

Project 5

Anna Stray Rongve
Knut Magnus Aasrud
Amund Midtgard Raniseth

December 2, 2019

1 Theory

1.1 The Earth-Sun system

Before starting coding with object orientation, we will look at the problem by simply using Eulers forward method and something starting with V.. In two dimensions we will have the following for the Earth-Sun system

The gravitational force F_G

$$F = \frac{GM_0M_E}{r^2} \quad (1)$$

where,

$$M_E = 6 \times 10^{24} \text{Kg}, M_0 = 2 \times 10^{30} \text{Kg} \text{ and } r = 1.5 \times 10^{11} \text{m}$$

In order to simplify we will use Astronomical units AU(bold) defined by r , which is the average distance between Earth and the sun.

The force acting on Earth is given by Newtons 2. law, here given in x- and y- direction

$$\frac{d^2x}{dt^2} = \frac{F_x}{M_E}, \quad \frac{d^2y}{dt^2} = \frac{F_y}{M_E}$$

by using the following equalities

$$x = r \cos(\theta), \quad y = r \sin(\theta), \quad \text{and} \quad r = \sqrt{x^2 + y^2}$$

we obtain

$$F_x = -\frac{GM_0M_E}{r^2} \cos(\theta) = -\frac{GM_0M_E}{r^3} x \quad (2)$$

$$F_y = -\frac{GM_0M_E}{r^2} \sin(\theta) = -\frac{GM_0M_E}{r^3} y, \quad (3)$$

First order coupled differential equations:

$$\frac{dv_x}{dt} = -\frac{GM_0}{r^3} x \frac{dx}{dt} = v_x, \frac{dv_y}{dt} = -\frac{GM_0}{r^3} y \frac{dy}{dt} = v_y, \quad (4)$$

Introducing astronomical units: $1 \text{ AU} = 1.5 \times 10^{11}$, $r = 1\text{AU}$ $\frac{M_e v^2}{r} = F = \frac{GM_0M_E}{r^2}$,
have that $GM_0 = v^2 r$

Velocity of Earth, assuming it moves in circular motion: $v = 2\pi \text{AU} / \text{years} = 2\pi \text{AU} / \text{years}$

$$GM_0 = v^2 r = 4\pi^2 \frac{(\text{AU})^2}{\text{years}^2}$$

Bulding code- discretized equations: $v_{x,i+1} = v_{x,i} - h \frac{4\pi^2}{r_i^3} x_i$,

$$x_{i+1} = x_i + h v_{x,i},$$

$$v_{y,i+1} = v_{y,i} - h \frac{4\pi^2}{r_i^3} y_i,$$

$$y_{i+1} = y_i + h v_{y,i},$$

2 Results

3 Appendix