Assignment 1 textbook answers better copy

1.

$x(n) = \delta(n+3) + u(n) - u(n-2)$

$h(n) = u(n) + 0.6^n u(n)$

discrete time so $x(n) = \delta(n+3) + \delta(n) + \delta(n-1)$

system $H_a(z) = (H(z) + z^2 H(z)) z^{-1}$

equivalent to

$h_a(n) = [h(n) + h(n-2)]$ Delayed 1

$= h(n-1) + h(n-3)$

δ's delay when convolved. x multiply

$y(n) = h_a(n) * x(n)$, since $x(n)$ is made of δ's

$y(n) = [h(n+2) + h(n+1) + h(n-2)] + [h(n) + h(n-3) + h(n-4)]$

$h(n) = u(n) + 0.6^n u(n)$

$\therefore$

$y(n) = u(n+2) + 0.6^{n+2} u(n+2) + u(n-1) + 0.6^{n-1} u(n-1) + u(n-2) + 0.6^{n-2} u(n-2) + u(n) + 0.6^n u(n) + u(n-3) + 0.6^{n-3} u(n-3) + u(n-4) + 0.6^{n-4} u(n-4)$

2. a) $x(n) = \{1, 4, 1\}$, $y(n) = \{1, 1, 1\}$

$r_{xx}(l) = x(l) * x(-l)$ $\therefore R_{xx}(z) = X(z) X(z^{-1})$ $X(z) = \sum_n x(n) z^{-n} = 1 + 4z^{-1} + z^{-2}$

$X(z^{-1}) = z^2\{1 + 4z^{-1} + z^{-2}\}$ $\therefore R_{xx}(z) = z^2\{1 + 8z^{-1} + 18z^{-2} + 8z^{-3} + z^{-4}\}$

$z^{-2}\ z^{-1}\ z^0$

1 4 1

1 4 1

1 4 1

4 16 4

1 4 1

1 8 18 8 1

$z^{-4}\ z^{-3}\ z^{-2}\ z^{-1}\ z^0$

$r_{xx}(l) = \{1, 8, \underset{\uparrow}{18}, 8, 1\}$ $\rho_{xx}(l) = \frac{r_{xx}(l)}{r_{xx}(0)} = \{\frac{1}{18}, \frac{8}{18}, 1, \frac{8}{18}, \frac{1}{18}\}$

b) $r_{yy}(l) = y(l) * y(-l)$ $Y(z) = 1 + z^{-1} + z^{-2}$, $Y(z^{-1}) = z^2\{1 + z^{-1} + z^{-2}\}$

$R_{yy}(z) = z^2\{1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}\}$ $\therefore r_{yy}(l) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$R_{xy}(z) = z^2\{1 + 5z^{-1} + 6z^{-2} + 5z^{-3} + 1z^{-4}\}$ $\therefore r_{xy}(l) = \{1, 5, \underset{\uparrow}{6}, 5, 1\}$

$R_{xy}(z) = X(z) Y(z^{-1})$

3. a) easy. $z^2+3z+5+3z^{-1}+z^{-2}$ by def

b) $\frac{1}{1-0.5z^{-1}} + \frac{z^{-1}}{(1-z^{-1})^2}$ by table

c) $u(n) - u(n-2) + 0.25^n u(n+1)$ discrete time & math

$= \delta(n) + \delta(n-1) + 4 \cdot 0.25^{n+1} u(n+1)$ $\quad 4 \cdot 0.25 = 1 \therefore 4 \cdot 0.25^{n+1} = 0.25^n$

$\Rightarrow 1 + z^{-1} + 4z\left(\frac{1}{1-0.25z^{-1}}\right)$

d) $x(n) = (a^n \sin(\omega_0 n))\, u(n)$ $|a| < 1$ tricky!

recall $\cos(x) = \frac{e^{jx}+e^{-jx}}{2}$ $\quad \sin(x) = \frac{e^{jx}-e^{-jx}}{2j}$

