36-402 Final Exam

Akaash Mungale

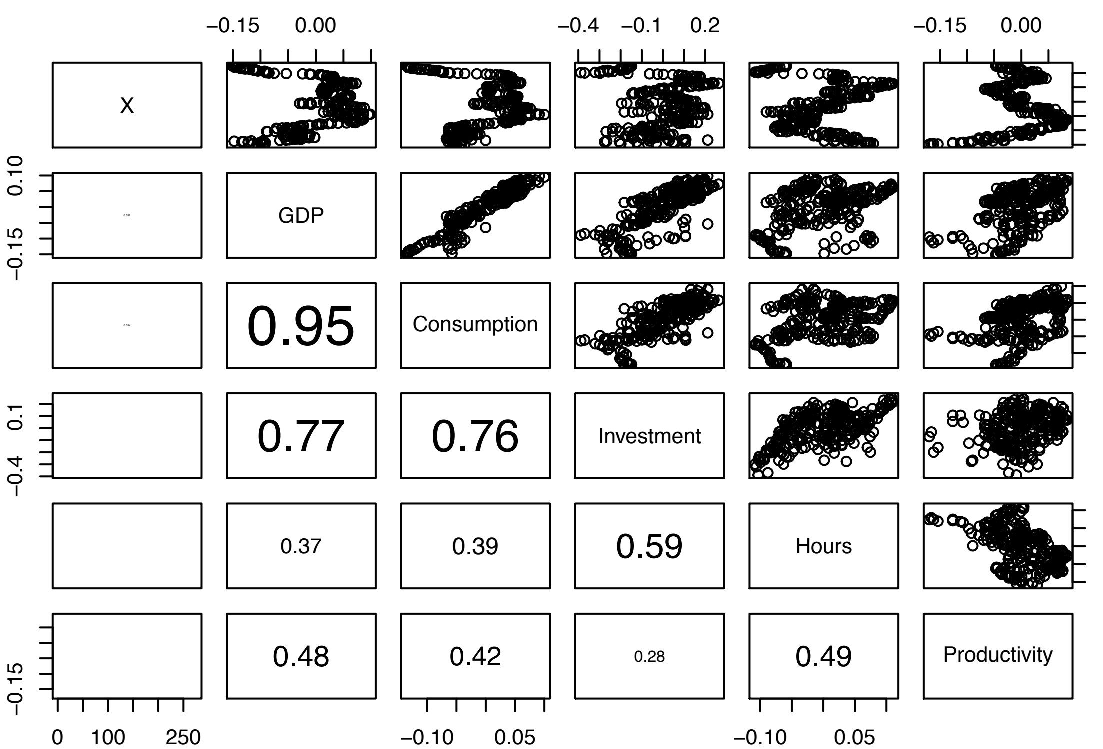
May 9, 2016

Introduction

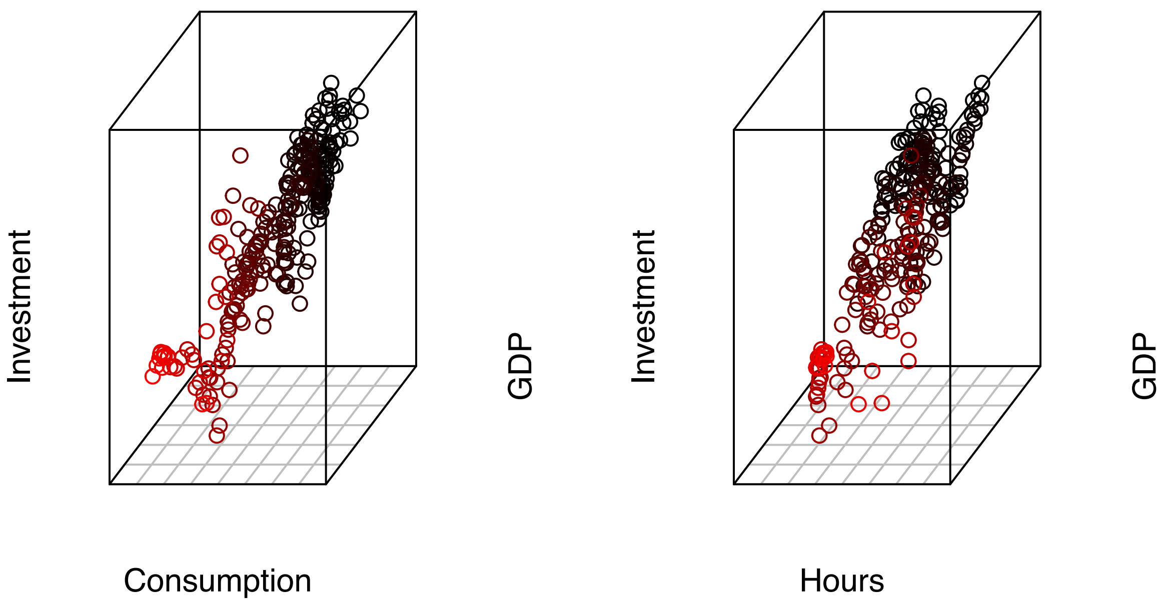
We are exploring the potential relationship between productivity, work, investment, GDP, and consumption. Specifically we will be testing the theory that when productivity rises people should work less and invest more to lead to higher consumption in the future (and vice versa). We will be working with time series of GDP, Consumption, Investment, Hours, and Productivity based on their logged fluctuations from the their long run economic trends. The data ranges from March 1947 to March 2016 - generally we will fit models from 1947 through 2005, and assess these modesl against the data from 2006 to 2016. Throughout the analysis, it is important to remember we are working with detrended fluctuations of each and not values.

