ELEC421 Take-home Assignment-1-- Brief Solution for textbook problems

Content focus: digital signal, correlation, z-transform, and LTI.
Due day: Oct. 08

Marking rules:

- For textbook-type problems: The brief solution is provided (together with the assignment). However the students are still required to submit the solutions for all 6 textbook-type problems to get full marks. The purpose is to 'force' the students to practice on such textbook problems similar to those in later exams.
- 20 points for Matlab problems: 20 = 8+12

Submission rules:

- For textbook-type problems: Submit the hard copy of your solutions in-class.
- For Matlab problems: Submit your solution file through Connect (with the file name like 'ELEC421_studentName_ID_HW1.pdf'). Please organize everything in a single .doc or .pdf file for the Matlab part; make sure to attach the codes in the end; and make sure that the results are calculated using Matlab and figures/tables are not drawn by hands.

1. We have

$$x(n) = \delta(n+3) + u(n) - u(n-2) = \delta(n+3) + \delta(n) + \delta(n-1)$$

The overall $\mathrm{H}_{all}(z) = (H(z) + H(z)z^{-2})z^{-1}$,

so equivallently, we have $h_{all}(n) = h(n-1) + h(n-3)$

$$\therefore\ y(n) = h_{all}(n) * x(n) = (h(n-1) + h(n-3)) * (\delta(n+3) + \delta(n) + \delta(n-1))$$

$$= (h(n+2) + h(n-1) + h(n-2)) + (h(n) + h(n-3) + h(n-4))$$

$$with\ h(n) = u(n) + 0.6^n u(n)$$

Tip: use the definition that $h(n) = h(n) * \delta(n)$

2. For auto-correlation (and cross-correlation) calculations, we can either do it in the time-domain by using the correlation definition; or we can do it in the z-domain by using the relationship between correlation and convolution.

Note: For signals with finite length, z-domain is easier.

Recall: $r_{xy}(l) = x(l) * y(-l)$; so we have $R_{xx}(Z)=X(z)X(z^{-1})$ and $R_{xy}(Z)=X(z)Y(z^{-1})$

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)y(n-l) \quad l = 0, \pm 1, \pm 2, \ldots$$

(a) $X(z) = \sum_n x(n)z^{-n} = 1 + 4z^{-1} + z^{-2}$, so we have

$R_{xx}(Z)=X(z)X(z^{-1}) = z^2+8z+18+8z^{-1}+z^{-2}$, therefore in the time-domain, we have

$r_{xx}(l)=\boldsymbol{Z}^{-1}\{R_{xx}(z)\}=\{1, 8, \mathbf{18}, 8, 1\}$, for the shifting index l=-2,-1,0,1 and 2 correspondingly.

The normlized auto-correlation sequence $\rho_{xx}(l) = r_{xx}(l)/ r_{xx}(0)= \{1/18, 4/9, \mathbf{1}, 4/9, 1/18\}$.

(b) Similarly, we use $r_{yy}(l)=\boldsymbol{Z}^{-1}\{R_{yy}(z)\}= \boldsymbol{Z}^{-1}\{Y(z)Y(z^{-1})\}=\{ 1, 2, 3, 2, 1\}$ for l=-2,-1,…,2.

We use $r_{xy}(l)=\boldsymbol{Z}^{-1}\{R_{xy}(z)\}= \boldsymbol{Z}^{-1}\{X(z)Y(z^{-1})\}=\{1,5,\mathbf{6},5,1\}$ for l=-3,-2,…,2.

3. Note: the relationship between $X(z)$ and $X(\omega)$ is $z \equiv e^{j\omega}$.
– For DTFT, $X(\omega) \equiv X(e^{j\omega})$.
– Important properties: shifting; convolution; $nx(n) \leftrightarrow -zdX(z)/dz$, etc.
– Typical example: $a^n u(n) \leftrightarrow 1/(1-az^{-1})$, with ROC $|z| \geq |a|$, and $-a^n u(-n-1) \leftrightarrow 1/(1-az^{-1})$, with ROC $|z| \leq |a|$

(a) Using the definition $X(z) = \sum_n x(n)z^{-n}$, we have
$X(z) = z^2 + 3z + 5 + 3z^{-1} + z^{-2}$, and
$X(\omega) = e^{j2\omega} + 3j\omega + 5 + 3e^{-j\omega} + e^{-j2\omega}$.

