

# 1 Results

## 1.1 Gauss-Legendre

Solving our integral with Legendre polynomials gives unstable results for  $N \in [-5, 5]$  as seen in table 1. Though with a carefull choise of  $N = 27$  and integration limits  $a = -2.9$  and  $b = 2.9$  our results are precice with 4 leading digits after the decimal point.

The results from our Legendre (and Laguerre 1.2) integration program are found at: (main.exe)

| Legendre |                      |          |
|----------|----------------------|----------|
| N        | Approximate integral | Error    |
| 11       | 0.297447             | 0.104681 |
| 15       | 0.315863             | 0.123098 |
| 21       | 0.268075             | 0.075310 |
| 25       | 0.240135             | 0.047370 |
| 27       | 0.229623             | 0.036858 |
| 27*      | 0.192725             | 0.000039 |

Table 1: Values of the integral for different N's, calculated with Gauss-Legendre. Integration limits are  $x \in [-5, 5]$ . \*: Special case with integration limits  $x \in [-2.9, 2.9]$

## 1.2 Gauss-Laguerre

Improving our algorithm using Legendre polynomials for angles and Laguerre polynomials for radial parts improved accuracy and stability of our results. An increase in  $N \in [-5, 5]$  from  $N = 11$  to  $N = 15$  also gives an increase in precision, though for and higer increase the accuracy decrease slightly, which is shown in table 2.

| Laguerre |                      |          |
|----------|----------------------|----------|
| N        | Approximate integral | Error    |
| 11       | 0.183021             | 0.009743 |
| 15       | 0.193285             | 0.000520 |
| 21       | 0.194807             | 0.002050 |
| 25       | 0.194804             | 0.002030 |
| 27       | 0.194795             | 0.002029 |

Table 2: Values of the integral for different N's, calculated with Gauss-Laguerre. Integration limits are  $x \in [-5, 5]$ .

## 1.3 Monte Carlo

### 1.3.1 Naïve approach

The results from our Monte Carlo integration program (`main.exe`), are listed in table 3.

| Naïve Monte Carlo |                      |                    |              |
|-------------------|----------------------|--------------------|--------------|
| N                 | Approximate integral | Standard deviation | Error        |
| $10^5$            | 0.21953065           | 0.154683           | 0.026764935  |
| $10^6$            | 0.14149215           | 0.0368397          | 0.051273556  |
| $10^7$            | 0.16704012           | 0.023165           | 0.025725592  |
| $10^8$            | 0.17903453           | 0.00936631         | 0.013731177  |
| $10^9$            | 0.19105511           | 0.0041004          | 0.0017106036 |

Table 3: Results from running Monte Carlo with cartesian coordinates and integration limits  $x \in [-5, 5]$  - our approximation of infinity.

For higher  $N$ 's, the approximated integral get closer to the actual value and the standard deviation decreases. The error ( $|\text{Exact} - \text{Approximated}|$ ) does however not match up with the standard deviation, and oscillates a bit up and down, despite having a trend of decreasing.

### 1.3.2 Importance sampling

The results from our Monte Carlo integration program (`main.exe`), are listed in table 4.

| Improved Monte Carlo |                      |                    |              |
|----------------------|----------------------|--------------------|--------------|
| N                    | Approximate integral | Standard deviation | Error        |
| $10^5$               | 0.13773907           | 0.284624           | 0.055026645  |
| $10^6$               | 0.19068327           | 0.405372           | 0.0020824368 |
| $10^7$               | 0.2075781            | 0.381901           | 0.014812393  |
| $10^8$               | 0.19459392           | 0.092418           | 0.001828214  |
| $10^9$               | 0.20918288           | 0.0646068          | 0.016417166  |

Table 4: Results from running Monte Carlo with importance sampling along the exponential distribution and using spherical coordinates.

The improved Monte Carlo integration gets within a small error margin for smaller  $N$ 's than the naïve, However, it over- and undershoots randomly. The trend is that the standard deviation decreases, but does not match up with the error ( $|\text{Exact} - \text{Approximated}|$ ).

## 1.4 Paralellization

Our paralellization results was achieved using a quad core Intel Core i5-8250U processor with 6MB cache at 1.6GHz base clock, which boosted to 3.4GHz during testing. Thermal throttling was avoided. The memory was 4GB 2133MHz LPDDR3 soldered on board. See table 5

We also ran this test on an octa-core processor with memory of 8GB 2400MHz (12.5% faster), and achieved an additional speedup compared to the abovementioned computer. See table 6

For runtime inputs the number of samples was set to  $10^8$ , with an approximation of infity of  $\lambda = 5$ .

| Runtime with different optimizations |              |     |          |                 |
|--------------------------------------|--------------|-----|----------|-----------------|
| Compile flags                        | -O3 -fopenMP | -O3 | -fopenmp | No optimization |
| Naive MC                             | 12s          | 31s | 71s      | 173s            |
| Improved MC                          | 15s          | 38s | 79s      | 200s            |

Table 5: Shows the time spent on the same calculations with different compile parameters on a quad core processor. ( $N = 10^8, \lambda = 5$ )

| Runtime with optimization on octa-core |              |                             |
|----------------------------------------|--------------|-----------------------------|
| Compile flags                          | -O3 -fopenMP | % faster than the quad-core |
| Naive MC                               | 8s           | 50%                         |
| Improved MC                            | 11s          | 36%                         |

Table 6: Shows the time spent on the Monte-Carlo calculations on an octa-core system. ( $N = 10^8, \lambda = 5$ )