Prelimary Examinations

We first consider a pairs plot with correlations - we cannot conclude anything from this plot but it gives a visual representation of the relationship between all the variables. We can see that GDP shows a generally increasing trend with each of the other variables, and that there is no clear trend between time and the other variables. The highest correlation is between GDP and Consumption, and the next higest between GDP and Investment - this makes sense since GDP is partially defined as the sum of Consumption and Investment. We are also interested in the relationship between productivity and the other variables: we see weaker trends. The generally increasing trend between productivity and GDP is intuitive. However, it is interesting to see a weak decreasing trend between productivity and hours worked and a weak increasing trend in Productivity and Consumption.



We are cautious in identifying correlations because detrending data can create correlations that did not exist in the original data. Instead, we use these observations to identify potential interaction terms we can formally test in the form of models. We further consider the relationship of potential interaction terms by plotting them in a 3 dimensional space, to see if there is an interacting effect between two predictor variables against GDP as the third variable. Two of the interesting plots are shown below: the colors from red to black help label the increasing values in GDP fluctions.



1. Predict GDP by Values at time t

We want to fit a model for GDP using the data through 2005. We will consider categories of models: 1. A simple linear multiple regression model, because if the model actually fits well it is easy to work with, intrepret, and predict with. 2. An additive model to consider higher order predictor variable terms, while still maintaining a level interpretability. 3. A completely non parametric model.