(b) Using the properties of the z-transform and $a^n u(n) \leftrightarrow 1/(1-az^{-1})$, we have
$X(z) = \frac{1}{1-0.5z^{-1}} + \frac{z^{-1}}{((1-z^{-1}))^2}$, with ROC: $|z| \geq 1$. Having $z \equiv e^{j\omega}$, we have
$X(z) = \frac{1}{1-0.5e^{-j\omega}} + \frac{e^{-j\omega}}{((1-e^{-j\omega}))^2}$.

(c) We can write $x_1(n) = u(n) - u(n-2) = \delta(n) + \delta(n-1)$, and $x_2(n) = 4 \times 0.25^{n+1} u(n+1)$. Using the time shifting property, we have $X(z) = (1 + z^{-1}) + 4z/(1-0.25z^{-1})$, with ROC: $|z| \geq 0.25$. Similarly, we have
$X(\omega) = (1 + e^{-j\omega}) + 4e^{j\omega}/(1-0.25e^{-j\omega})$.

(d) we can write $x(n) = -a^n 0.5j(e^{j\omega_0 n} - e^{-j\omega_0 n})u(n)$. So we have
$X(z) = -0.5j\{\frac{1}{1-ae^{j\omega_0}z^{-1}} - \frac{1}{1-ae^{-j\omega_0}z^{-1}}\}$. Similarly, using the frequency shifting property of DTFT, we have
$X(\omega) = \frac{1}{2j}\{\frac{1}{1-ae^{-j(\omega-\omega_0)}} - \frac{1}{1-ae^{-j(\omega+\omega_0)}}\})$.

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.(a) Using the property $nx(n) \leftrightarrow -zdX(z)/dz \equiv Y(z) = -1/(1-z^{-1})$, since the ROC is $|z| \leq 1$ we have $y(n) = u(-n-1) \equiv nx(n)$, therefore
$x(n) = u(-n-1)/n$.
(b) We can write $X(z) = \frac{-1/2}{1-0.1z^{-1}} + \frac{3/2}{1-0.3z^{-1}}$. So:
If ROC is $|z| \geq 0.3$, we have $x(n) = -\frac{1}{2}0.1^n u(n) + \frac{3}{2}0.3^n u(n)$.
If ROC is $0.1 \leq |z| \leq 0.3$, we have $x(n) = -\frac{1}{2}0.1^n u(n) - \frac{3}{2}0.3^n u(-n-1)$.
If ROC is $|z| \leq 0.1$, we have $x(n) = \frac{1}{2}0.1^n u(-n-1) - \frac{3}{2}0.3^n u(-n-1)$.

(c)

$$nx[n] \Leftrightarrow -z\frac{d}{dz}X(z)$$

$$x[n-n_0] \Leftrightarrow z^{-n_0}X(z)$$

$$X(z) = \frac{3z^{-3}}{(1-\frac{1}{4}z^{-1})^2} = 12z^{-2}\left[-z\frac{d}{dz}\left(\frac{1}{1-\frac{1}{4}z^{-1}}\right)\right]$$

$x[n]$ is left-sided. Therefore, $X(z)$ corresponds to:

$$x[n] = -12(n-2)\left(\frac{1}{4}\right)^{n-2}u[-n+1]$$

5.

. Based on the figure, the difference equation is $y(n) = 1/2y(n-1) + 2x(n) + x(n-1)$. So we have the z-domain equation as $Y(z) = 1/2z^{-1}Y(z) + 2X(z) + z^{-1}X(z)$. Using $X(z) = 1/(1-z^{-1})$, we have $Y(z) = \frac{2+z^{-1}}{1-0.5z^{-1}}X(z) = \frac{-4}{1-0.5z^{-1}} + \frac{6}{1-z^{-1}}$. Therefore the time domain response is $y(n) = -4(1/2)^n u(n) + 6u(n)$.

