Project 1 b)

We have a linear set of equations $\mathbf{Au = f}$ The matrix A can be rewritten as

$\mathbf{A}=\left[\begin{matrix}b_1 & c_1 & 0 & \cdots & \cdots & \cdots\a_2 & b_2 & c_2 & 0 & &\0 & a_3 &b_3 & c_3 & 0 &\\vdots&\vdots & \ddots& \ddots&\ddots &\vdots\0 & & & a_{n-1} & b_{n-1} & c_{n-1}\0 & & & & a_{n} & b_n\end{matrix}\right]$

where $a_, , b , \textrm{and} , c$ are one-dimensional arrays with lenght $1:n$. $a_i = c_i = -1/h^2 \textrm{and} , b_i = 2$

Then we can write the linear set of equations as

$\mathbf{A}=\left[\begin{matrix}b_1 & c_1 & 0 & \cdots & \cdots & \cdots\a_2 & b_2 & c_2 & 0 & &\0 & a_3 &b_3 & c_3 & 0 &\\vdots&\vdots & \ddots& \ddots&\ddots &\vdots\0 & & & a_{n-1} & b_{n-1} & c_{n-1}\0 & & & & a_n & b_n\end{matrix}\right] \left[\begin{matrix}u_1\u_2\ \cdots\\cdots\\cdots\u_n\end{matrix}\right] = \left[\begin{matrix}f_1\f_2\ \cdots\\cdots\\cdots\f_n\end{matrix}\right]$

In the 4×4 case you will get

$\mathbf{A}=\left[\begin{matrix}b_1 & c_1 & 0 & 0 \a_2 & b_2 & c_2 & 0 &\0 & a_3 &b_3 & c_3 &\ 0 & 0 & a_4 & b_4\end{matrix}\right]$

If you apply Gaussian elimination by $\textrm{II}- \frac{a_1\cdot \textrm{I}}{b_1}$ you will get

$\mathbf{A}=\left[\begin{matrix}b_1 & c_1 & 0 & 0 \0 & b_2-\frac{a_2c_1}{b_1} & c_2 & 0 &\0 & a_3 &b_3 & c_3 &\ 0 & 0 & a_4 & b_4\end{matrix}\right]$

Then we put $\tilde b_2 = b_2-\frac{a_2c_1}{b_1}$, wich gives

$\mathbf{A}=\left[\begin{matrix}b_1 & c_1 & 0 & 0 \0 & \tilde b_2 & c_2 & 0 &\0 & a_3 &b_3 & c_3 &\ 0 & 0 & a_4 & b_4\end{matrix}\right]$

If we do the same for row III and IV we will get

$\mathbf{A}=\left[\begin{matrix}b_1 & c_1 & 0 & 0 \0 & \tilde b_2 & c_2 & 0 &\0 & 0 & \tilde b_3 & c_3 &\ 0 & 0 & 0 & \tilde b_4\end{matrix}\right]$