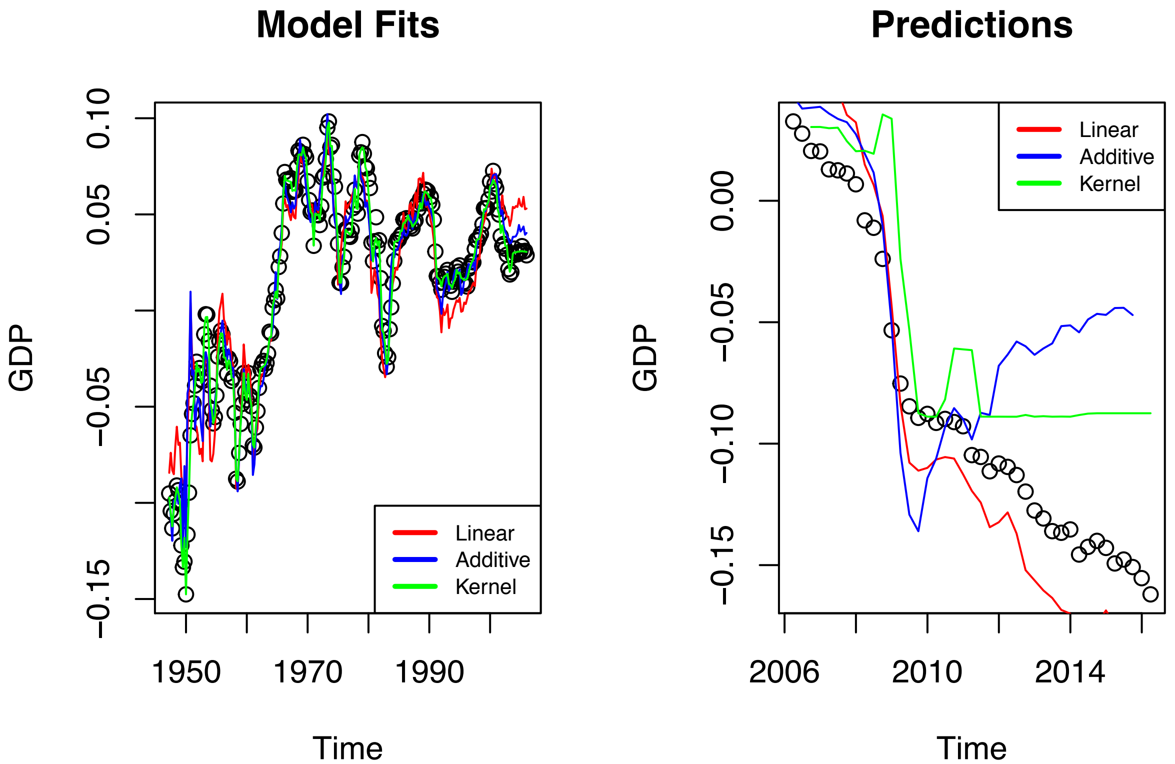
We start by considering a basic linear multiple regression model. We then aslo consider the model with a potential Investment and Hours interaction term as considered above. We consider the difference in the two models with ANOVA using a Chi Squared test, and we fail to reject the null hypothesis with a p-value of 0.1629: we do not include the interaction term. We repeat with other potential interaction term, Consumption and Investment, and reject the null hypothesis with a p-value of 3.152e-06. Therefore, we decide to keep the model with this interaction term. This model gives us consumption as the most important predictor of GDP fluctuation, with a coefficient of 0.918474. This is intuitive given the definition of GDP.

This model has a leave one out cross validated MSE of 0.00034, compared to 0.00037 for the basic linear model without any interaction terms.

Next, we fit the additive model. We keep the interaction term, since the additive model will be returning functions of each predictor and GDP - if there was a relationship between the interaction term and GDP in a linear model then a function of that interaction could still have a relationship with GDP. However, we decide to remove this interaction after an ANOVA test in model difference with a p-value 0.07. We measure uncertainty by the prediction error critereon of the GAM, Genearlized Cross Validation: the error is 0.00021.

Finally, we consider a completely non parametric model, using Kernel Regression. Note we do not include interaction terms here because Kerenl Regression accounts for interactions between the variables. To measure uncertainity, we use fval staistic in the bandwiths component of our Kernel model, because it gives the leave one out cross validated MSE for the best set of bandwiths used for the model. This MSE is 0.00017.

Now we have three regression models for GDP as a funciton of the other four variables at time t: a basic linear model with interaction between Consumption and Investment, a general additive model without interaction, and a kernel regression model. The fitted values against the original values are shown in the plot below.

