ELEC421 Assignment-2
Due day: Oct. 19
Covered contents: DTFT, DFT
Focus: DFT, its properties and its effects

Marking rules:

- For textbook-type problems: The brief solution is provided (together with the assignment). However the students are still required to submit the solutions for all textbook-type problems to get full marks. The purpose is to 'force' the students to practice on such textbook problems similar to those in later exams.
- 14 points for Matlab problems: 14 = 7+7

Submission rules:

- For textbook-type problems: Submit the hard copy of your solutions in-class.
- For Matlab problems: Submit your solution file through Connect (with the file name like 'ELEC421_studentName_ID_HW2.pdf'). Please organize everything in a single .doc or .pdf file for the Matlab part; make sure to attach the codes in the end; and make sure that the results are calculated using Matlab and figures/tables are not drawn by hands.

1. Computer the Fourier transform of the following signals.
   (a) $x(n) = \delta(n) + \delta(n-2) + u(n) - u(n-2)$
   (b) $x(n) = \{-1,2,3,2,1\}$
   (c) $x(n) = a^n \sin(\omega_0 n)\, u(n)$, $|a|<1$
   (d) A signal x(n) has the Fourier transform X(w), determine the Fourier transform of $y(n) = x(n)\sin(\omega_0 n) + x(n-1)$, where $X(\omega) = \dfrac{1}{1-0.8e^{-j\omega}}$.

2. Consider an LTI system, described by
   $$y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n)$$
   (a) Determine the impulse response, h(n), of the system
   (b) Sketch roughly the magnitude response $|H(\omega)|$ of this system.
   (c) What is the response of the system to the input signal $0.4^n u(n)$?
   (d) Is this system stable?

3. For the sequence, N=6. $x_1(n) = \cos(\frac{2\pi}{N}n) + \delta(n)$, and $x_2(n) = u(n) - \delta(n-1)$, $0 \le n \le N-1$.
   (a) Suppose N=6. Determine the N-point DFT of $x_1(n)$.
   (b) Determine the 2N-point DFT of $x_1(n)$ by zero-padding first. What is their relationship?
   (c) Determine the N-point circular convolution $x_1(n)\ \text{Ⓝ}\ x_2(n)$

4. Determine the 8-point DFTs of the following signals
   (a) $x(n) = \{1,0,1,0,0,0,0,0\}$.
   (b) $x(n) = a^n$, $|a|<1$, $0<=n<=7$.

5. Consider the sequence $x_1(n) = \{\underset{\uparrow}{1}, 1, 0, 0\}$ and $x_2(n) = \{\underset{\uparrow}{1}, 1, 3, 6\}$.

   (a) Given the 4-point DFT of the sequence x1(n), compute the DFT of the sequence $y(n) = \{\underset{\uparrow}{1},0, 0, 1\}$.

   (b) Determine a sequence y(n) such that $Y(k)=X_1(k)X_2(k)$.

   (c) Calculate the linear convolution $x_1(n)*x_2(n)$ by using DFT.

6. An LTI system has the impulse response in the frequency domain as

$$H(e^{j\omega}) = \frac{1 - 1.25e^{-j\omega}}{1 - 0.8e^{-j\omega}} = 1 - \frac{0.45e^{-j\omega}}{1 - 0.8e^{-j\omega}}.$$

   (a) Specify the difference equation that is satisfied by the input x(n) and the output y(n).
   (b) Determine the impulse respose h(n).
   (c) Show that H(w) is an all-pass filter (i.e., $|H(w)|^2=C$) and determine the constant C.

Matlab Assignment:

General Policy

- Though the students are allowed to discuss with each other, each student should work *individually and independently* on Matlab implementation and report writing.
- A brief report is required, with free form. Contents to be included in the report: results (e.g. figures) and brief discussions of results, and also the codes at the end.
- If a student has a very special situation requiring an unavoidable late submission, she/he should inform the instructor before the due day as early as possible.

Problem 1: DFT leakage, noisy signal, spectrum
Note: You might want to study the reading material on Spectrum Analysing using DFT.
Suppose the signal of interest x(t) consists of 3 sinusoid components, i.e.
$x(t) = \sin(2\pi f_1 t) + \sin(2\pi f_2 t) + 2\sin(2\pi f_3 t)$ with the three frequencies f1=5 kHz, f2=5.5 kHz, and f3=10 kHz. Assume the sampling frequency is f_s=32 kHz.
(a) Write down x(n), and plot x(n) for n=0, 1, …, N-1, with N =32, 64, 128 respectively.
(b) What is the corresponding frequency resolution? For N=32, 64, and 128, plot the corresponding magnitude spectrums of {x(n)}. Comment on the results.
(c) For N=32, zero-padding {x(n)} to be with length 128 first, then plot the corresponding magnitude spectrum. Comment on the results when compared with N=128 in (b).

(d) Study the effects of Gaussian white noise (WGN). Generate a zero-mean white Gaussian noise sequence (use ‘randn’ in Matlab) with variance 1. For N=32, plot the noise sequence, the signal sequence of (a), and the result of adding the two signals. Repeat (a) and (b) by using the above noisy signal.

(e) Repeat the steps in (d) but for a WGN signal of mean 1 and variance of 10. Comments on your results.

(f) For the noisy signal as in (d) with N= 64 and 128, study the effects of different windows (e.g., Hamming, Hann and Blackman). Comments on your results.

Problem 2: STFT for speech signal spectral analysis

Download the “dft.wav” file from the course website, and figure out some information of the .wav file by using ‘wavread’ command in Matlab. E.g. find out the sampling rate and the length of the recording (in second); hear the speech by using either ‘sound’ or ‘wavplay’ command in Matlab.

(a) Demonstrate the time and frequency domain representations of the speech signals. Download the data file ‘wordSample.mat’, which is part of the above speech data with the same sampling rate, and load it into Matlab (e.g. load wordSample.mat). Plot the data, plot the autocorrelation sequence.

(b) Plot the STFT-based time-varying spectrum of the speech signal, using the data in ‘wordSample.mat’. Make the window length (i.e. the length of each segment) to be 22.5 milliseconds (i.e. the window length N=Fs*22.5/1000).