

1 Theory

1.1 Ising model

The Ising model is applied for the study of phase transitions at finite temperatures for magnetic systems. Energy is expressed as:

$$E = -J \sum_{\langle kl \rangle}^N s_k s_l \quad s_k = \pm 1 \quad (1)$$

N is the number of spins and J is a constant expressing the interaction between neighboring spins. The sum is over the nearest neighbours only, indicated by $\langle kl \rangle$ in the above equation. For $J < 0$ it is energetically favorable for neighboring spins to align. Leading to, at low temperatures, T, spontaneous magnetisation.

A probability distribution is needed in order to calculate the mean energy $\langle E \rangle$ and magnetization $\langle M \rangle$ at a given temperature. The distribution is given by:

$$P_i(\beta) = \frac{1}{M} \sum_{i=1}^M s_k s_l \exp -\beta E_i, \quad (2)$$

where M is all the microstates, P_i is the probability of having the system in a state/configuration i.