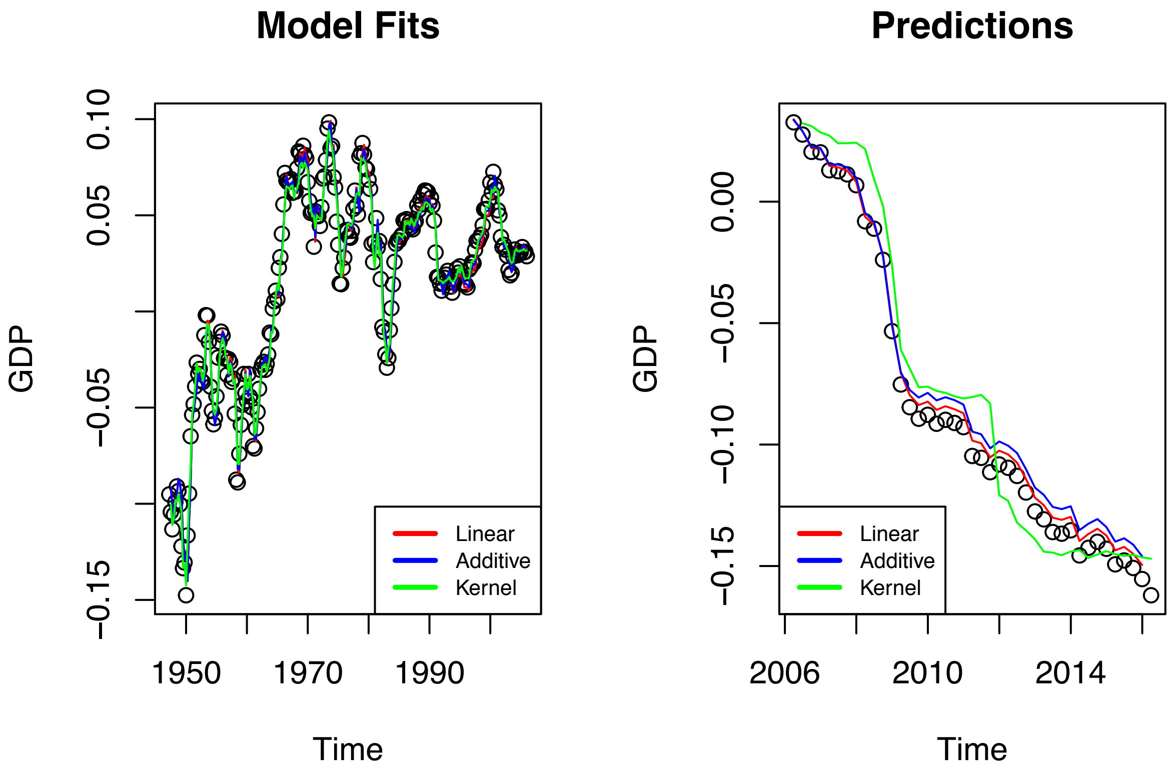


We can see that each of the models generally models the original data. We look at measures of uncertainty, but it is clear that Kernel Regression gives the best model in this case, which is consistent with what we found with our measures of uncertainty. This makes sense, since the model's structure is based on the data: we can't infer any relative relationships between the predictors and GDP.

However, as shown above, our non parametric model is the worst at predicting the future GDP fluctuations, and almost always overestimates. The linear model seems to best predict the future quarters not used in genearting it. We can compare in model MSEs using residuals to see their relative performance in predicting GDP over the two different time periods. The in model MSE for the linear model roughly doubles from 0.0003204592 to 0.0006716428 (through 2005 and after respectively). For the Kernel model, the MSE increases from 5.867195e-06 to 0.00114382, which is almost 200 times larger. So although the Kernel model is the best for the time periods used to build the model, it is much more innacurate for predicting future models and its inaccuracy increases by several orders of magnitude. After 2010, all the predictions deviate from the true values, so we still keep our non parametric model overall.

2. Predict GDP by Previous Quarter, excluding Productivity

We follow the approach of our previous question, considering the three categories of models, but instead regress GDP at time t on all the variables except productivty at time t-1. We keep the same potential interaction terms, and using the same ANOVA Chi Squared test methods, we find that we do not improve the linear model with interaction terms, but we do improve the GAM by including the interaction between Investment and Consumption. Using the same measures of uncertainity and justifications from part 1, for the three respective models we get MSE values of 9.3e-05, 9.1e-05, and 9.8e-05. Our general additive model has the lowest MSE. We consider the models' fits and predictions on the data in the graphs below, and it is evident that our non parametric regression is no longer clearly the best fit to the data.



When using the previous quarter to predict the current quarter GDP, all three fits improve significantly. Interestingly, our predictions also improve significantly. This is evident just by comparing the plots from the previous models. We also see that our simple linear and general additive models perform the best in general, both backed by our MSE statistics and plots.

This tells us that GDP can be predicted more effectively by data from the previous quarter (including the previous quarter's GDP) than it can just from Consumption, Investment, Hours, and Productivity levels for the quarter we are examining.

Of cousre we would prefer to use our simple linear model, but we formally test it against our additive to see if it is truly better. To do this, we bootstrap to find a confidence interval for the difference in MSEs between the two models. Note, we cannot directly compare the GCV error for the GAM and the LOOCV MSE for the linear model. Instead we recompute the MSE for each model on the entire data set to account for both model fit and predictions. We cannot randomly resample from the data either, because clearly as shown through our predictions, the values are highly time dependent. Instead we will block bootstrap with block lengths of 24 quarters.

