table 4: Results from running Monte Carlo with importance sampling along the exponential distribution and using spherical coordinates.

The improved Monte Carlo integration gets within a small error margin for smaller N's than the naïve, However, it over- and undershoots randomly. The trend is that the standard deviation decreases, but does not match up with the error ($|\text{Exact} - \text{Approximated}|$).

1.4 Paralellization

Our paralellization results was achieved using a quad core Intel Core i5-8250U processor with 6MB cache at 1.6GHz base clock, which boosted to 3.4GHz during testing. Thermal throttling was avoided. The memory was 4GB 2133MHz LPDDR3 soldered on board. See table 5

We also ran this test on an octa-core processor with memory of 8GB 2400MHz (12.5% faster), and achieved an additional speedup compared to the abovementioned computer. See table 6

For runtime imputs the number of samples was set to 10^8, with an approximation of infity of $\lambda = 5$.

Runtime with different optimizations				
Compile flags	-O3 -fopenMP	-O3	-fopenmp	No optimization
Naive MC	12s	31s	71s	173s
Improved MC	15s	38s	79s	200s

Table 5: Shows the time spent on the same calculations with different compile parameters on a quad core processor.($N = 10^8, \lambda = 5$)

Runtime with optimization on octa-core		
Compile flags	-O3 -fopenMP	% faster than the quad-core
Naive MC	8s	50%
Improved MC	11s	36%

Table 6: Shows the time spent on the Monte-Carlo calculations on an octa-core system.($N = 10^8, \lambda = 5$)