$x(n) = -a^n 0.5j\left(e^{j\omega_0 n} - e^{-j\omega_0 n}\right) u(n)$

$X(z) = -0.5j\left[\frac{1}{1-ae^{j\omega_0}z^{-1}} - \frac{1}{1-ae^{-j\omega_0}z^{-1}}\right]$

4.

a) $X(z) = \log(z-1)$, $|z|<1$, $n x(n) \Leftrightarrow -z\frac{dX(z)}{dz}$

Say $y(n) = n x(n)$ $\therefore Y(z) = -z\frac{d}{dz}[\log(z-1)] = -z\frac{1}{z-1}$

$= -z \cdot \frac{z^{-1}}{1-z^{-1}} = \frac{-1}{1-z^{-1}} \Leftrightarrow -u(n) = y(n)$, $y(n) = n x(n) \therefore x(n) = \frac{-u(n)}{n}$

but $|z|<1 \therefore u(n) \to -u(-n-1) \Rightarrow x(n) = \frac{u(-n-1)}{n}$

b) $X(z) = \frac{1}{1-0.4z^{-1}+0.03z^{-2}}$ $\quad \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{0.4 \pm \sqrt{0.4^2-4(0.03)}}{2} = 0.3, 0.1$

$X(z) = \frac{1}{(1-0.3z^{-1})(1-0.1z^{-1})} = \frac{a_1}{1-0.3z^{-1}} + \frac{a_2}{1-0.1z^{-1}} \Rightarrow 1 = a_1(1-0.1z^{-1}) + a_2(1-0.3z^{-1})$

@ $z=0.3$ we see $a_1 = \frac{1}{1-\frac{0.1}{0.3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

Want all regions.

@ $z=0.1$ $a_2 = \frac{1}{1-\frac{0.3}{0.1}} = -\frac{1}{2}$

$|z|>0.3$: $x(n) = -\frac{1}{2}(0.1)^n u(n) + \frac{3}{2}(0.3)^n u(n)$

$0.3>|z|>0.1$: $x(n) = -\frac{1}{2}(0.1)^n u(n) - \frac{3}{2}(0.3)^n u(-n-1)$

$0.1>|z|$: $x(n) = \frac{1}{2}(0.1)^n u(-n-1) - \frac{3}{2}(0.3)^n u(-n-1)$

$X(z) = \frac{3/2}{1-0.3z^{-1}} - \frac{\frac{1}{2}}{1-0.1z^{-1}}$

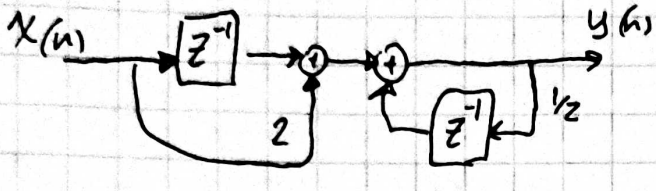
4 c) $X(z) = \dfrac{3z^{-3}}{(1-0.25z^{-1})^2}$ $\quad n a^n u(n) \Leftrightarrow \dfrac{az^{-1}}{(1-az^{-1})^2}$, $x(n-k) \Leftrightarrow z^{-k}X(z)$

$3z^{-3} = 0.25z^{-1} \cdot 12z^{-2}$ $\quad \therefore X(z) = 12z^{-2}\dfrac{0.25z^{-1}}{(1-0.25z^{-1})^2}$

$\therefore x(n) = 12(n-2)(0.25)^{(n-2)}u(n-2)$ But left sided so $|z| < |a|$

$\therefore x(n) = -12(n-2)(0.25)^{(n-2)}u(-n+1)$ $\quad \rightarrow (-(n-2)-1) = -n+1$

5.