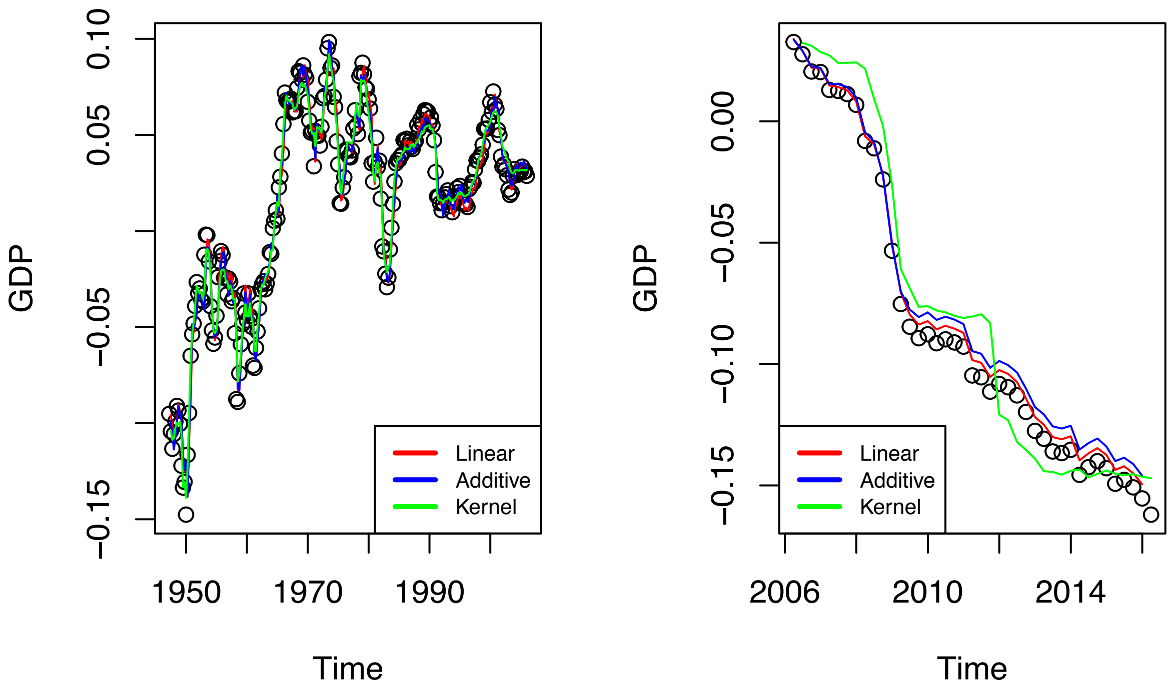
Specifically we 1. simulate the data through 2005 500 times 2. refit our linear and additive models to each 3. calculate the MSEs based on their predictions for the entire dataset 4. return (MSE linear model) - (MSE additive model) to see if the linear model truly performs worse. The summary statistics for the 500 MSE differences are given below.

