

Limits and Derivatives Formulas

1. Limits

Properties

if $\lim_{x \rightarrow a} f(x) = l$ and $\lim_{x \rightarrow a} g(x) = m$, then

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = l \pm m$$

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = l \cdot m$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{l}{m} \text{ where } m \neq 0$$

$$\lim_{x \rightarrow a} c \cdot f(x) = c \cdot l$$

$$\lim_{x \rightarrow a} \frac{1}{f(x)} = \frac{1}{l} \text{ where } l \neq 0$$

Formulas

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

$$\lim_{x \rightarrow \infty} (1 + n)^{\frac{1}{n}} = e$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$$

$$\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$$

2. Common Derivatives

Basic Properties and Formulas

$$(cf)' = cf'(x)$$

$$(f \pm g)' = f'(x) + g'(x)$$

Product rule

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

Quotient rule

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

Power rule

$$\frac{d}{dx} (x^n) = nx^{n-1}$$

Chain rule

$$\frac{d}{dx} (f(g(x))) = f'(g(x))g'(x)$$

Common Derivatives

$$\frac{d}{dx} (c) = 0$$

$$\frac{d}{dx} (x) = 1$$

$$\frac{d}{dx} (\sin x) = \cos x$$

$$\frac{d}{dx} (\cos x) = -\sin x$$

$$\frac{d}{dx} (\tan x) = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{d}{dx} (\sec x) = \sec x \tan x$$

$$\frac{d}{dx} (\csc x) = -\csc x \cot x$$

$$\frac{d}{dx} (\cot x) = -\frac{1}{\sin^2 x} = -\csc^2 x$$

$$\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx} (a^x) = a^x \ln a$$

$$\frac{d}{dx} (e^x) = e^x$$

$$\frac{d}{dx} (\ln x) = \frac{1}{x}, x > 0$$

$$\frac{d}{dx} (\ln|x|) = \frac{1}{x}, x \neq 0$$

$$\frac{d}{dx} (\log_a x) = \frac{1}{x \ln a}, x > 0$$

3. Higher-order Derivatives

Definitions and properties

Second derivative

$$f'' = \frac{d}{dx} \left(\frac{dy}{dx} \right) - \frac{d^2 y}{dx^2}$$

Higher-Order derivative

$$f^{(n)} = \left(f^{(n-1)} \right)'$$

$$(f + g)^{(n)} = f^{(n)} + g^{(n)}$$

$$(f - g)^{(n)} = f^{(n)} - g^{(n)}$$

Leibniz's Formulas

$$(f \cdot g)'' = f'' \cdot g + 2f' \cdot g' + f \cdot g''$$

$$(f \cdot g)''' = f''' \cdot g + 3f'' \cdot g' + 3f' \cdot g'' + f \cdot g'''$$

$$(f \cdot g)^{(n)} = f^{(n)} g + n f^{(n-1)} g' + \frac{n(n-1)}{1 \cdot 2} f^{(n-2)} g'' + \dots + f g^{(n)}$$

Important Formulas

$$(x^m)^{(n)} = \frac{m!}{(m-n)!} x^{m-n}$$

$$(x^n)^{(n)} = n!$$

$$(\log_a x)^{(n)} = \frac{(-1)^{n-1} (n-1)!}{x^n \cdot \ln a}$$

$$(\ln x)^{(n)} = \frac{(-1)^{n-1} (n-1)!}{x^n}$$

$$(a^x)^{(n)} = a^x \ln^n a$$

$$(e^x)^{(n)} = e^x$$

$$(a^{mx})^{(n)} = m^n a^{mx} \ln^n a$$

$$(\sin x)^{(n)} = \sin \left(x + \frac{n\pi}{2} \right)$$

$$(\cos x)^{(n)} = \cos \left(x + \frac{n\pi}{2} \right)$$