UBC ELEC421 Homework Assignment-3

Focus: AR process, Linear prediction, Wiener filter
Matlab practice: digital filter design, integer filter, AR process, linear prediction
Due day: Nov. 19

Marking rules:

- For textbook-type problems: The brief solution is provided (together with the assignment). However the students are still required to submit the solutions for all textbook-type problems to get full marks. The purpose is to 'force' the students to practice on such textbook problems similar to those in later exams.
- **34 points** for Matlab problems: 34 (=8+8+8+10) points
- **All students are supposed to finish the assignment INDEPENDENTLY.**

Submission rules:

- For textbook-type problems: Submit the hard copy in-class or e-copy through Canvas.
- For Matlab problems: Submit your solution file through Canvas (with the file name like 'ELEC421_studentName_ID_HW3.pdf'). Please organize everything in a single .doc or .pdf file for the Matlab part; make sure to attach the codes in the end; and make sure that the results are calculated using Matlab and figures/tables are not drawn by hands.

Problem 1: Suppose a process {x(n)} is characterized by the autocorrelation sequence

$$\gamma_{xx}(0)=\frac{4}{3},\gamma_{xx}(1)=\frac{2}{3},\gamma_{xx}(2)=\frac{1}{3},\gamma_{xx}(3)=\frac{1}{6}.$$

(a) Use the autocorrelations and calculate an AR(2) model for this process.
(b) Substituting these results into the Yule-Walker equation for k=0 to estimate the white noise variance. Then compute the power spectrum based on the AR(2) model.
(c) Repeat the above calculation for an AR(3) model. Comment on your results.

Problem 2: A process {x(n)} is characterized by the autocorrelation sequence

$$\gamma_{xx}(0)=\frac{4}{3},\gamma_{xx}(1)=\frac{2}{3},\gamma_{xx}(2)=\frac{1}{3},\gamma_{xx}(3)=\frac{1}{6}.$$

(a) Consider the one-step forward predictor of order 2. Determine the prediction coefficients.
(b) Calculate the resulting MMSE (minimum mean-square error) for this 2nd-order predictor.
(c) Write down the difference equation.

Problem 3: Wiener filter

A zero-mean stationary process {s(n)} is characterized by the autocorrelation sequence

$$\gamma_{ss}(0)=1,\gamma_{ss}(1)=\frac{1}{2},\gamma_{ss}(2)=\frac{1}{8},\gamma_{ss}(3)=\frac{1}{64}.$$

a.) Let us assume that the observed signal x(n) can be expressed as x(n) = 2 s(n) + w(n), where s(n) is a process described above, and {w(n)} is a white noise sequence with variance $\sigma_w^2=1$. Assume that {w(n)} is uncorrelated to {s(n)}. Determine the autocorrelation sequence $\{\gamma_{xx}(m)\}$ of the process {x(n)} for m=0,1, 2, 3.

b.) Consider the signal smoothing application. Based on the signal {x(n)}, we want to design a FIR Wiener filter of length M=2 to estimate s(n+1). Write down the expression of the output y(n) in terms of the FIR filter of length M=2 with coefficients {h(0),h(1)}.

c.) Calculate the solution for this optimum FIR Wiener filter in b.), and calculate the corresponding minimum MSE (i.e. $MMSE_2$).

d.) In the above problem, suppose the observed signal x(n) can be expressed as x(n) = s(n) + s(n-1) + w(n). We want to use x(n) and x(n+1) to estimate s(n). Write down the expression of the output y(n) with coefficients {h(0),h(1)}. Calculate the solution for this optimum Wiener filter.

Matlab Practice:

General Policy: Though the students are allowed to discuss with each other, each student should work *individually and independently* on Matlab implementation and report writing (if needed).

Problem 1: FIR digital filter design. (You can use 'fir1' command.)

1 Design an FIR linear phase, digital filter approximating the ideal frequency response

$$H_d(\omega) = \begin{cases} 1, & \text{for } |\omega| \le \frac{\pi}{6} \\ 0, & \text{for } \frac{\pi}{6} < |\omega| \le \pi \end{cases}$$

(a) Determine the coefficients of a 25-tap filter based on the window method with a rectangular window.
(b) Determine and plot the magnitude and phase response of the filter.
(c) Repeat parts (a) and (b) using the Hamming window.
(d) Repeat parts (a) and (b) using a Bartlett window.

Problem 2: FIR filter design using windows

7.16. We wish to design an FIR lowpass filter satisfying the specifications

$$0.98 < H(e^{j\omega}) < 1.02, \qquad 0 \le |\omega| \le 0.63\pi,$$
$$-0.15 < H(e^{j\omega}) < 0.15, \qquad 0.65\pi \le |\omega| \le \pi,$$

by applying a Kaiser window to the impulse response $h_d[n]$ for the ideal discrete-time lowpass filter with cutoff $\omega_c = 0.64\pi$. Find the values of β and M required to satisfy this specification.

Problem 3: Integer filters

7.14 For a filter with the following transfer function, what is the (a) amplitude response, (b) phase response, (c) difference equation?

$$H(z) = \frac{1 - z^{-8}}{1 + z^{-2}}$$

7.15 A digital filter has the following transfer function. (a) What traditional filter type best describes this filter? (b) Draw its pole-zero plot. (c) Calculate its amplitude response. (d) What is its difference equation?

$$H(z) = \frac{(1 - z^{-8})^2}{(1 + z^{-2})^2}$$

Problem 4: AR processes

The power spectrum of an EMG signal typically spans in the range of 20-400Hz. When the muscle fatigues, the spectrum of its EMG signal increases at low frequencies (e.g. below 100 Hz) and decreases at high frequencies. Some researches proposed that they could determine muscle fatigue by calculating and AR(2) model of an EMG signal and looking for changes in the parameters of this model. In the following table are the calculated auto-correlation values for the diaphragm EMG from one subject before and during fatigue of the diaphragm muscle.

Lag	0	1	2	3	4	5	6	7
Rest	1	0.83	0.47	0.08	-0.22	-0.37	-0.39	-0.26
Fatigue	1	0.9	0.66	0.36	0.07	-0.17	-0.32	-0.37

Auto-correlation values.

(a) Calculate the AR(4) or AR(3) models of both conditions, then compute the power spectrums correspondingly. Comment on your results.

(b) Calculate the AR(2) parameters for both condition. Do the changes in AR(2) parameters qualitatively reflect the expected changes in the power spectrum?