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## 9.278e-07 1.987e-05 2.726e-05 2.930e-05 3.720e-05 8.623e-05
```

Every single simulated linear model performs worse than the respective additive model, so in our analysis so far our best model for GDP at time t is usng an additive model, where GDP of the previous quarter is by far the most important predictor (given by ANOVA significance test, with a p-value $<$ 2e-16).

3. Predict GDP by Previous Quarter, including Productivity

We continue with our 3 models, but we now add the predictor Productivity to each model. The three respective uncertainty measurements change to 9.2e-05, 8.6e-05, and 9.1e-05. Although our non parametric model improves the most, our additive model still appears to be the best.

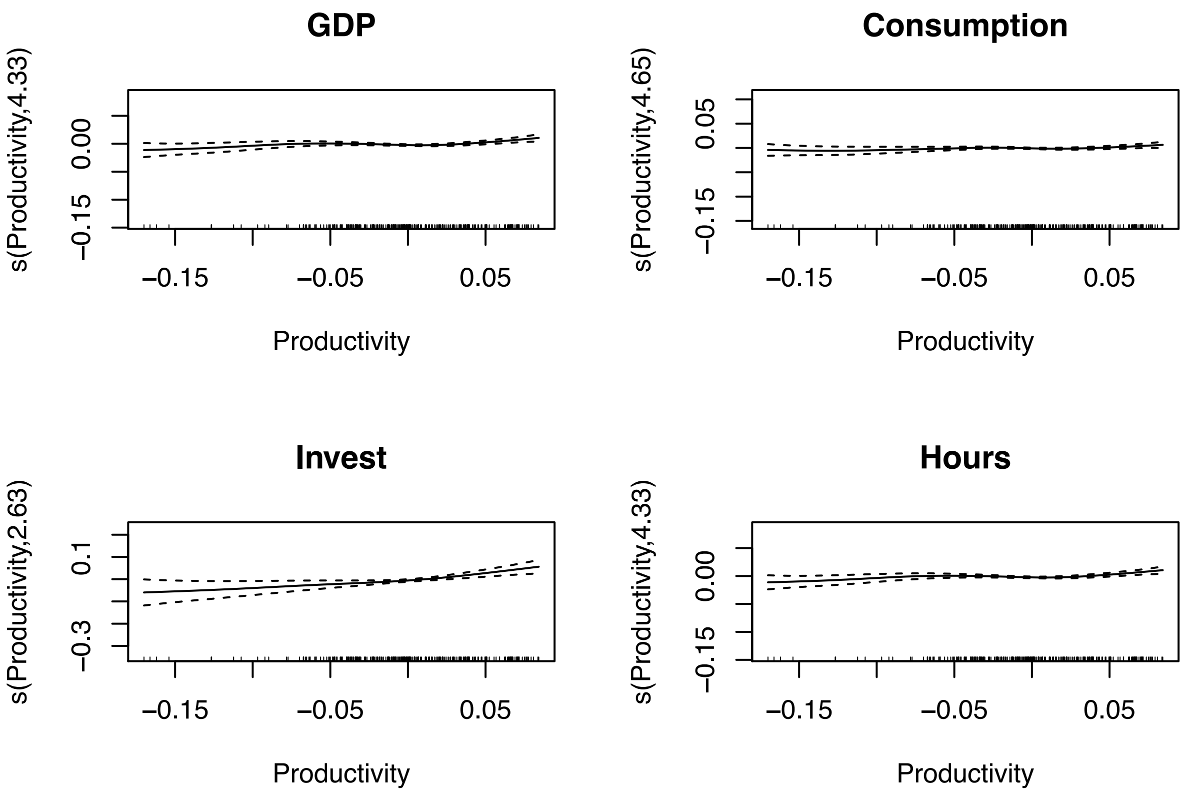


All of the models including productivity tend to fit the GDP fluctuations better. This tells us that productvity at time t-1 is a predictor of the GDP at time t. We confirm this through an ANOVA test of significance - productivity becomes the second most significant predictor with a p-value of 0.00094, while GDP remains most important with a p-value < 2e-16.

4. Additive Regressions for non productivity variables

We are examining the theory that productivity is an exogenous variable - so we fit four different additive models to predict GDP, Consumption, Investment, and Hours by the all variable values from the previous quarter. We consider uncertainty by the prediction error critereon of the GAM, GCV. These values are shown below, along with each of the partial response functions for Productivity, on the scale of Productivity.