I think response could mean impulse or step response

diff eqn easy

$y(n) = \frac{1}{2}y(n-1) + x(n-1) + 2x(n)$

$Y(z) = \frac{1}{2}z^{-1}Y(z) + z^{-1}X(z) + 2X(z)$

$Y(z)\left[1 - \frac{1}{2}z^{-1}\right] = X(z)\left[z^{-1} + 2\right]$

$Y(z) = X(z) \cdot \dfrac{2+z^{-1}}{1-\frac{1}{2}z^{-1}}$

impulse response $X(z) = 1$

$y(n) = 2(0.5)^n u(n) + (0.5)^{(n-1)} u(n-1)$

Step response

$X(z) = \dfrac{1}{1-z^{-1}}$ $\quad \dfrac{2+z^{-1}}{(1-z^{-1})(1-\frac{1}{2}z^{-1})} = \dfrac{a_1}{1-z^{-1}} + \dfrac{a_2}{1-\frac{1}{2}z^{-1}}$

$2+z^{-1} = a_1(1-\frac{1}{2}z^{-1}) + a_2(1-z^{-1})$ $\quad$ @ $z=1$, $a_1 = \dfrac{3}{0.5} = 6$

@ $z=\frac{1}{2}$, $a_2 = \dfrac{4}{-1} = -4$

$y(n) = 6u(n) + 4(0.5)^n u(n)$

6. a) $x(n) = u(n-1) + n\,u(n)$

$y(n) = 0.3y(n-1) - 0.02y(n-2) + 2x(n) + x(n-1)$

$Y(z) = 0.3z^{-1}Y(z) - 0.02z^{-2}Y(z) + 2X(z) + z^{-1}X(z)$

$\dfrac{0.3 \pm \sqrt{0.09 - 4(0.02)}}{2} = \dfrac{0.3 \pm 0.1}{2}$

$Y(z)\left[1 - 0.3z^{-1} + 0.02z^{-2}\right] = X(z)\left[2 + z^{-1}\right]$

$Y(z) = \left[z^{-1}\dfrac{1}{1-z^{-1}} + \dfrac{z^{-1}}{(1-z^{-1})^2}\right]\dfrac{2+z^{-1}}{1-0.3z^{-1}+0.02z^{-2}}$

$\left(\dfrac{z^{-1}(1-z^{-1}) + z^{-1}}{(1-z^{-1})^2}\right)\dfrac{2+z^{-1}}{(1-0.2z^{-1})(1-0.1z^{-1})} \Rightarrow \dfrac{2z^{-1}-z^{-2}}{(1-z^{-1})^2} \cdot \dfrac{(2+z^{-1})z^{-1}}{(1-0.2z^{-1})(1-0.1z^{-1})z^{-1}}$

$\dfrac{(2z^{-1}-z^{-2})(2z^{-1}+z^{-2})}{(1-z^{-1})^2(1-0.2z^{-1})(1-0.1z^{-1})z^{-1}} = z^{-1}(2-z^{-1})z^{-1}(2+z^{-1}) = z^{-2}(4-z^{-2})$

6 a) cont. $\frac{z^2(4-z^{-2})}{z^3(1-z)^2(1-0.2z^{-1})(1-0.1z^{-1})} = \frac{4z^{-1}-z^{-3}}{(1-z^{-1})^2(1-0.2z^{-1})(1-0.1z^{-1})}$

$\frac{a_1}{1-0.2z^{-1}} + \frac{a_2}{1-0.1z^{-1}} + \frac{a_3}{1-z^{-1}} + \frac{a_4}{(1-z^{-1})^2}$ Solving w. VA calc

$Y(z) = \frac{4.162z^{-3}}{(1-z^{-1})^2} + \frac{5.448z^{-1}}{1-z^{-1}} - \frac{2.625z^{-1}}{1-0.2z^{-1}} + \frac{1.185z^{-1}}{1-0.1z^{-1}}$

