assignment 4 textbook

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Problem 1: The Laplacian operator for normal/unrotated coordinates is defined as

$$\nabla^2 f = \frac{\vartheta^2 f}{\vartheta x^2} + \frac{\vartheta^2 f}{\vartheta y^2}$$

and by the following equation for rotated coordinates

$$\nabla^2 f = \frac{\vartheta^2 f}{\vartheta x'^2} + \frac{\vartheta^2 f}{\vartheta y'^2}$$

It is given that the relationship of coordinates for a rotation by angle θ is given by

$$x = x' cos\theta - y' sin\theta$$
$$y = x' sin\theta + y' cos\theta$$

where (x,y) are the unrotated and (x',y') are the rotated coordinates.

Show that the Laplacian as defined above is invariant to rotation.

$$x' = x \cos\theta + y \sin\theta$$

$$y' = -x \sin\theta + y \cos\theta$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'}\cos\theta - \frac{\partial f}{\partial y'}\sin\theta \,, \quad \frac{\partial f}{\partial y} = \frac{\partial f}{\partial x'}\sin\theta + \frac{\partial f}{\partial y'}\cos\theta$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left[\frac{\partial f}{\partial x'}\sin\theta + \frac{\partial f}{\partial y'}\cos\theta\right] = \left[\frac{\partial}{\partial x'}\sin\theta + \frac{\partial}{\partial y'}\cos\theta\right]\cdot\frac{\partial f}{\partial y}$$

$$= \frac{\partial^2 f}{\partial x'^2}\sin^2\theta + \cancel{\frac{\partial^2 f}{\partial y'\partial y}\sin\theta\cos\theta} + \cancel{\frac{\partial^2 f}{\partial x'\partial y}\cos\theta\sin\theta} + \frac{\partial^2 f}{\partial y'^2}\cos^2\theta$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left[\frac{\partial f}{\partial y'}\cos\theta - \frac{\partial f}{\partial y'}\sin\theta\right] = \left[\frac{\partial}{\partial x'}\cos\theta - \frac{\partial}{\partial y'}\sin\theta\right]\cdot\frac{\partial f}{\partial x}$$

$$= \frac{\partial^2 f}{\partial x'^2}\cos^2\theta - \cancel{\frac{\partial^2 f}{\partial x' \partial y}\cos\theta\sin\theta} - \cancel{\frac{\partial^2 f}{\partial y' \partial x}\cos\theta\sin\theta} + \frac{\partial^2 f}{\partial y'}\sin^2\theta$$

$$- \frac{\partial^2 f}{\partial x'^2}\cos\theta - \cancel{\frac{\partial^2 f}{\partial x' \partial y'}}\cos\theta\sin\theta - \cancel{\frac{\partial^2 f}{\partial y' \partial x'}}\cos\theta\sin\theta + \frac{\partial^2 f}{\partial y'^2}\sin\theta$$

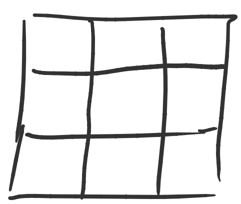
$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial x'}\overbrace{(\sin^2\theta + \cos^2\theta)}^{=1} + \frac{\partial^2 f}{\partial y'}\overbrace{(\sin^2\theta + \cos^2\theta)}^{=1}$$

$$\therefore \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2}$$

$\nabla^2 f$ is rotation invariant.

Problem 2. Assume an average 3x3 filter that uses the four closest neighbours and excludes the center point from the average.

a. Determine the filter in the frequency domain.
b. Is this a high-pass or a low-pass filter? Explain your answer.



a)
$$g(x,y) = \frac{1}{4}\left[f(x,y+1) + f(x+1,y) + f(x-1,y) + f(x,y-1)\right]$$

$$G(u,v) = \frac{1}{4}F(u,v)\left[e^{\frac{j2\pi v}{N}} + e^{-\frac{j2\pi v}{N}} + e^{\frac{j2\pi u}{N}} + e^{-\frac{j2\pi u}{N}}\right]$$

$$= H(u,v)\,F(u,v)$$

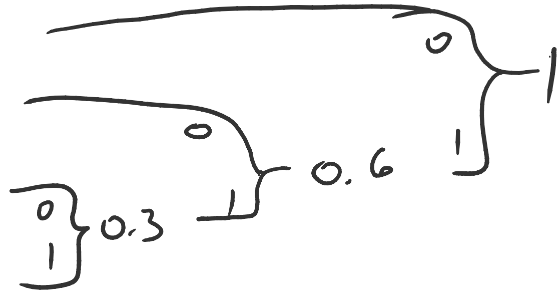
$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$H(u,v) = \frac{1}{2}\left[\cos\left(\frac{2\pi v}{N}\right) + \cos\left(\frac{2\pi u}{N}\right)\right]$$

b) Low pass filter, $H(u,v)$ are cosine functions, Mag is greatest @ $u,v=0$ and decreases as you increase up to $u,v = \frac{N}{2}$.

Problem 3. (Huffman Coding) Consider an alphabet A={a, b, c, d} with the probabilities P(a)=0.2, P(b)=0.1, P(c)=0.3 and P(d)=0.4. Design one possible Huffman code for this data source.

Huffman		
0	d	0.4
10	c	0.3
110	a	0.2
111	b	0.1



Problem 4. Consider the images shown below. The image on the right is obtained by low-pass filtering the image on the left, using a Gaussian low-pass filter, and then high-pass filtering the result with a Gaussian high-pass filter.

c. Explain why the center part of the finger ring on the filtered image appears bright and solid, given that the dominant characteristic of the filtered image consists of edges on the outer boundary of bones with a darker area in between. In other words, would you not expect the high-pass filter to render the constant area inside the ring dark, since a high-pass filter eliminates the dc term?

d. Explain if the result would have been different if we had first applied the high-pass filter and then the low-pass.


c. We do see the inner Part of the ring darken, The effect just isn't as noticable since the image underwent smoothing. The center area is averaged out. The image is so bright because the edges of the ring are so much higher that it creates a grey dominant averaged area.

d. we would get the same image as applying a LPF to a HPF is the same as applying a HPF to a LPF & resuts in a band pass filter.