```
##          GDP  Consumption       Invest        Hours
## 8.566900e-05 6.249737e-05 2.106035e-03 8.566900e-05
```



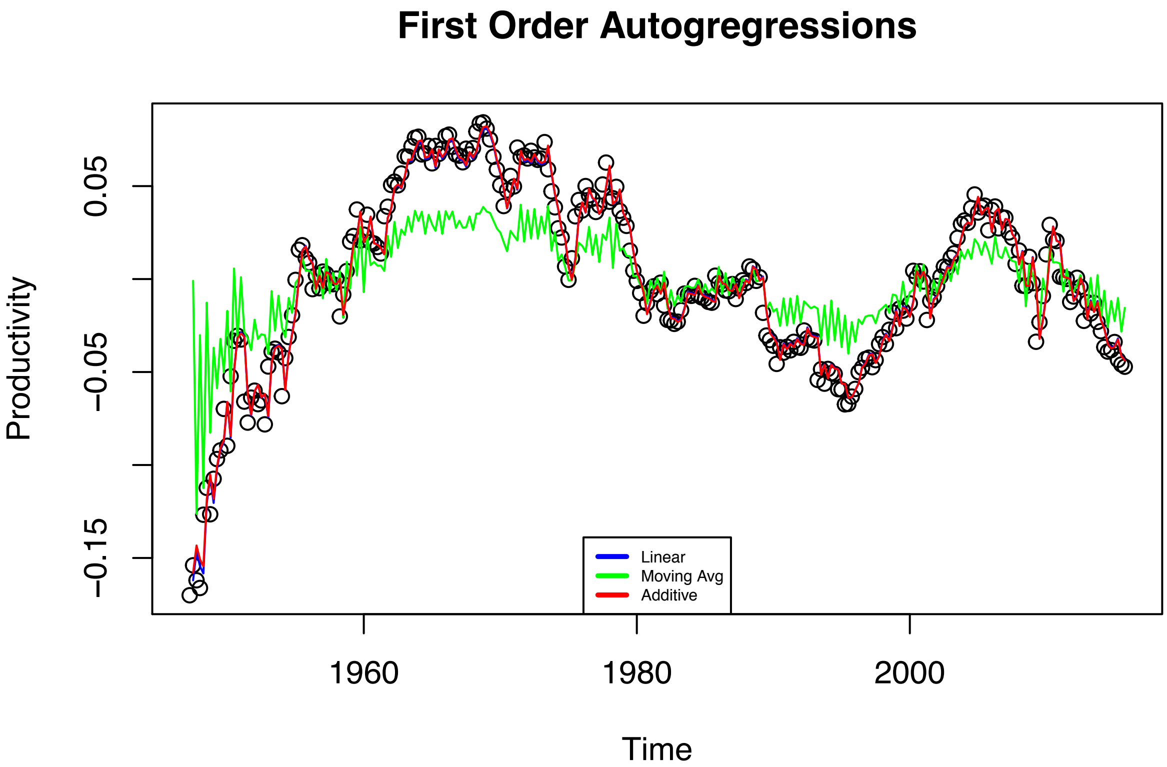
If productivity was truly an exogenous variable, each of the four models would show productivity of the previous quarter as a significant predictor of the current quarter's value. A partial plot centered around 0 would mean that productivity had little effect - so it seems like only the fluctuations in consumption are not determined by productivity. We also check this by hypothesis testing each of the four models. We find that Productivity is a significant predictor of GDP, Invest, and Hours, using a signifance level of 0.005.

So in our analysis so far, we would modify the theory that Productivity drives Consumption, but we do believe Productivity does matter in determining the other variables.

5. Auto regressive Model

We want to test and hopefully improve the theory's model for productivity by fitting a first order autoregressive model on the entire data set. We consider first order autoregression models and work within the Gaussian family because of our continuous positve and negative productivity values (the other families, e.g. binomial or Poisson would be inappropriate). The linear first order autoregressive model gives a MSE of 9.78e-05, and fits the data points well. We can consider this a good baseline model. We then try the first order autoregressive moving average model by both Least Squares Estimation and Maximum Likelihood estimation. The MLE does not perform well at all with an MSE of 0.0001, compared to the Least Squares first order autoregressive moving average model, which improves with an MSE of 9.77e-05. However, we can improve the baseline model more by considering the nonlinear first order additive model, for which we now get an MSE of 9.65e-05. We choose this model.

The theory model and our final are shown with the true Production fluctuations, also overlayed with the moving average curve.

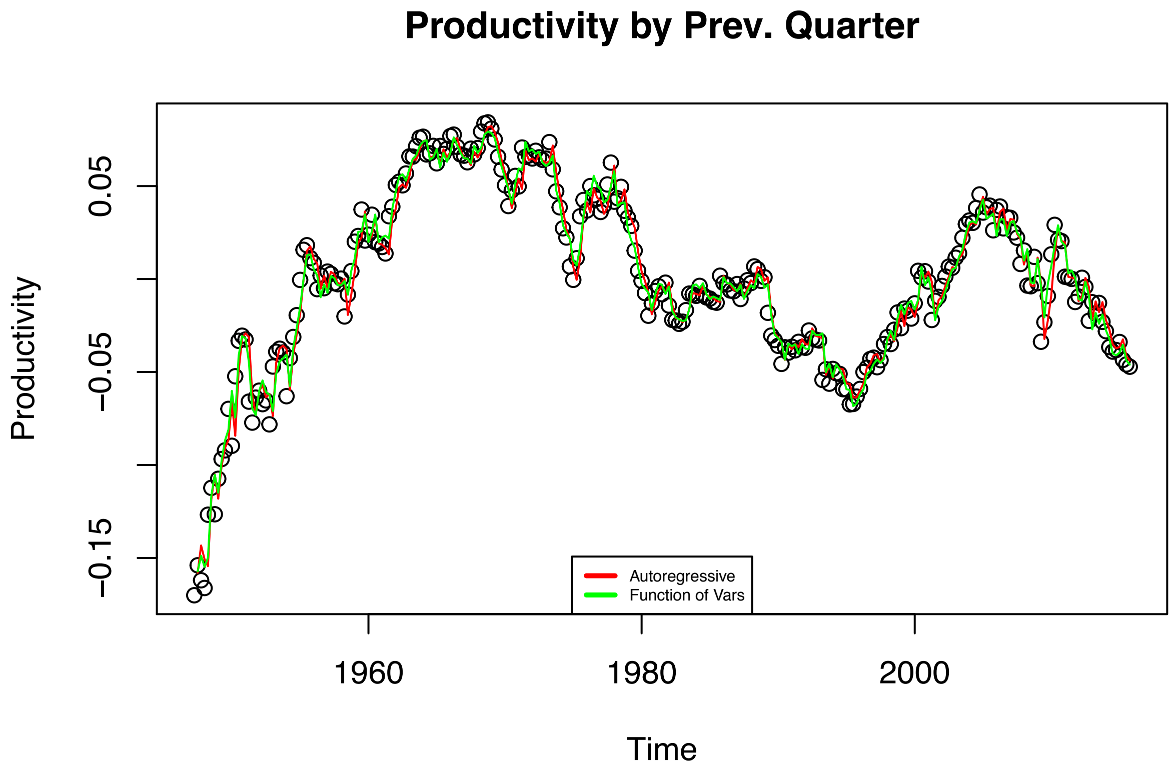


6. Predict Productivity based on all previous quarter data

We compare the purely autoregressive model for Productivity with a function estimating Productivity fluctuations by all our variable fluctuations from the previous quarter.

As per our method throughout this exam, we consider a simple linear model, an additive model, and a non parametric model, which give respective errors of 9.9e-05, 8.5e-05, and 8.5e-05.

Below is the plot of Productivty fluctations again below, now overlayed with our additive autoregressive model and our additive functional model.



We formally test for the difference in predictive capabilities using a block bootstrap of length 24. Again, we 1. simulate the data through 500 times 2. refit our autoregressive and functional models to each 3. return (MSE autoregressive model) - (MSE additive model) to see if the functional model truly performs better The summary statistics for the 500 MSE differences are given below.