6 (a): From $x(n) = u(n-1) + nu(n)$. We have $X(z) = \frac{z^{-1}}{1-z^{-1}} + \frac{z^{-1}}{(1-z^{-1})^2}$

From $y(n) = 0.3y(n-1) - 0.02y(n-2) + 2x(n) + x(n-1)$. We have

$$Y(z) = \frac{2+z^{-1}}{1-0.3z^{-1}+0.02z^{-2}}X(z)$$

$$= \frac{2z^{-1}+z^{-2}}{(1-0.1z^{-1})(1-0.2z^{-1})(1-z^{-1})} + \frac{2z^{-1}+z^{-2}}{(1-0.1z^{-1})(1-0.2z^{-1})(1-z^{-1})^2}$$

You can use Matlab to find y(n) .

(b) Sine $h(n) = 0.5^n u(n)$,

$$x(n) = u(n) - u(n-2) + \cos(\pi n/3)u(n) = \delta(n) + \delta(n-1) + \frac{1}{2}(e^{j\pi n/3} + e^{-j\pi n/3})u(n)$$

we have

$$H(z) = \frac{1}{1-0.5z^{-1}} = Y(z)/X(z)$$

$$X(z) = 1 + z^{-1} + \frac{1}{2}\left(\frac{1}{1-e^{j\pi/3}z^{-1}} + \frac{1}{1-e^{-j\pi/3}z^{-1}}\right)$$

Therefore we have
$$Y(z)=X(z)H(z)=(1+z^{-1})\frac{1}{1-0.5z^{-1}}+\frac{1}{2}\left(\frac{1}{1-e^{j\pi/3}z^{-1}}+\frac{1}{1-e^{-j\pi/3}z^{-1}}\right)\frac{1}{1-0.5z^{-1}}$$
$$=(1+z^{-1})\frac{1}{1-0.5z^{-1}}+\frac{0.5-0.29j}{1-e^{j\pi/3}z^{-1}}+\frac{0.5+0.29j}{1-e^{-j\pi/3}z^{-1}}$$

And thus we have $y(n)=0.5^{n}u(n)+0.5^{n-1}u(n-1)+\cos(\pi n/3)u(n)+0.58\sin(\pi n/3)u(n)$

(c) From the difference equation, we have $H(z)=\dfrac{b}{1-0.6z^{-1}}$

For z=1 (corresponding to w=0 in the frequency domain), $|H(1)|=|\dfrac{b}{1-0.6}|=1$, so b =0.4

Matlab Assignments Tips:

1. Main notes:

- The relationship between a digital signal and an analog signal is x_digital(n)≡x_analog(nT).
- The relationship between a true analog freq. component f_a and the corresponding "digital" freq. component f_d is f_a=f_d+k*Fs. It means that, after sampling at Fs, the true f_a freq. component will appear as a f_d freq. component.

2. Correlation analysis of EEG channels: The main idea is to use correlation as the simplest linear measure to represent the dependency patterns between brain regions (since here each EEG node represents a specific brain region).

   A higher correlation coefficient between nodes 1 and 2 means a higher dependency between brain regions 1 and 2. If overall R(k,j) is larger in the normal subject, it indicates that the brain regions are better coordinated in the normal subject. If the summation of R(k,j) over j is larger, it intuitively means that the node (brain region) k is overall more independent with other regions and thus is considered more important for the particular task.

   Basically, any features (e.g. the eigenvalue) extracted from the matrix R can be used to represent the overall difference of the dependency patterns between brain regions. Analysis based on the matrix R can intuitively tell us how different brain regions and how different connection patterns are recruited for motor tasks in normal and PD subjects.

   Please make sure to organize your results in a good format and provides comments on your results when applicable.