$y(n) = 4.167(n-1)u(n-1) + 5.44u(n-1) - 2.625(0.2)^{(n-1)}u(n-1) + 1.185(0.1)^{n-1}u(n-1)$

b) $h(n) = 0.5^n u(n)$, $x(n) = u(n) - u(n-2) + \cos(\frac{\pi n}{3})u(n)$

$y(n) \overset{F}{\leftrightarrow} H(z)X(z)$ $x(n) = \delta(n) + \delta(n-1) + \cos(\frac{\pi n}{3})u(n)$

$H(z) = \frac{1}{1-0.5z^{-1}}$, $X(z) = 1 + z^{-1} + \frac{1 - z^{-1}\cos(\frac{\pi}{3})}{1 - 2z^{-1}\cos\frac{\pi}{3} + z^{-2}}$ $\cos(\frac{\pi}{3}) = 0.5$

$Y(z) = \frac{1+z^{-1}}{1-0.5z^{-1}} + \frac{1-0.5z^{-1}}{1-z^{-1}+z^{-2}(1-0.5z^{-1})}$

$\cos(\frac{\pi}{3}n)u(n) = \frac{1}{2}(e^{j\frac{\pi}{3}n} + e^{-j\frac{\pi}{3}n})u(n) \Rightarrow \frac{1}{2}\left\{\frac{1}{1-e^{j\frac{\pi}{3}}z^{-1}} + \frac{1}{1-e^{-j\frac{\pi}{3}}z^{-1}}\right\}$

$\frac{1}{1-z^{-1}+z^{-2}} = \frac{1}{(1-e^{j\frac{\pi}{3}}z^{-1})(1-e^{-j\frac{\pi}{3}}z^{-1})} = \frac{a_1}{1-e^{j\frac{\pi}{3}}z^{-1}} + \frac{a_2}{1-e^{-j\frac{\pi}{3}}z^{-1}}$

$a_1 = \frac{1}{1-e^{j\frac{\pi}{3}}} = \frac{1}{1-(0.5+0.87j)} = 0.5 + 0.289j$

$a_2 = \frac{1}{1-e^{-j\frac{\pi}{3}}} + \frac{1}{1-(0.5+\frac{\sqrt{3}}{2}j)} \cdot \frac{1.5+\frac{\sqrt{3}}{2}j}{1.5+\frac{\sqrt{3}}{2}j} \Rightarrow \frac{1.5+\frac{\sqrt{3}}{2}j}{1.5^2+\frac{3}{4}} = 0.5 + \frac{\sqrt{3}}{6}j = 0.5 + 0.29j$

$Y(z) = \frac{1+z^{-1}}{1-0.5z^{-1}} + \frac{0.5-0.29j}{1-e^{j\frac{\pi}{3}}z^{-1}} + \frac{0.5+0.29j}{1-e^{-j\frac{\pi}{3}}z^{-1}}$ split $\frac{0.5}{1-e^{j\frac{\pi}{3}}z^{-1}} + \frac{0.5}{1-e^{-j\frac{\pi}{3}}z^{-1}} + \frac{0.29j}{1-e^{j\frac{\pi}{3}}z^{-1}} - \frac{0.29j}{1-e^{-j\frac{\pi}{3}}z^{-1}}$ (cos, sin)

$0.29j = -\frac{0.58}{2j}$

$y(n) = 0.5^n u(n) + 0.5^{(n-1)}u(n-1) + \cos(e^{j\frac{\pi}{3}n})\delta(n) - 0.58\sin(e^{j\frac{\pi}{3}n})u(n)$

c) $y(n) = 0.6y(n-1) + bx(n)$ $|H(z)| = 1$ for $z = 1$ $H(z) = \frac{Y(z)}{X(z)} = \frac{b}{1-0.6z^{-1}}$

$Y(z) = 0.6z^{-1}Y(z) + bX(z) \therefore Y(z) = X(z)\frac{b}{1-0.6z^{-1}}$

$\left|\frac{b}{1-0.6z^{-1}}\right| = 1$ for $z=1$ $\therefore b = 0.4$