```
## [1] 0.482
```

```
##       Min.    1st Qu.     Median       Mean    3rd Qu.       Max.
## -1.694e-04 -4.081e-05 -3.105e-06 -2.280e-06  3.377e-05  2.089e-04
```

The statistic 0.482 tells us that less than half of the trials in our bootstrap gave additive autoregressive models with higher MSE than the additive functional models. As we can also see from the range of differences, the MSE differences were close to 0. Given this information, we cannot conclude that the Autoregressive model performs any worse than the functional model.

7. Productivity as an Exogenous Variable

Let's examine the entire analysis to consider whether "exogenous changes in productivity are the main driver of the macroeconomic fluctuations".

Through this analysis, we have found evidence in support of this statement, with certain exceptions.

In the first part of our analysis, we found that GDP was not best predicted by variables in the same quarter, including productivity. When we moved onto the second and third parts of our analysis we found that GDP was much better predicted by data from the previous quarters, and the predictions improved once we added in Productivity. We supported this claim with a hypothesis test indicating that the previous quarter Productivity was a significant predictor of GDP, but second to the previous quarter GDP.

So up to this point, we found that for a given quarter, Productivity of the previous quarter was a driver for the current quarter's GDP, but we would argue that it was not the main driver.

In the following part we fit similar additive models for the other variables and conducted hypothesis tests again. We found that for any quarter, the previous quarter's Productivity was also a significant predictor in Investment and Hours Worked but not Consumption. Again, our findings slightly differ from the theory.

In the final two parts of our analysis, we consider what drives Productivity. Through our modeling and bootstrap tests, we could not argue previous quarter fluctations other than Productivity helped predict the current quarter's Productivity. This leads us to believe that Productivity is an exogenous variable, as we certainly found that it affects some of the other variables.

In conclusion, we differ from the theory in that we found Productivity is not necessarily the main driver of the other variables, but we agree that it is a driver and that it predicts other variables, while the other variables do not predict Productivity.